Space-Time Adaptive Processing Fundamentals

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An Inspiring Paradigm…

This presentation largely follows the notation and development originally presented in the seminal J. Ward's technical report on STAP:

[0] J. Ward, *"Space-Time Adaptive Processing for Airborne Radar", Technical Report 1015, Lincoln Laboratory, MIT, Dec. 1994.*

All MatLab files used for this lecture may be downloaded from:
http://www.mathworks.com/matlabcentral/fileexchange/47750-space-time-adaptive-processing-for-airborne-radar-by-j-ward--tech-report-1015-

Please do not forget to provide your rating and/or comments/feedback about this coding effort.

STAP Fundamentals – Contents

Space-Time Adaptive Processing (STAP)
What it really is?

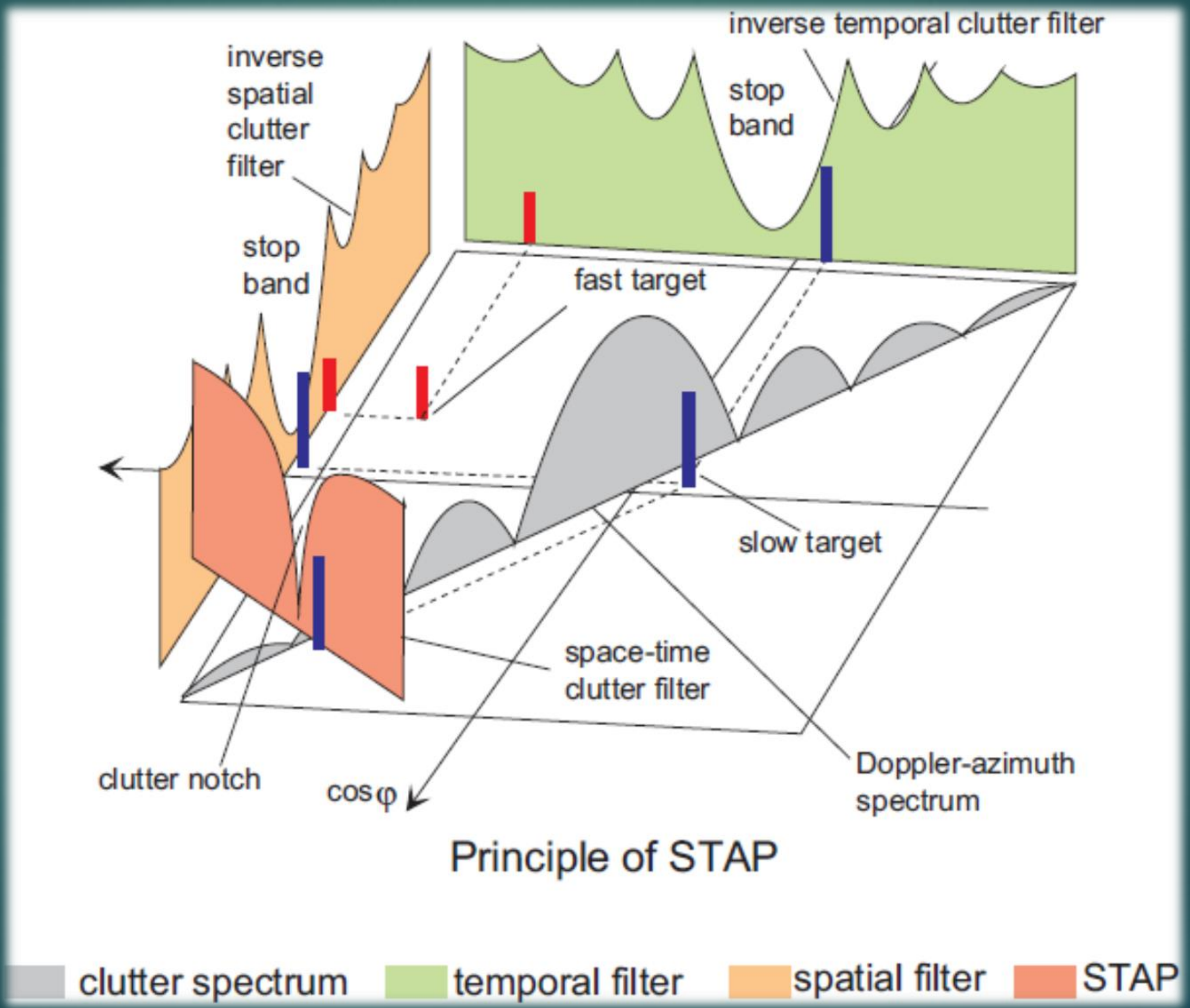


Figure adopted from [1]

STAP Fundamentals – General Architecture

- A STAP processor is defined as a linear filter that combines all the samples from the range gate of interest to produce a scalar output.
- Alternatively, it utilizes the spatial samples from the array antenna elements and the temporal samples provided by the successive pulses of a multiple-pulse waveform.
- It is represented by an *MN*-dimensional complex weight vector **w**:

$$z = \mathbf{w}^H \boldsymbol{\chi}$$

is the scalar processor output.

STAP Fundamentals – General Architecture

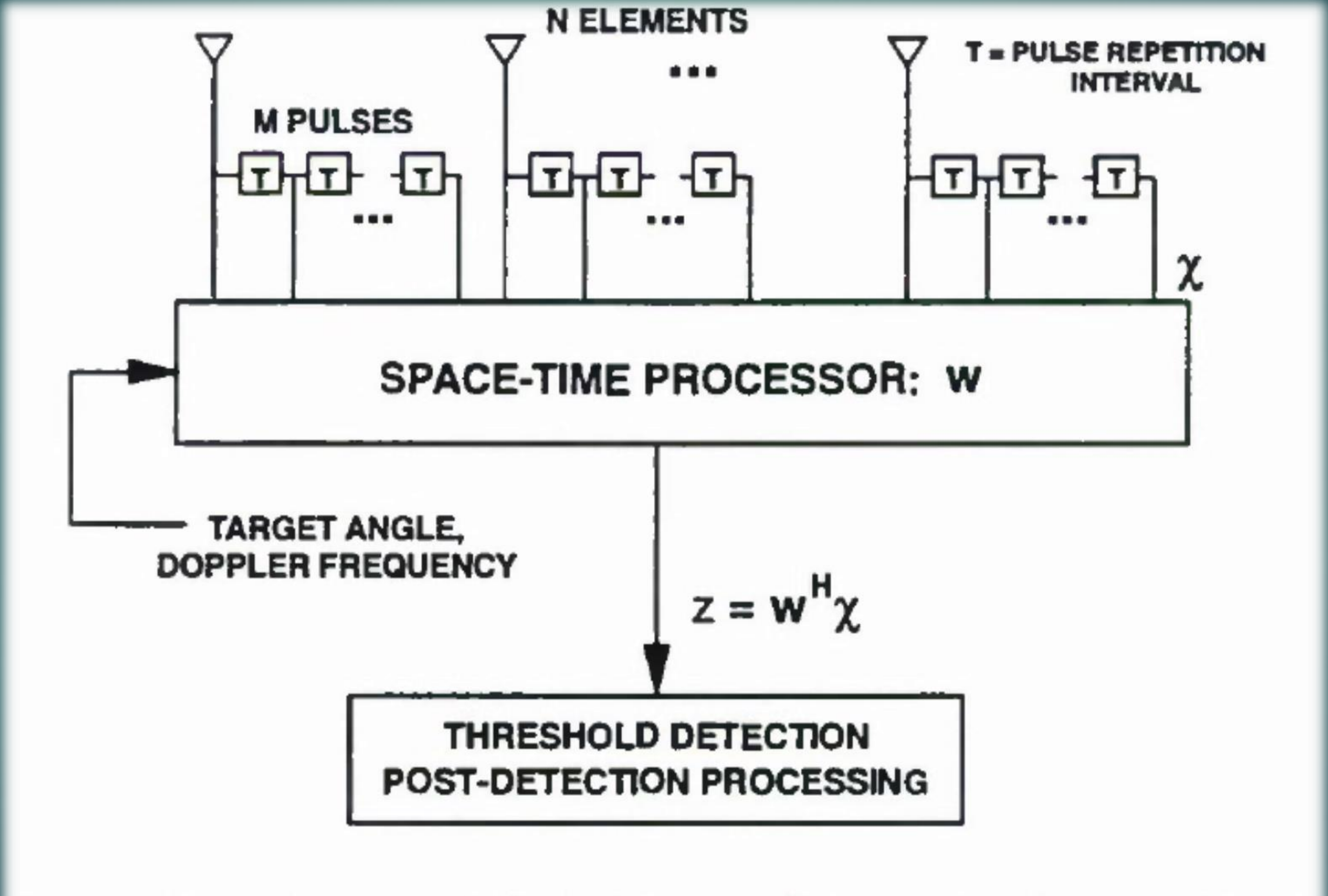


Figure 20. A general block diagram for a space-time processor.

