# Multi-Object Modelling of Clutter Processes

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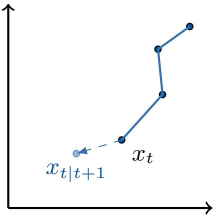
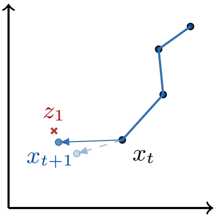
# Outline

# Modelling: Why bother?




Prediction



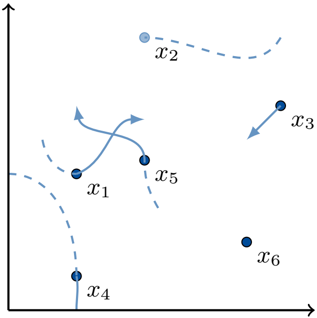

Update

Bayesian filtering

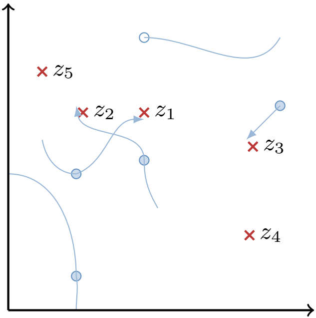
Good models ...

- make Bayesian filtering feasible to compute
- help to estimate the target state more accurately
- can represent information concisely with only a few parameters

## Multi-object filtering using the PHD filter




Object state space




Measurement space

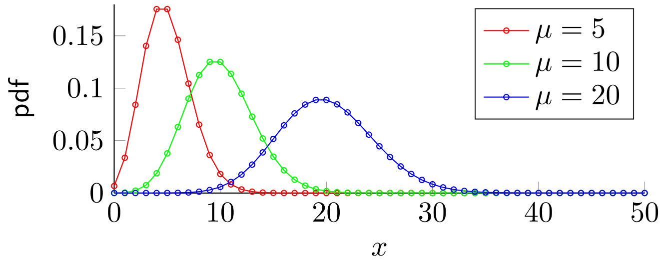
Challenges:

- *Localisation*: inaccurate sensors, different object behaviours
- *Association*: which measurement belongs to which object?
- *Cardinality*: # objects $\longleftrightarrow$ # measurements

Probabilistic solution: Probability Hypothesis Density (PHD) filter [Mah03, VM06]

- *Localisation*: uncertainty in position through spatial distribution $s$
- *Association*: association likelihood $\ell(z|x)$
- *Cardinality*: **Point processes** for detection, survival, clutter, and persisting and new-born targets

## Choice of the PHD filter: Poisson model

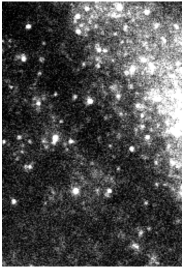



Poisson probability density function (pdf) for different means $\mu$

## Properties

- The Poisson model is easy:
  - one parameter only
  - mean equals variance
  - easy equations, exponential form
- Poisson behaviour is found in many applications

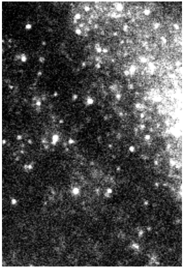
## Non-Poisson noise: an example



Total Internal Reflection Microscopy (TIRF)

Pixel width/height: 106 nm

## Non-Poisson noise: an example



PHD filter assumes Poisson noise

$\rightarrow$ cannot cope with bursts of clutter!

# Different point processes

Three examples:

- Bernoulli point process
- Poisson point process
- Negative binomial point process

## Definition: Point process

- Goal: describe a population of points in a target space $\mathcal{X}$
- Both the target number and locations are random
- Point process $\Phi$: assigns probabilities to any number and configuration of targets in $\mathcal{X}$:
$$(\Omega, \mathcal{F}, \mathbb{P}) \xrightarrow{\Phi} \Big(E_{\mathcal{X}}, \mathcal{B}(E_{\mathcal{X}})\Big) \tag{1}$$
where $(\Omega, \mathcal{F}, \mathbb{P})$ is a probability space and $\Big(E_{\mathcal{X}}, \mathcal{B}(E_{\mathcal{X}})\Big)$ a measurable space

## Definition: Probability Generating Functionals (PGFLs)

Describe the probability density function $p_\Phi$ of the point process $\Phi$ for any object number $n$ by an infinite sum:

$$G_\Phi(h) = \sum_{n \geq 0} \int_{\mathcal{X}^n} \left[ \prod_{i=1}^{n} h(x_i) \right] p_\Phi(x_{1:n}) \mathrm{d}(x_{1:n}) \tag{2}$$

where $h$ is any test function.

