Array Radar Signal Environment

ILIAS S. KONSOULAS, MSc, CEng, PMP, MIET

HELLENIC AIR FORCE

ekonsoul@yahoo.com

An Inspiring Paradigm…

This presentation largely follows the notation and development originally presented in the seminal J. Ward's technical report on STAP:

[0] *J. Ward, "Space-Time Adaptive Processing for Airborne Radar", Technical Report 1015, Lincoln Laboratory, MIT, Dec. 1994.*

All MatLab files used for this lecture may be downloaded from:
http://www.mathworks.com/matlabcentral/fileexchange/47750-space-time-adaptive-processing-for-airborne-radar-by-j-ward--tech-report-1015-

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Array Radar Signal Environment - Topics

- Radar System Description & Signal Models
  - Platform Geometry
  - The CPI Datacube
  - Target, Noise, Jamming and Clutter Signal Models
- Clutter Space-Time Covariance Matrix
  - Morphology, Rank, Eigenspecrum
  - Impact of Velocity Misalignment on Rank
  - Impact of Element Backlobe Power on Rank
  - Impact of Intrinsic Clutter Motion (ICM)
  - Clutter Fourier Power Spectrum
  - Clutter Minimum Variance Spectrum
  - Clutter Range-Doppler Dependence
- Signal Modeling Summary
- References

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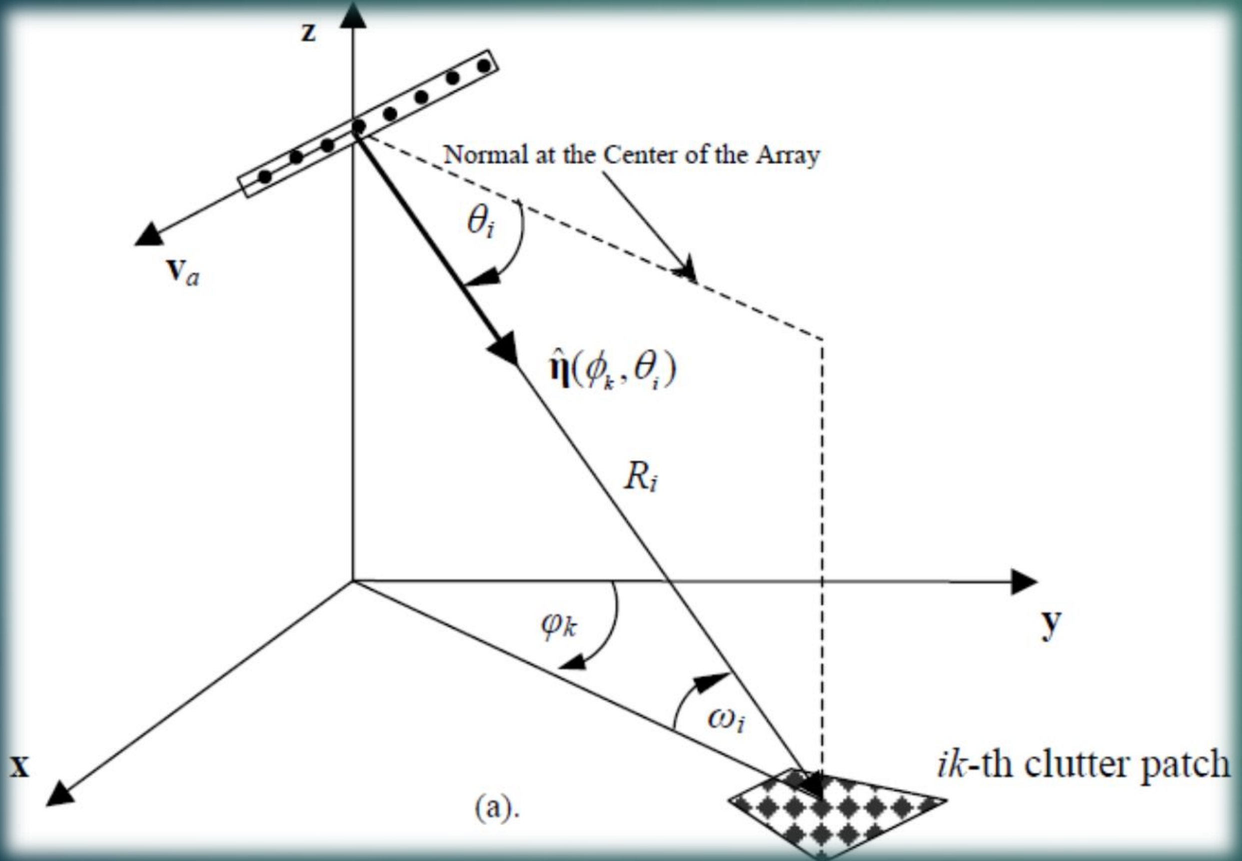
Platform Geometry



(a).

$$\hat{\boldsymbol{\eta}}(\phi,\theta) = \cos\theta\sin\phi\,\hat{\mathbf{i}} + \cos\theta\cos\phi\,\hat{\mathbf{j}} + \sin\theta\,\hat{\mathbf{k}}$$

$\mathbf{d} = d\,\hat{\mathbf{i}}$ for the side-looking antenna

$\mathbf{d} = d\,\hat{\mathbf{j}}$ for the forward looking antenna configuration, where d is the inter-element distance set equal to $\lambda/2$.

The *n*-th array element is located at:

$$\mathbf{r}_n = n\mathbf{d}\,. \qquad n = -N/2, \ldots, N/2 \ (N \text{ even})$$

