



# Partially Adaptive STAP

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# Taxonomy of Partially STAP Algorithms

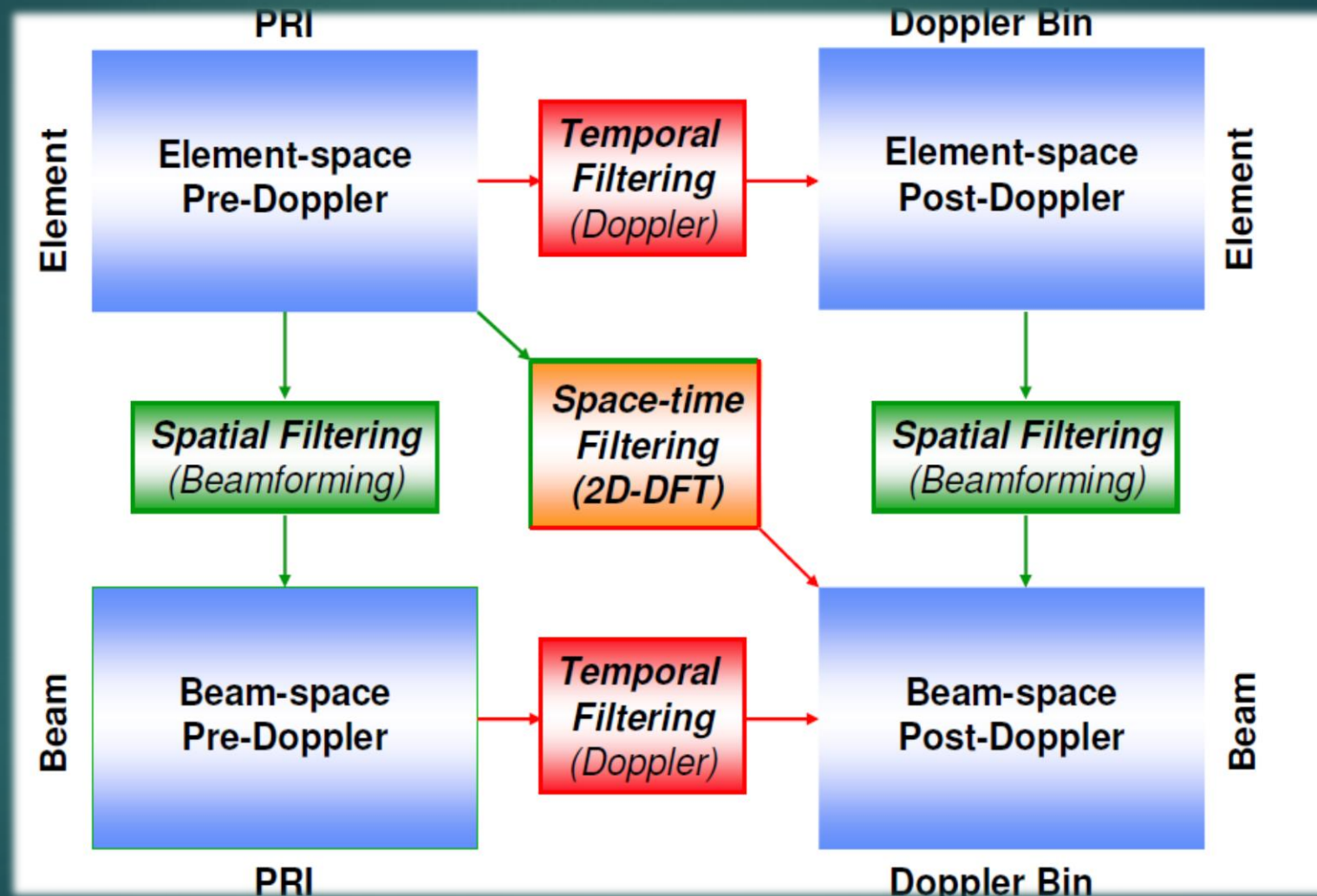
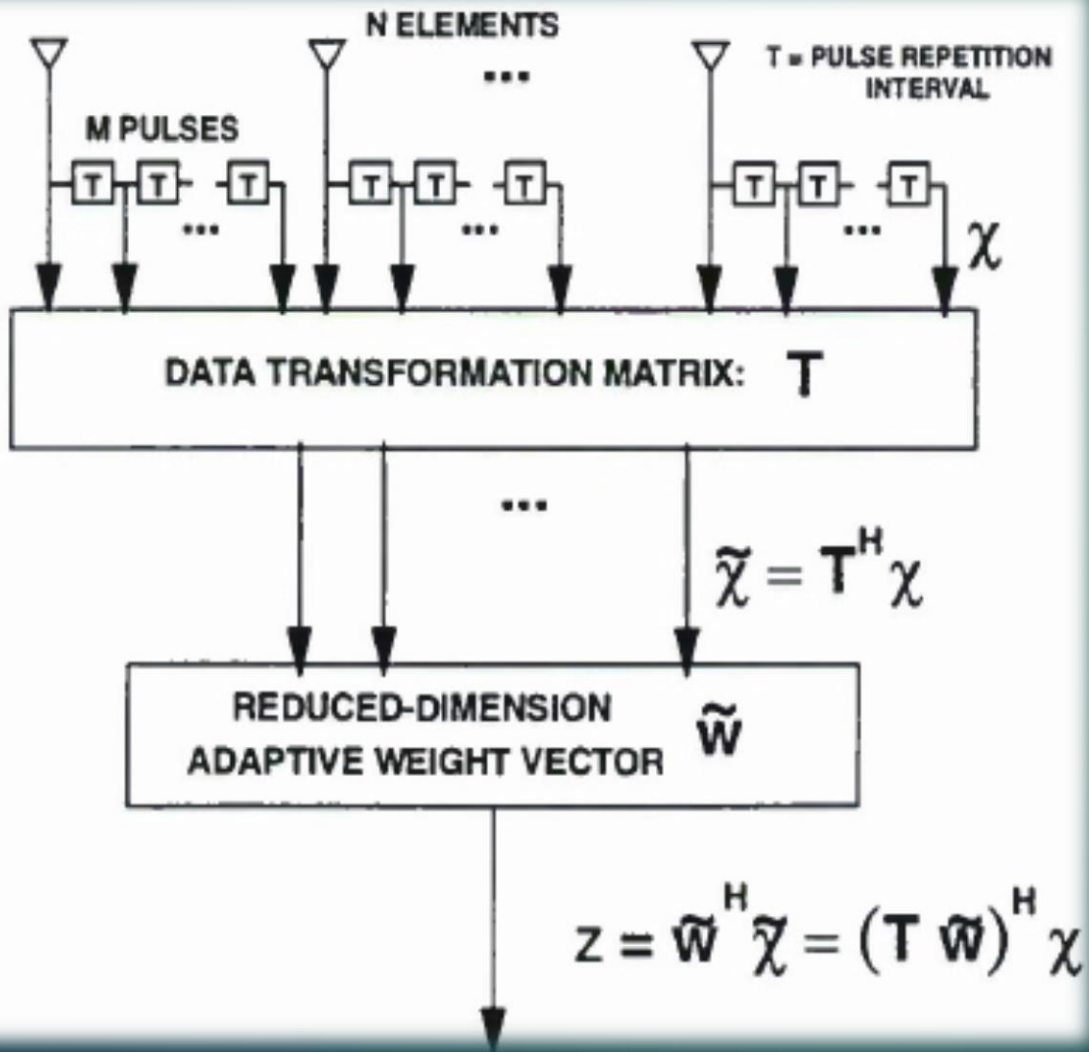


