Exploiting Sparsity for Underwater Acoustic Source Localisation and Array Optimisation

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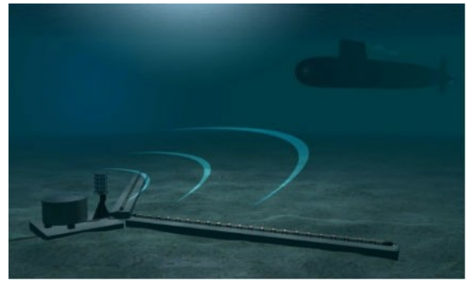
Contents

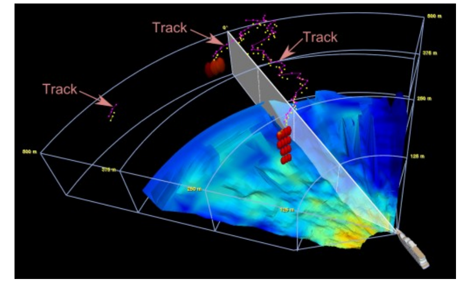
Acoustic Array System

Application

Non-destructive test



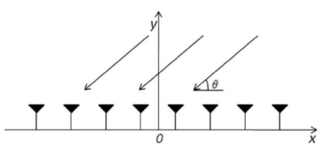
Underwater detection



Target tracking

Useful information collected by sensors → Direction of Arrival (DoA) estimation

Full array



Linear array

→ Spatial sparsity optimisation

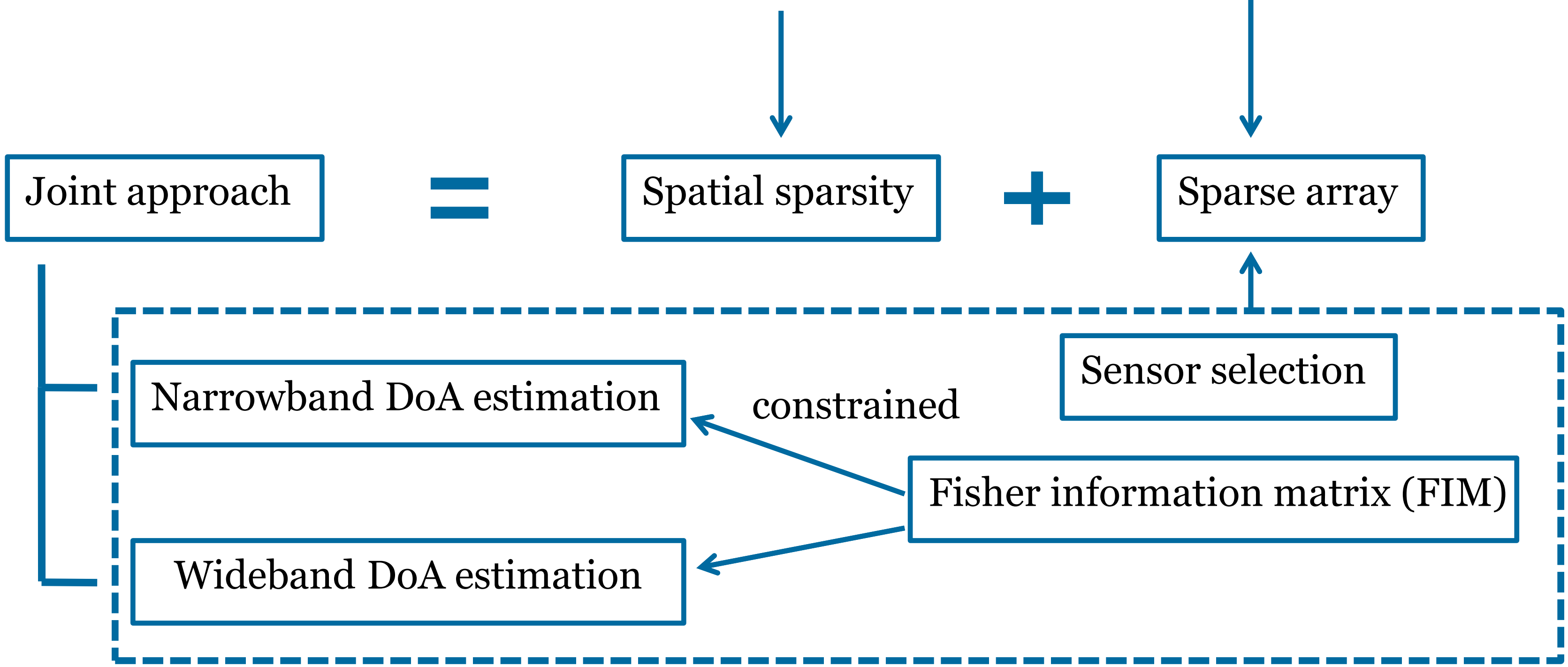
Sparse array

Constraints

- Computation resources
- Communication bandwidth
- Energy consumption

Research Aim

Motivated by the problems mentioned before, we are interested in using as few sensors as possible at each sampling time to implement spatial sparsity optimisation for DoA estimation.



Signal Model

Source signal

$$y_k = Ax_k + n_k$$

$y_k : (y_{k_1}, y_{k_2}, \cdots, y_{k_N})$ at each time step k where N is the number of sensors.

$x_k : (x_{k_1}, x_{k_2}, \cdots, x_{k_M})$ where M is the number of potential source directions.

A contains vectors from -90 to +90 degrees with size of $N \times M$.

n_k is the noise signal.

Desired beam response

$$p = [p(\Omega, \theta_1), p(\Omega, \theta_2), \cdots, p(\Omega, \theta_M)]$$

where $p \in C^{1\times M}$ is the vector holding the desired beam response ($y^H A$) at the sampled angular points θ_m for the frequency of interest Ω.

Compressed Sensing Based Sparse Array

The idea behind Compressed Sensing (CS) is to break through the limitation of sampling rate (twice the frequency of interest) at the same time to recover the signal.

It is ideal to find the minimum number of sensors which still achieve an exact match to a desired beam response.

$$\begin{aligned} \min \quad & \|w\|_1 \\ \text{subject to} \quad & \|p - w^H A\|_2 \leq \alpha \end{aligned} \qquad (1)$$

where $w \in C^N$ is the coefficient vector of the array, $p \in C^M$ is the desired beam response, $\alpha \in \Re^+$ is a threshold measuring the similarity between the designed response and thedesired response, $(\cdot)^H$ is a Hermitian operator, $\|\cdot\|_1$ and $\|\cdot\|_2$ are respectively the ℓ_1 and ℓ_2 norm of their arguments.

