Broadband Processing and Beamforming

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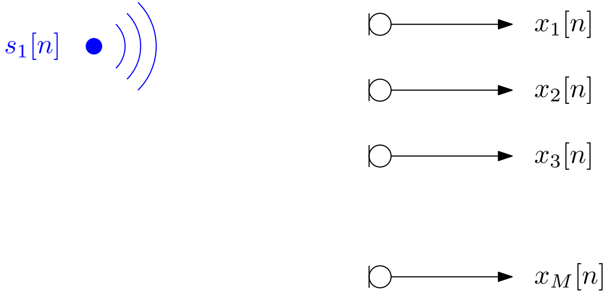
Presentation Overview

1. Sensor array and steering vectors
2. Narrowband case:
   – Minimum variance distrotionless response beamformer
   – Generalised sidelobe canceller
3. Broadband case: standard solution
4. Polynomial space-time covariance matrix
5. Polynomial MVDR / GSC formulation
6. Example
7. Conclusions

Sensor Array and Steering Vectors

- Scenario with sensor array and far-field sources:



- for the narrowband case, the source signals arrive with delays, expressed by phase shifts in a steering vector
- data model:

$$\mathbf{x}[n] =$$

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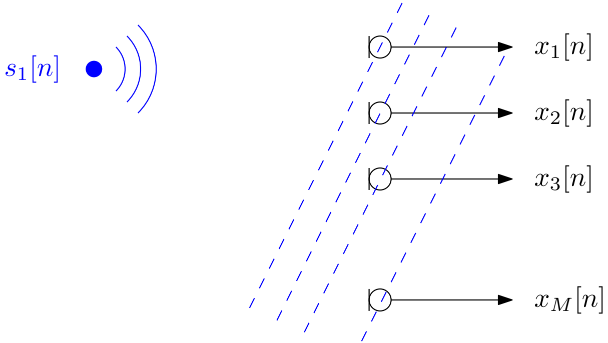

- for the narrowband case, the source signals arrive with delays, expressed by phase shifts in a steering vector $\mathbf{a}_1$
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$$\mathbf{x}[n] = s_1[n] \cdot \mathbf{a}_1$$

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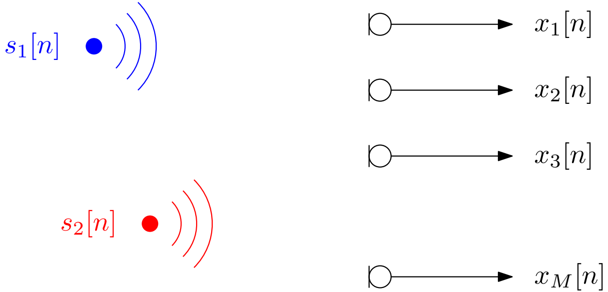

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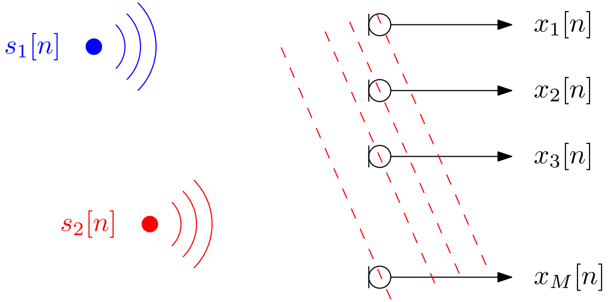

- for the narrowband case, the source signals arrive with delays, expressed by phase shifts in a steering vector $\mathbf{a}_1$, $\mathbf{a}_2$
- data model:

$$\mathbf{x}[n] = s_1[n] \cdot \mathbf{a}_1 + s_1[n] \cdot \mathbf{a}_2$$

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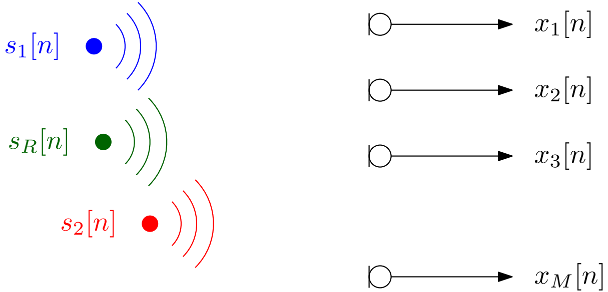