STAP Fundamentals – General Architecture

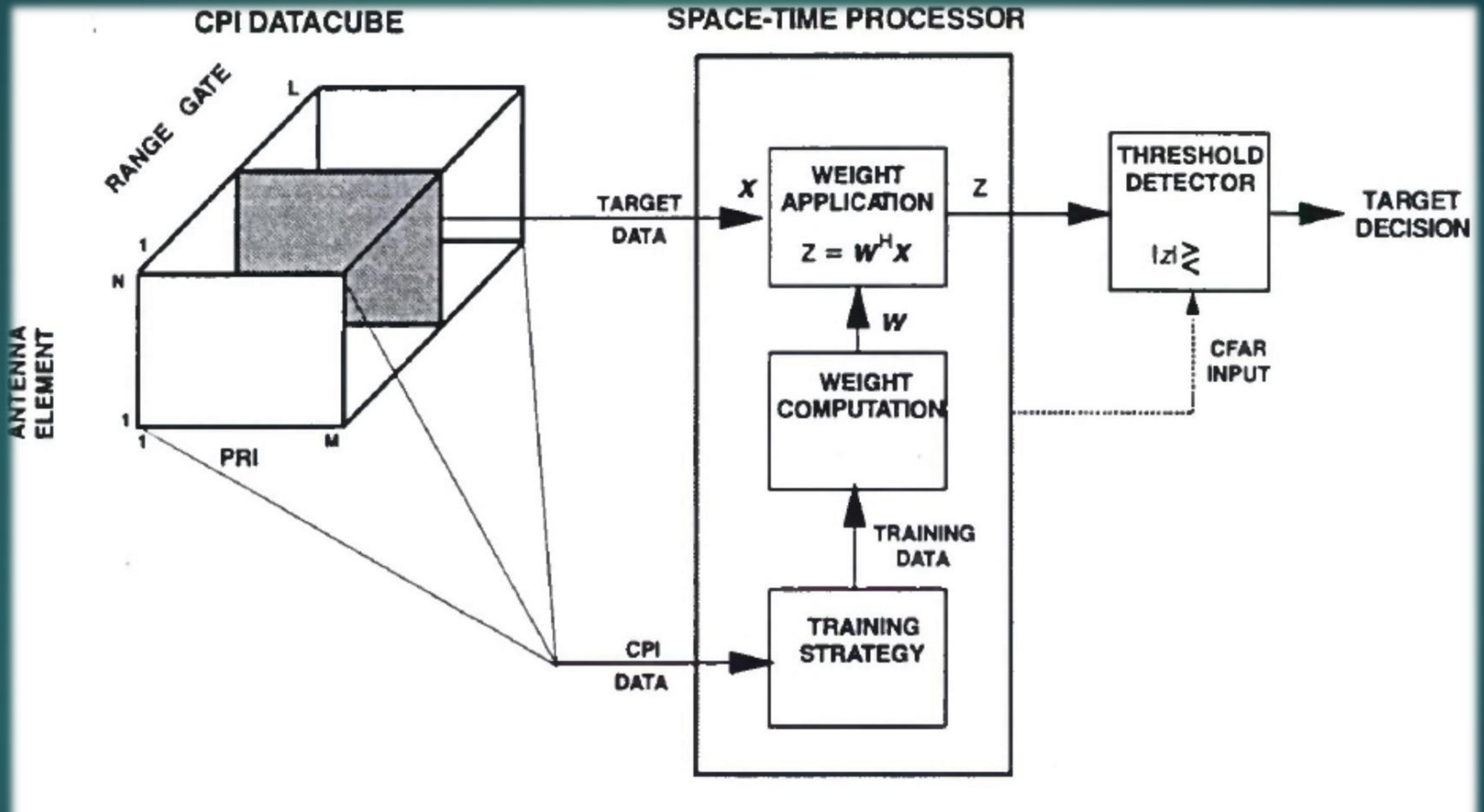


Figure 21. Data-domain view of space-time adaptive processing.

STAP Fundamentals – General Architecture

- The weight vector must be determined in a data-adaptive way from radar returns.
- The goal of the training strategy is to obtain the best estimate of the interference that exists at the range gate under test. Data from several range gates near the gate of interest are used.
- The training set must be updated in accordance with the non-stationarity of interference.

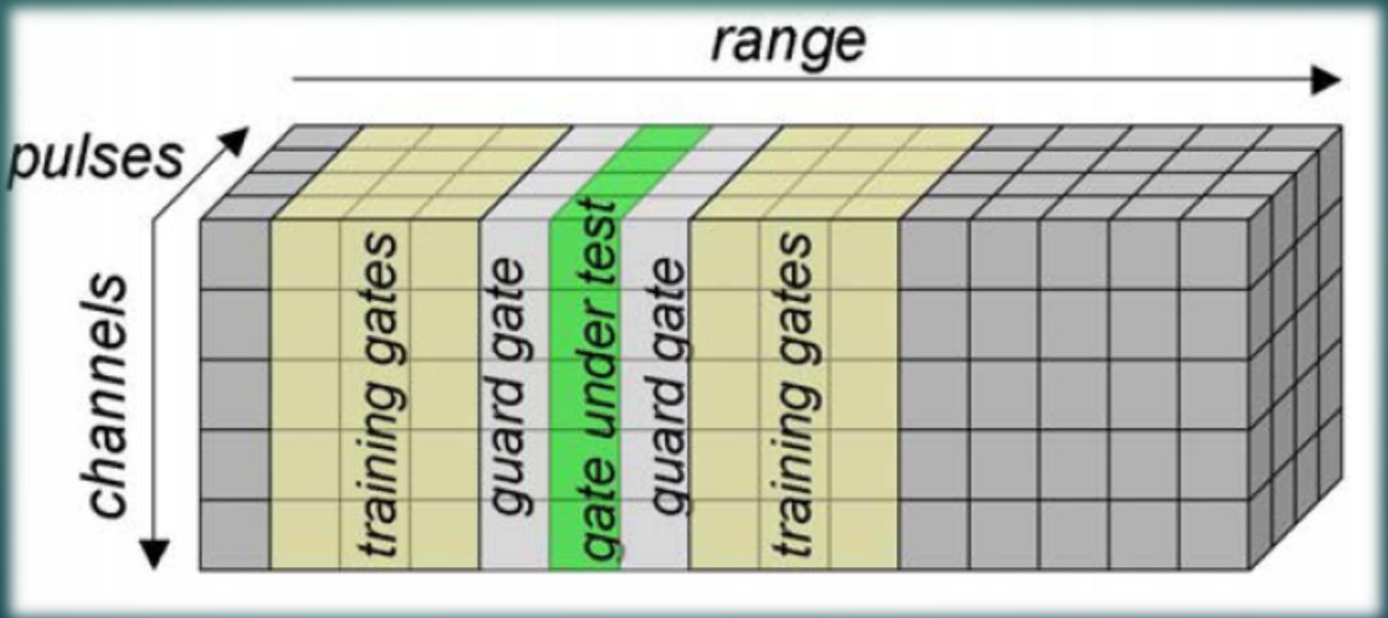
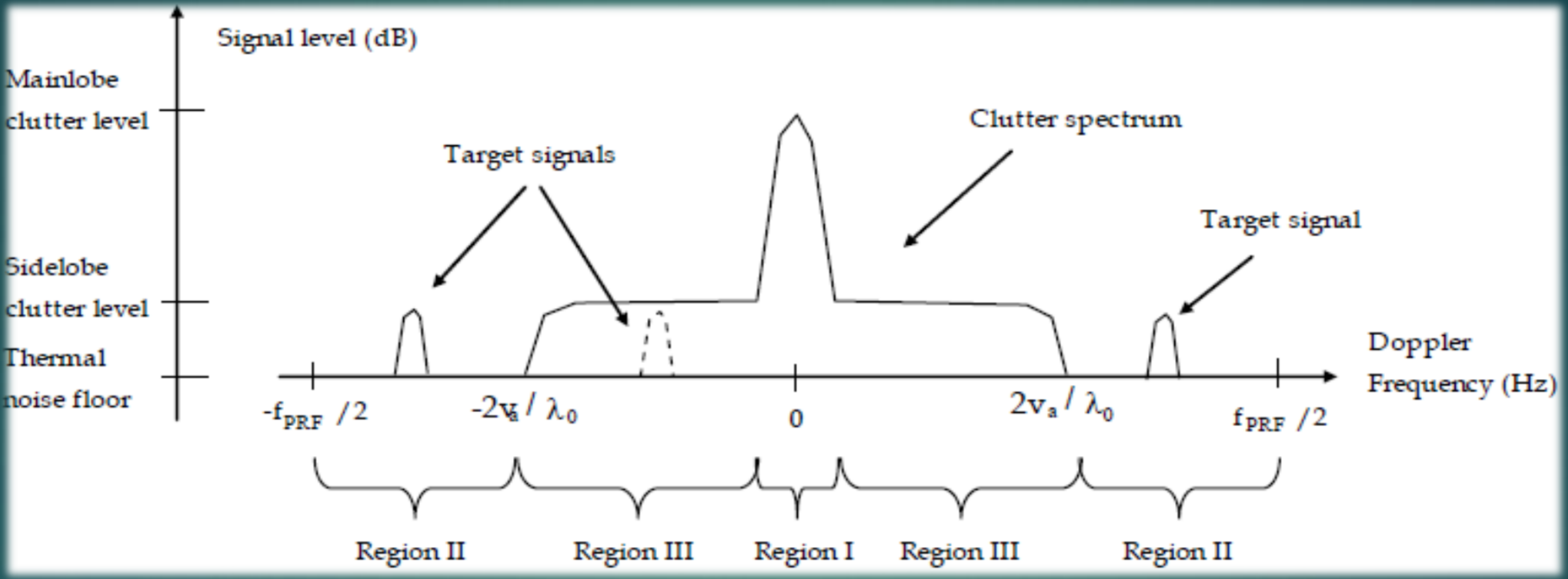


Figure Adopted from [6]

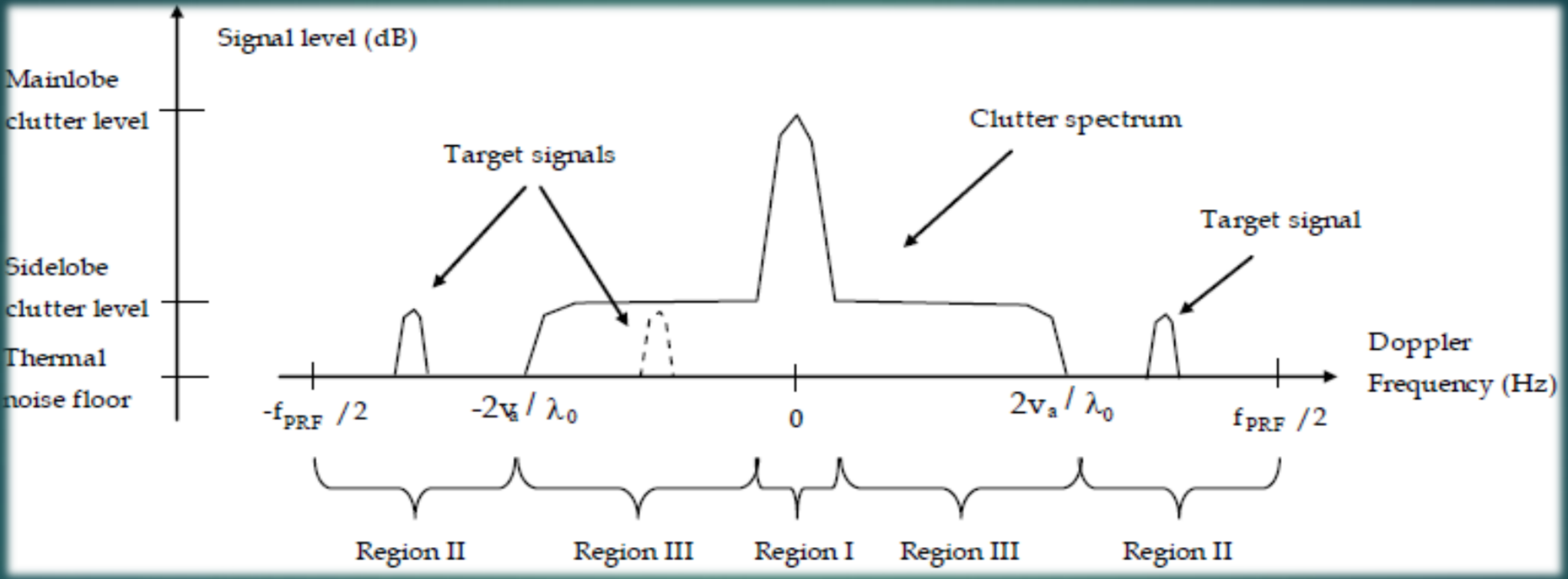
Clutter Suppression by STAP



Region I: The Mainlobe Doppler Region.