Spatial Sparsity based DoA Estimation

Different from the sparse array problem, in spatial sparsity based DoA estimation, the aim is to find the weight that corresponds to the source direction based on a full array. This can be achieved by a sequential Bayesian technique based on the least absolute shrinkage and selection operator (LASSO).

$$\min \quad \|y_k - Ax_k\|_2^2 + \mu \|x_k\|_1 \tag{2}$$

where for the signal y_k at time step k and source activation vector x_k, the cost function to be minimised is given. μ is a regularization parameter.

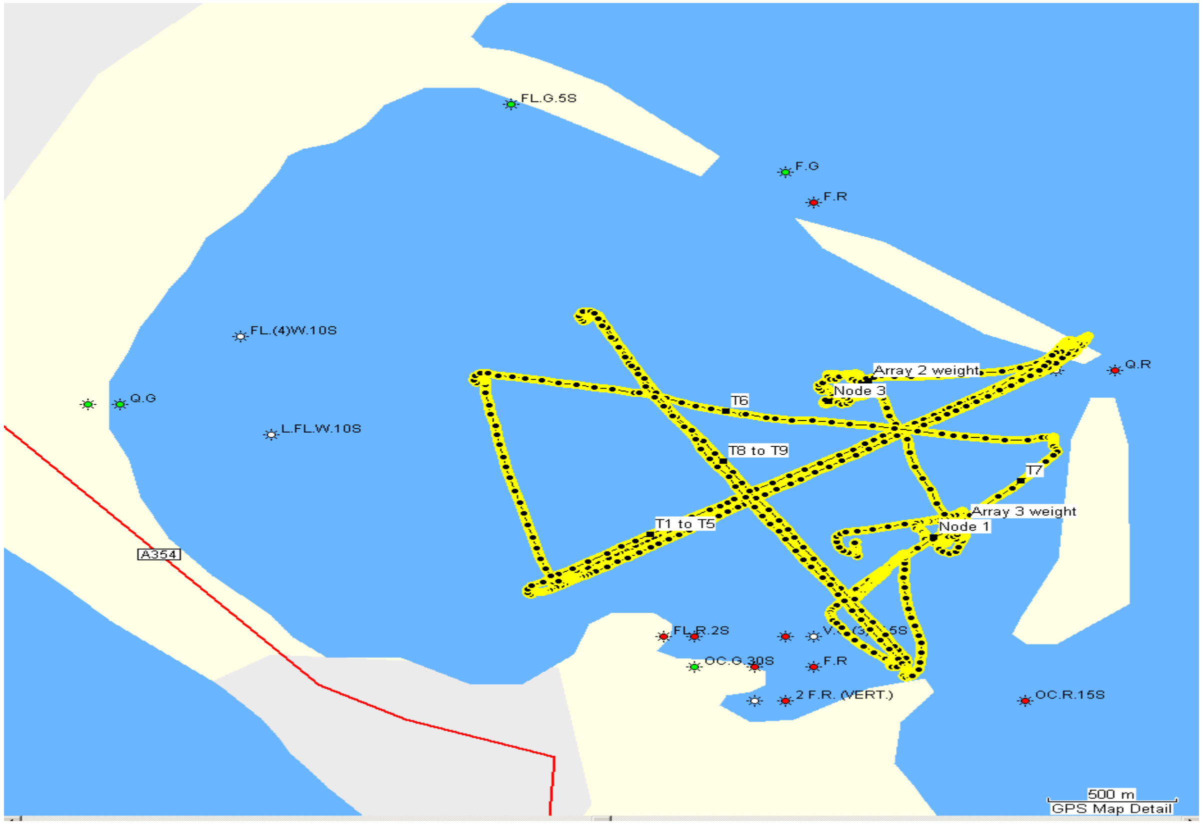
A Sequential Sparse Bayesian Method

- Extends sparse reconstruction methods to sequential data
- Extends the classic Bayesian approach to a sequential MAP estimation of the signal over time.
- Sparsity constraint is enforced with a Laplacian like prior at each time step.
- An adaptive LASSO cost function is minimised at each time step

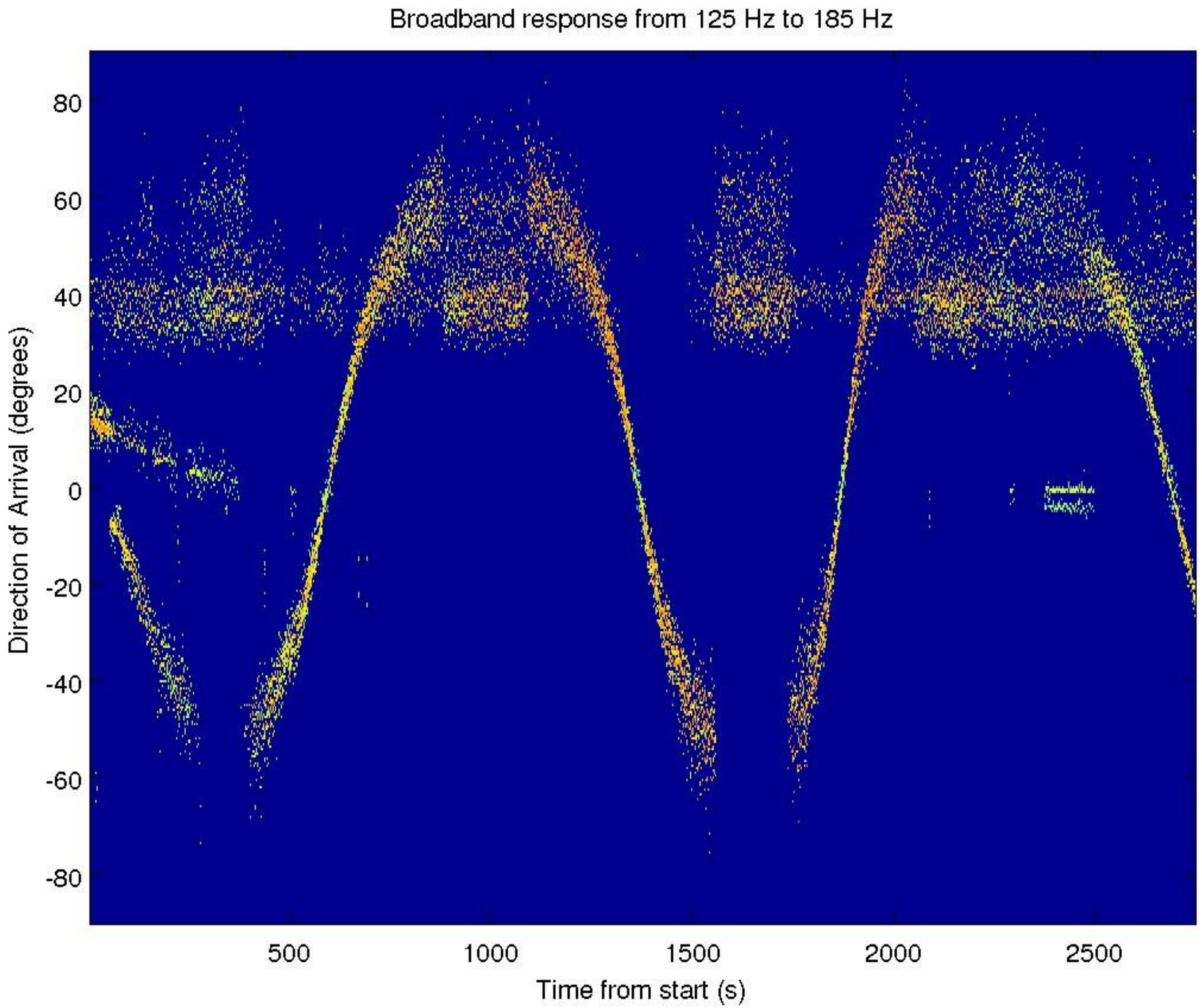
C. Mecklenbruker, P. Gerstoft, A. Panahi, M. Viberg, “Sequential Bayesian Sparse Signal Reconstruction using Array Data,” *IEEE Transactions on Signal Processing*, vol. 61, no. 24, pp. 6344 - 6354, 2013.