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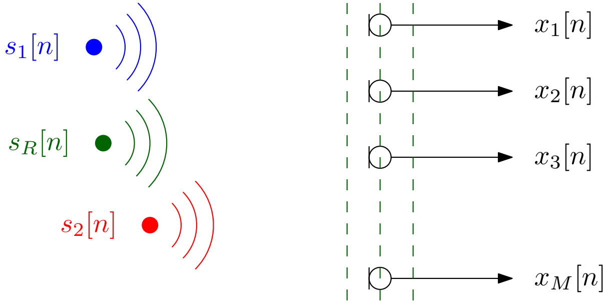

- for the narrowband case, the source signals arrive with delays, expressed by phase shifts in a steering vector $\mathbf{a}_1$, $\mathbf{a}_2$, $\ldots \mathbf{a}_R$;
- data model:

$$\mathbf{x}[n] = s_1[n] \cdot \mathbf{a}_1 + s_1[n] \cdot \mathbf{a}_2 + \cdots + s_R[n] \cdot \mathbf{a}_R = \sum_{r=1}^{R} s_r[n] \cdot \mathbf{a}_r$$

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Steering Vector

- A signal $s[n]$ arriving at the array can be characterised by the delays of its wavefront (neglecting attenuation):

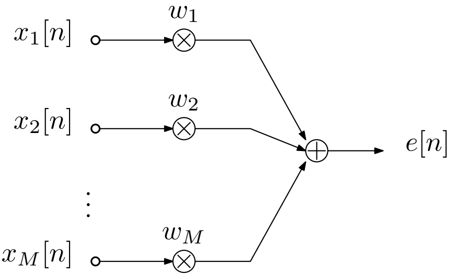
$$\begin{bmatrix} x_0[n] \\ x_1[n] \\ \vdots \\ x_{M-1}[n] \end{bmatrix} = \begin{bmatrix} s[n-\tau_0] \\ s[n-\tau_1] \\ \vdots \\ s[n-\tau_{M-1}] \end{bmatrix} = \begin{bmatrix} f[n-\tau_0] \\ f[n-\tau_1] \\ \vdots \\ f[n-\tau_{M-1}] \end{bmatrix} * s[n] \circ\!\!-\!\!\bullet\ \mathbf{a}_\vartheta(z)S(z)$$

- if evaluated at a narrowband normalised angular frequency Ω_i, the time delays τ_m in the broadband steering vector $\mathbf{a}_\vartheta(z)$ collapse to phase shifts in the narrowband steering vector $\mathbf{a}_{\vartheta,\Omega_i}$,

$$\mathbf{a}_{\vartheta,\Omega_i} = \mathbf{a}_\vartheta(z)\big|_{z=e^{j\Omega_i}} = \begin{bmatrix} e^{-j\tau_0\Omega_i} \\ e^{-j\tau_1\Omega_i} \\ \vdots \\ e^{-j\tau_{M-1}\Omega_i} \end{bmatrix} .$$

Narrowband Minimum Variance Distortionless Response Beamformer

- Scenario: an array of M sensors receives data $\mathbf{x}[n]$, containing a desired signal with frequency Ω_s and angle of arrival ϑ_s, corrupted by interferers;
- a narrowband beamformer applies a single coefficient to every of the M sensor signals:



Narrowband MVDR Problem

- Recall the space-time covariance matrix:

$$\mathbf{R}[\tau] = \mathcal{E}\{\mathbf{x}[n]\mathbf{x}^{\mathrm{H}}[n-\tau]\}$$

- the MVDR beamformer minimises the output power of the beamformer:

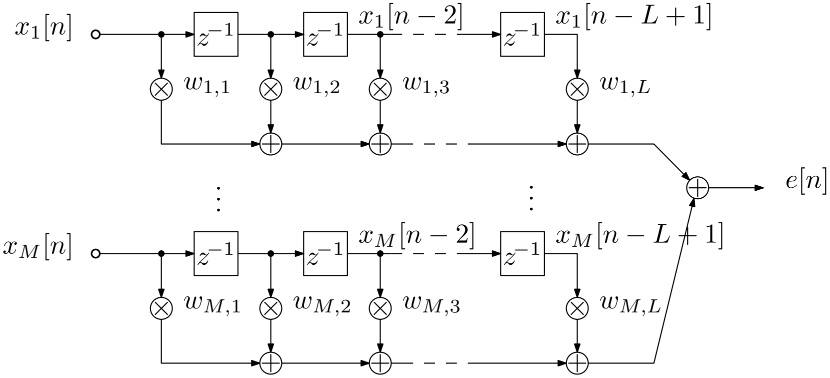
$$\min_{\mathbf{w}} \mathcal{E}\{|e[n]|^2\} = \min_{\mathbf{w}} \mathbf{w}^{\mathrm{H}}\mathbf{R}[0]\mathbf{w} \quad (1)$$