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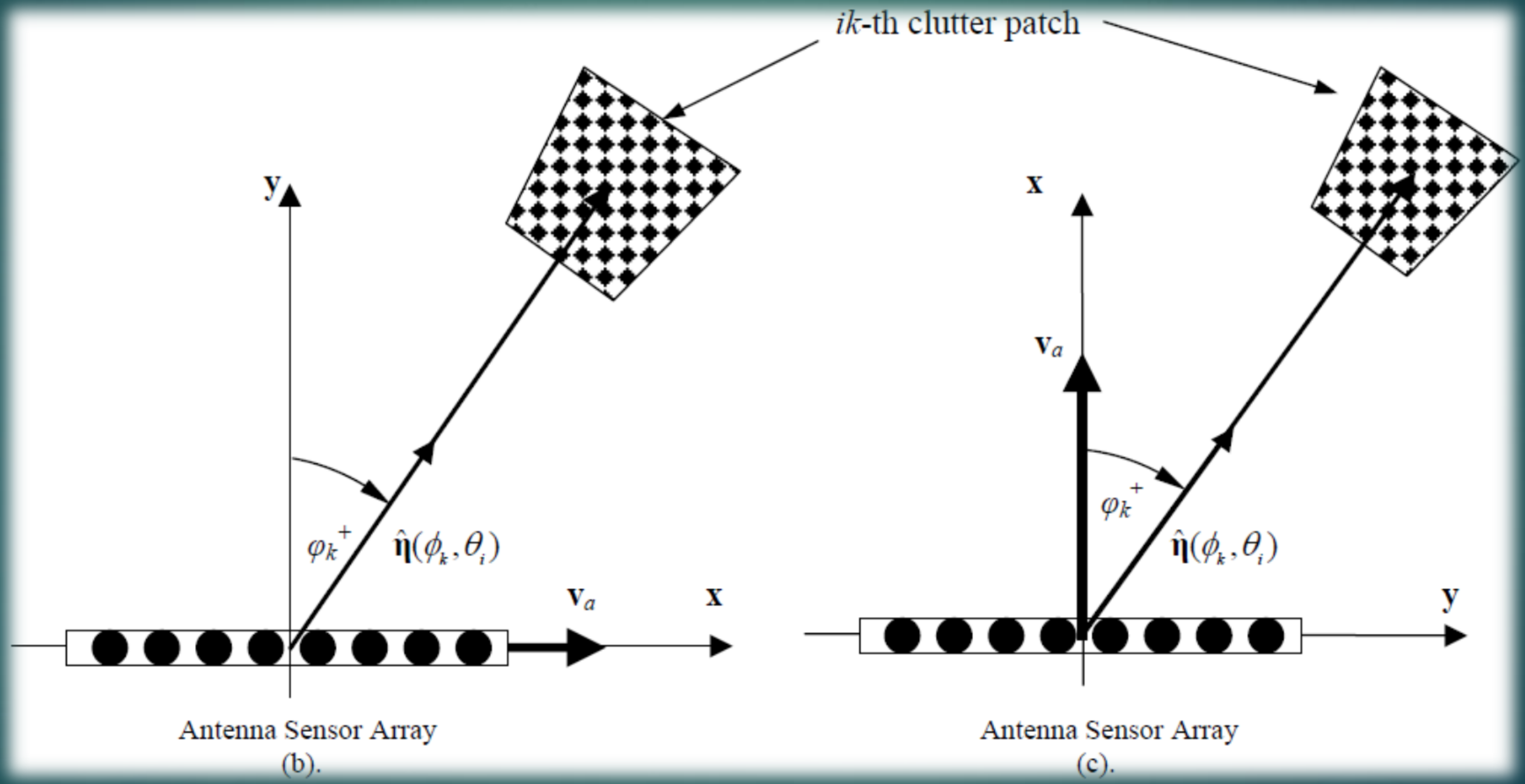
Top View



Side-Looking Airborne Radar (SLAR) Forward-Looking Airborne Radar (FLAR)

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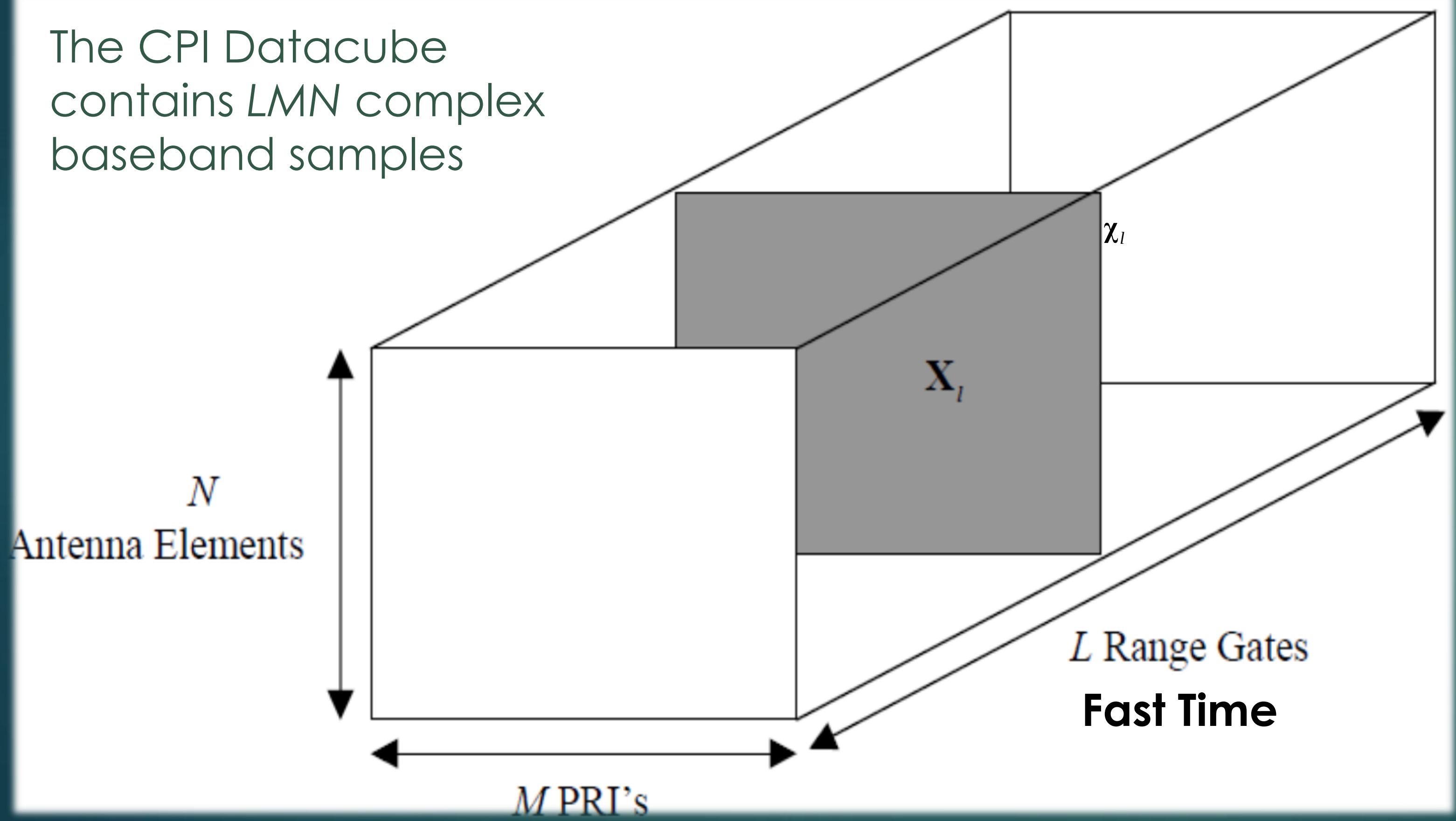
The CPI Datacube

The CPI Datacube contains *LMN* complex baseband samples



Slow Time

$$\mathbf{X}_l = [\mathbf{x}_{0,l}, \mathbf{x}_{1,l}, \ldots, \mathbf{x}_{M-1,l}]$$