Portland03 Dataset

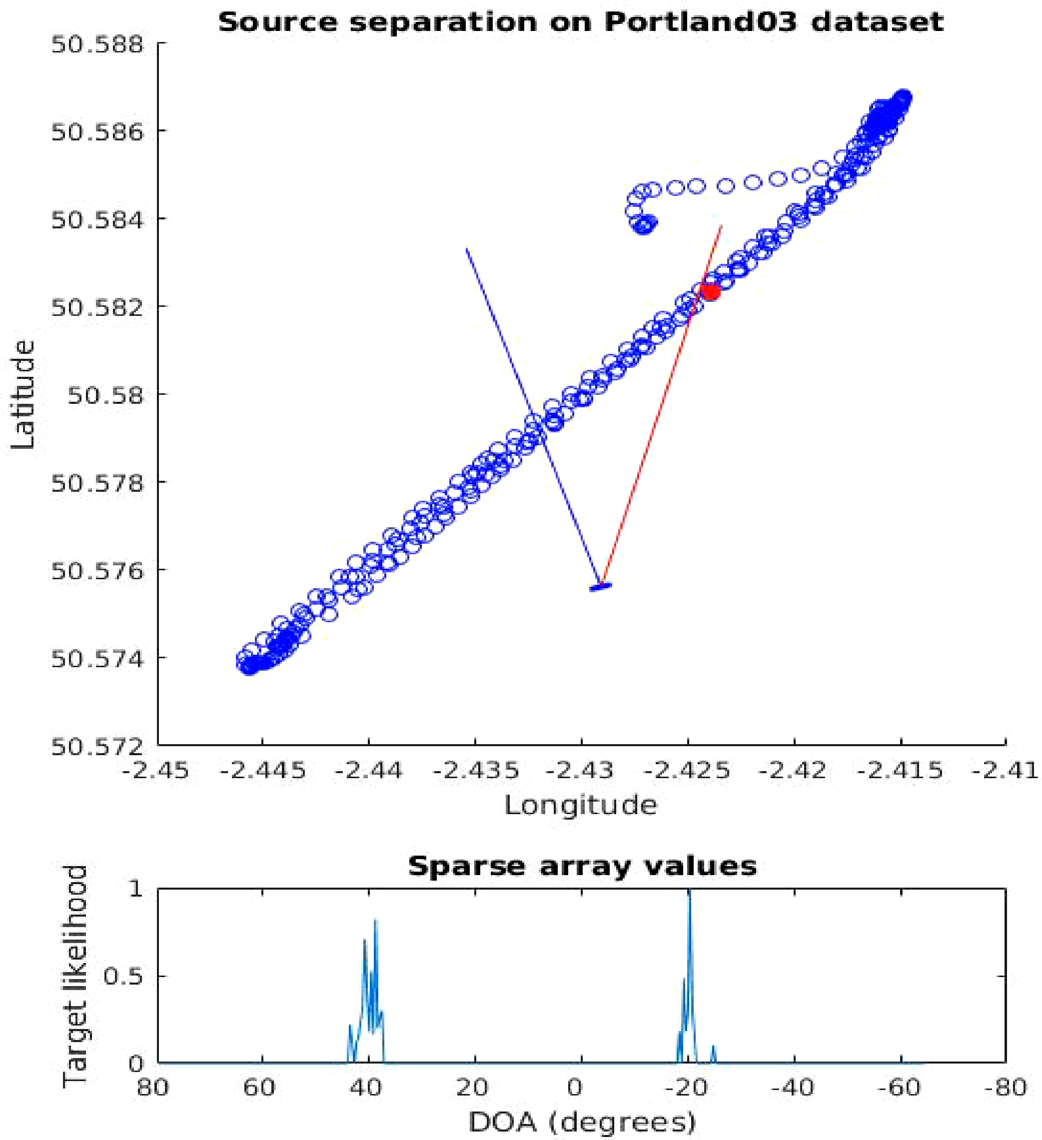
- Collected in 2003 in Portland harbour
- One moving target on nine different trajectories at a constant speed
- Two 32 element linear hydrophone arrays