$$\text{s.t.} \quad \mathbf{a}^{\mathrm{H}}(\vartheta_{\mathrm{s}}, \Omega_{\mathrm{s}})\mathbf{w} = 1 \,, \quad (2)$$

- this is subject to protecting the signal of interest by a constraint in look direction ϑ_{s};
- the steering vector $\mathbf{a}_{\vartheta_{\mathrm{s}},\Omega_{\mathrm{s}}}$ defines the signal of interest's parameters.

Broadband MVDR Beamformer

- Each sensor is followed by a tap delay line of dimension L, giving a total of ML coefficients in a vector $\mathbf{v} \in \mathbb{C}^{ML}$



Broadband MVDR Beamformer

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- A larger input vector $\mathbf{x}_n \in \mathbb{C}^{ML}$ is generated, also including lags;
- the general approach is similar to the narrowband system, minimising the power of $e[n] = \mathbf{v}^{\mathrm{H}}\mathbf{x}_n$;
- however, we require several constraint equations to protect the signal of interest, e.g.

$$\mathbf{C} = [\mathbf{a}(\vartheta_{\mathrm{s}}, \Omega_0), \;\; \mathbf{a}(\vartheta_{\mathrm{s}}, \Omega_1) \; \ldots \; \mathbf{a}(\vartheta_{\mathrm{s}}, \Omega_{L-1})] \tag{3}$$

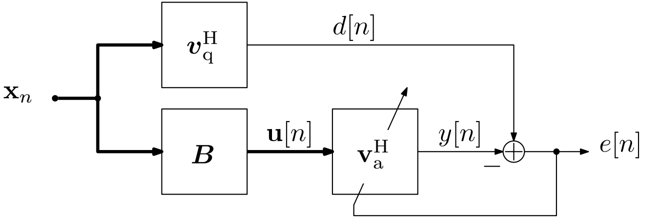
- these L constraints pin down the response to unit gain at L separate points in frequency:

$$\mathbf{C}^{\mathrm{H}}\mathbf{v} = \mathbf{1} \; ; \tag{4}$$

- generally $\mathbf{C} \in \mathbb{C}^{ML \times L}$, but simplifications can be applied if the look direction is towards broadside.

Generalised Sidelobe Canceller

- A quiescent beamformer $\mathbf{v}_{\mathrm{q}} = \left(\mathbf{C}^{\mathrm{H}}\right)^{\dagger} \mathbf{1} \in \mathbb{C}^{ML}$ picks the signal of interest;
- the quiescent beamformer is optimal for AWGN but generally passes structured interference;
- the output of the blocking matrix $\mathbf{B}$ contains interference only, which requires $[\mathbf{BC}]$ to be unitary; hence $\mathbf{B} \in \mathbb{C}^{ML\times(M-1)L}$;
- an adaptive noise canceller $\mathbf{v}_{\mathrm{a}} \in \mathbb{C}^{(M-1)L}$ aims to remove the residual interference:



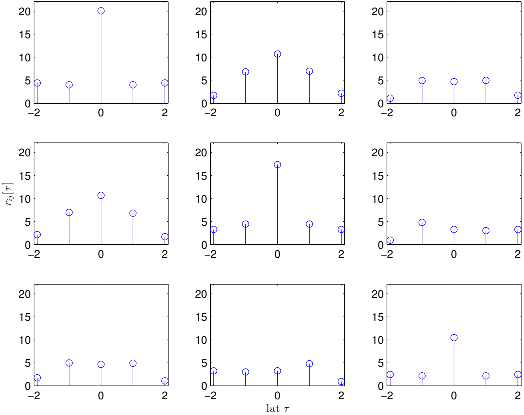
- note: all dimensions are determined by $\{M, L\}$.