## Bernoulli Point Process (parameter $p$, spatial distribution $s$)

- Binary point process: either no target or one target
- Target exists with probability $p$, distributed according to $s(\cdot)$
- PGFL:

$$G_{\text{Bernoulli}}(h) = \underbrace{(1-p)}_{\text{no target}} + \underbrace{p \int_{\mathcal{X}} h(x)s(x)\mathrm{d}x}_{\text{one target}} \tag{3}$$

## Poisson Point Process (intensity $\mu$)

- Poisson distributed target number with parameter $\mu(\mathcal{X})$
- Targets independently and identically distributed (i.i.d.) according to the normalised intensity $\frac{\mu(\cdot)}{\mu(\mathcal{X})}$
- PGFL:

$$G_{\text{Poisson}}(h) = \exp\left(\int_{\mathcal{X}} [h(x) - 1]\mu(x)\mathrm{d}x\right) \tag{4}$$

## Negative Binomial Point Process (parameters $\alpha$, $\beta$, spatial distribution $s$)

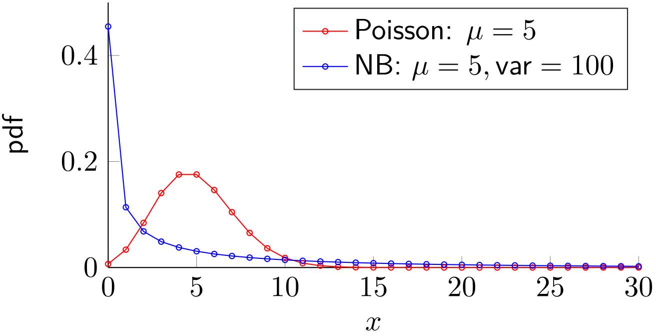
- Negative binomial distributed target number with parameters $\alpha, \beta \in \mathbb{R}_+$
- Targets i.i.d. according to $s(\cdot)$
- PGFL:

$$G_{\mathsf{NB}}(h) = \left(1 + \frac{1}{\beta}\int_{\mathcal{X}}[1 - h(x)]s(x)\mathrm{d}x\right)^{-\alpha} \tag{5}$$

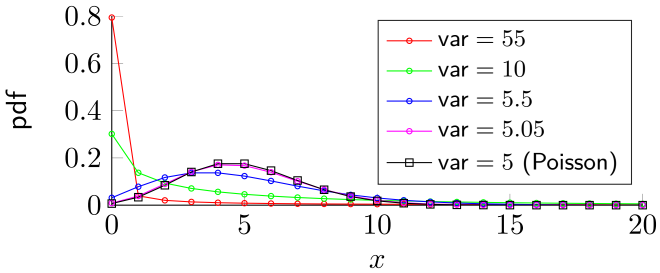
- Target number is characterised by mean $\mu$ and variance var:

$$\mu = \frac{\alpha}{\beta}, \quad \mathsf{var} = \mu\left(1 + \frac{1}{\beta}\right). \tag{6}$$

## Poisson vs. negative binomial distribution



## NB $\rightarrow$ Poisson!




Negative binomial distributions with mean $\mu = 5$ and decreasing variance

# The PHD filter [Mah03, VM06]

## Filter assumptions

- All observations are made independently
- Survival and detection processes are Bernoulli processes
- Predicted and false alarm processes are Poisson processes

## The recursion at time $k \to k+1$ in terms of PGFLs

Prediction PGFL:

$$G_{k+1|k}(h) = G_k\Big(\underbrace{G_{\mathrm{s}}(h)}_{\text{Bernoulli}}\Big)G_{\mathrm{b}}(h) \tag{7}$$

Update PGFL:

$$G_{k+1}(g,h) = \underbrace{G_{k+1|k}}_{\text{Poisson}}\Big(\underbrace{G_{\mathrm{d}}(h)}_{\text{Bernoulli}}\Big)\underbrace{G_{\mathrm{c}}(g)}_{\text{Poisson}} \tag{8}$$

## The recursion at time $k \to k+1$ in terms of intensity

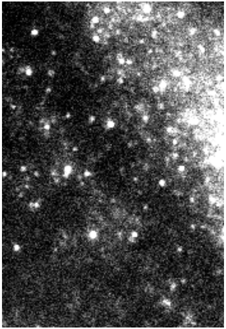
PHD filter prediction:

$$\mu_{k+1|k}(x) = \int_{\mathcal{X}} p_{\mathrm{s}}(y) f_{k+1|k}(x|y) \mu_k(y) \mathrm{d}y + \mu_{\mathrm{b}}(x) \tag{9}$$

PHD filter update:

$$\begin{aligned} \mu_{k+1}(x|Z) &= (1 - p_{\mathrm{d}}(x)) \mu_{k+1|k}(x) \\ &\quad + \sum_{z \in Z} \frac{p_{\mathrm{d}}(x) \ell(z|x) \mu_{k+1|k}(x)}{\int_{\mathcal{X}} p_{\mathrm{d}}(y) l(z|y) \mu_{\Phi}(y) \mathrm{d}y + \mu_{\mathrm{c}}(z)} \end{aligned} \tag{10}$$

## Limitation



- All observations are made independently
- Survival and detection processes are Bernoulli processes
- Predicted and false alarm processes are Poisson processes

$\rightarrow$ does not work in the example!