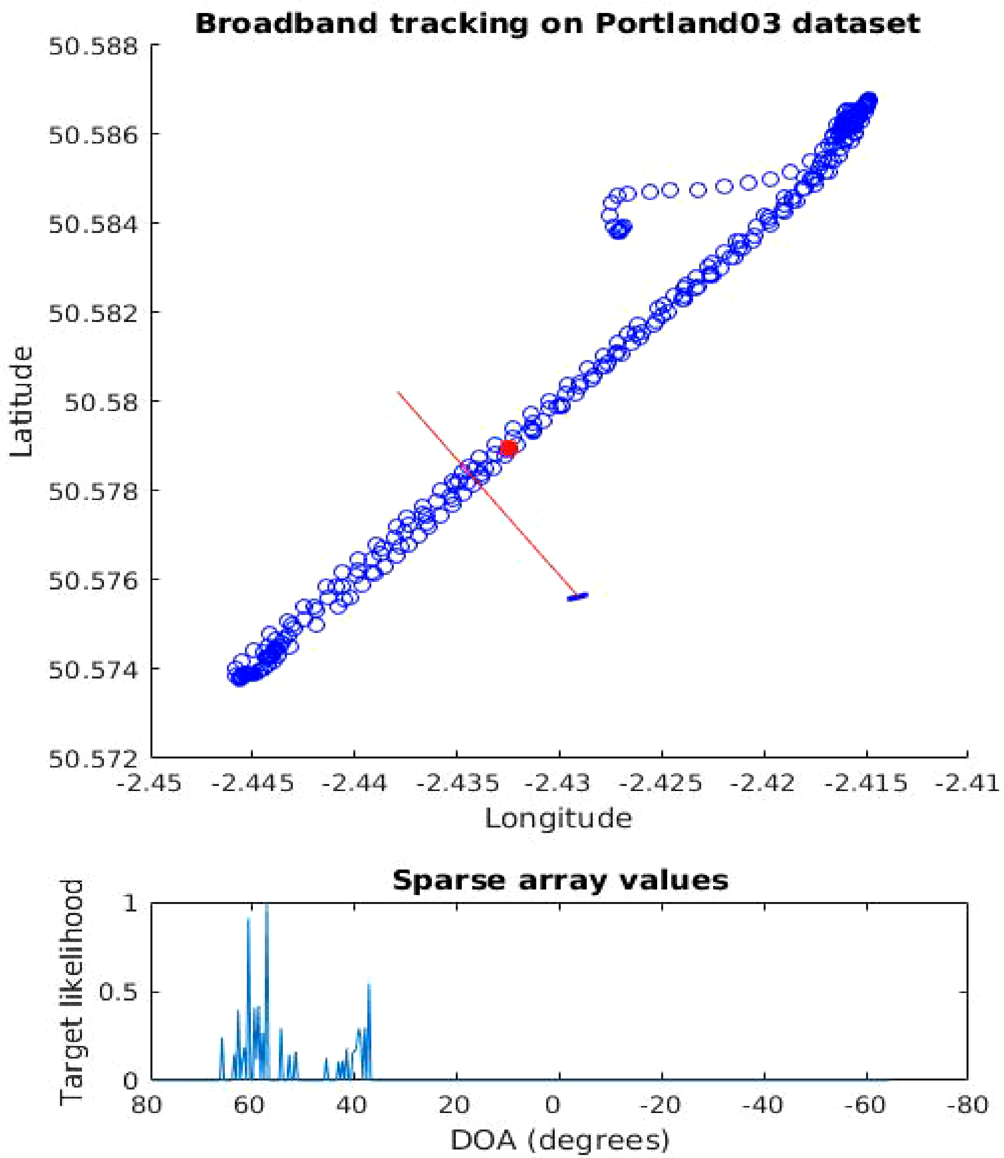
Portland03 Dataset – DoA Estimation Results



Portland03 Dataset – Demo 1



Portland03 Dataset – Demo 2



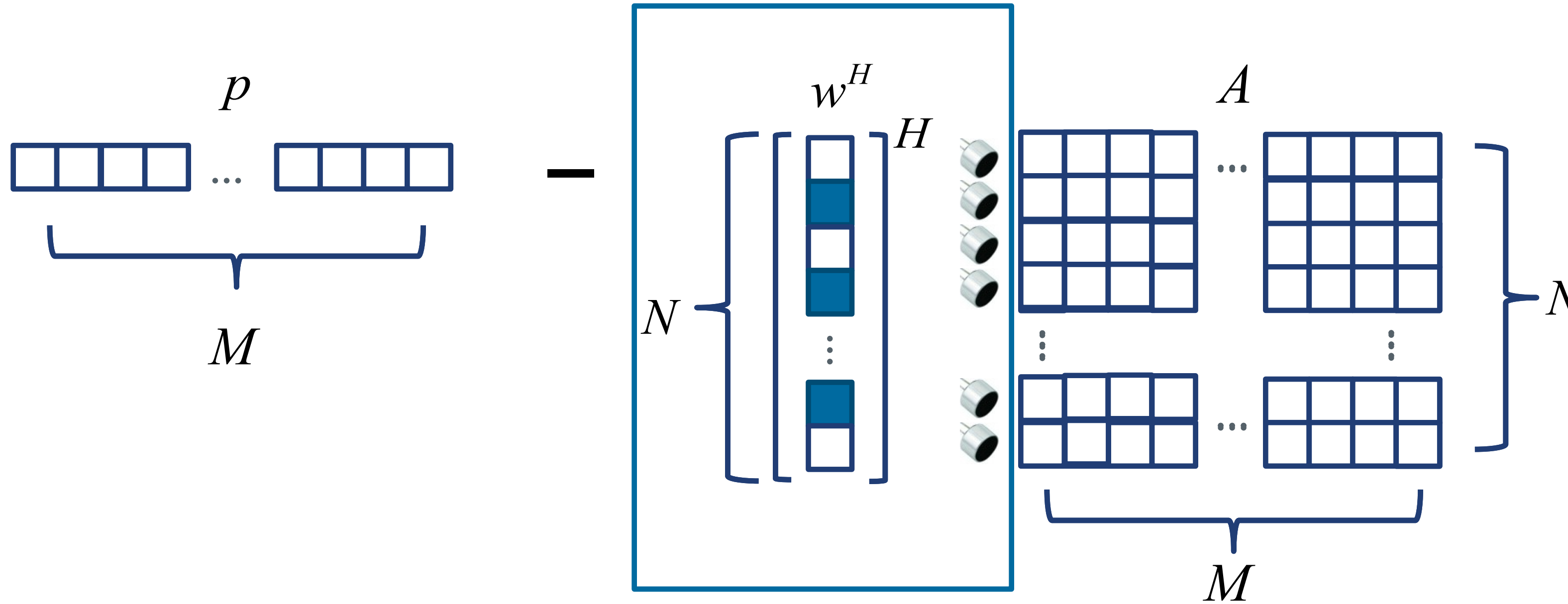
JASSO for Narrowband DoA estimation

Compressed sensing (CS) based sparse array

Minimisation of the ℓ_1 norm of vector weight coefficients, i.e. $w = w_R + w_I i$

$$\min \quad \|w_R\|_1 + \|w_I\|_1$$
$$\text{subject to} \quad \|p - (w_R + w_I i)^H A\|_2 \le \alpha \qquad (3)$$

where p is the vector holding the desired beam response, α is a constraint.