Space-Time Covariance Matrix

- If delays must be considered, the (space-time) covariance matrix must capture the lag τ:

$$\mathbf{R}[\tau] = \mathcal{E}\{\mathbf{x}[n] \cdot \mathbf{x}^{\mathrm{H}}[n-\tau]\}$$

- $\mathbf{R}[\tau]$ contains auto- and cross-correlation sequences:



Cross Spectral Density Matrix

- z-transform of the space-time covariance matrix is given by

$$\mathbf{R}[\tau] = \mathcal{E}\{\mathbf{x}_n \mathbf{x}_{n-\tau}^{\mathrm{H}}\} \quad \circ\!\!-\!\!\bullet \quad \boldsymbol{R}(z) = \sum_l S_l(z)\mathbf{a}_{\vartheta_l}(z)\mathbf{a}_{\vartheta_l}^{\mathrm{P}}(z) + \sigma_N^2 \mathbf{I}$$

  with ϑ_l the direction of arrival and $S_l(z)$ the PSD of the lth source;

- $\boldsymbol{R}(z)$ is the cross spectral density (CSD) matrix; this matrix is parahermitian,

$$\boldsymbol{R}(z) = \boldsymbol{R}^{\mathrm{P}}(z) = \boldsymbol{R}^{\mathrm{H}}(z^{-1}) \quad ;$$

- the instantaneous covariance matrix (no lag parameter τ)

$$\mathbf{R} = \mathcal{E}\{\mathbf{x}_n \mathbf{x}_n^{\mathrm{H}}\} = \mathbf{R}[0]$$

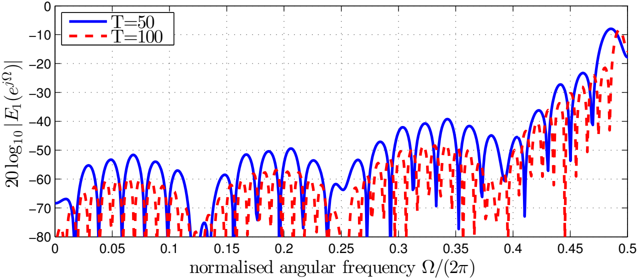
Polynomial Matrix MVDR Formulation

- Power spectral density of beamformer output:
  $R_e(z) = \mathbf{w}^{\mathrm{P}}(z)\mathbf{R}(z)\mathbf{w}(z)$
- proposed broadband MVDR beamformer formulation:

$$\min_{\mathbf{w}(z)} \oint_{|z|=1} R_e(z)\frac{\mathrm{d}z}{z} \quad (5)$$

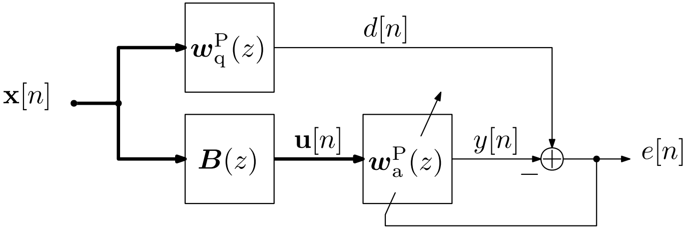
$$\text{s.t.} \quad \mathbf{a}^{\mathrm{P}}(\vartheta_{\mathrm{s}}, z)\mathbf{w}(z) = F(z)\ . \quad (6)$$

- precision of broadband steering vector, $|\mathbf{a}^{\mathrm{P}}(\vartheta_{\mathrm{s}}, z)\mathbf{a}(\vartheta_{\mathrm{s}}, z) - 1|$, depends on the length T of the fractional delay filter:



Generalised Sidelobe Canceller

- The broadband GSC now uses polynomial vector and matrix formulations:



- the quiescent vector is generated from the constraints,

$$\mathbf{w}_{\mathrm{q}}(z) = \mathbf{a}(\vartheta_{\mathrm{s}}, z) \quad ;$$

- the blocking matrix $\boldsymbol{B}(z)$ has to be orthonormal to $\mathbf{w}_{\mathrm{q}}(z)$ and only pass interference.