# A PHD filter alternative [SDHC16]

## Filter assumptions

- All observations are made independently
- Survival and detection processes are Bernoulli processes
- Predicted process is a Poisson process
- The false alarm process is a negative binomial process

## The recursion at time $k \to k+1$ in terms of PGFLs

Prediction PGFL:

$$G_{k+1|k}(h) = G_k\big( \underbrace{G_\mathrm{s}(h)}_{\text{Bernoulli}} \big) G_\mathrm{b}(h) \tag{11}$$

Update PGFL:

$$G_{k+1}(g,h) = \underbrace{G_{k+1|k}}_{\text{Poisson}} \big( \underbrace{G_\mathrm{d}(h)}_{\text{Bernoulli}} \big) \underbrace{G_\mathrm{c}(g)}_{\text{NB!}} \tag{12}$$

## The recursion at time $k \to k+1$ in terms of intensity

Prediction (the same as for the PHD filter):

$$\mu_{k+1|k}(x) = \int_{\mathcal{X}} p_{\mathrm{s}}(y) f_{k+1|k}(x|y) \mu_k(y) \mathrm{d}y + \mu_{\mathrm{b}}(x) \tag{13}$$

New update intensity:

$$\begin{aligned} \mu_{k+1}(x|Z) = {} & (1 - p_{\mathrm{d}}(x)) \mu_{k+1|k}(x) \\ & + \sum_{z \in Z} \frac{p_{\mathrm{d}}(x) \ell(z|x) \mu_{k+1|k}(x)}{s_{\mathrm{c}}(z)} l(Z \setminus \{z\}) \end{aligned} \tag{14}$$

## Both filters in comparison (update)

PHD filter update:

$$\mu_{k+1}(x|Z) = (1 - p_{\mathrm{d}}(x))\mu_{k+1|k}(x) + \sum_{z \in Z} \frac{p_{\mathrm{d}}(x)\ell(z|x)\mu_{k+1|k}(x)}{\int_{\mathcal{X}} p_{\mathrm{d}}(y) l(z|y)\mu_{\Phi}(y)\mathrm{d}y + \mu_{\mathrm{c}}(z)}$$

New filter update:

$$\mu_{k+1}(x|Z) = (1 - p_{\mathrm{d}}(x))\mu_{k+1|k}(x) + \sum_{z \in Z} \frac{p_{\mathrm{d}}(x)\ell(z|x)\mu_{k+1|k}(x)}{s_{\mathrm{c}}(z)} l(Z \setminus \{z\})$$

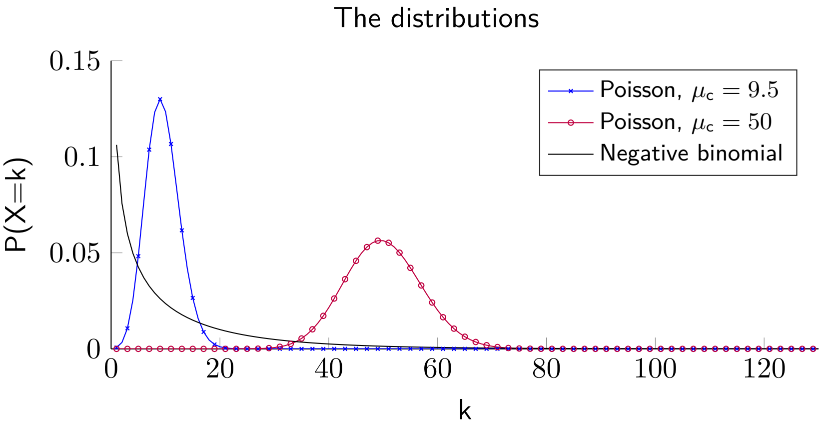
# Experiments

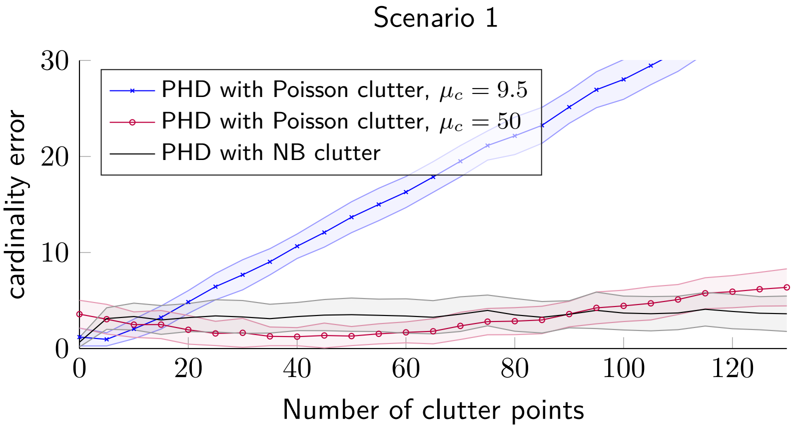
Simulation settings:

- Field of view: $(50\,\mathrm{m})^2$
- $p_\mathrm{s} = 0.99$, acceleration noise $q = 0.01\,\mathrm{m\,s^{-2}}$
- i.i.d. targets, standard deviation of initial target velocity: $0.5\,\mathrm{m\,s^{-1}}$
- $p_\mathrm{d} = 0.9$, measurement noise per dimension: $0.5\,\mathrm{m}$
- Birth intensity: Poisson, $\mu_\mathrm{b} = 0.5$

Scenario 1:

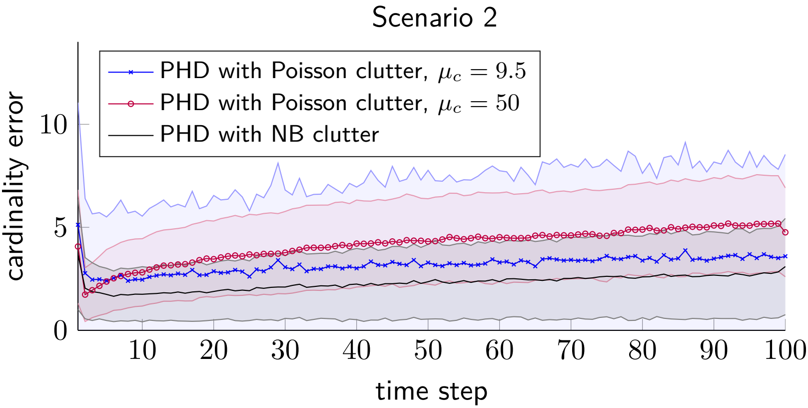
- $100$ Monte Carlo runs, $15$ time steps
- $9$ clutter points at times $1$–$14$
- varying number of clutter points at time $15$: 0,1,2,...130
- Poisson PHD: $\mu_\mathrm{c} = 9.5$ and $50$
- Negative binomial PHD: $\mu_\mathrm{c} = 9.5$ and $\mathrm{var}_\mathrm{c} = 190$


The distributions
0.15
0.1
0.05
0
P(X=k)
0
20
40
60
80
100
120
k
Poisson, $\mu_c = 9.5$
Poisson, $\mu_c = 50$
Negative binomial


Scenario 1
30
20
10
0
0
20
40
60
80
100
120
cardinality error
Number of clutter points
PHD with Poisson clutter, $\mu_c = 9.5$
PHD with Poisson clutter, $\mu_c = 50$
PHD with NB clutter

Scenario 2:

- $500$ Monte Carlo runs, $100$ time steps
- NB distributed clutter points with mean $9.5$ and variance $190$
- Poisson PHD: $\mu_{\mathrm{c}} = 9.5$ and $50$
- Negative binomial PHD: $\mu_{\mathrm{c}} = 9.5$ and $\mathrm{var}_{\mathrm{c}} = 190$


Scenario 2
PHD with Poisson clutter, $\mu_c = 9.5$
PHD with Poisson clutter, $\mu_c = 50$
PHD with NB clutter
cardinality error
0
5
10
10
20
30
40
50
60
70
80
90
100
time step

# Conclusion

- Good models are important because they facilitate Bayesian filtering
- Restrictive models cannot cope with out-of-model behaviour
- Negative binomial model generalises Poisson model
  $\rightarrow$ more flexibility to describe noise
  $\rightarrow$ more robustness against outliers in clutter cardinality

# References


[Mah03] Ronald PS Mahler. Multitarget Bayes filtering via first-order multitarget moments. *Aerospace and Electronic Systems, IEEE Transactions on*, 39(4):1152–1178, 2003.

[SDHC16] Isabel Schlangen, Emmanuel D. Delande, Jérémie Houssineau, and Daniel Clark. A PHD filter with negative binomial clutter. In *Accepted to: International Conference on Information Fusion (FUSION)*, 2016.

[VM06] Ba-Ngu Vo and Wing-Kin Ma. The Gaussian mixture probability hypothesis density filter. *Signal Processing, IEEE Transactions on*, 54(11):4091–4104, 2006.

# Questions?