The detection scenario often assumes that the target is in the mainlobe direction, but has a different Doppler frequency from the mainlobe clutter. Therefore, the detection in this Doppler region is either not considered, or alternatively the target signal has to be so strong to exceed the mainlobe clutter so that it can be detected. [7]

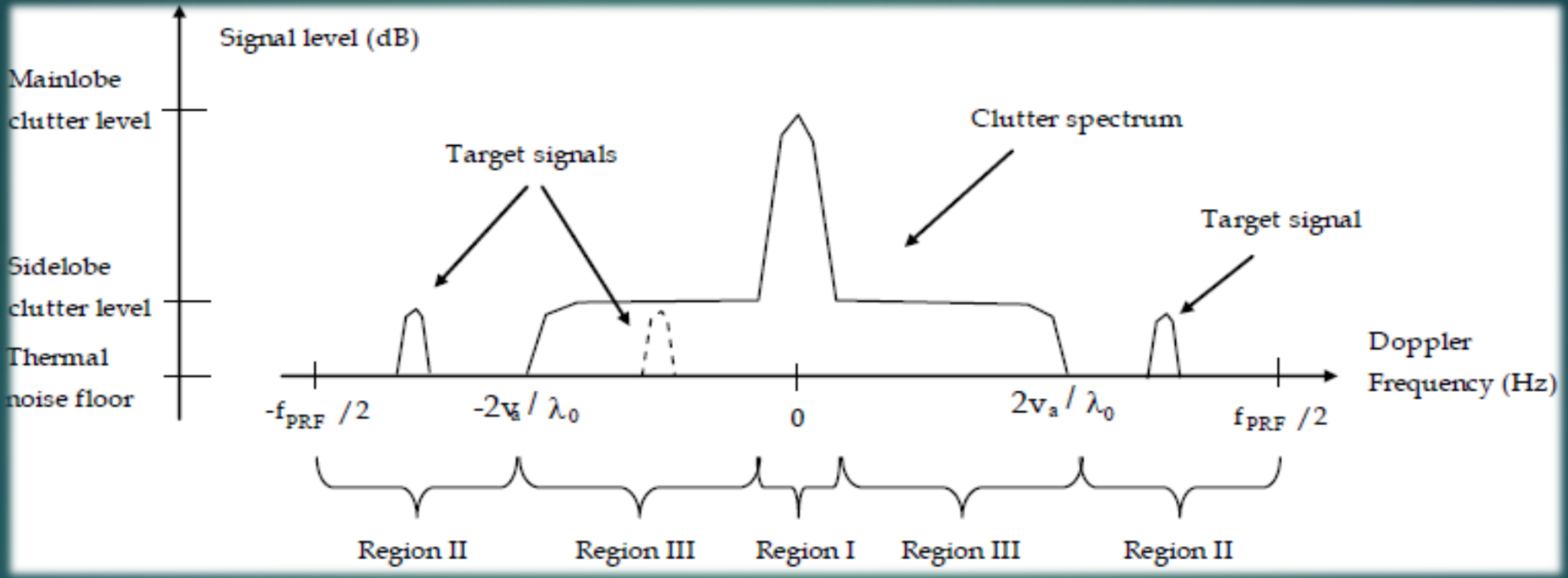
Clutter Suppression by STAP



Region II: The Clutter-Free Region, $\left[-\frac{f_{PRF}}{2}, -\frac{2v_a}{\lambda_0}\right]$ and $\left[\frac{2v_a}{\lambda_0}, \frac{f_{PRF}}{2},\right]$

Target signals in this region only encounter competition with thermal noise. The detection is relatively easy, and a standard windowed discrete Fourier transform (DFT) processing is generally sufficient. [7]

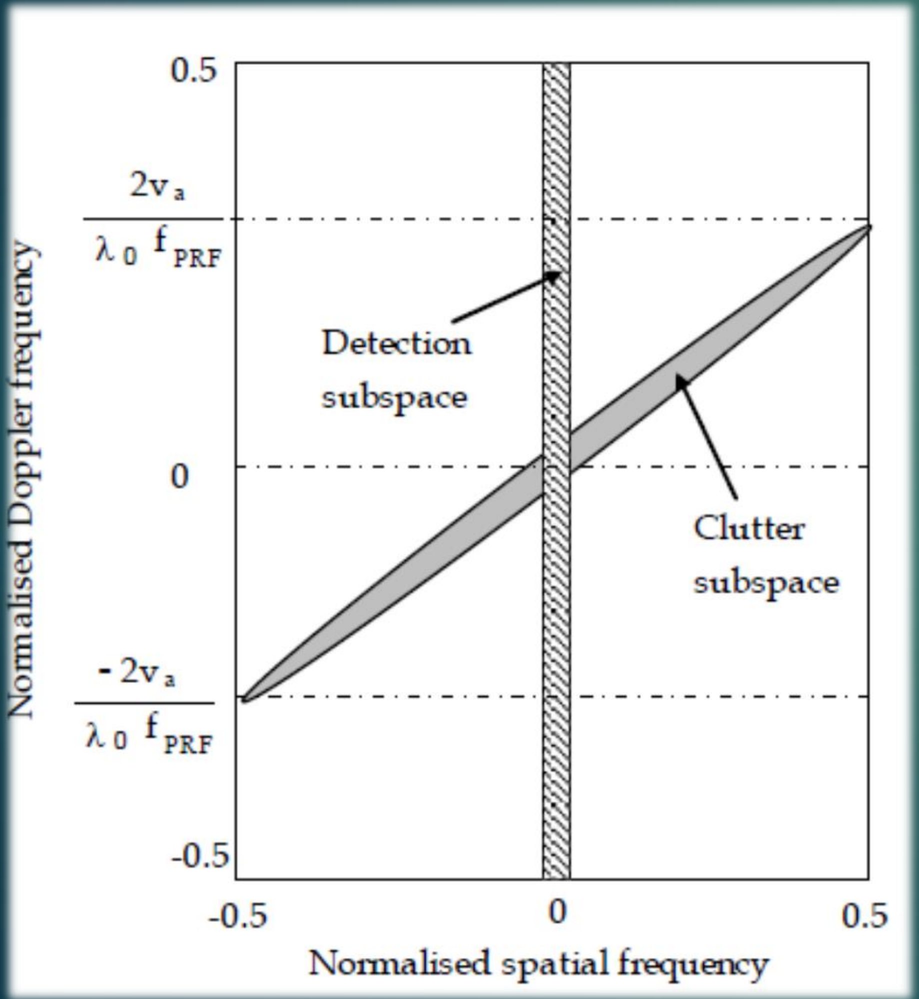
Clutter Suppression by STAP



Region III: The Sidelobe Clutter Region

All detection algorithms for airborne radars focus at this region to improve the subclutter visibility (SCV). Algorithms such as STAP are able to fully suppress the clutter in this region if all conditions STAP requires are met, and hence detect target signals embedded in the clutter. As shown in the figure, a target embedded in clutter that would not be detectable without clutter suppression, becomes detectable once the clutter is suppressed. [7]

Why STAP “Works as Advertised”



- The detection subspace and the clutter subspace are well separated on the azimuth-Doppler plane, and the overlay only happens at the point where the target has the same Doppler as the mainlobe clutter (the detection direction). [7]

- Therefore, if we can design a processor that suppresses clutter at a specific spatial frequency, then targets having the same Doppler frequency and different spatial frequency will no longer be masked by the clutter and become detectable. [7]

STAP Fundamentals – Critical Issues [7]

I) Demands a significant number of training samples to estimate the interference + noise covariance matrix in a real environment (non-stationary).

II) STAP requires the inverse of the covariance matrix for construction of the optimal weights. The number of operational counts for matrix inversion is in an order of the cube of the dimension of the matrix. Current airborne radars collect a CPI dwell in a fraction of a second, which means that the real-time radar has to process the data at the same rate. The current computer is generally not capable for such a fast response.

III) The sample data is supposed to be target-free. Without prior knowledge, it is difficult in real-time to satisfy this criterion. If sample data is contaminated, the performance of STAP degrades significantly.

STAP Fundamentals – Algorithm Taxonomy [7]

- Classic, Fully Adaptive STAP
- Reduced Dimension (or Partially Adaptive) STAP
- Rank-Reduced STAP
  - Principle Component Method
  - Multistage Wiener Filter (MWF)
- Parametric Adaptive Matched Filter (PAMF)
- Adaptive Displaced Center Antenna (ADPCA)
- Knowledge-Aided STAP (KA-STAP) (rather an architecture than an algorithm)
- Pseudo-STAP Variants
  - Pre-Built STAP
  - Eigencanceller
- 3D-STAP Data Processing

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STAP Fundamentals – Fully Adaptive STAP

Brennan and Reed [2] proved in a 1973 paper that the optimum space-time filter is:

$$\mathbf{w} = \gamma \mathbf{R}_u^{-1} \mathbf{v}_t$$

Where $\mathbf{R}_u = E\{\boldsymbol{\chi}_u \boldsymbol{\chi}_u^H\}$ is the interference-plus-noise covariance matrix.

Criterion	Mathematical Formulation	Constant Value
Maximum SINR	$\max\limits_{\mathbf{w}} \frac{\lvert \mathbf{w}^H \mathbf{v} \rvert^2}{\mathbf{w}^H \mathbf{R} \mathbf{w}}$	$\gamma \neq 0$
Maximum P_D while maintaining CFAR P_{FA}	$\max\limits_{\mathbf{w}} P_D(\mathbf{w})$ subject to $P_{FA} = const.$	$\gamma = \frac{1}{(\mathbf{v}^H \mathbf{R}^{-1} \mathbf{v})^{1/2}}$
Minimum output power subject to unit gain constraint in look direction	$\min\limits_{\mathbf{w}} (\mathbf{w}^H \mathbf{R} \mathbf{w})$ subject to $\mathbf{w}^H \mathbf{v} = 1$	$\gamma = \frac{1}{\mathbf{v}^H \mathbf{R}^{-1} \mathbf{v}}$

STAP Fundamentals – Fully Adaptive STAP

- Suboptimum fully adaptive STAP processor:

$$\mathbf{w} = \mathbf{R}_u^{-1}\mathbf{g}_t$$

- $\mathbf{g}_t$ is not the target steering vector but a desired weight vector that includes tapering to obtain low sidelobes.
  - $\mathbf{t}_a$ is an N×1 vector containing the desired low-sidelobe angle response.
  - $\mathbf{t}_b$ is an M×1 vector containing the desired Doppler response.
  - $\mathbf{t} = \mathbf{t}_b \otimes \mathbf{t}_a$ is a separable space-time window sequence.
- The desired vector $\mathbf{g}_t = \mathbf{t} \odot \mathbf{v}_t$ is not strictly optimum in any sense.
- This processor is termed as “Tapered Fully Adaptive”.