Figure adopted from [4]

Algorithms classified by domain where adaptive processing occurs.

# Partially Adaptive STAP Fundamentals



- Reduced dimensionality comes from an  $MN \times D$  preprocessor matrix  $\mathbf{T}$ :  

$$\tilde{\chi} = \mathbf{T}^H \chi = \alpha \tilde{\mathbf{v}}_t + \tilde{\chi}_u \text{ is } D \times 1 \text{ where } D \ll MN.$$
- After data transformation, the  $D \times 1$  adaptive weight vector is computed:

$$\tilde{\mathbf{w}} = \tilde{\mathbf{R}}_u^{-1} \tilde{\mathbf{g}}_t$$

Where

$$\tilde{\mathbf{R}}_u = E\{\tilde{\chi}_u \tilde{\chi}_u^H\} = \mathbf{T}^H \mathbf{R}_u \mathbf{T}$$

is the  $D \times D$  covariance matrix of the transformed data.

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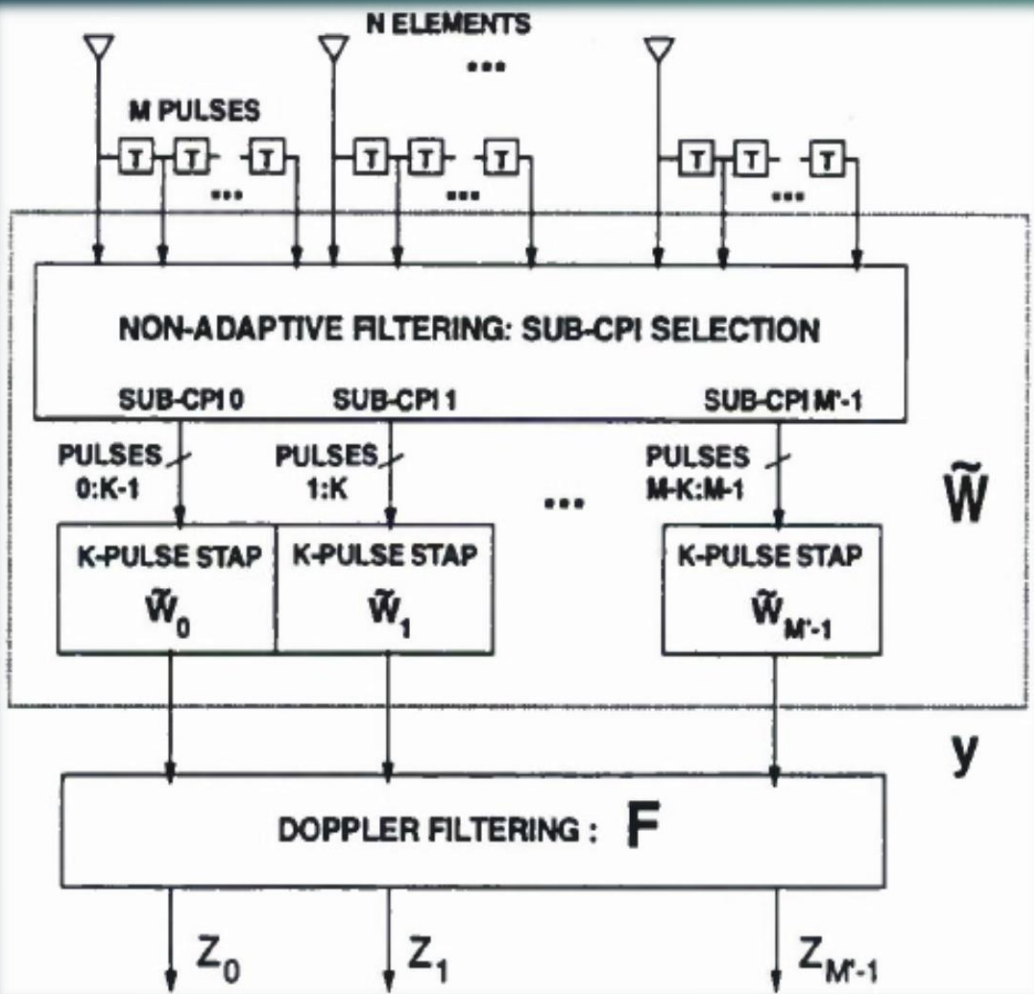
## Case I: Pre-Doppler Processing “Adapt then Filter”

- ▶ Adaptively Combine returns from few (2 or 3) pulses at a time.
- ▶ Retain Full Spatial Adaptivity.
- ▶ Adaptive temporal and spatial processing is followed by a fixed, non-adaptive, Doppler filter bank providing coherent integration over the full CPI.
- ▶ Preprocessor Matrix is:  $\mathbf{T} = \mathbf{J}_p \otimes \mathbf{I}_N$ ,

where  $\mathbf{J}_p = \begin{bmatrix} \mathbf{0}_{p \times K} \\ \mathbf{I}_K \\ \mathbf{0}_{(M-K-p) \times K} \end{bmatrix}$  is an  $M \times K$  pulse selector matrix.

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## Case I: Pre-Doppler Processing “Adapt then Filter”



- using more than one pulse provides temporal adaptivity required for clutter cancellation.
- Full spatial adaptivity provides the means to null jamming at the same time.
- Adaptive (temporal & spatial) processing is followed by a non-adaptive, Doppler filter bank providing coherent integration over the full CPI and the means for velocity estimation.