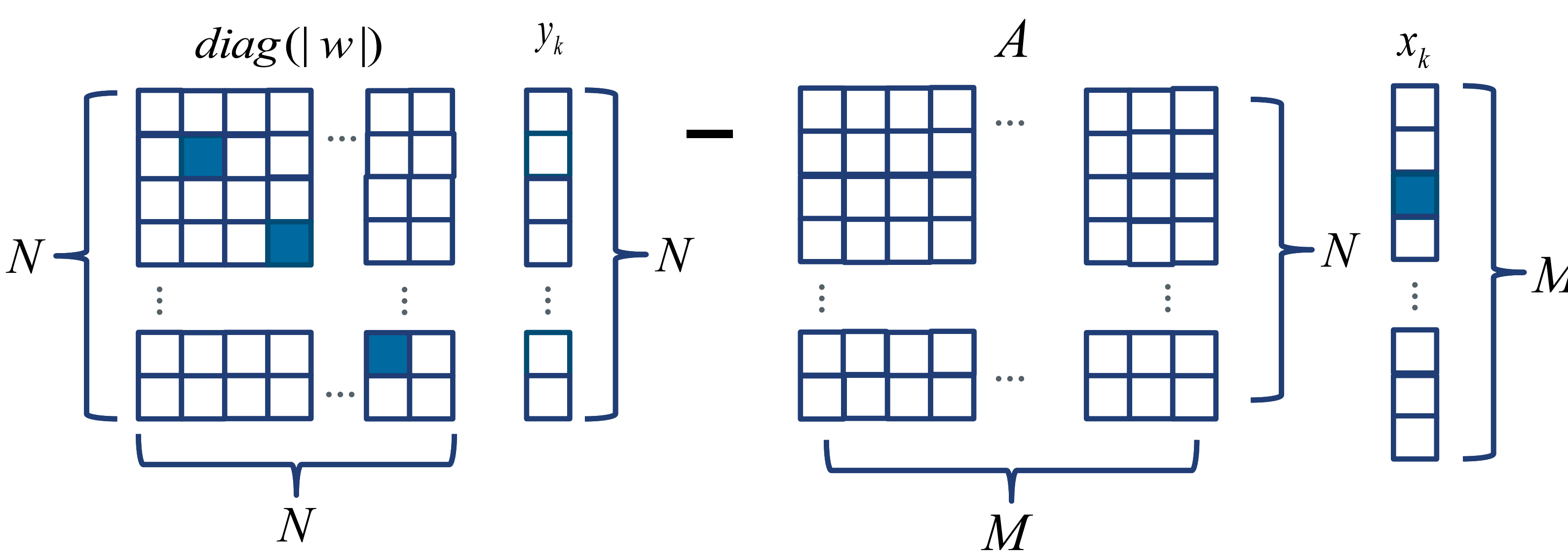
JASSO for Narrowband DoA estimation

Spatial sparsity optimisation using LASSO

The LASSO function promoting spatial sparsity is defined by

$$\min \quad \left\| \boxed{diag(|w|)\, y_k} - A x_k \right\|_2^2 + \mu \left\| D x_k \right\|_1 \tag{4}$$

where μ is to control sparsity, D is the matrix holding the coefficients to the source activity in the source space. Both cost functions (1) and (2) are optimised by the CVX toolbox in Matlab.



JASSO for Narrowband DoA estimation

Implementation and experiment results[1]

Input: observed signal y_k
Output: weight coefficient for sensors: w
spatial sparsity: x_k and estimated DoA: p
Initialisation: generate $p \in C^{1\times M}$ at random degrees
Run:
for $k = 1,2,3\cdots$
for $k = 1,2$
optimise (3) to obtain w_R and w_I
$w = w_R + w_I i$
optimise (4) to obtain x_k
reconstruct $p_{reconstruct} = (Ax_k)^H A$
$p_{reconstruct} = p$
end
end

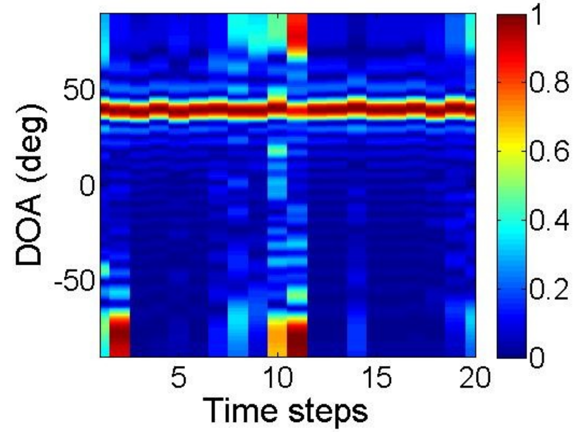


Figure 1. Narrowband DoA estimations for stationary source (SNR=20dB), 37/100 active sensors

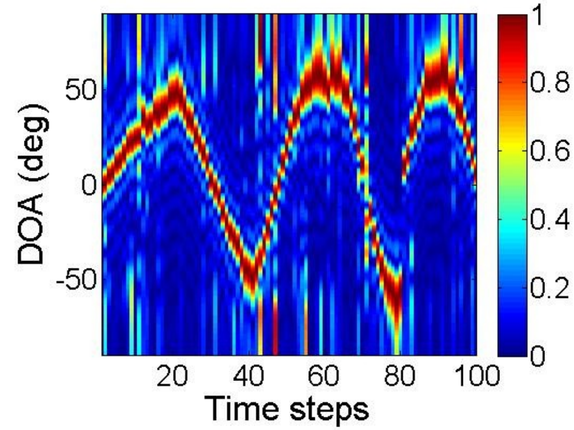


Figure 2. Narrowband DoA estimations for moving source (SNR=20dB), 22/100 active sensors

M. Chen, M. Barnard, and W. Wang. "Joint Array and Spatial Sparsity Based Optimisation for DoA Estimation." in Sensor Signal Processing for Defence (SSPD), IEEE, 2016.

JASSO for Wideband DoA estimation

Source signal

$$y_k(j) = A(j)x_k(j) + n_k(j)$$

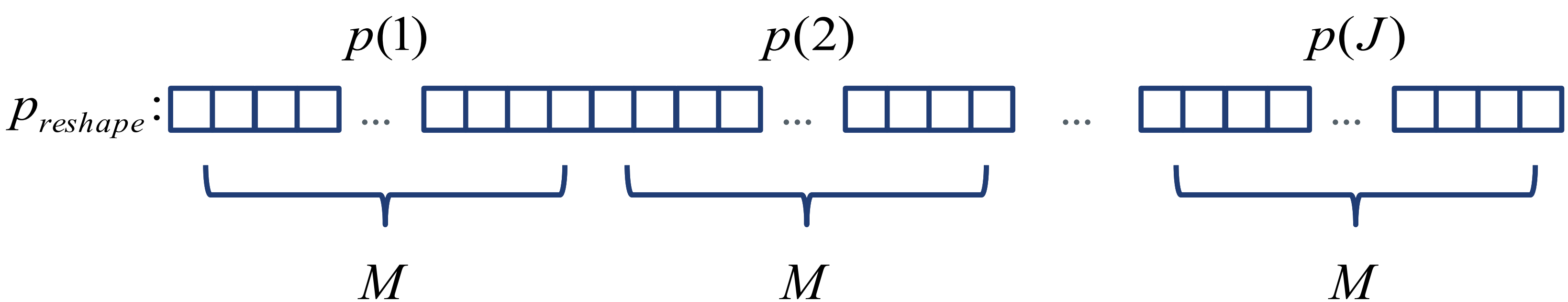
$y_k(j):(y_{k_1}(j), y_{k_2}(j), \cdots, y_{k_N}(j))^T$ at each time step k where N is the number of sensors and $j = 1,2,\cdots J$ is the index of the frequency band Ω_j .