N×*M* matrix consisting of all the spatial snapshots for all *M* pulses of the CPI at the range gate of interest (*l*-th gate)

$$\chi_l = \mathrm{vect}(\mathbf{X}_l) = [\mathbf{x}_{0,l}; \mathbf{x}_{1,l}; \ldots; \mathbf{x}_{M-1,l};]$$

Data from a single range gate *l* as an *MN*×1 vector $\boldsymbol{\chi}_l$, called a *space - time snapshot* by stacking the columns of $\mathbf{X}_l$

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Target Signal

It can be shown [0] that the output signal of the *n*-th antenna element after the *m*-th pulse due to a moving point target is

$$x_{m,n} = a_t \, e^{j2\pi n \vartheta_t} \, e^{j2\pi m \varpi_t}, \qquad \begin{array}{l} n = 0, 1, \ldots, N-1 \\ m = 0, 1, \ldots, M-1 \end{array}$$

$a_t = \alpha_r \exp(j\theta_r)$ is the target's complex random amplitude.

$$\vartheta_t = \frac{\hat{\boldsymbol{\eta}}(\phi_t, \theta_t) \cdot \mathbf{d}}{\lambda}$$

is the target's **spatial frequency**.

$$\varpi_t = \frac{f_t}{f_r}$$

Is the target's **normalized Doppler frequency**.

f_r is radar's PRF and f_t is target's Doppler frequency defined as:

$$f_t = \frac{2}{\lambda}(\mathbf{v}_a - \mathbf{v}_{tgt}) \cdot \hat{\boldsymbol{\eta}}(\phi_t, \theta_t)$$

The target signal-to-noise ratio (SNR) for a single pulse at a single element's channel output is:

$$\xi_t = \frac{P_t G_t(\phi_t, \theta_t) \, g_t(\phi_t, \theta_t) \, \lambda^2 \sigma_t}{(4\pi)^3 N R_t^4}$$

$$p_t = E\{|a_t|^2\} = \sigma^2 \xi_t$$

$$\alpha_r = \sqrt{\sigma^2 \xi_t}$$

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Target Signal

Define the $N \times 1$ **spatial steering vector**:

$$\mathbf{a}(\vartheta_t) = [1;\ e^{j2\pi\vartheta_t};\ e^{j2\pi 2\vartheta_t};\ \ldots\ ;e^{j2\pi(N-1)\vartheta_t}\ ]$$

And the $M \times 1$ **temporal steering vector**:

$$\mathbf{b}(\varpi_t) = [1;\ e^{j2\pi\varpi_t};\ e^{j2\pi 2\varpi_t};\ \ldots\ ;e^{j2\pi(N-1)\varpi_t}\ ]$$

It can be shown that the target data can be assembled in the form of a space-time snapshot:

$$\begin{aligned}\chi_t &= a_t[\mathbf{a}(\vartheta_t);\ \mathbf{a}(\vartheta_t)e^{j2\pi\varpi_t};\ \mathbf{a}(\vartheta_t)e^{j2\pi 2\varpi_t};\ \ldots\ ;\ \mathbf{a}(\vartheta_t)e^{j2\pi(M-1)\varpi_t}\ ] \\ &= a_t\,\mathbf{b}(\varpi_t)\otimes\mathbf{a}(\vartheta_t),\end{aligned}$$

$\otimes$ is the Kronecker matrix product operator.

The $MN \times 1$ composite space-time steering vector

$$\mathbf{v}(\vartheta,\varpi) = \mathbf{b}(\varpi)\otimes\mathbf{a}(\vartheta)$$

is the response of a unit amplitude target at spatial frequency ϑ_t and normalized Doppler frequency ϖ_t .

Therefore, we can model the target's contribution in the received data snapshot as:

$$\chi_t = a_t\mathbf{v}_t = a_t\mathbf{v}(\vartheta_t,\varpi_t)$$

where $\mathbf{v}_t$ is also called target's space-time steering vector.

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Noise Signal

- The noise processes on each element are mutually uncorrelated.
- Noise samples on a single element taken at time instants separated by a multiple of the PRI are temporally uncorrelated.
- Let $z_{m,n}$ be the noise sample on the n-th element for the m-th PRI. The above assumptions mean:

$$E\{z_{n_1,m}\, z^*_{n_2,m}\} = \sigma^2 \delta_{n_1-n_2}$$
$$E\{z_{n,m_1}\, z^*_{n,m_2}\} = \sigma^2 \delta_{m_1-m_2}$$

- Therefore, the noise components of the space-time covariance matrix is a scaled identity matrix:

$$\mathbf{R}_n = E\{\chi_n \chi_n^H\} = \sigma^2 \mathbf{I}_{MN}$$

- Where $\sigma^2 = N_0 B$ is the noise power at a single element. For the following analysis we set $\sigma^2 = 1$.

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Jamming Signal

- $J_0 = \frac{S_j g(\theta_j, \varphi_j)\lambda_0^2}{(4\pi)^2 R_j^2 L_r}$ is the j-th jammer's PSD.
- S_j is th j-th jammer's Effective Radiated Power Density (ERPD) in Watts/Hz.
- The j-th jammer space-time snapshot is: $\boldsymbol{\chi}_j = \mathbf{b}_j \otimes \mathbf{a}_j$, $\mathbf{b}_\mathrm{j} = [b_0, b_1, \dots, b_{M-1}]^T$ is a random vector of jammer uncorrelated amplitudes at all CPI pulses.
- $\mathbf{a}_j = \mathbf{a}(\varphi_j, \theta_j)$ is the j-th jammer's spatial steering vector.
- $\mathbf{A}_j = [\boldsymbol{a}(\varphi_1, \theta_1), \boldsymbol{a}(\varphi_2, \theta_2), \dots, \boldsymbol{a}(\varphi_J, \theta_J)]$ is a $N\times J$ matrix of the jammer spatial steering vectors.
- The j-th jammer's spatial covariance matrix is: $\boldsymbol{\Phi}_j = \boldsymbol{\sigma}^2 \xi_j \mathbf{a}_j \mathbf{a}_j^H$.
- For multiple jamming sources: $\boldsymbol{\Phi}_j = \mathbf{A}_j \boldsymbol{\Xi}_j \mathbf{A}_j^H$ where $\boldsymbol{\Xi}_j$ is the $J\times J$ jammer source covariance matrix.
- The jamming space-time covariance matrix: $\mathbf{R}_j = E\{\boldsymbol{\chi}_j \boldsymbol{\chi}_j^H\} = \mathbf{I}_M \otimes \boldsymbol{\Phi}_j = \mathbf{I}_M \otimes (\mathbf{A}_j \boldsymbol{\Xi}_j \mathbf{A}_j^H)$.
- $\mathrm{rank}(\boldsymbol{\Phi}_j) = J$.
- $\mathrm{rank}(\mathbf{R}_j) = MJ$.