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## Case II: Post-Doppler Processing

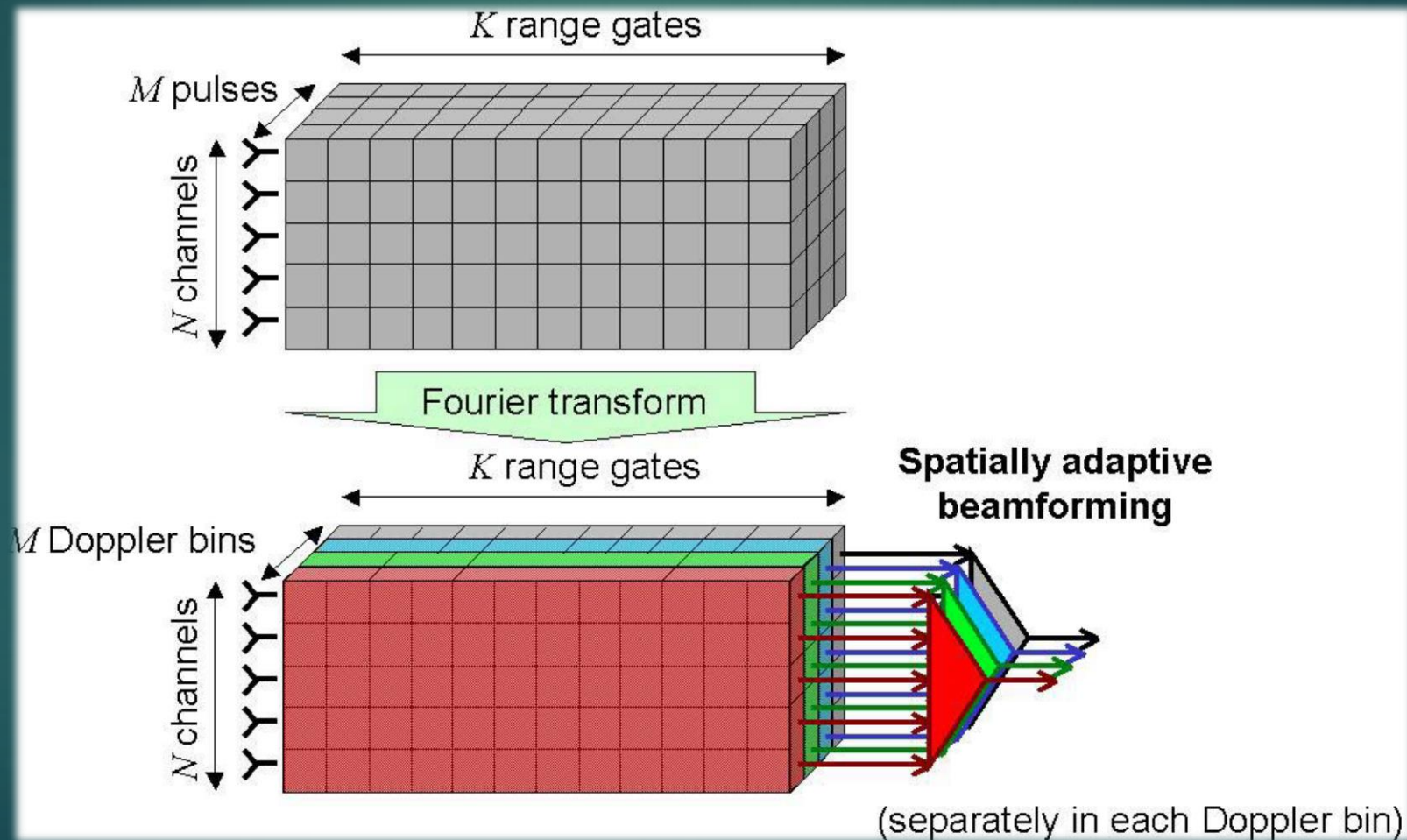


Figure adopted from [5].

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## Case II: Post-Doppler Processing

- ▶ Otherwise termed “*Factored Post-Doppler*” or “*Factored STAP*”.
- ▶ Apply a single non-adaptive Doppler filter bank on each element.
- ▶ Perform adaptive spatial beamforming separately within each Doppler bin.
- ▶ It is assumed that Doppler filtering suppresses mainlobe clutter non-adaptively => Poor Results. (Up-to this point no temporal adaptivity).
- ▶ Performance Improvement: Multiwindow post-Doppler STAP

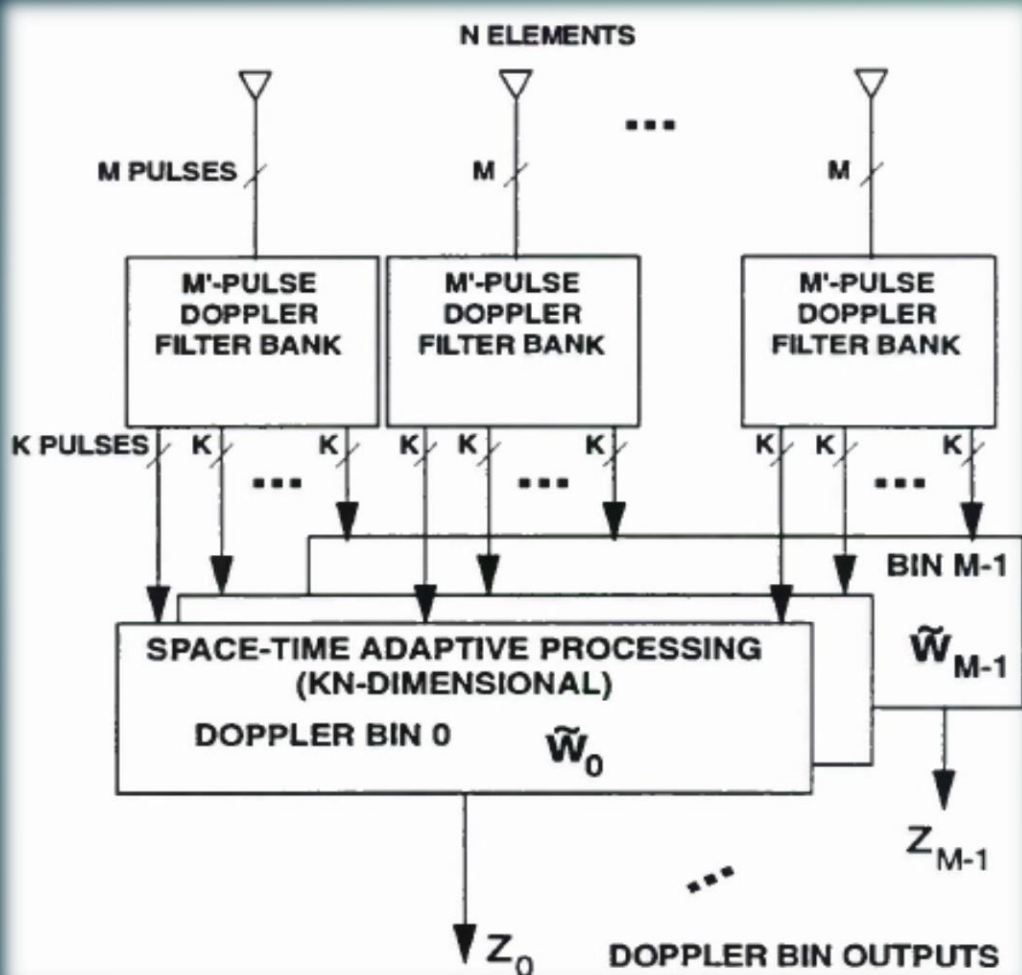
Add temporal adaptivity by employing adaptive combination of multiple Doppler bins from each element. Depending on the method of construction of the Doppler filter bank matrix  $\tilde{\mathbf{F}}_m$ , we may further discriminate between 2 methods:

- i) **PRI-Staggered Post-Doppler** (“Filter then Adapt” by Brennan & Staudaher [1])
- ii) **Adjacent-bin Post-Doppler** (“Extended Factored STAP” by DiPietro [2]).

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## Case II: Post-Doppler Processing *“Filter then Adapt”*

### PRI-Staggered Post-Doppler STAP



- Each element has a filterbank of  $M'$ -pulse Doppler filters that produce  $K$  output pulses for each filter (Doppler bin).
- For each Doppler bin an adaptive processor combines the  $K$  pulses from each element to produce the output signal for that bin.



# ELEMENT SPACE STAP

## Case II: Post-Doppler Processing “Filter then Adapt”

► PRI-Staggered Post-Doppler Preprocessor :  $\mathbf{T}_m = \tilde{\mathbf{F}}_m \otimes \mathbf{I}_N$ ,

- $\tilde{\mathbf{F}}_m$  is an  $M \times K$  matrix whose columns form a set of  $M$ -pulse FIR Doppler filters applied to the incoming signals on each element.
- For PRI-Staggered Post-Doppler STAP:

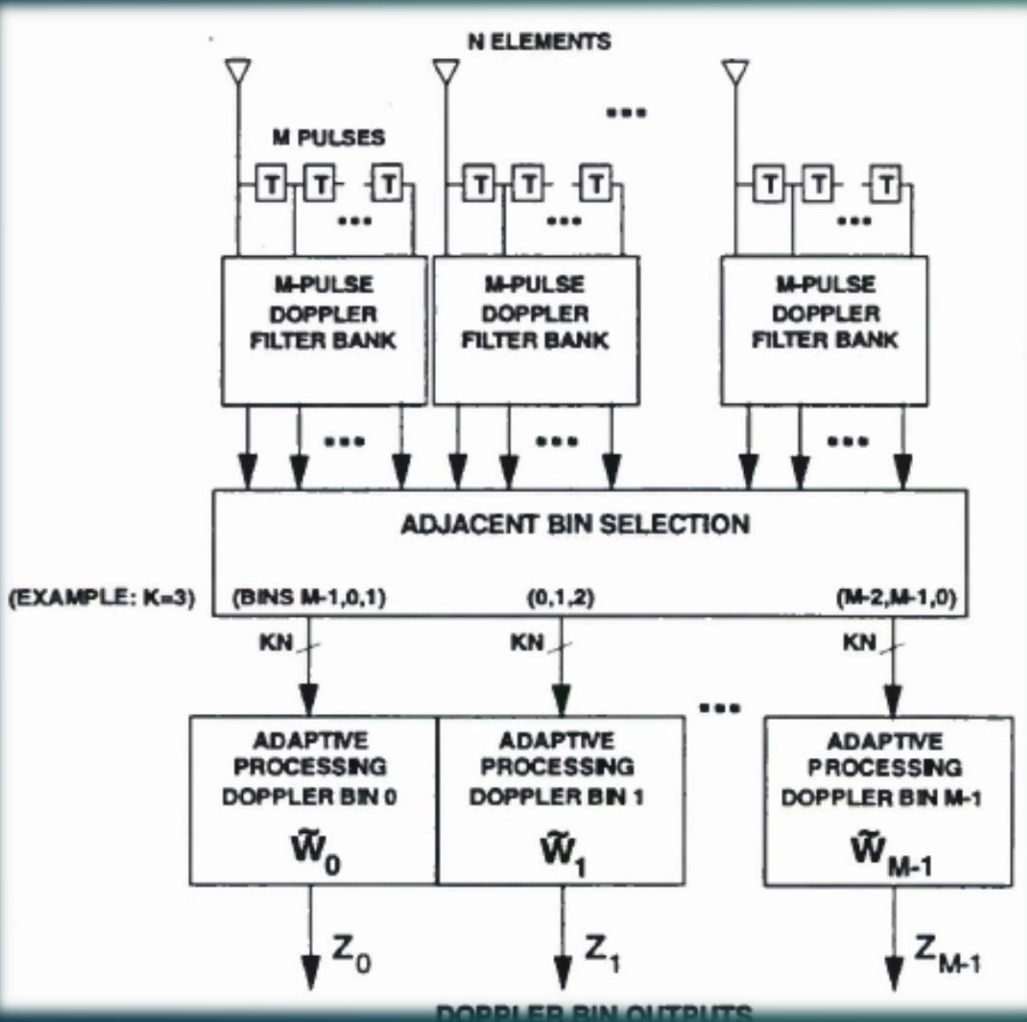
$$\tilde{\mathbf{F}}_m = \text{Toeplitz}([\mathbf{f}_m ; \mathbf{0}_{(K-1) \times 1}], [f_{m0} ; \mathbf{0}_{1 \times (K-1)}])$$

- $\mathbf{f}_m = \mathbf{t}_f \odot \mathbf{u}_m$  is the  $m$ -th bin Doppler filter impulse response.

$m$ : index of the target Doppler bin of interest

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## Case II: Post-Doppler Processing “Factored STAP”



### Adjacent-Bin Post-Doppler STAP

- A single length  $M$  Doppler filterbank is present on each element.
- The  $m$ -th Doppler bin output is formed by adaptively combining the spatial samples from a cluster of  $K$  adjacent Doppler bins centered at the  $m$ -th bin center frequency.