STAP Fundamentals – Fully Adaptive STAP

- Fully Adaptive STAP requires the solution of an *MN*-dimensional linear system.

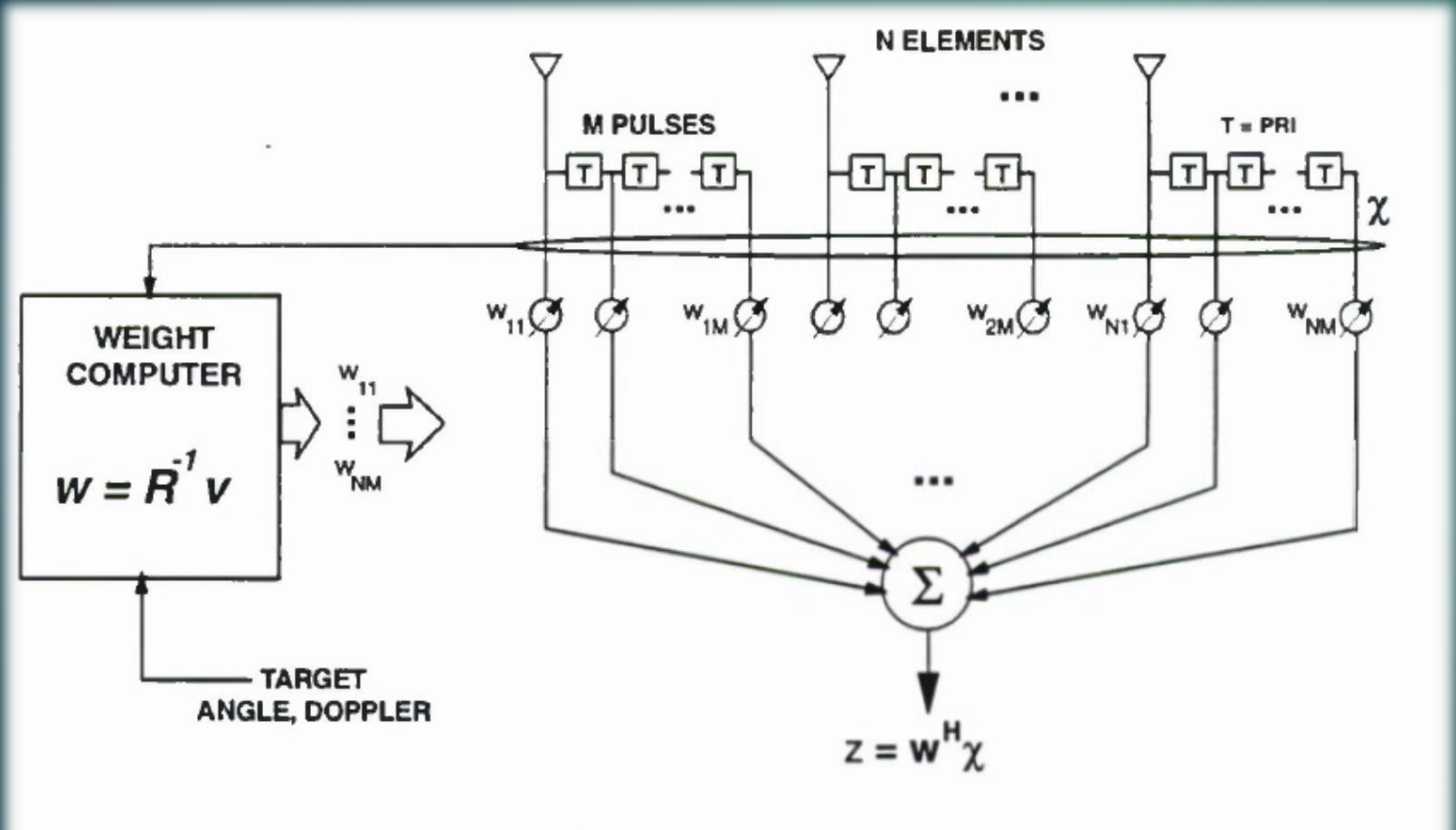


Figure 22. Fully adaptive space-time processing

- Dimensionality of problem *MN* can be very large: 10^2 to 10^5.
- Interference Covariance Matrix is unknown a priori and therefore it must be estimated from data in real-time!
- Also target steering vector not known a-priori, therefore there is a very large search space of all possible azimuth angles and Doppler frequencies.

STAP Fundamentals – Performance Metrics

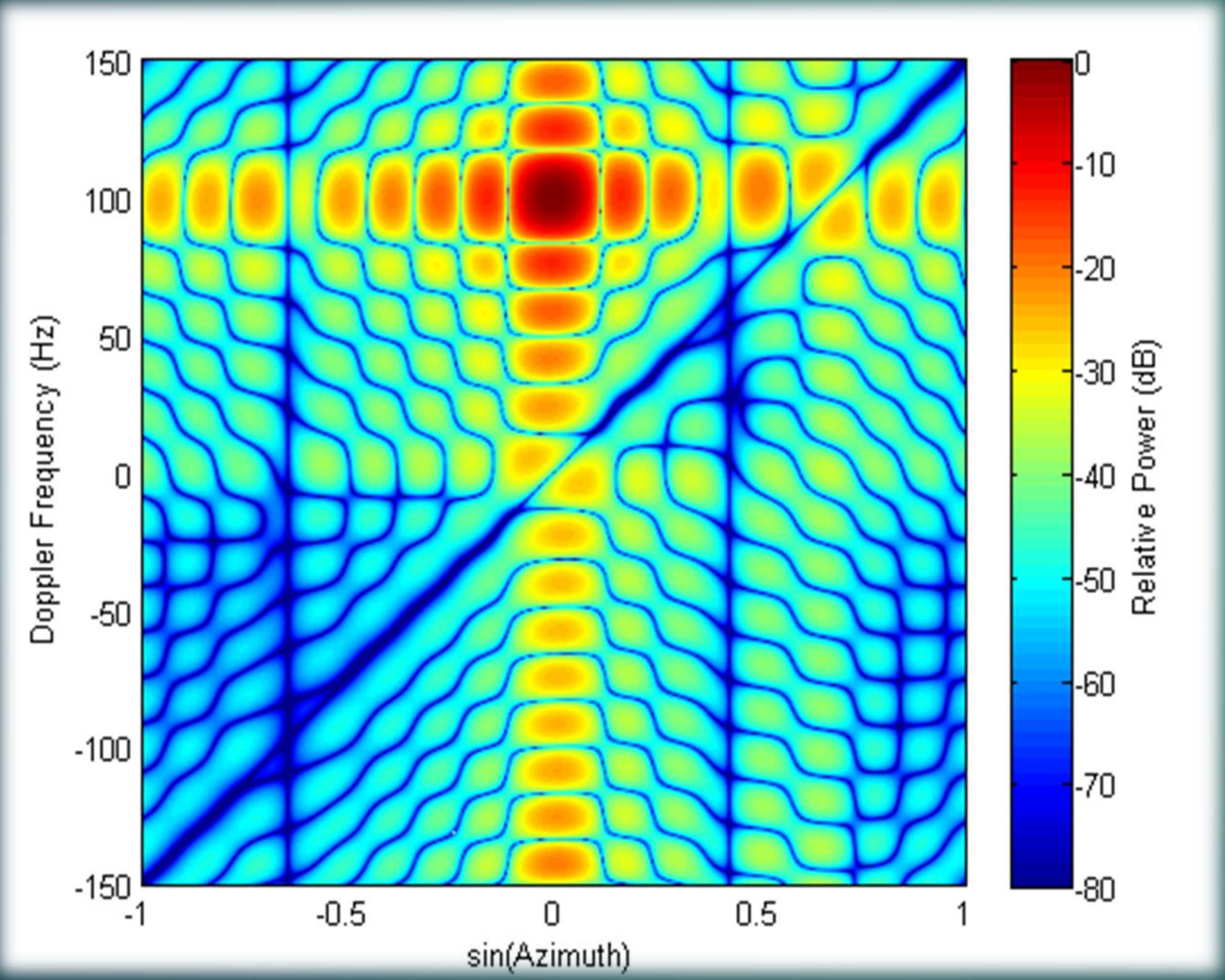
- The Adaptive Pattern is the two-dimensional frequency response of a STAP filter (processor) as a function of angle and Doppler frequency:

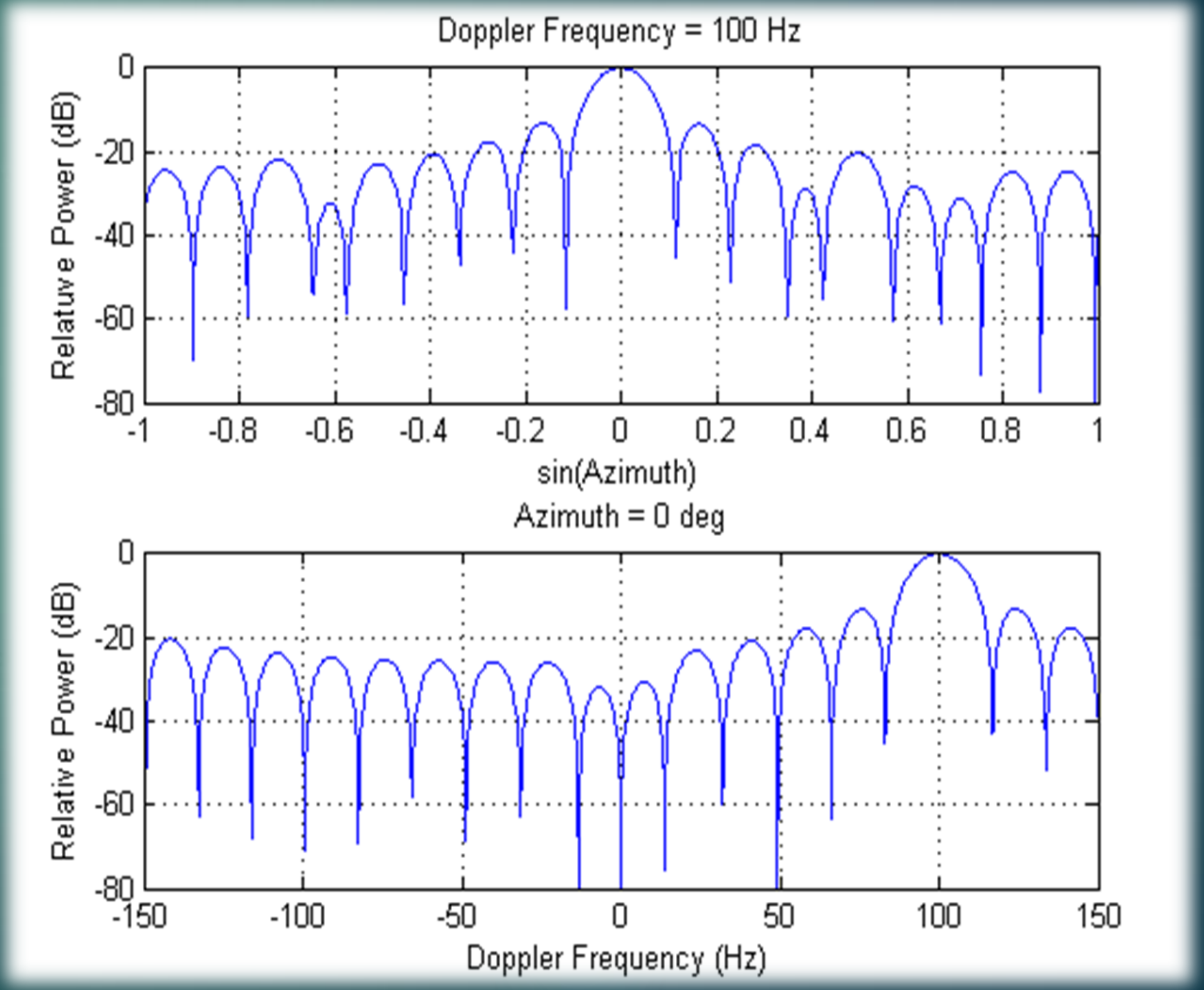
$$P_{\mathbf{w}}(\vartheta,\varpi) = |\mathbf{w}^H \mathbf{v}(\vartheta,\varpi)|^2 = \left|\mathbf{v}_t^H(\vartheta_t,\varpi_t)\mathbf{R}_u^{-1}\mathbf{v}(\vartheta,\varpi)\right|^2$$