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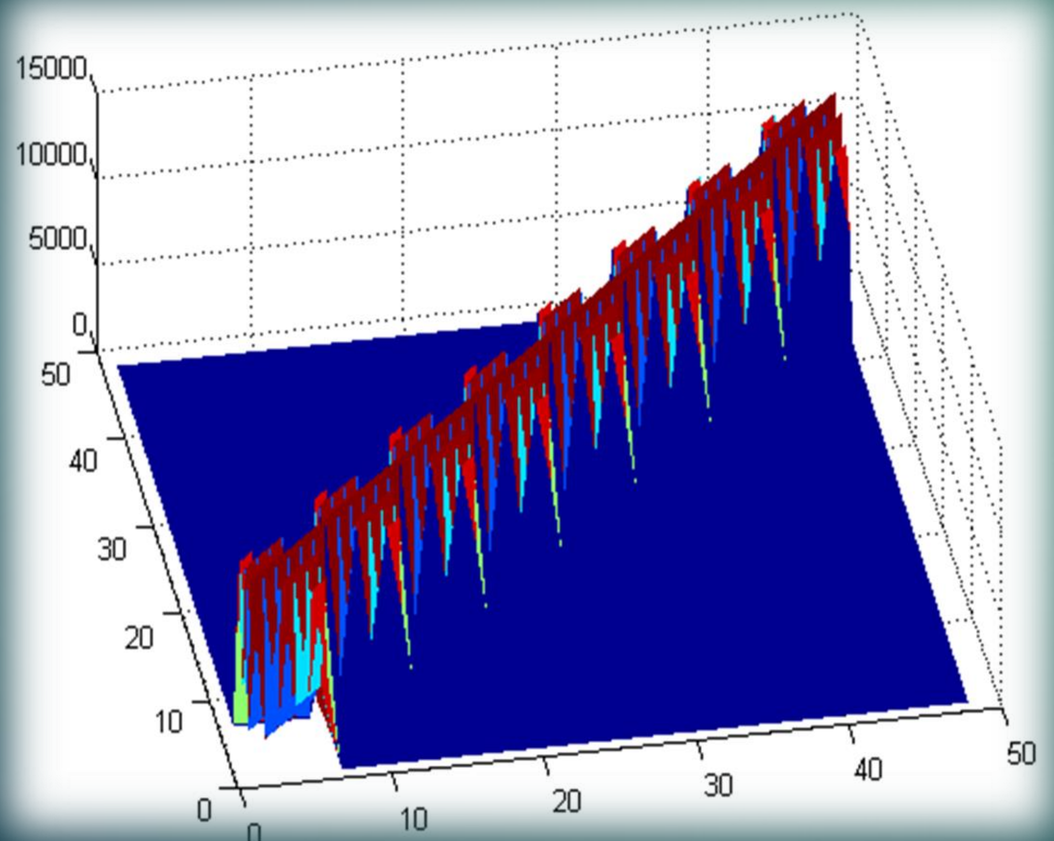
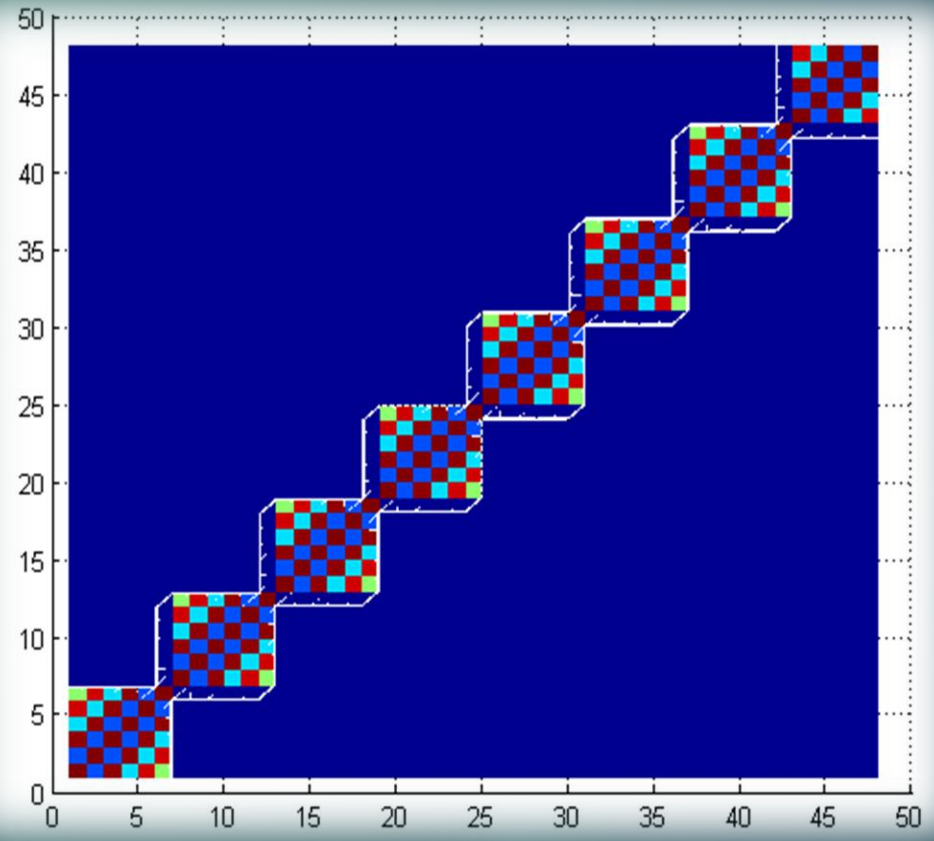
Jamming Space-Time Covariance Matrix





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Clutter Signal

- Defined as echoes from earth surface scatterers of no tactical significance.
- It is the most complicated source of interference because it is distributed both in angle and range.
- Exhibits Doppler spread due to platform movement. The Clutter Doppler frequency is:
  - Constant with range for a side-looking array.
  - Range-dependent for a forward looking array.
- In general, the clutter component consists of the superposition of returns from all ambiguous ranges within the radar horizon.
- We will demonstrate a model for the ground clutter component of the space-time snapshot **for a given range gate** and study the properties of the clutter space-time covariance matrix.
- We will also demonstrate the effects of velocity misalignment and Intrinsic Clutter Motion (ICM) on the rank of the clutter space-time covariance matrix.