# ELEMENT SPACE STAP

## Case II: Post-Doppler Processing “Factored STAP”

- **Adjacent-Bin Post-Doppler Preprocessor** :  $\mathbf{T}_m = \tilde{\mathbf{F}}_m \otimes \mathbf{I}_N$ ,
  - $\tilde{\mathbf{F}}_m$  is an  $M \times K$  matrix whose columns form a set of  $M$ -pulse FIR Doppler filters applied to the incoming signals on each element.
  - For Adjacent Beam Post-Doppler:

$$\tilde{\mathbf{F}}_m = [\mathbf{f}_{m-P}, \dots, \mathbf{f}_m, \dots, \mathbf{f}_{m+P}]$$

where the  $m$ -th bin output adaptively combines signal from Doppler bins  $m-P, \dots, m+P$ .

- $\mathbf{f}_m = \mathbf{t}_f \odot \mathbf{u}_m$  is the  $m$ -th bin Doppler filter impulse response.

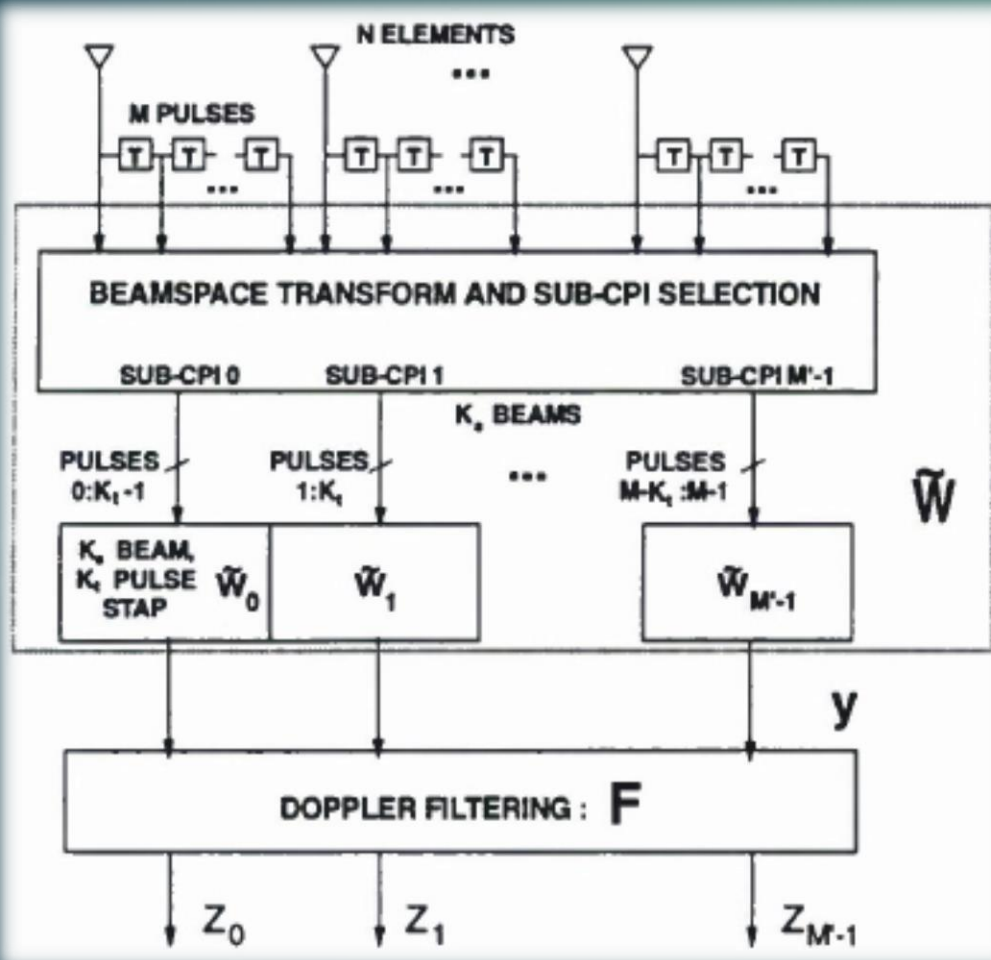
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## Case III: Pre-Doppler Processing

- ▶ Element outputs are preprocessed (non-adaptively) with an  $N \times K_s$  Beamformer matrix  $\tilde{\mathbf{G}}$ , to produce a small number of  $K_s$  beam outputs.
- ▶ Next, only the beam outputs from a small,  $K_t$ -pulse, sub-CPI are adaptively processed at a time.
- ▶ The adaptive problem dimensionality reduces to:  $K = K_s \times K_t \ll N \times M$ .
- ▶ A separate adaptive problem is solved for each sub-CPI and the outputs are coherently processed with an  $M' = M - K_t + 1$  pulse Doppler filter bank.
- ▶ Depending on the method of construction of the Beamformer matrix  $\tilde{\mathbf{G}}$ , we further discriminate between 2 methods:
  - i) **Displaced-Beam pre-Doppler** (or Displaced Phase Center pre-Doppler).
  - ii) **Adjacent-Beam pre-Doppler**.

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## Case III: Pre-Doppler Processing



### Displaced-Beam pre-Doppler

- Dual of PRI-Staggered post-Doppler STAP.
- A single  $N \times K_s$  Beamformer Matrix is applied to each element producing  $K_s$  Output Beams.
- For each Beam, an adaptive processor combines the  $K_t$  output pulses to produce the output signal for that sub-CPI.
- Finally, the outputs are coherently processed with an non-adaptive  $M'$  pulse Doppler filter bank  $F$ .