Blocking Matrix Design

- The blocking matrix can be obtained by polynomial matrix completion: $[\boldsymbol{B}(z)\ \mathbf{w}_{\mathrm{q}}(z)]$ must be paraunitary;
- this can be achieved by calculating a polynomial EVD of the rank one parahermitian matrix

$$\mathbf{w}_{\mathrm{q}}(z)\mathbf{w}_{\mathrm{q}}^{\mathrm{P}}(z) = \boldsymbol{Q}^{\mathrm{P}}(z)\boldsymbol{\Gamma}(z)\boldsymbol{Q}(z)$$

- rank one and orthonormality of $\mathbf{w}_{\mathrm{q}}(z)$ means that

$$\boldsymbol{\Gamma}(z) \approx \mathrm{diag}\{1,\ 0,\ \ldots\ 0\} \quad ;$$

- the parauntary $\boldsymbol{Q}^{\mathrm{P}}(z) = [\boldsymbol{q}_1(z)\ \ldots\ \boldsymbol{q}_M(z)]$ is the completed matrix;
- block matrix is

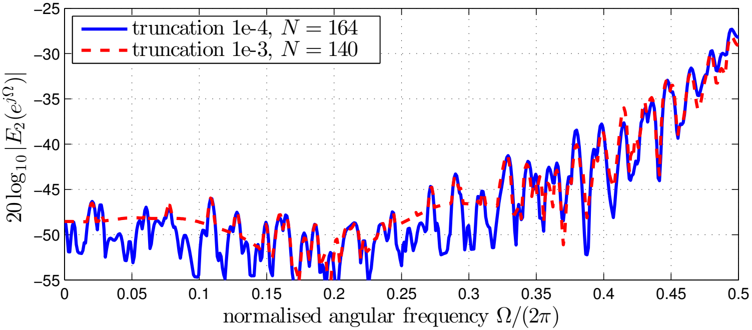
$$\boldsymbol{B}(z) = [\boldsymbol{q}_2(z)\ \ldots\ \boldsymbol{q}_M(z)]$$

of order N that is typically greater than T.

Blocking Matrix Accuracy

- PEVD is not unique w.r.t. $\boldsymbol{Q}(z)$, and it is important to find a representation that minimises the order N;
- numerical techniques use trunction of $\boldsymbol{Q}(z)$;
- measuring the maximum leakage of the signal of interest through the blocking matrix:

$$E_2(e^{j\Omega}) = \max_{m \in [2,M]} \|\mathbf{w}_{\mathrm{q}}^{\mathrm{P}}(z)\boldsymbol{q}_m(z)|_{z=e^{j\Omega}}\|_2$$



Computational Cost

- With M sensors and a TDL length of L, the complexity of a standard beamformer is dominated by the blocking matrix;
- in the proposed design, $\mathbf{w}_{\mathrm{a}} \in \mathbb{C}^{M-1}$ has degree L;
- the quiescent vector $\mathbf{w}_{\mathrm{q}}(z) \in \mathbb{C}^{M}$ has degree T;
- the blocking matrix $\boldsymbol{B}(z) \in \mathbb{C}^{(M-1)\times M}$ has degree N;
- cost comparison in multiply-accumulates (MACs):

component	GSC cost	
	polynomial	standard
quiescent beamformer	MT	ML
blocking matrix	$M(M-1)N$	$M(M-1)L^2$
adaptive filter (NLMS)	$2(M-1)L$	$2(M-1)L$

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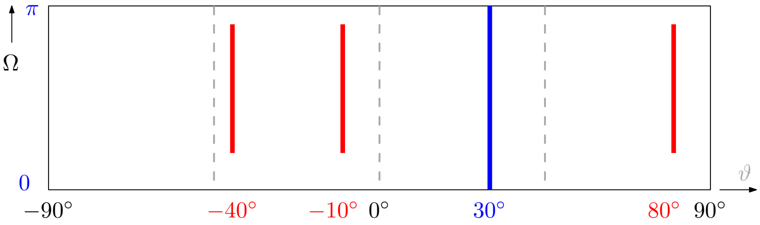
Example