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Clutter Signal – Spectral Elements

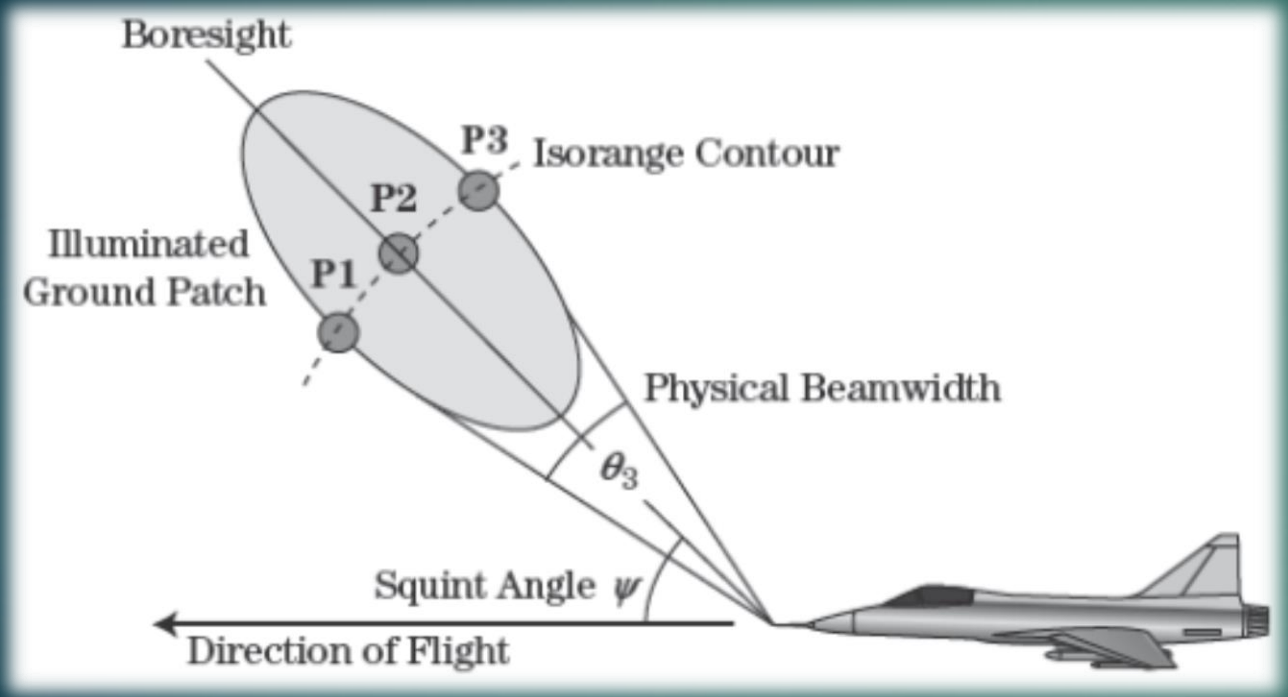

Figure adopted from Chapter 8, page 297 of [1].

- Main Lobe Clutter (MLC) Doppler Spread:
  $B_{MLC} \cong \frac{2v\theta_3}{\lambda}\sin\psi$, (3-dB Bandwidth).
- Main Lobe plus Side Lobe Clutter (SLC) Spread:
  $B_{MLC+SLC} = \frac{4v}{\lambda}$, Total Clutter BW.
- For scatterers distributed at a certain range *Ri* (corresponding to elevation angle *θi*) the clutter BW extends [3]:

$$B_D = \left[-\frac{2v_a}{\lambda}\cos\theta_i ,\ \frac{2v_a}{\lambda}\cos\theta_i \right]$$

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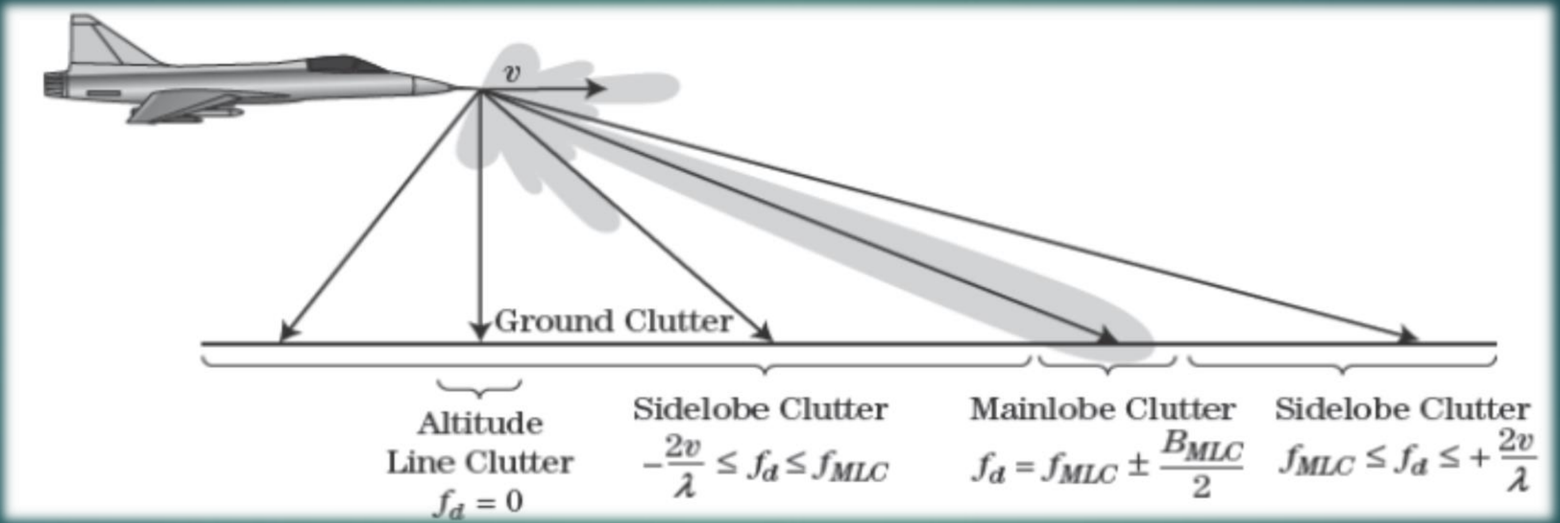
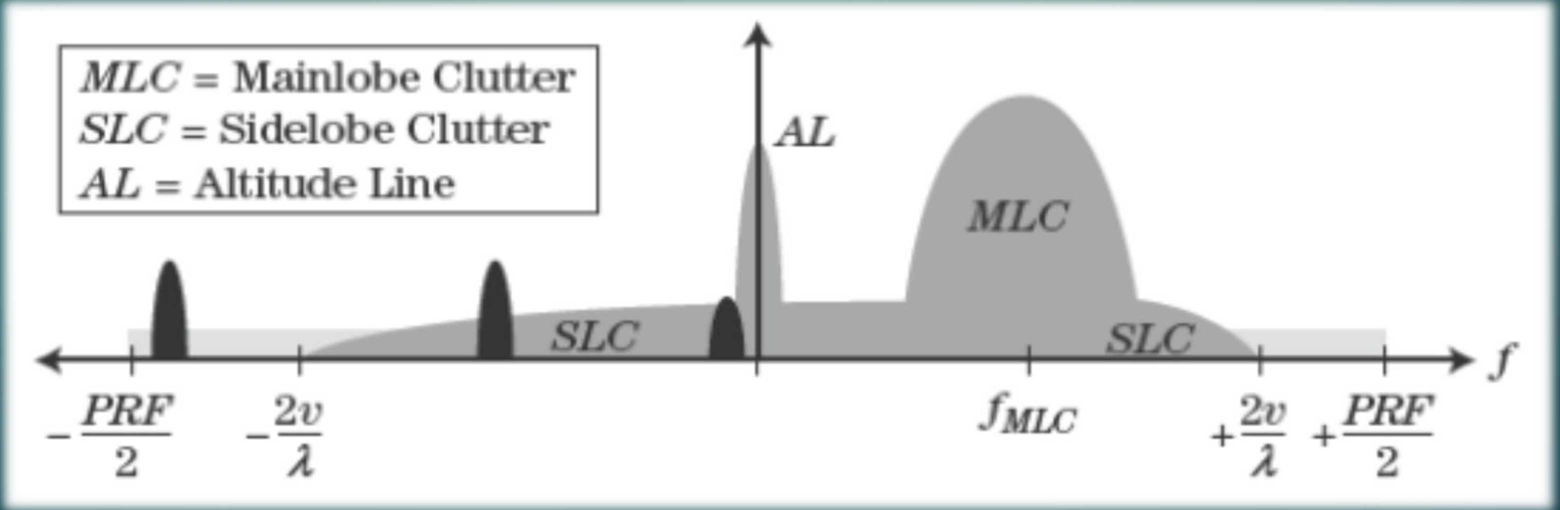
Clutter Signal – Spectral Elements