$x_k:(x_{k_1}, x_{k_2}, \cdots, x_{k_M})^T$ where M is the number of potential source directions.

$n_k(j)$ is a random noise vector at the $j-th$ frequency band.

Desired beam response

$$p_{reshape} = [p(\Omega_1, \theta_1), \cdots, p(\Omega_1, \theta_M), p(\Omega_2, \theta_1), \cdots, p(\Omega_2, \theta_M), \cdots, p(\Omega_J, \theta_1), \cdots, p(\Omega_J, \theta_M)]$$

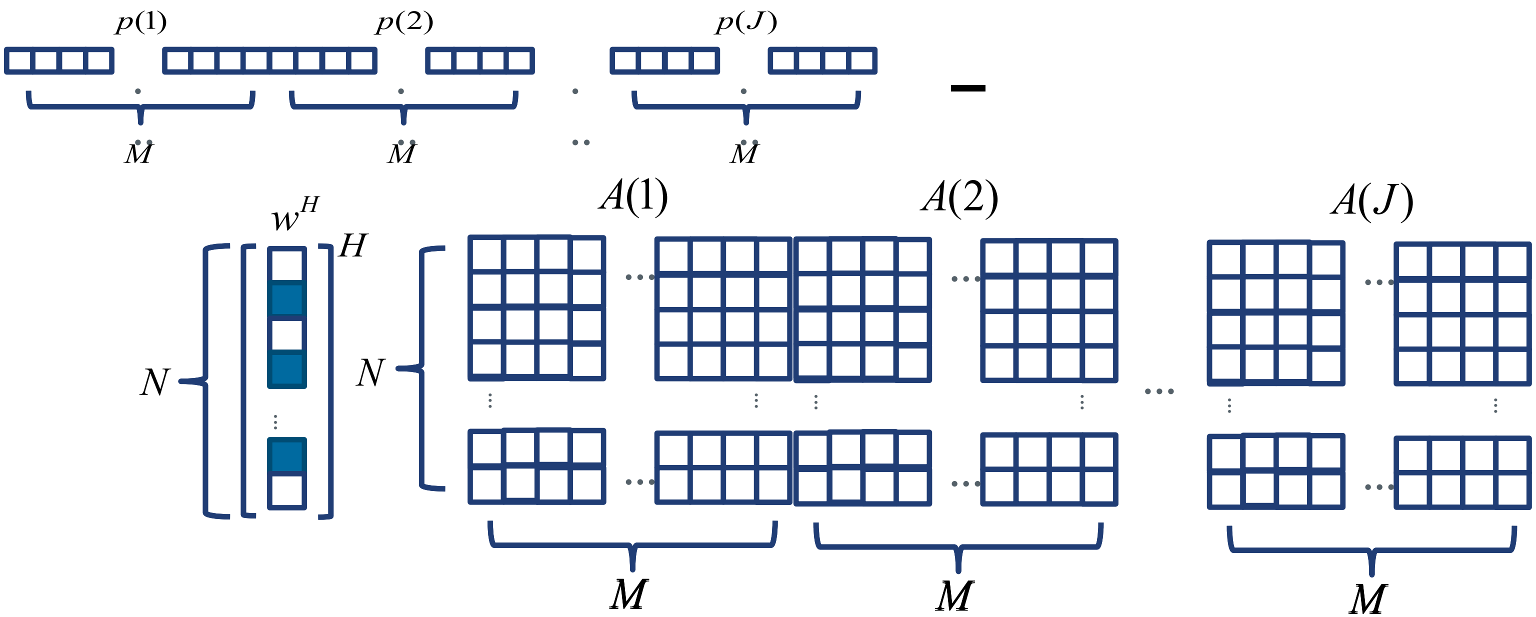


Compressed sensing (CS) based sparse array

For the CS-based sparse wideband array optimisation, the cost function (1) with complex vector weight coefficients, i.e. $w = w_R + w_I i$, is modified as

$$\begin{aligned} &\min \quad \|w_R\|_1 + \|w_I\|_1 \\ &\text{subject to} \quad \left\| p_{reshape} - (w_R + w_I i)^H A_{array} \right\|_2 \le \alpha \end{aligned} \qquad (5)$$

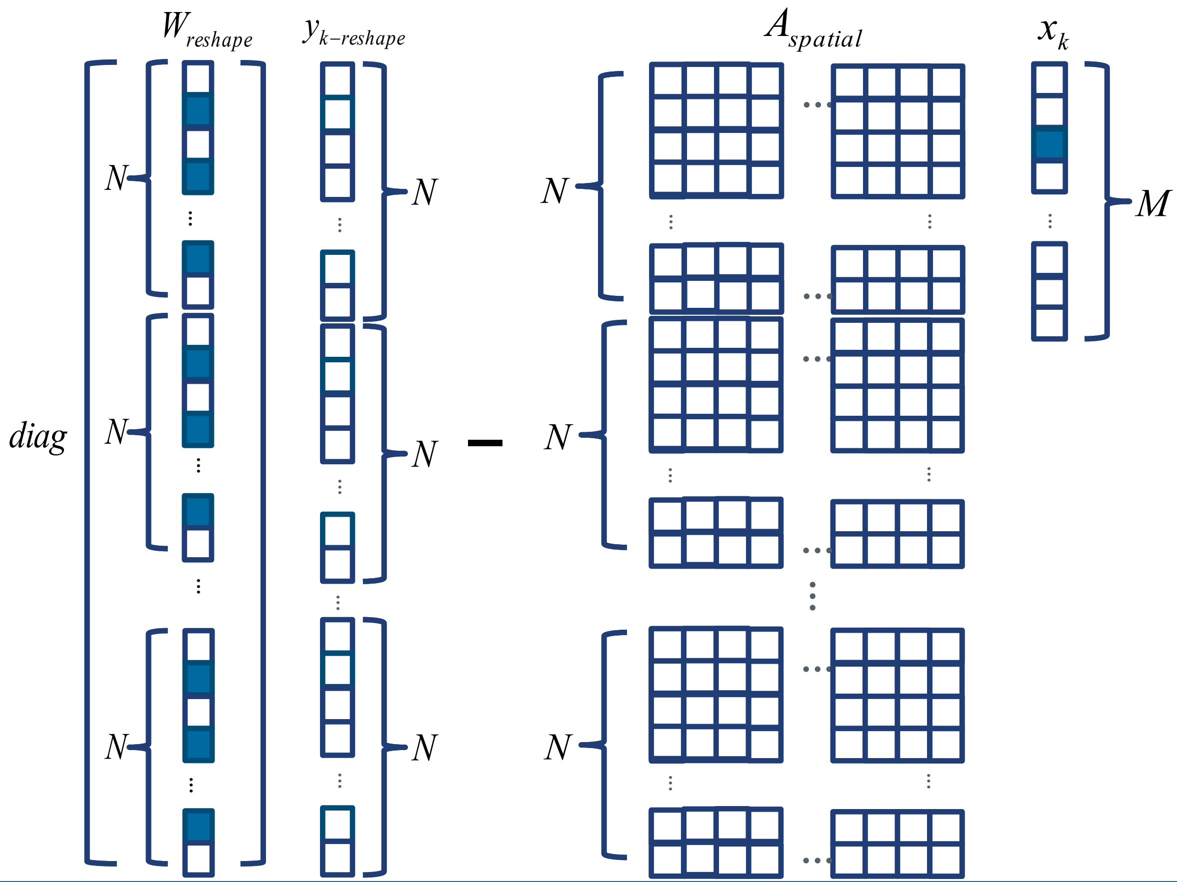
where α is a constraint.