- By its definition the pattern is the squared modulus of a 2D-Inverse Fourier Transform of the weight vector.
- Ideally, it presents nulls in the directions of interference (clutter & jamming) and high gain at the presumed Doppler frequency and angle of the target.

STAP Fundamentals – Performance Metrics

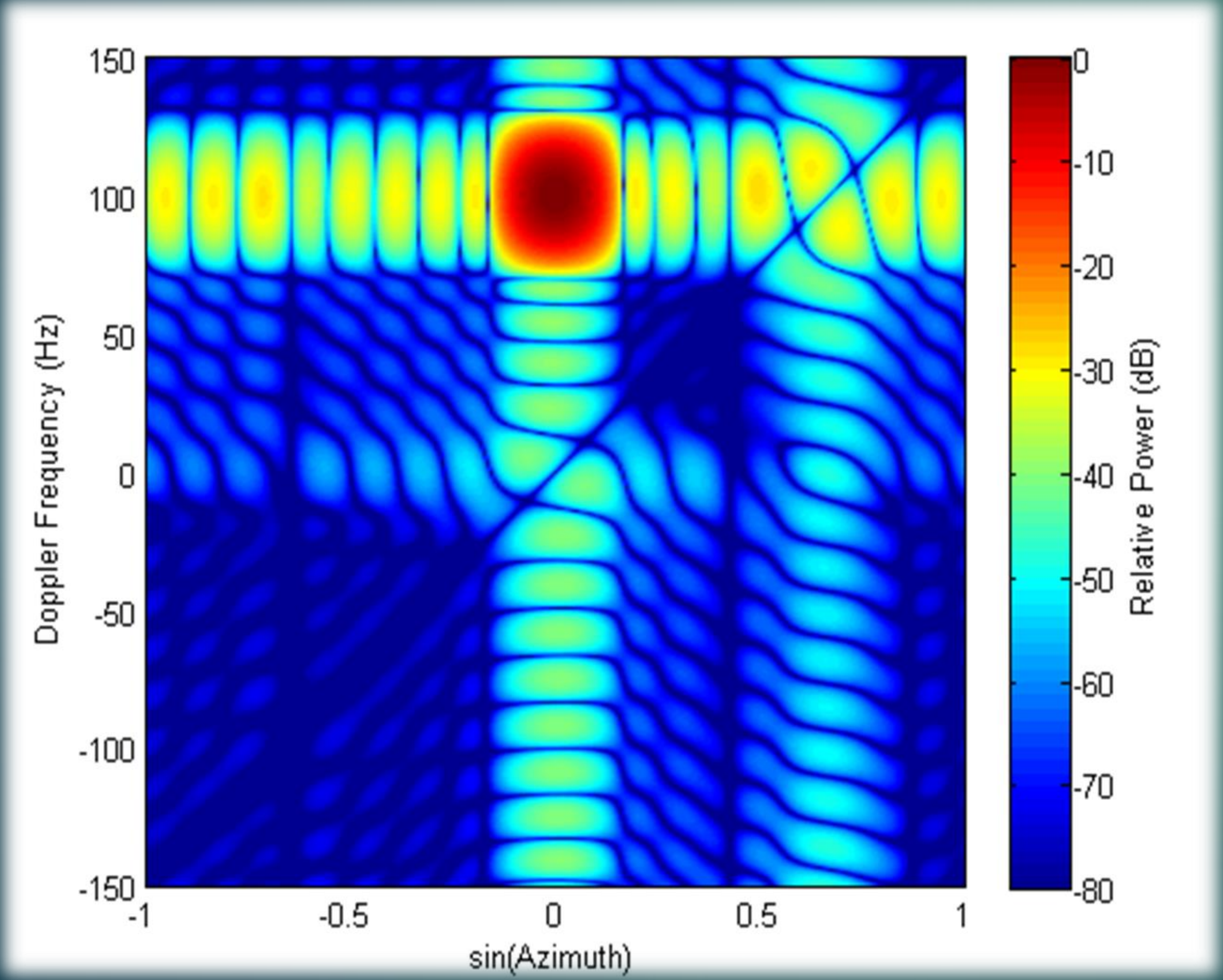
Adapted Pattern for the Untapered, Fully Adaptive Case

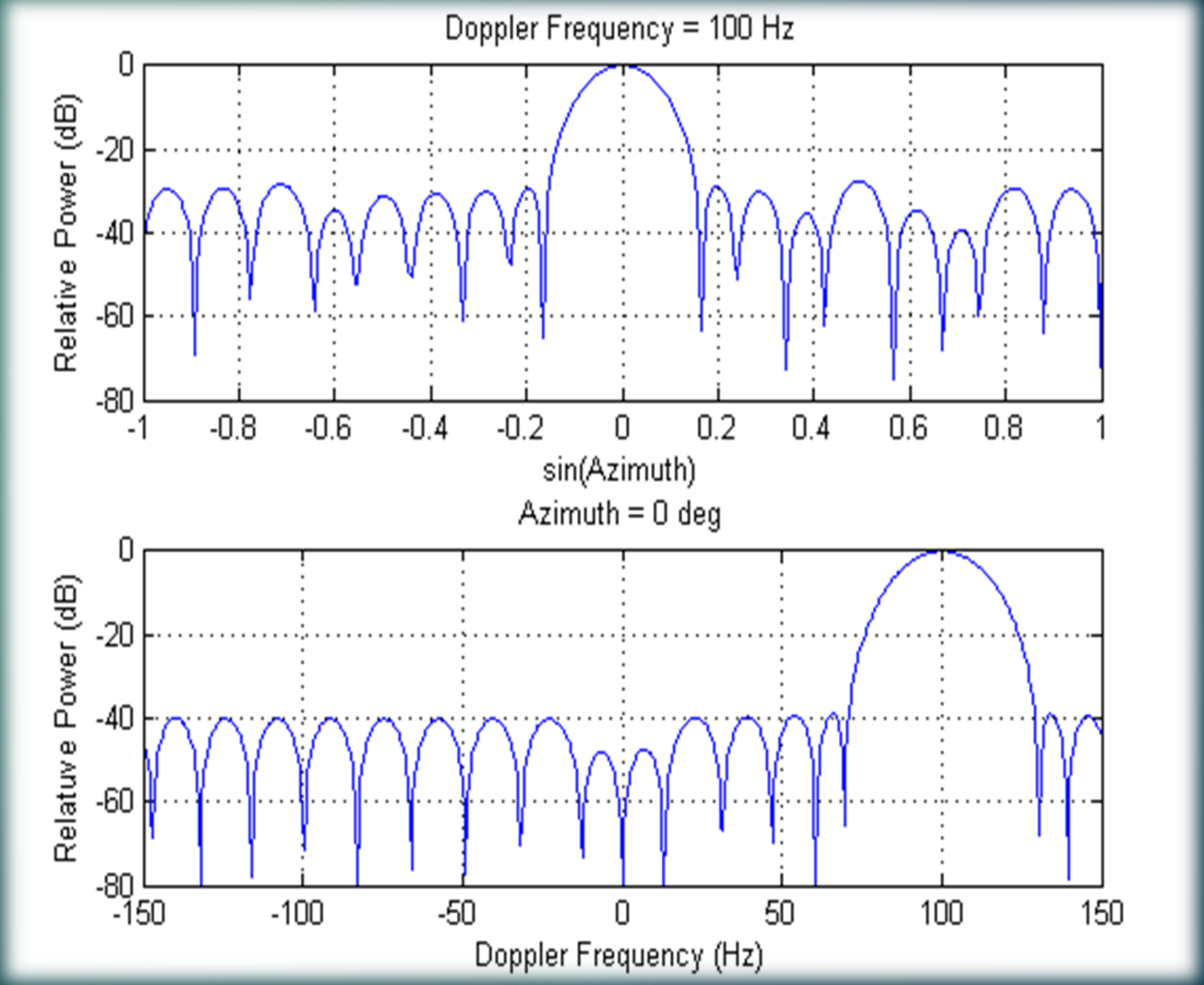




STAP Fundamentals – Performance Metrics

Tapered Fully Adapted Pattern.
Application of a spatial 30-dB Chebyshev and a temporal 40-dB Chebyshev tapers.





STAP Fundamentals – Performance Metrics

- Divide the output signal into target and interference-plus-noise components:

$$z = z_t + z_u = a_t \mathbf{w}^H \mathbf{v}_t + \mathbf{w}^H \boldsymbol{\chi}_u$$

- Signal-to-Interference-plus-Noise Ratio (SINR): $\mathrm{SINR} = \dfrac{p_t}{p_u} = \dfrac{\sigma^2 \xi_t \left|\mathbf{w}^H \mathbf{v}_t\right|^2}{\mathbf{w}^H \mathbf{R}_u \mathbf{w}}$

$p_t = E\{|z_t|^2\}$ is the output target power.

$p_u = E\{|z_u|^2\}$ is the output interference-plus-noise power.