Both figures come from Chapter 8, page 298 of [1].

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Clutter Signal – Side Looking Airborne Radar (SLAR)

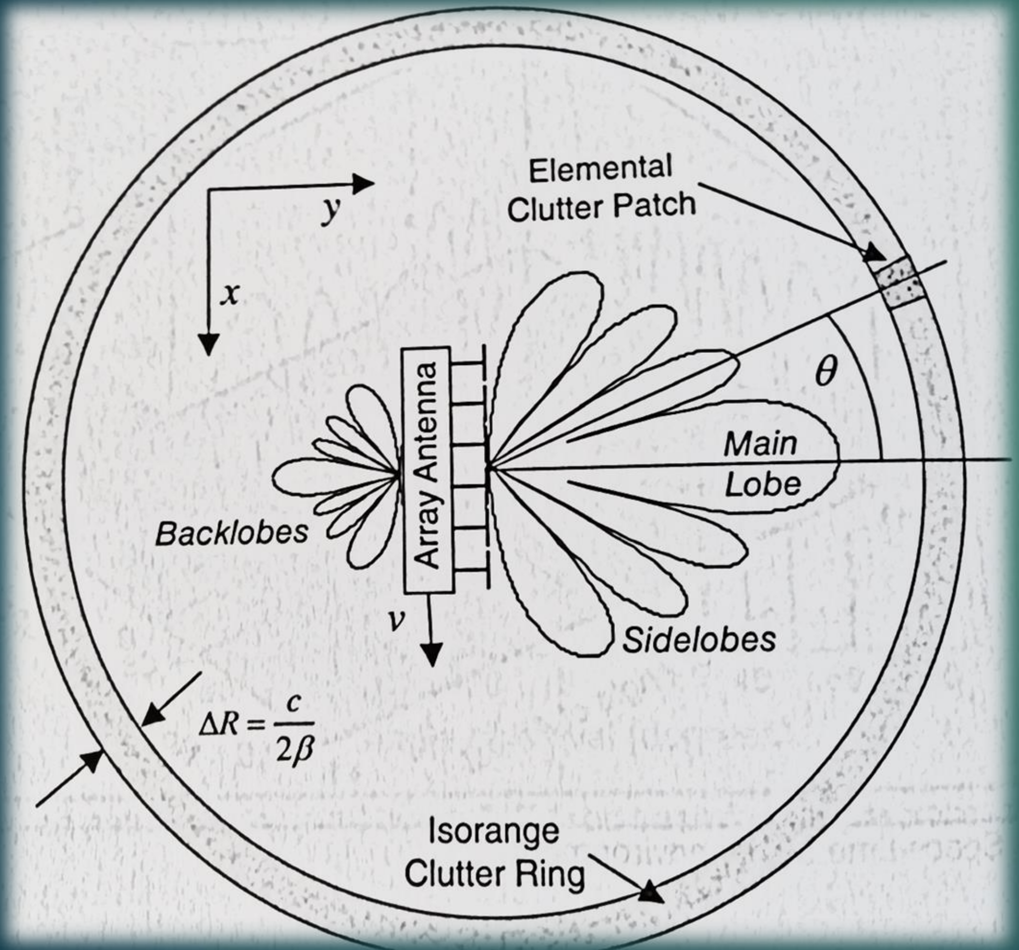

Image adopted from [2], Chapter 9, Page 528

- Range Resolution is $\Delta R = \frac{c}{2B}$.
- The clutter return from each range gate is modelled as the superposition of a large number N_c of independent clutter sources, evenly distributed in azimuth around the radar antenna.
- Each patch represents an effective area bounded in azimuth by the granularity of the angle sampling $\Delta\varphi = \frac{2\pi}{Nc}$ and in range by the range resolution ΔR.
- Grazing angle $\omega_i = \theta_i$ (flat earth assumption).
- Patch Area = $R_i \, \Delta\varphi \, \Delta R \, sec\omega_i$