Spatial sparsity optimisation using LASSO

After obtaining the suitable w, the LASSO function (2) is then modified as

$$\min \left\| W_{reshape} y_{k-reshape} - A_{spatial} x_k \right\|_2^2 + \mu \left\| D x_k \right\|_1 \quad (6)$$



JASSO for Wideband DoA estimation

Implementation

Input: observed signal y_k
Output: weight coefficients for sensors: w and estimated DoA: p
Initialisation: generate $p_{reshape} \in C^{1\times(MJ)}$ at 0 degrees,
form $A_{array} \in C^{N\times(MJ)}$
form $A_{spatial} \in C^{(NJ)\times M}$
Run:
for $k = 1,2,3,\cdots$
for $kk = 1,2$
optimise (5) to obtain w_R and w_I
$w = w_R + w_I i$
optimise (6) to obtain x_k
calculate $x_{k-repeat} = [x_k, \cdots, x_k]^T$
reconstruct $p = (A_{array} x_{k-repeat})^H A_{array}$
$p_{reshape} = p$
end
end

Experment setup

Implement wideband DoA estimations of stationary source and moving source.

The first input beam response is initialised at 0 degrees based on a Chebyshev window function. The wideband frequency of the sources is divided into bands at 1000 Hz, 1100 Hz, 1200 Hz, 1300 Hz, and 1400 Hz. A linear array with 300 sensors is used and the inter-sensor spacing is 0.05λ. The maximum running time-step is $K = 20$.

The Mean Square Errors (MSEs) for sparse array optimisation and spatial sparsity based optimisation are used as the performance index, which are defined according to functions as

$$MSE_{array} = 20\log_{10}\left(\frac{\| p_{reshape} - w^H A_{array} \|_2^2}{M}\right) dB, \quad MSE_{spatial} = 20\log_{10}\left(\frac{\| y_k^H A - p \|_2^2}{M}\right) dB$$

M. Chen, W. Wang, M. Barnard, and J.A. Chambers, "Wideband DoA Estimation Based on Joint Optimisation of Array and Spatial Sparsity", in *Proc. European Signal Processing Conference* (EUSIPCO 2017), Kos Island, Greece, August 28- September 2, 2017. (accepted)

JASSO for Wideband DoA estimation

Experiment results

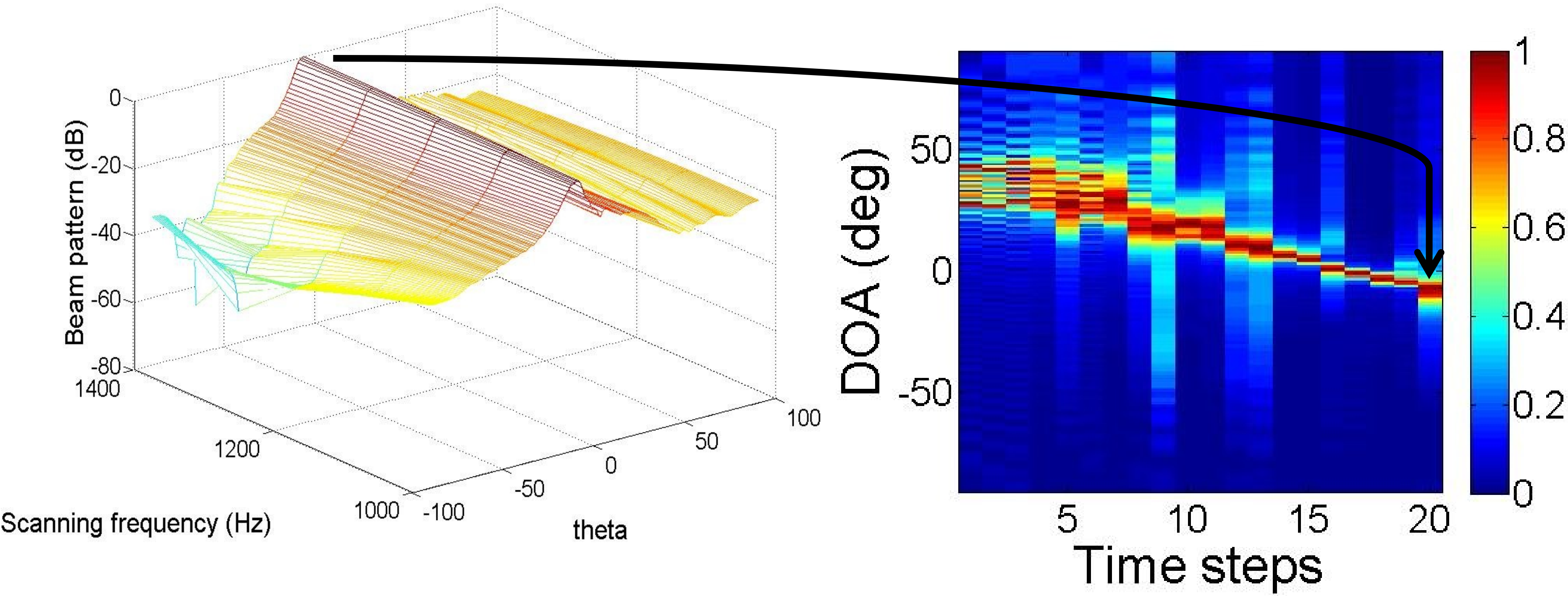


Figure 3. Wideband DoA estimations for the moving source, the 3D graph illustrates the simulation result at the 20-th time step and the spectrogram is the DoA estimation for the last frequency band, 69/300 active sensors.