- Substitution of the optimum weight vector $\mathbf{w} = \mathbf{R}_u^{-1} \mathbf{v}_t$ leads to the <u>optimum SINR</u>:

$$\mathrm{SINR}_o = \sigma^2 \xi_t \mathbf{v}_t^H \mathbf{R}_u^{-1} \mathbf{v}_t$$

- Substitution of the tapered fully adaptive processor $\mathbf{w} = \mathbf{R}_u^{-1} \mathbf{g}_t$ produces a <u>suboptimum SINR</u> :

$$\mathrm{SINR}_{sub} = \frac{\sigma^2 \xi_t |\mathbf{g}_t^H \mathbf{R}_u^{-1} \mathbf{v}_t|^2}{\mathbf{g}_t^H \mathbf{R}_u^{-1} \mathbf{g}_t}$$

STAP Fundamentals – Performance Metrics

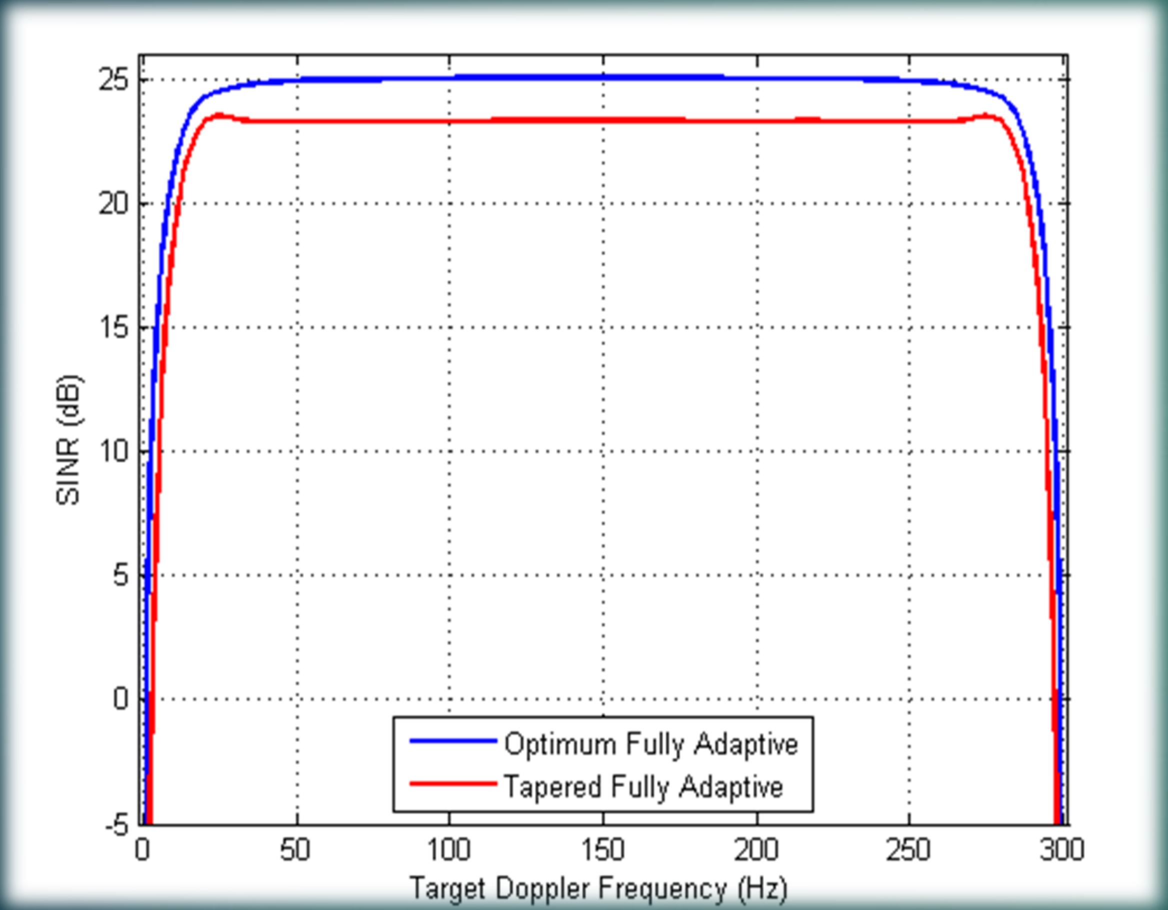
- Since the target's velocity is unknown a priori, the interest in SINR performance is a function of target's Doppler frequency.
- We hold the target's angle fixed and vary the target's Doppler frequency and compute a new adaptive weight vector for each Doppler frequency.
- Substitution of the optimum weight vector $\mathbf{w}(\varpi) = \mathbf{R}_u^{-1}\mathbf{v}_t(\varpi)$ leads to the optimum SINR:

$$\mathrm{SINR}_o(\varpi) = \sigma^2 \xi_t \mathbf{v}_t^H(\varpi)\mathbf{R}_u^{-1}\mathbf{v}_t(\varpi)$$

- Substitution of the tapered fully adaptive processor $\mathbf{w}(\varpi) = \mathbf{R}_u^{-1}\mathbf{g}_t(\varpi)$ produces a

suboptimum SINR :
$$\mathrm{SINR}_{sub}(\varpi) = \frac{\sigma^2 \xi_t \left|\mathbf{g}_t^H(\varpi)\mathbf{R}_u^{-1}\mathbf{v}_t(\varpi)\right|^2}{\mathbf{g}_t^H(\varpi)\mathbf{R}_u^{-1}\mathbf{g}_t(\varpi)}$$

STAP Fundamentals – Performance Metrics



- The input SNR is taken to be 0 dB.
- In the presence of interference optimum fully adaptive provides near maximum gain on target while suppressing both clutter and jamming well below the thermal noise.
- When the target is near 0Hz or 300Hz the SINR is very low because in those cases the target is close to the mainlobe clutter both in Doppler and angle.
- Naturally performance degrades as the target falls into the response null that the processor places on mainlobe clutter.
- The tapered fully adaptive performance is 1.8 dB lower than the optimum.

SINR for the Optimum and Tapered Fully Adaptive STAP.

STAP Fundamentals – Performance Metrics

- In a noise only environment the optimum processor is the space-time matched filter:
$$\mathbf{w} = \mathbf{v}_t$$
- The optimum output signal-to-noise ratio *SNRo* is: $\mathrm{SNR}_o = MN\xi_t$
- The SINR loss L_{SINR} of a space-time processor is defined to be its performance compared to this matched filter SNR in an interference-free environment:

$$\mathrm{L}_{SINR}(\varpi) = \frac{SINR(\varpi)}{SNR_o}$$

- It is useful because it incorporates many of the performance loss factors in a single quantity.

STAP Fundamentals – Performance Metrics

- The SINR Improvement Factor (IFSINR) is defined as:

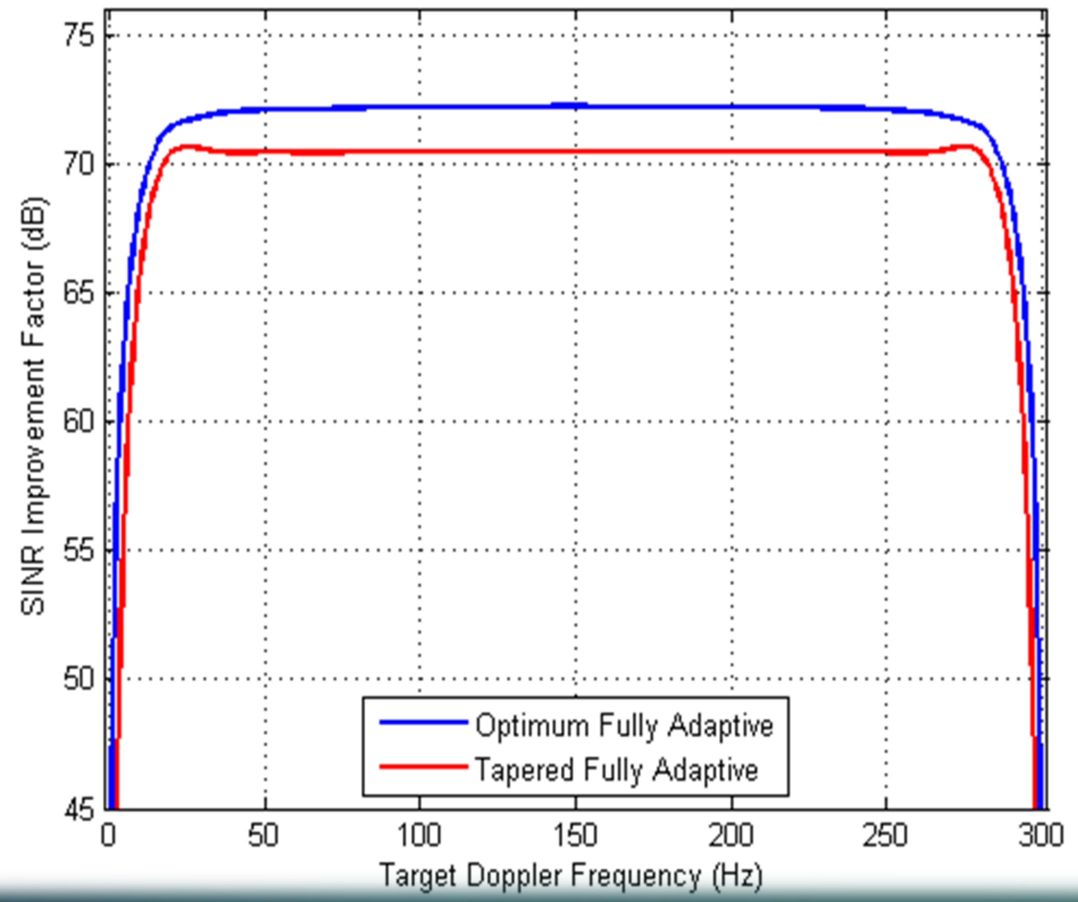
$$\mathrm{IF}_{SINR}(\varpi) = \frac{SINR_{out}(\varpi)}{SINR_{in}}$$

Where $SINR_{in} = \frac{\xi_t}{1+\xi_c+\xi_j}$ is the SINR on a single element for a single pulse.

- $SINR_{in}$ is usually a very small quantity.
- The IFSINR is typically large and <u>increases as the interference becomes stronger</u>.
- The IFSINR includes the amount of interference rejection and the coherent gain on target due to receive beamforming and Doppler filtering.
- When interference is strong and the STAP performs optimally the following holds:

$$\mathrm{IF}_{SINR}(\varpi) = MN\left(1 + \xi_c + \xi_j\right) \cong MN\left(\xi_c + \xi_j\right)$$

STAP Fundamentals – Performance Metrics



SINR Improvement Factor for the Optimum and Tapered Fully Adaptive STAP.

- Over the center of the Doppler space the optimum SINR achieves an IF_{SINR} of 72.2 dB.
- The input interference-to-noise ratio $\xi_i = \xi_c + \xi_j$ is 47.1 dB with CNR = 46.66 dB and JNR = 37.58 dB.
- Also, 10log(MN) = 25.1 dB.
- Therefore: 72.2 = 47.1 + 25.1 and the approximate relation applies.