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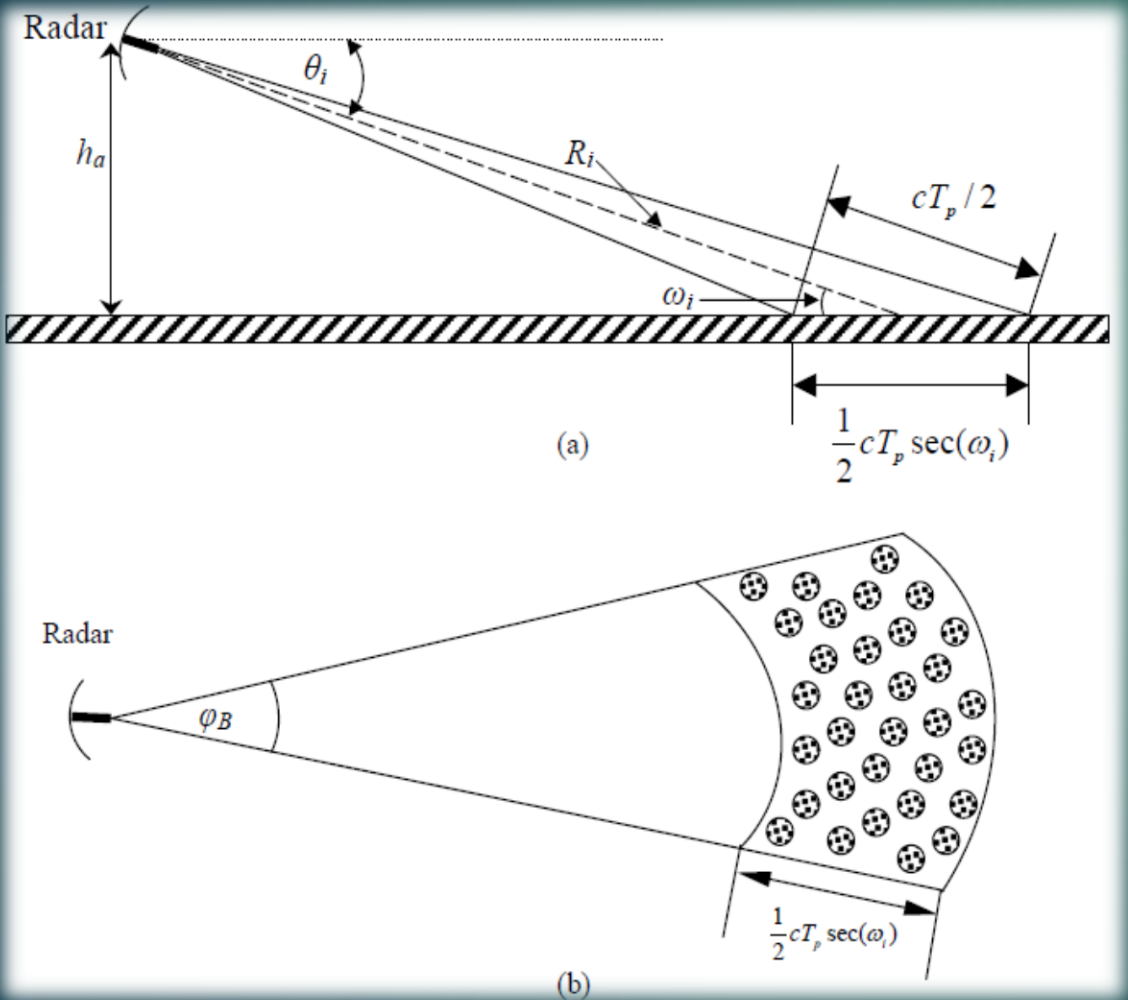
Clutter Signal – Side Looking Airborne Radar (SLAR)



- The spatial frequency of the *ik*-th clutter patch is:

$$\vartheta_{ik} = \frac{\hat{\boldsymbol{\eta}}(\theta_i, \varphi_k) \cdot \mathbf{d}}{\boldsymbol{\lambda}_0} = \frac{d}{\boldsymbol{\lambda}_0} cos\theta_i \, sin\varphi_k$$

The clutter component of the space-time snapshot is:

$$\chi_c = \sum_{k=1}^{N_c} a_{ik} \mathbf{v}_{ik}$$

a_{ik} is the random amplitude from the *ik*-th clutter patch.

(returns from a single range gate).

$$\mathbf{v}_{ik} = \mathbf{v}(\vartheta_{ik}, \varpi_{ik}) = \mathbf{b}(\varpi_{ik}) \otimes \mathbf{a}(\vartheta_{ik})$$

is the space-time steering vector of the *ik*-th clutter patch.

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Clutter Signal – Side Looking Airborne Radar (SLAR)

- The Doppler frequency of the *ik*-th clutter patch is:

$$f_c(\theta_i, \varphi_k) = \frac{2\hat{\mathbf{\eta}}(\theta_i,\varphi_k)\cdot\mathbf{v}_a}{\lambda_0} = \frac{2v_a}{\boldsymbol{\lambda_0}} cos\theta_i \; sin\varphi_k$$

- The normalized Doppler frequency is:

$$\varpi_{ik} = f_c(\theta_i, \varphi_k)T_r = \left(\frac{2v_aT_r}{d}\right)\vartheta_{ik}$$

- $\beta = \frac{2v_aT_r}{d}$ is the slope of the clutter locus line on the $(\vartheta_{ik}, \varpi_{ik})$ plane.

β represents the number of half-inter-element spacings $d/2$ traversed by the platform during one PRI.

For d = λ/2: <u>β is equivalently the number of times the clutter Doppler spectrum aliases into the unambiguous Doppler space</u>.

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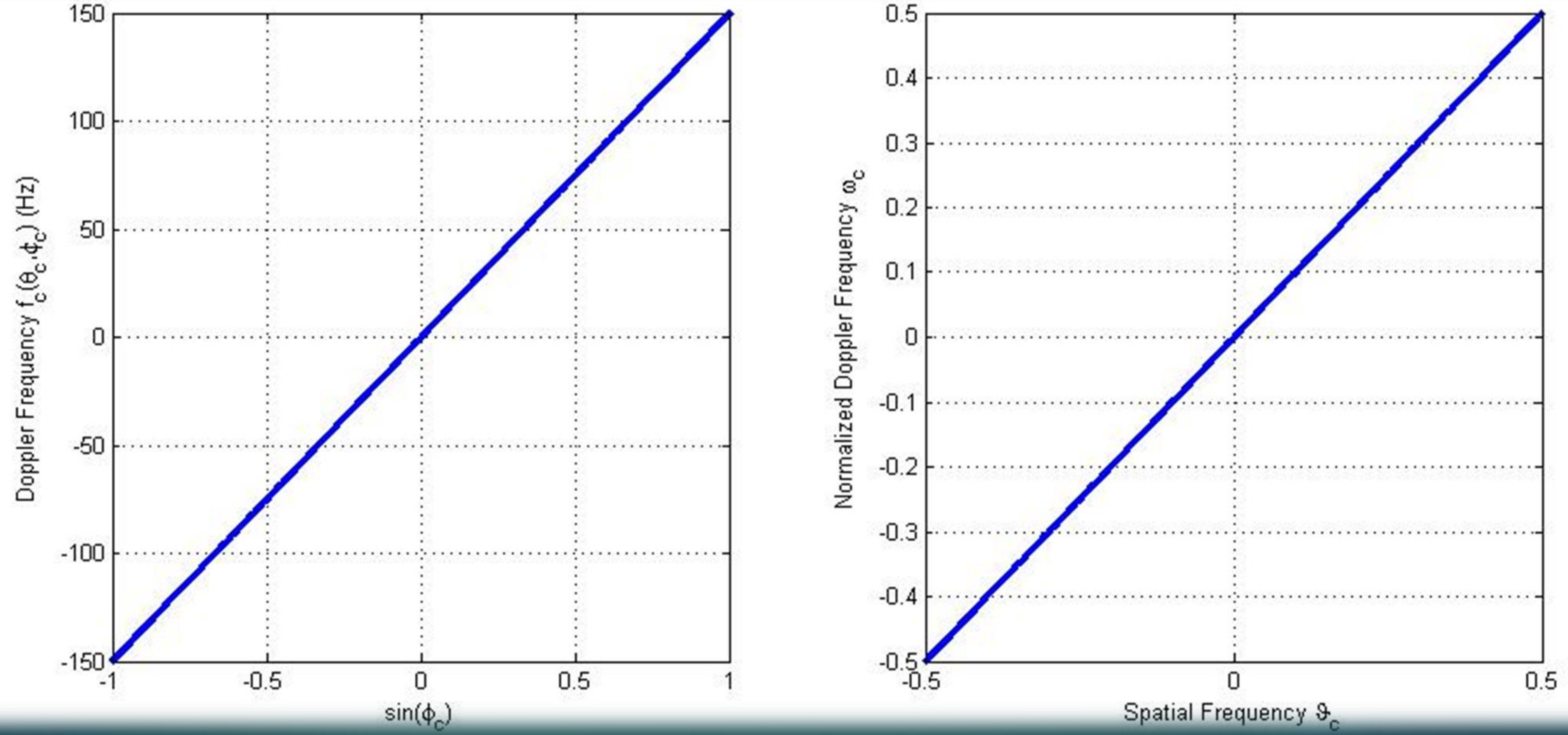
Clutter Signal – Side Looking Airborne Radar (SLAR)