# BEAMSPACE STAP

## Case III: Pre-Doppler Processing

► Displaced-Beam pre-Doppler Preprocessor :  $\mathbf{T}_p = \mathbf{J}_p \otimes \tilde{\mathbf{G}}$ ,

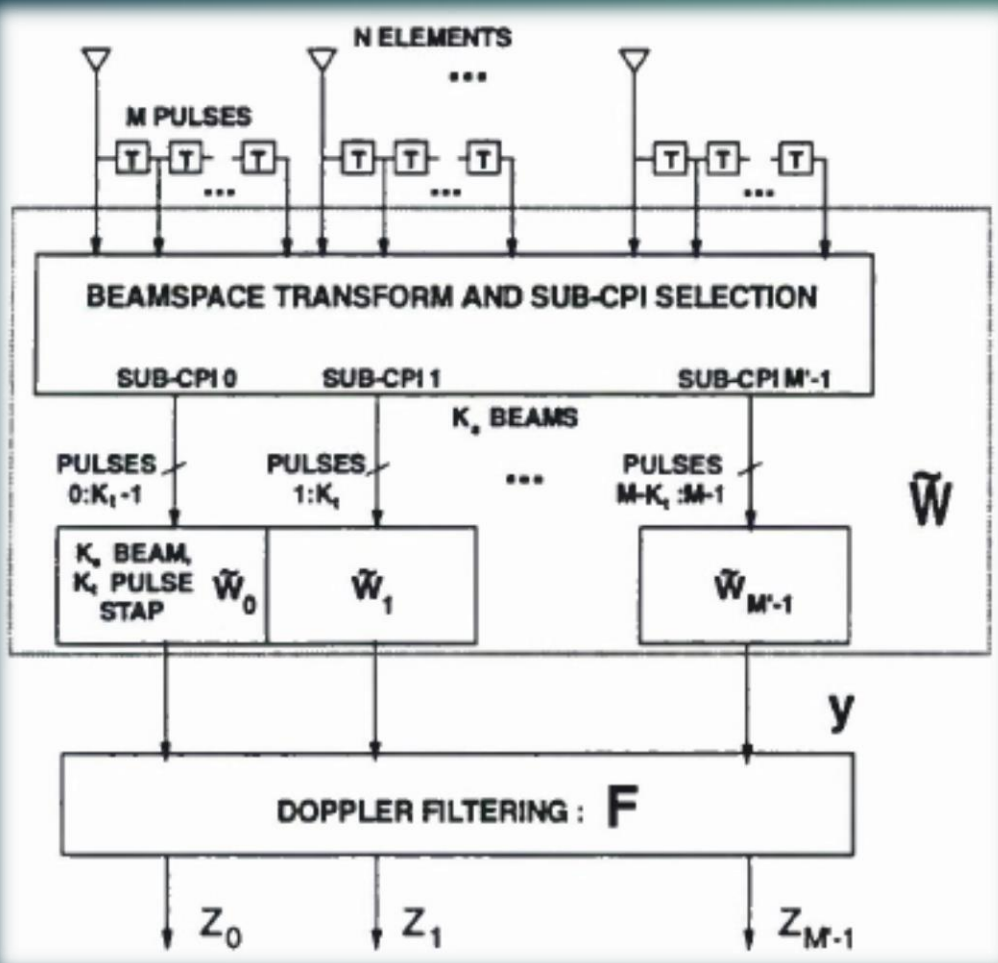
- where  $\mathbf{J}_p = \begin{bmatrix} \mathbf{0}_{p \times K} \\ \mathbf{I}_K \\ \mathbf{0}_{(M-K-p) \times K} \end{bmatrix}$  is an  $M \times K$  pulse selector matrix.
- For Displaced-Beam pre-Doppler STAP:

$$\tilde{\mathbf{G}} = \text{Toeplitz}([\mathbf{g} ; \mathbf{0}_{(K_s-1) \times 1}], [g_0 ; \mathbf{0}_{1 \times (K_s-1)}])$$

- $\mathbf{g}$  is a  $N' \times 1$  beamformer vector.
- Same Form of Beamformer Matrix as with DPCA.

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## Case III: Pre-Doppler Processing



### Adjacent-Beam pre-Doppler

- Dual of Adjacent-Bin post-Doppler STAP.
- Utilizes a set of adjacent beams each using the full aperture.

# BEAMSPACE STAP

## Case III: Pre-Doppler Processing

► **Adjacent-Beam pre-Doppler Preprocessor** :  $\mathbf{T}_p = \mathbf{J}_p \otimes \tilde{\mathbf{G}}$ ,

- where  $\mathbf{J}_p = \begin{bmatrix} \mathbf{0}_{p \times K} \\ \mathbf{I}_K \\ \mathbf{0}_{(M-K-p) \times K} \end{bmatrix}$  is an  $M \times K$  pulse selector matrix.
- For Adjacent-Beam pre-Doppler STAP:

$$\tilde{\mathbf{G}} = \mathbf{G}\mathbf{J}$$

- $\mathbf{G}$  is a  $N \times N$  beamformer matrix whose columns are beamformers steered to different angles.
- $\mathbf{J}$  is a  $N \times K_s$  selector matrix that chooses the columns of  $\mathbf{G}$  so as to form a cluster of adjacent beams centered around the transmit direction.

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## Case IV: Post-Doppler Processing