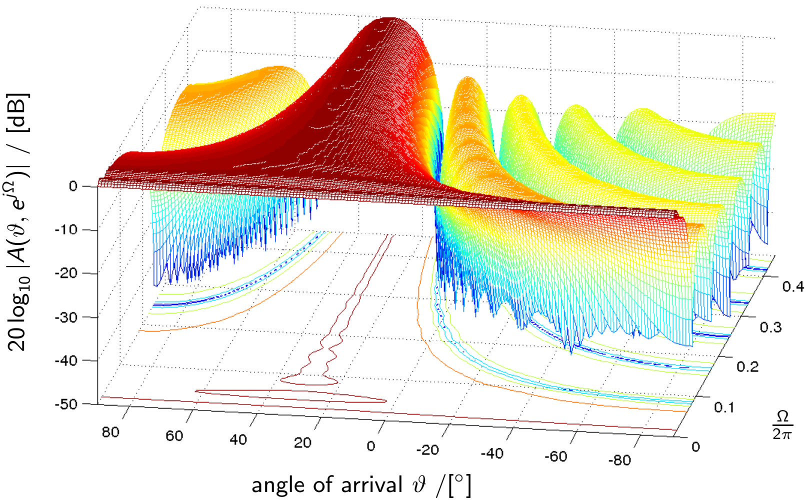
- We assume a signal of interest from $\vartheta = 30°$;
- three interferers with angles $\vartheta_i \in \{-40°, -10°, 80°\}$ active over the frequency range $\Omega = 2\pi \cdot [0.1;\ 0.45]$ at signal to interference ratio of -40 dB;



- $M = 8$ element linear uniform array is also corrupted by spatially and temporally white additive Gaussian noise at 20 dB SNR;
- parameters: $L = 175$, $T = 50$, and $N = 140$;
- cost per iteration: 10.7 kMACs (proposed) versus 1.72 MMACs (standard).

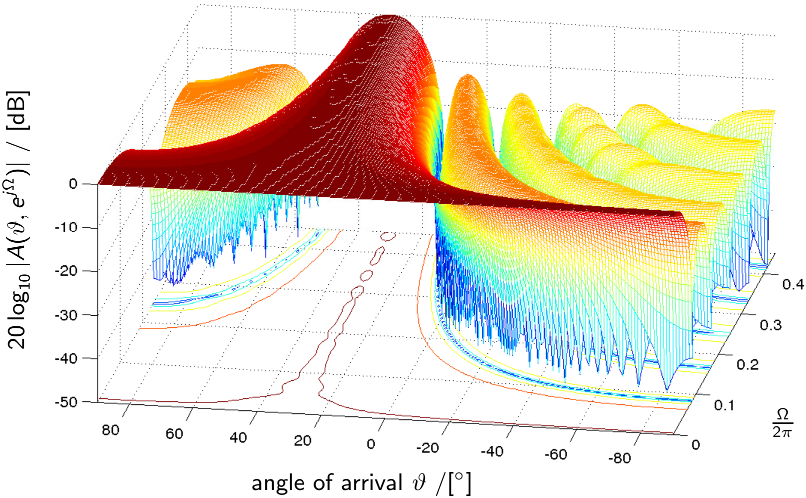
Quiescent Beamformer

- Directivity pattern of quiescent standard broadband beamformer:



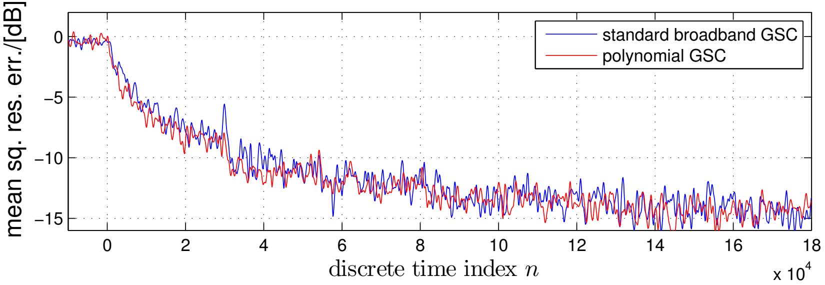
Quiescent Beamformer

- Directivity pattern of quiescent proposed broadband beamformer:



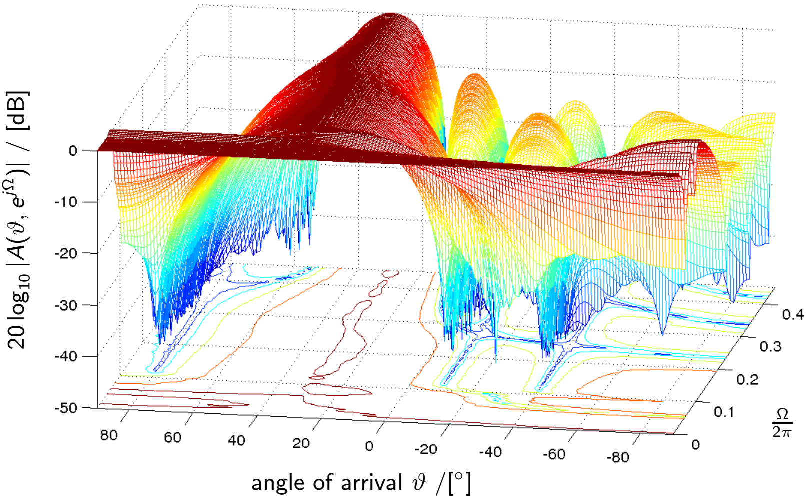
Adaptation

- Convergence curves of the two broadband beamformers, showing the residual mean squared error (i.e. beamformer output minus signal of interest):



Adapted Beamformer

- Directivity pattern of adapted proposed broadband beamformer:



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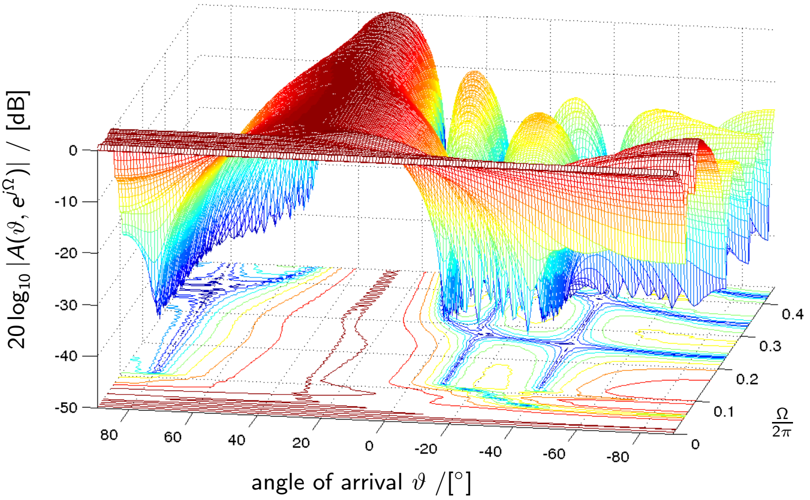
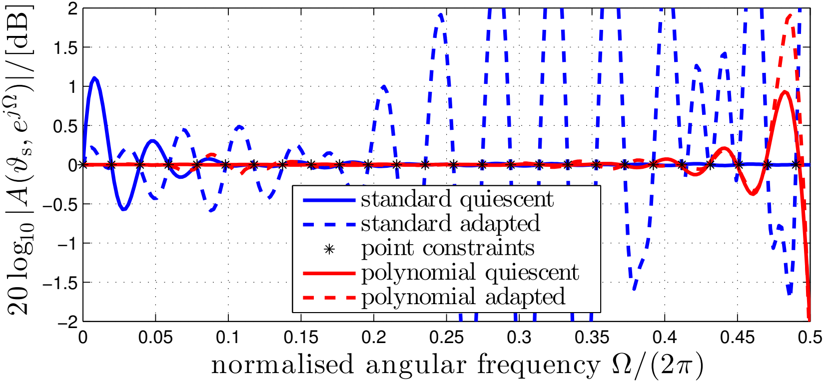


Gain in Look Direction

- Gain in look direction $\vartheta_s = 30°$ before and after adaptation:



- due to signal leakage, the standard broadband beamformer after adaptation only maintains the point constraints but deviates elsewhere.

Conclusions

- Broadband beamformers usually assume pre-steering such that the signal of interest lies at broadside;
- this is not always given, and difficult for arbitary array geometries;
- the proposed beamformer using a polynomial matrix formulation can implement abitrary constraints;
- the performance for such constraints is better in terms of the accuracy of the directivity pattern;
- because the proposed design decouples the complexities of the coefficient vector, the quiescent vector and block matrix, and the adaptive process, the cost is significantly lower than for a standard broadband adaptive beamformer;
- if interested in the discussed methods and algorithms, please download the free Matlab PEVD toolbox from

http://pevd-toolbox.eee.strath.ac.uk