JASSO for Wideband DoA estimation

Experiment results

We also compare the joint approach with a baseline which only considers the spatial sparsity-based optimisation using a full array. Only noiseless signals at frequency bands of 1000 Hz and 1100 Hz were processed and the total running time $K = 100$.

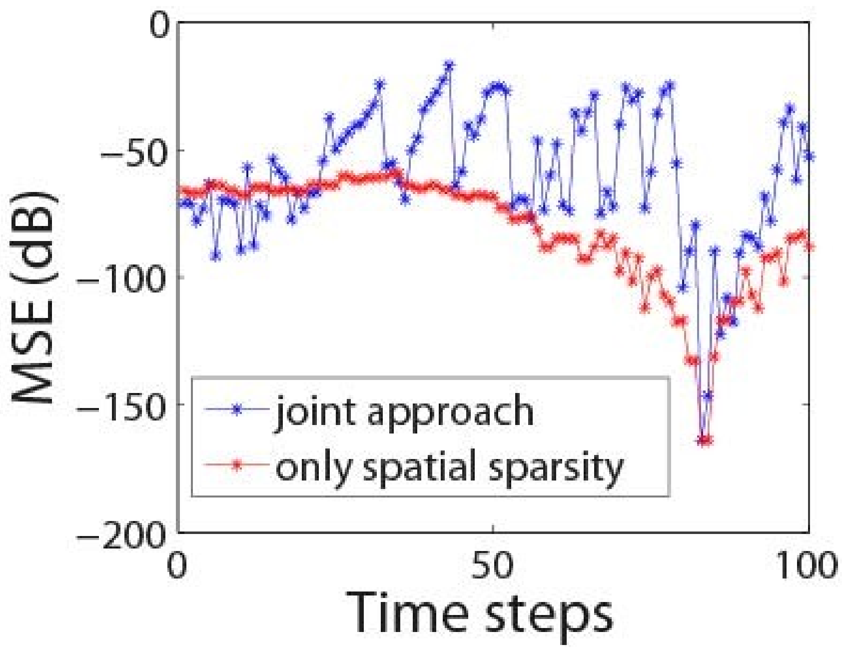


Figure 4. $MSE_{spatial}$ for the moving source without noise.

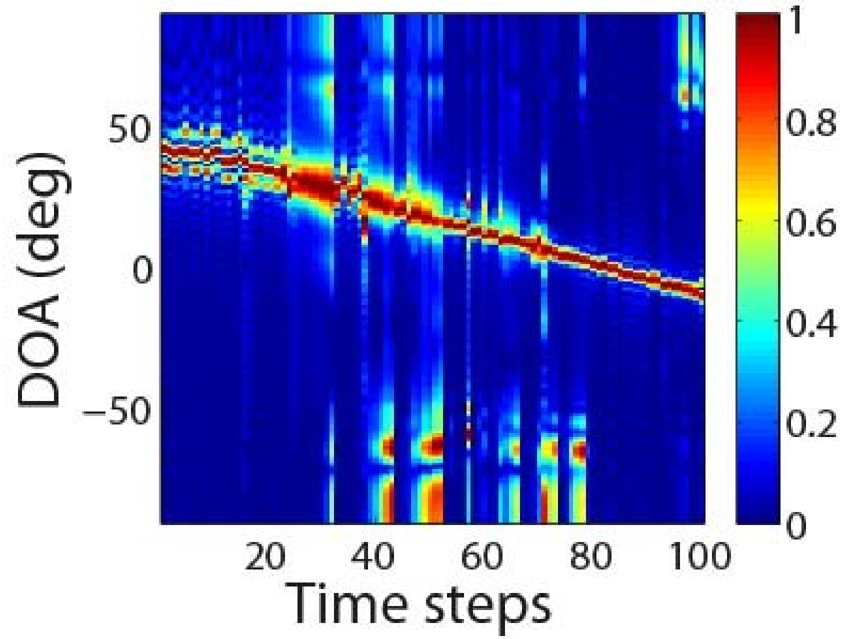


Figure 5. DoA estimated for the moving source without noise at the frequency band of 1100 Hz, 59/300 active sensors

Co-Prime Sampling Arrays

Detecting more sources than the number of sensors

Nested array

Enhancing the degrees-of freedom (DoFs)

Causing mutual coupling

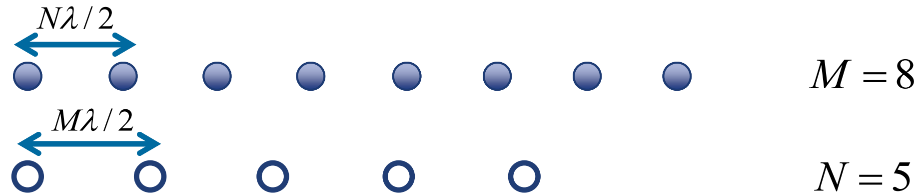
Improved

Co-Prime sampling arrrays

Less mutual coupling, using more sensors

Sensor reduction strategy

Original co-prime arrays:

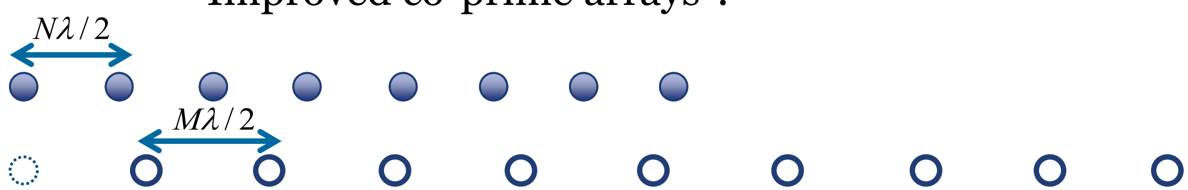


Difference co-array: $Mn - Nm, 0 \le n \le N-1, 0 \le m \le M-1$

Correlations

DoA estimation

Improved co-prime arrays[2]:



Difference co-array:

$Mn - Nm, 1 \le n \le 2N-1, 0 \le m \le M-1$

More DoFs but more sensors

P. Vaidyanathan and P. Pal. "Sparse Sensing With Co-Prime Samplers and Arrays," IEEE TSP, vol. 59, no. 2, Feb 2011.

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Conclusions

- A sparse Bayesian algorithm has been presented and tested on the Portland03 dataset and shows good performance for source localisation for this challenging dataset.
- A two-step iterative algorithm is proposed for narrowband and wideband DoA estimation based on joint optimisation of array and spatial sparsity. The results evaluated for both stationary sources and moving sources show that the proposed algorithm can maintain performance while using a reduced number of sensor for source localisation. This can be useful for joint source localisation and sensor selection.
- Sparse co-prime array based on Chinese remainder theorem is an interesting and emerging direction for the next stage of work.

End

Thank you