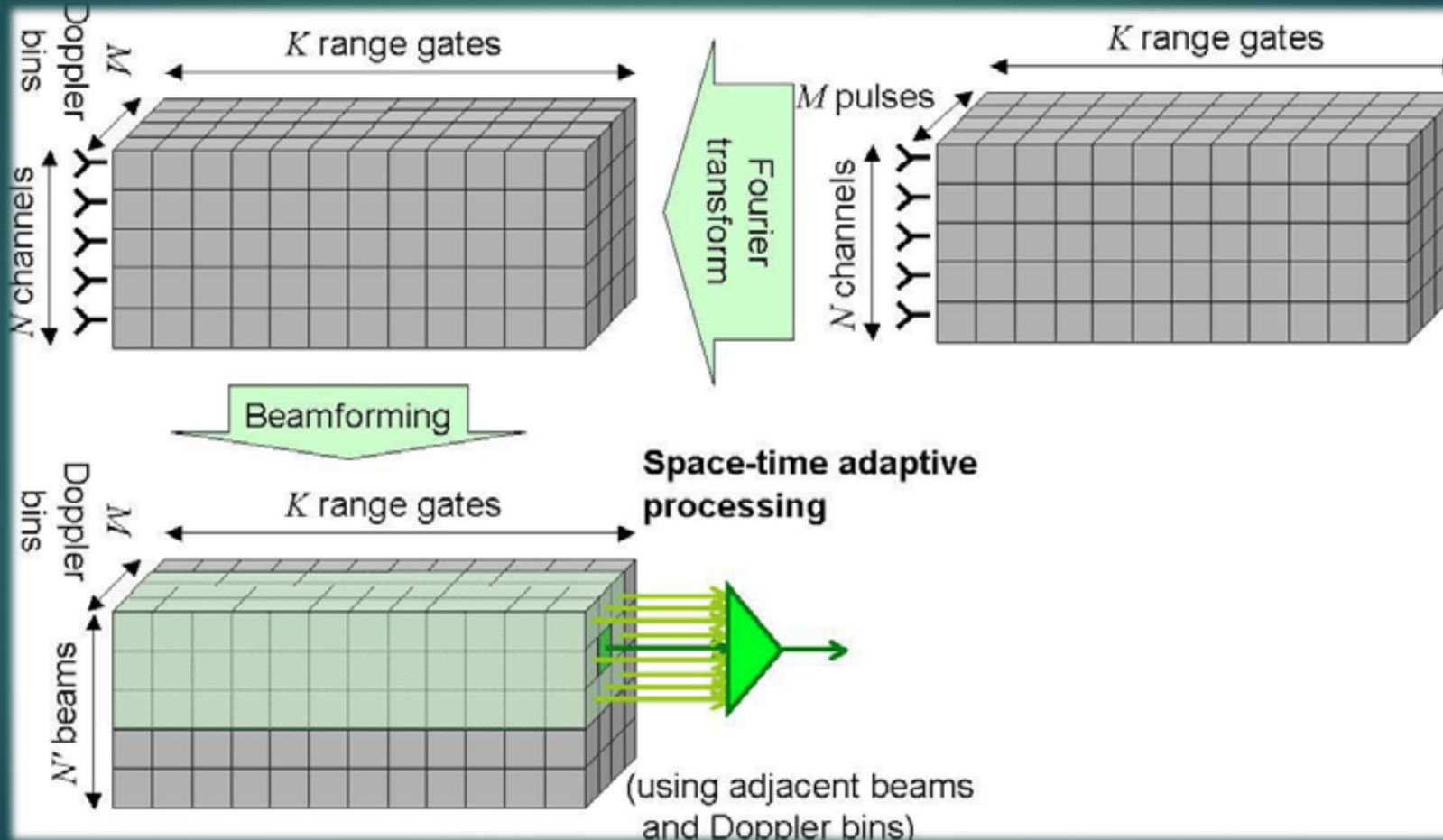


Figure adopted from [5].

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## Case IV: Post-Doppler Processing

- ▶ Preprocessing with a bank of small space-time filters.
- ▶ Cascade spatial beamformers on each pulse with Doppler filters on each beam.
- ▶ The filtered signals are then adaptively combined to produce Doppler bin outputs. This is repeated for every Doppler bin.
- ▶ Combined beamforming and Doppler filtering provides substantial suppression of portions of interference, localizing it prior to adaptation.
- ▶ Depending on the way of construction of the preprocessor filtering matrix  $\mathbf{T}_m$  we can further discriminate between 2 methods:
  - i) **Displaced Filter Beamspace post-Doppler**:  $\mathbf{T}_m = \tilde{\mathbf{F}}_m \otimes \tilde{\mathbf{G}}$
  - ii) **Adjacent Filter Beamspace post-Doppler**:  $\mathbf{T}_m = (\mathbf{F} \otimes \mathbf{G})\mathbf{J}_m$  by Cai & Wang [3].

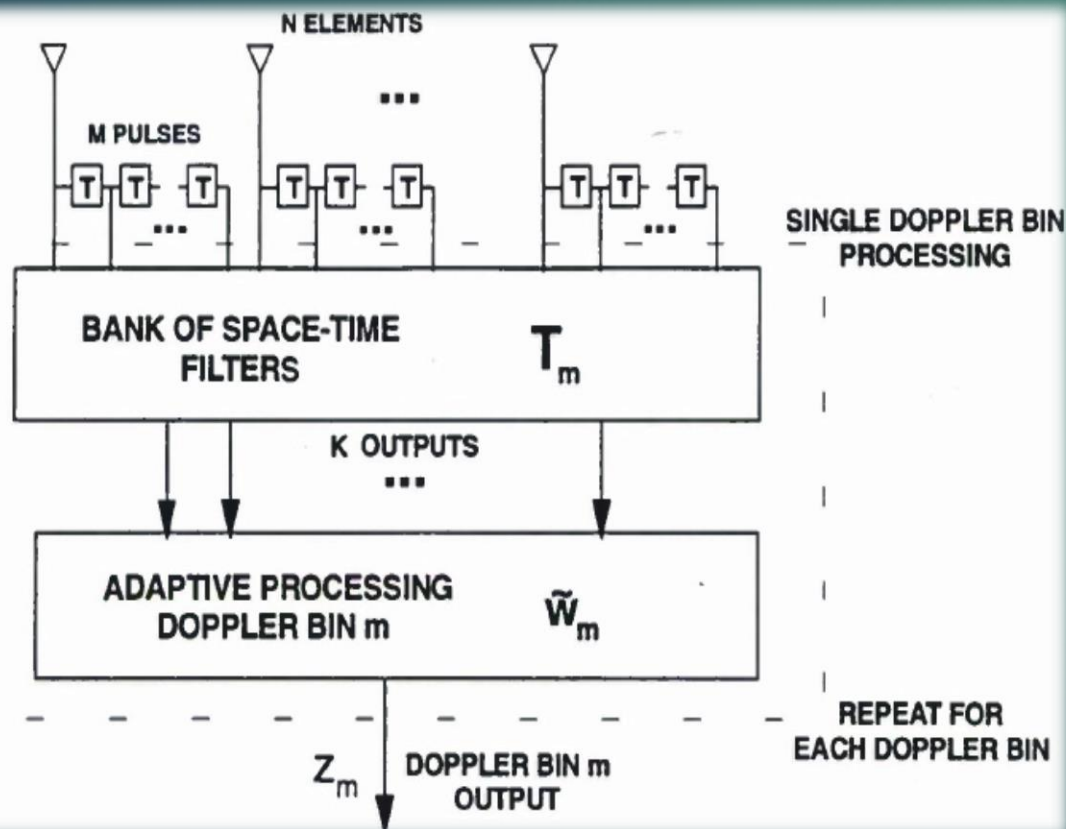


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## Case IV: Post-Doppler Processing