STAP Fundamentals – Performance Metrics

- SINR performance can be used to derive performance metrics that describe the velocity coverage provided by a STAP algorithm.
- Define the acceptable SINR performance as a SINR loss of $L_{SINR} = x = L_0$.
- The minimum detectable velocity (MDV) is defined as the velocity closest to that mainlobe clutter at which acceptable SINR loss is achieved.
- If $MDD_{-}(x)$ and $MDD_{+}(x)$ are the Doppler frequencies below and above the mainlobe clutter Doppler at which the acceptable SINR loss is achieved, the Minimum Detectable Doppler (MDD) is defined as:

$$MDD(x) = \frac{1}{2}(MDD_{+}(x) - MDD_{-}(x)), \qquad MDV(x) = \frac{\lambda}{2} MDD(x)$$

STAP Fundamentals – Performance Metrics

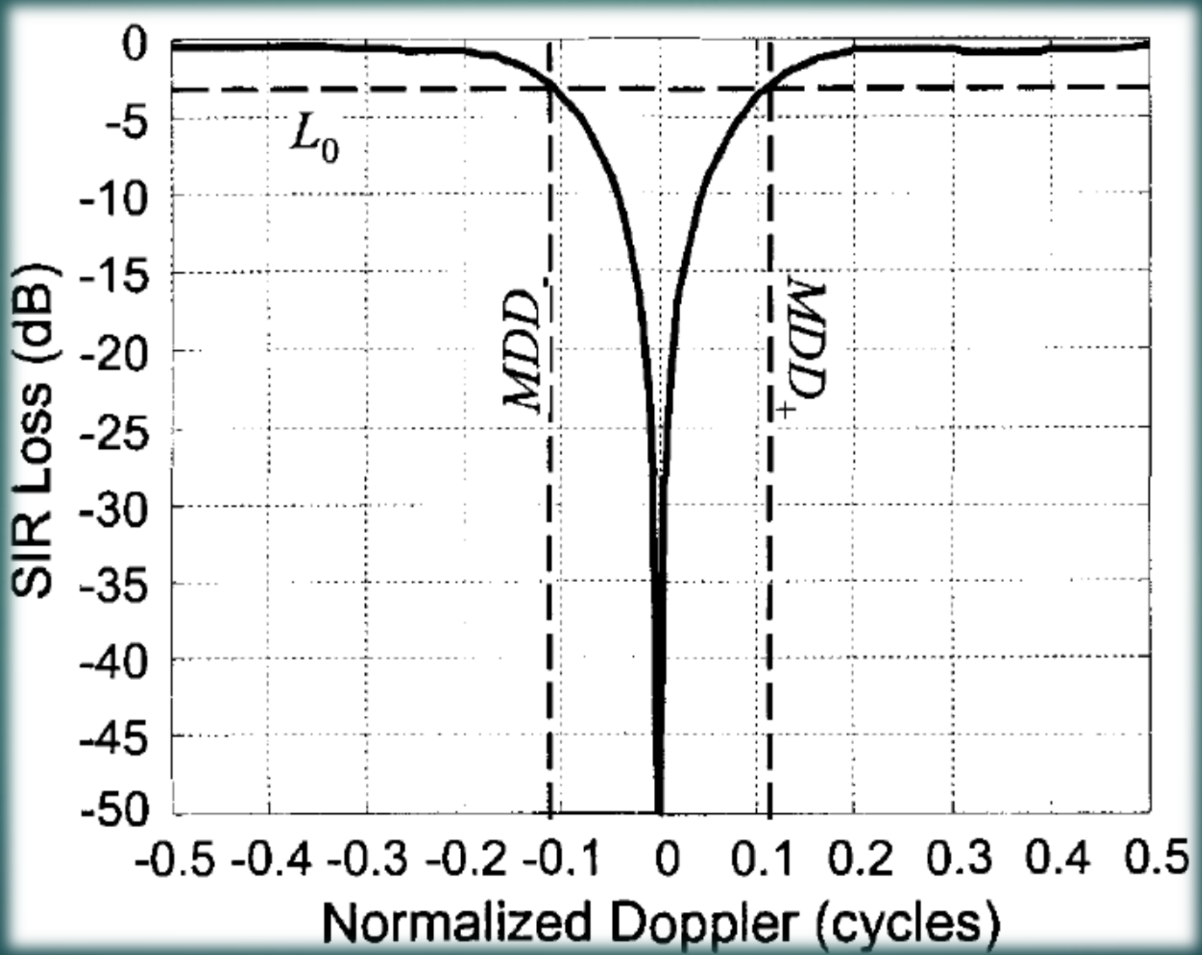


Image adopted from [3], Ch. 9, page 537.

STAP Fundamentals – Sample Matrix Inversion

- In practice $\mathbf{R}_u$ must be estimated from a finite set of available data.
- In the SMI approach, $\mathbf{R}_u$ is replaced by its *sample covariance matrix estimate* $\hat{\mathbf{R}}_u$:

$$\hat{\mathbf{R}}_u = \frac{1}{K}\sum_{k=1}^{K} \boldsymbol{\chi}_k \boldsymbol{\chi}_k^H$$

- The training samples $\boldsymbol{\chi}_k$ arise from range gates before and after the range gate of interest.
- The SMI weight vector is computed by:

$$\mathbf{w} = \hat{\mathbf{R}}_u^{-1}\mathbf{g}_t$$

- Because the covariance matrix is estimated, the SMI weight vector is suboptimum.

STAP Fundamentals – Sample Matrix Inversion

- We have already defined the SINR assuming perfectly Known Covariance Matrix (KC):

$$\mathrm{SINR}_{KC} = \frac{\sigma^2 \xi_t |\mathbf{g}_t^H \mathbf{R}_u^{-1} \mathbf{v}_t|^2}{\mathbf{g}_t^H \mathbf{R}_u^{-1} \mathbf{g}_t}$$

- The SINR obtained with the SMI weight vector is:

$$\mathrm{SINR}_{SMI} = \frac{\sigma^2 \xi_t \left|\mathbf{g}_t^H \widehat{\mathbf{R}}_u^{-1} \mathbf{v}_t\right|^2}{\mathbf{g}_t^H \widehat{\mathbf{R}}_u^{-1} \mathbf{R}_u \widehat{\mathbf{R}}_u^{-1} \mathbf{g}_t}$$

Which is a RV which depends heavily on the number of snapshots *K* used for the covariance matrix estimation.

- Define a new RV to be the loss of performance due to covariance estimation: $\rho = \frac{\mathrm{SINR}_{SMI}}{\mathrm{SINR}_{KC}}$
- It was found by Boroson [4] and Kelly [5] that with a matched steering vector $\mathbf{g}_t = \mathbf{v}_t$, ρ is a beta RV with expected value:

$$E\{\rho\} = \frac{K + 2 - \mathrm{N}_{dof}}{K + 1}$$

STAP Fundamentals – Sample Matrix Inversion

- Therefore, the expected loss of performance is independent of the interference scenario and depends only on the number of samples *K* and the weight vector dimension.
- From previous equation it is evident that for effective performance in a stationary environment, we need $2N_{dof}$ to $5N_{dof}$ independent snapshots for covariance estimation.
- This theory can be used to include effects of covariance estimation in any of the SINR-derived performance metrics. For example the SINR loss equation:

$$L_{SINR}(\varpi) = \frac{SINR(\varpi)}{SNR_o}$$

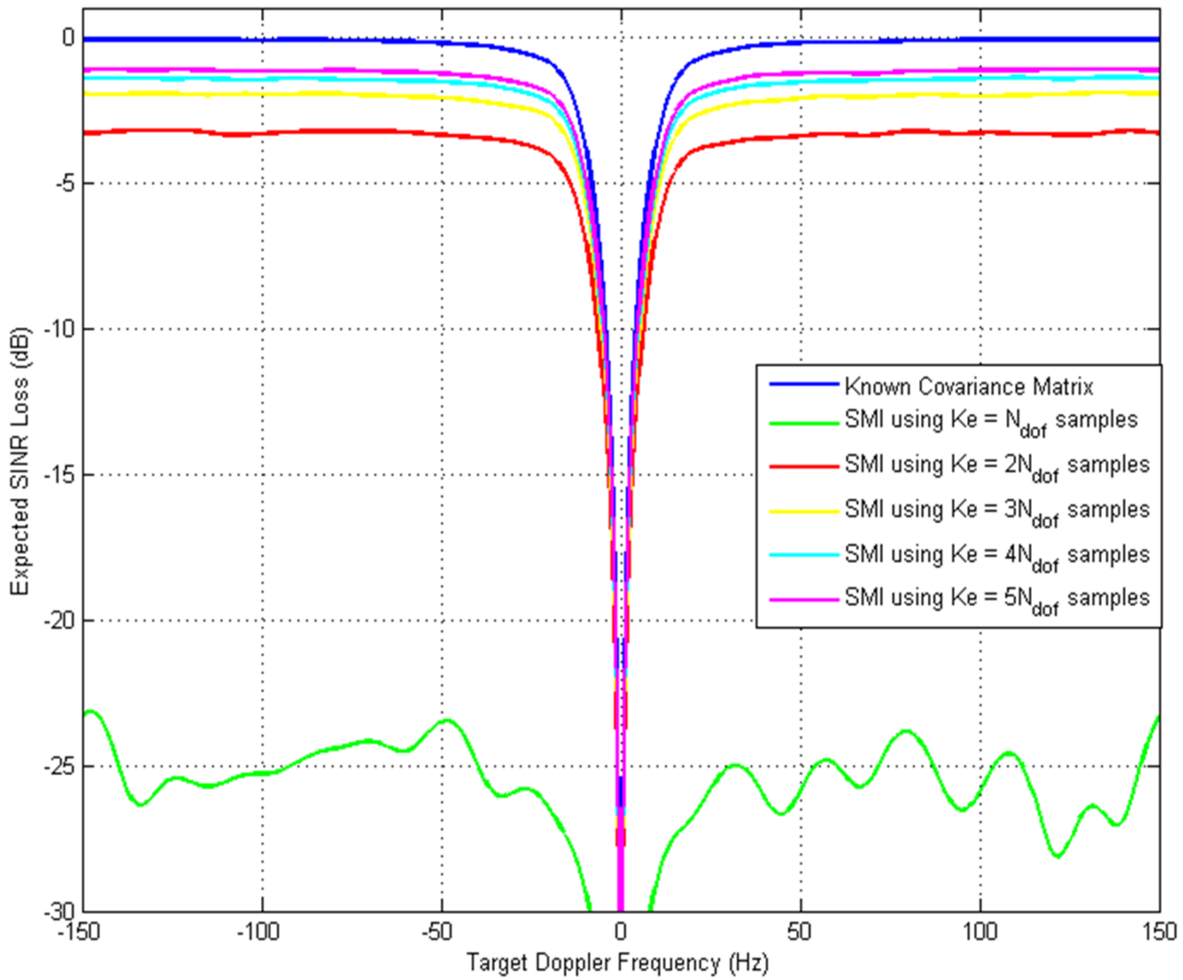
may be modified as:

STAP Fundamentals – Sample Matrix Inversion

- $\mathrm{L}_{SINR}(\varpi, K) = E\left[\frac{SINR_{SMI}}{SNR_o}\right] = E\left[\frac{SINR_{SMI}}{SNR_{KC}} \frac{SINR_{KC}}{SNR_o}\right] = E\{\rho\}\ \mathrm{L}_{SINR}(\varpi, \infty)$

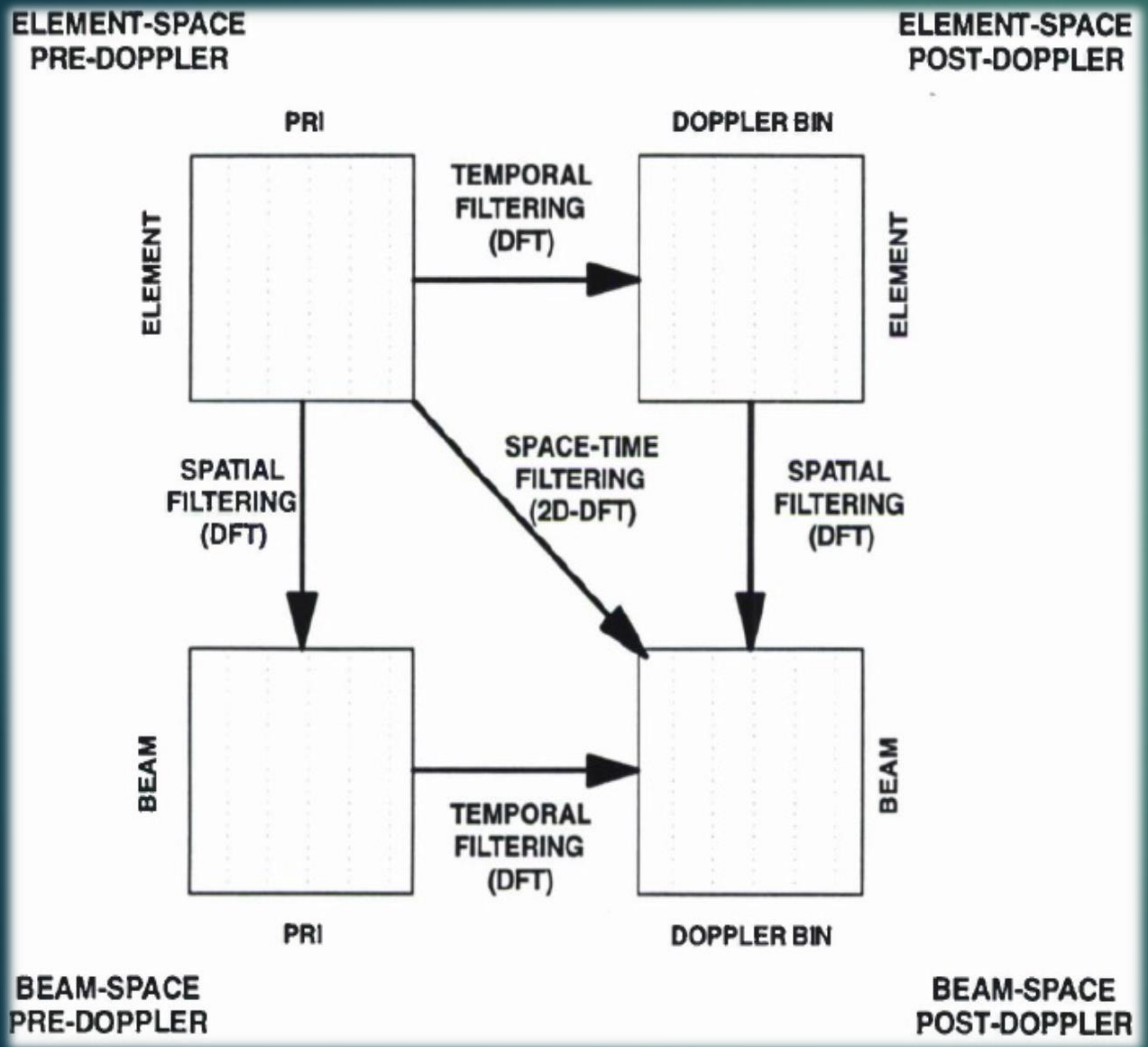
Where $\mathrm{L}_{SINR}(\varpi, \infty) = \frac{SINR_{KC}}{SNR_o}$, i.e. SINR loss with perfect covariance.

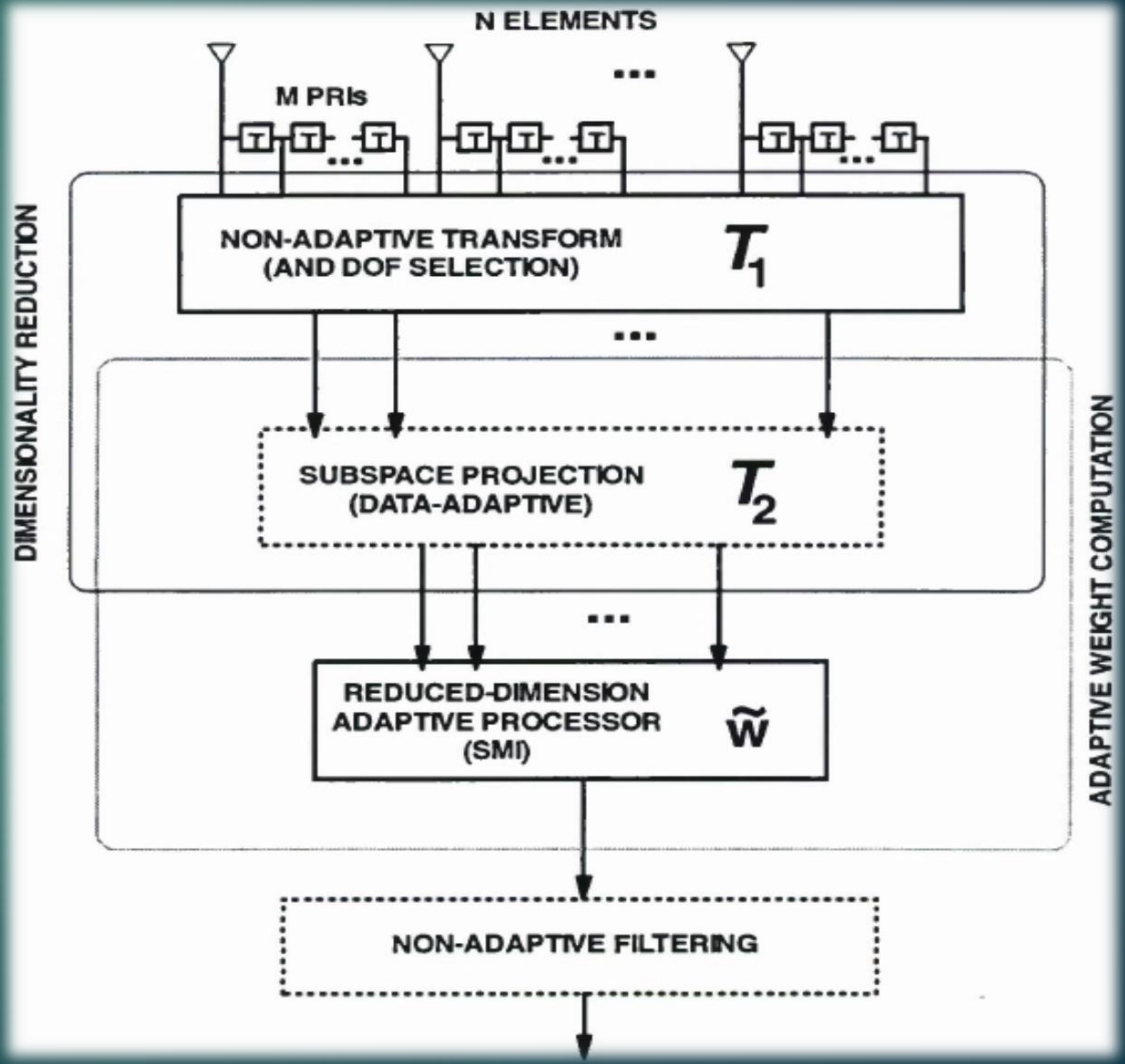
STAP Fundamentals – Sample Matrix Inversion



- At least 2Ndof-3 = 645 Samples are required to reach 3-dB below the optimum SINR loss.
- By reducing the weight dimensionality the performance for a fixed number of snapshots can be dramatically improved.
- Non-homogeneity of clutter in range, combined with the clutter power and elevation angle dependence on range reduce the number of range gates over which the clutter is effectively stationary.
- The need for adequate covariance estimation is a major factor for reduced-dimension STAP.
- The presence of a strong target signal in the snapshots can dramatically increase then number of samples required for a specified level of performance.

Reduced Dimension STAP Algorithms – A taxonomy





Spatial-Temporal Inseparable Covariance Matrix

- A space-time covariance matrix $\mathbf{C}$ is called <u>*spatial-temporal separable*</u> if it can be expressed as a Kronecker product: $\mathbf{C} = \mathbf{C}_s \otimes \mathbf{C}_t$.
- For example, the jamming interference covariance matrix $\mathbf{R}_j = E\{\boldsymbol{\chi}_j \boldsymbol{\chi}_j^H\} = \mathbf{I}_M \otimes \boldsymbol{\Phi}_j$ (where $\boldsymbol{\Phi}_j$ is the jamming sources spatial covariance matrix), is spatial–temporal separable.
- If a space-time covariance matrix is separable, then the corresponding optimal weight vector is also separable and the output of the optimal processor cay be written as:

$$\mathbf{y} = \mathbf{w}^H \boldsymbol{\chi} = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} w_n^* w_m \chi_{nm}$$

- In another words, for a temporal-spatial separable problem, finding an MN×1 optimal weighting vector can be simplified into finding an M×1 temporal optimal weighting vector and an N×1 spatial optimal weighting vector <u>in two discrete steps</u>.

Spatial-Temporal Inseparable Covariance Matrix

- Ground clutter is spatially and temporally correlated as both the clutter intensity and Doppler are a function of elevation & azimuth angles. As a result, the covariance matrix of the ground clutter cannot in general be written as a Kronecker product and hence it is spatial-temporal inseparable.
- Since the dominant part of the covariance matrix of interest is the ground clutter, therefore, in general the covariance matrix of interference is spatial-temporal inseparable.
- This means that any space-time cascaded processors, such as factored time-space (FTS), factored space-time (FST) and majority of dimension-reduced algorithms discussed in next lecture, are unavoidably associated with some SINR loss.
- In essence, all reduced dimension cascaded processors that process data in space-domain and time domain separately, imply the covariance matrix to be spatial-temporal separable which is against the nature of the covariance matrix.

References

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