### Displaced Filter Beamspace post-Doppler

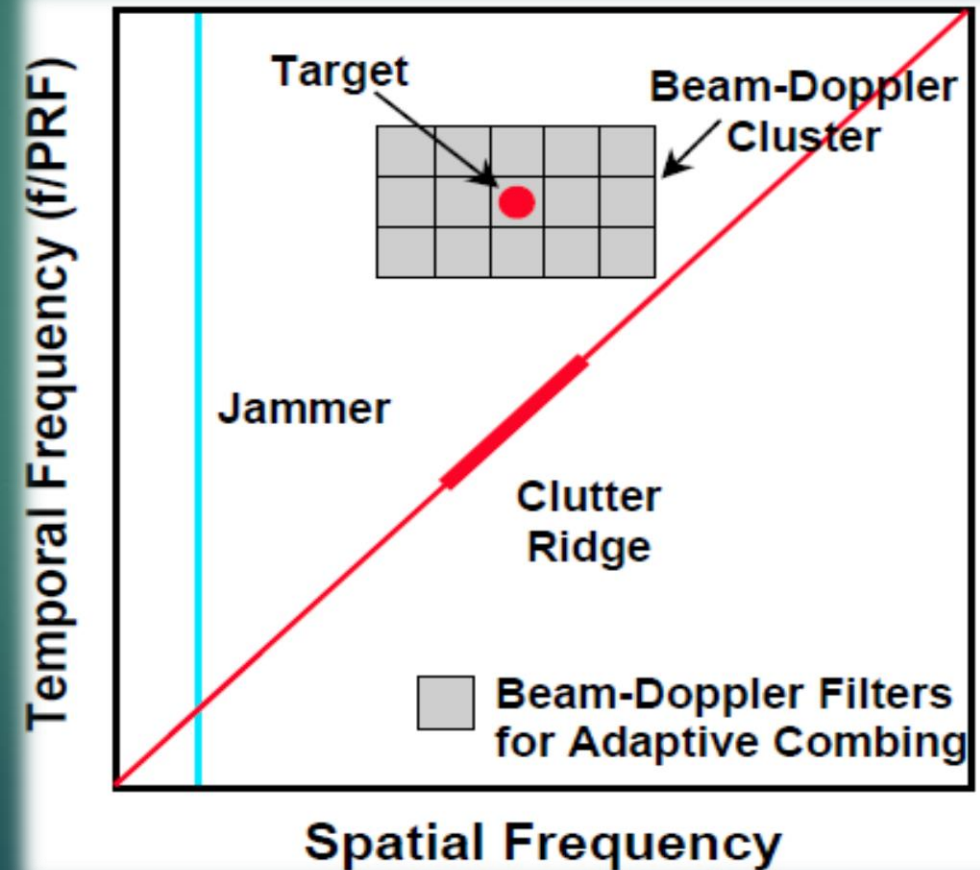
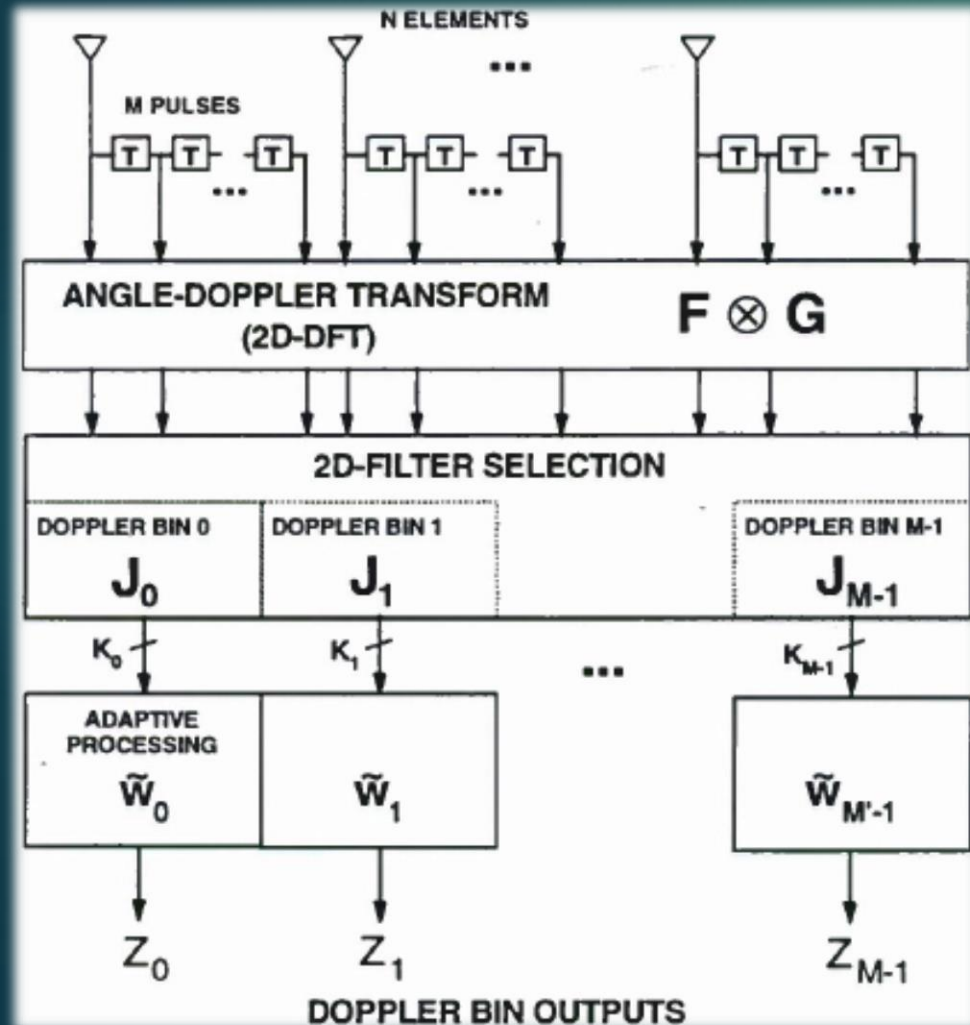
- This preprocessor is *separable* because it may be implemented by cascading multiple beamformers on each pulse with multiple Doppler filters on each beam (or vice versa).



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## Case IV: Post-Doppler Processing

Adjacent Filter Beamspace post-Doppler



# Partially Adaptive STAP

## References

- [1] L.E. Brennan, F.M. Staudaher, “Subclutter Visibility Demonstration”, Technical Report RL-TR-92-21, Adaptive Sensors Incorporated, March 1992.
- [2] R. DiPietro, “Extended Factored Space-Time Adaptive Processing for Airborne Radar Systems”, Proceedings of the 26<sup>th</sup> Asilomar Conference on Signals, Systems and Computing, Pacific Grove, CA, Oct. 1992, pp 425-430.
- [3] H. Wang, L. Cai, “On Adaptive Spatial-Temporal Processing for Airborne Surveillance Radar Systems”, *IEEE Trans. Aerospace and Electr. Sys.*, vol. 30, no.3, July 1994, pp 660–669.
- [4] Y. Dong, “Overview of STAP Algorithms”, DSTO-TN-0992, March 2011.
- [5] Bürger W., “Space-Time Adaptive Processing: Algorithms”, In *Advanced Radar Signal and Data Processing*, Educational Notes RTO-EN-SET-086, Paper 7.