The β=1 Clutter Ridge for SLAR, PRF = 300 Hz



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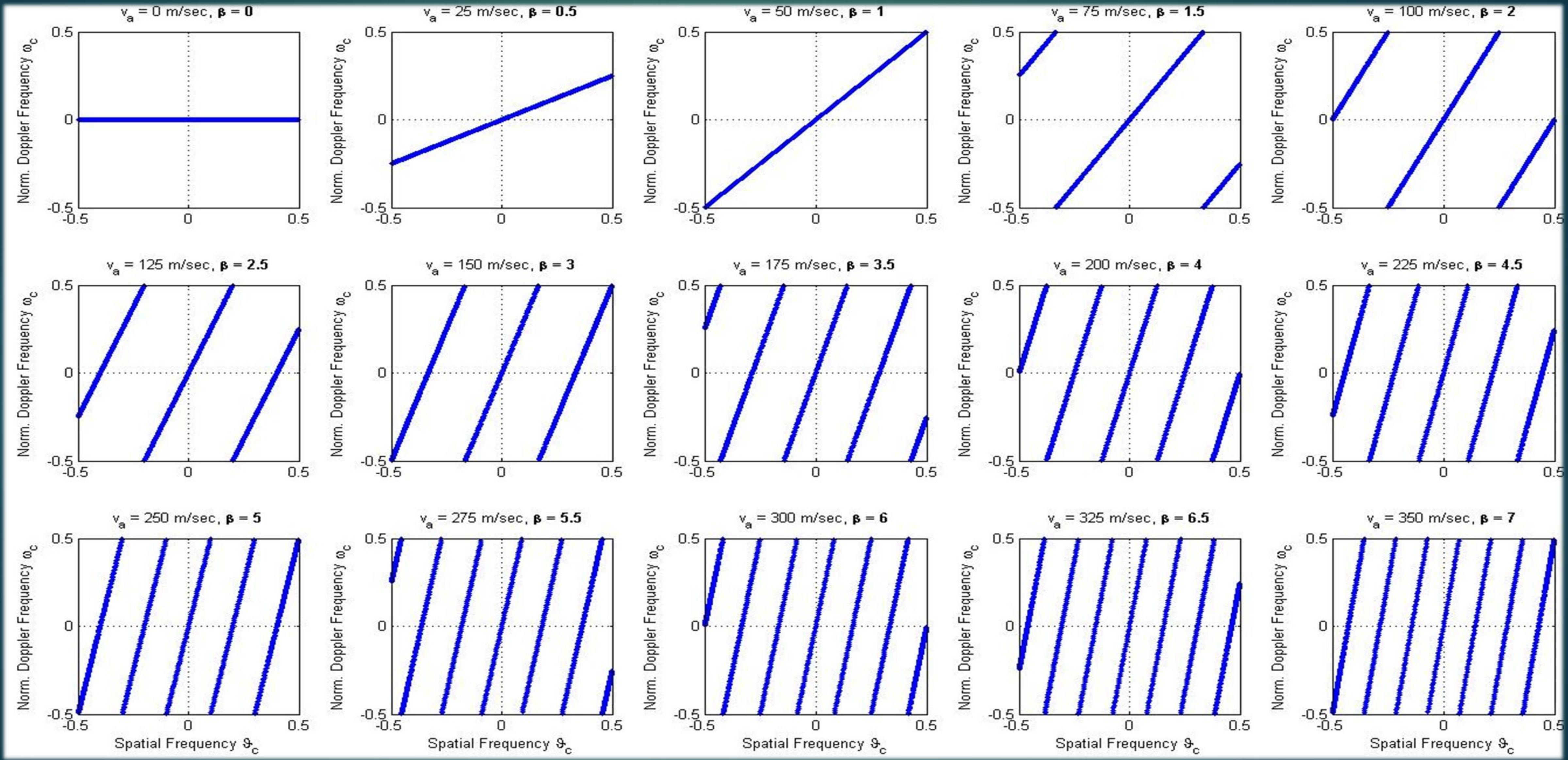
Clutter Signal – Side Looking Airborne Radar (SLAR)



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Clutter Signal – Side Looking Airborne Radar (SLAR)

- The clutter signal-to-noise ratio (SNR) due to the *ik*-th patch, for a single pulse at a single element's channel output is:

$$\xi_{ik} = \frac{P_t G_t(\phi_k, \theta_i) g_t(\phi_k, \theta_i) \lambda^2 \sigma_{ik}}{(4\pi)^3 N R_i^4}$$

- The effective RCS of the *ik*-th patch is:

$$\sigma_{ik} = \sigma_0(\phi_k, \theta_i) A_c \qquad \sigma_0 = \gamma \sin \omega_i$$

The *constant gamma model* was employed for this study.

Patch Area = $A_c = R_i \, \Delta\varphi \, \Delta R \, sec\omega_i$

- Returns from different clutter patches are uncorrelated:

$$E\{|a_{ik}|^2\} = \sigma^2 \xi_{ik}$$

$$E\{a_{ik} a_{jl}^*\} = \sigma^2 \xi_{ik} \delta_{i-j} \delta_{k-l}$$

- The CNR per element per pulse is:

$$\xi_c = \frac{\mathbf{R}_c(1,1)}{\sigma^2} = \frac{1}{\sigma^2} \sum_{k=1} p_{ck}$$

- According to the radar equation, the backscattered Clutter Signal Power decreases with range: $\xi_{ik} \propto 1/R_i^3$.

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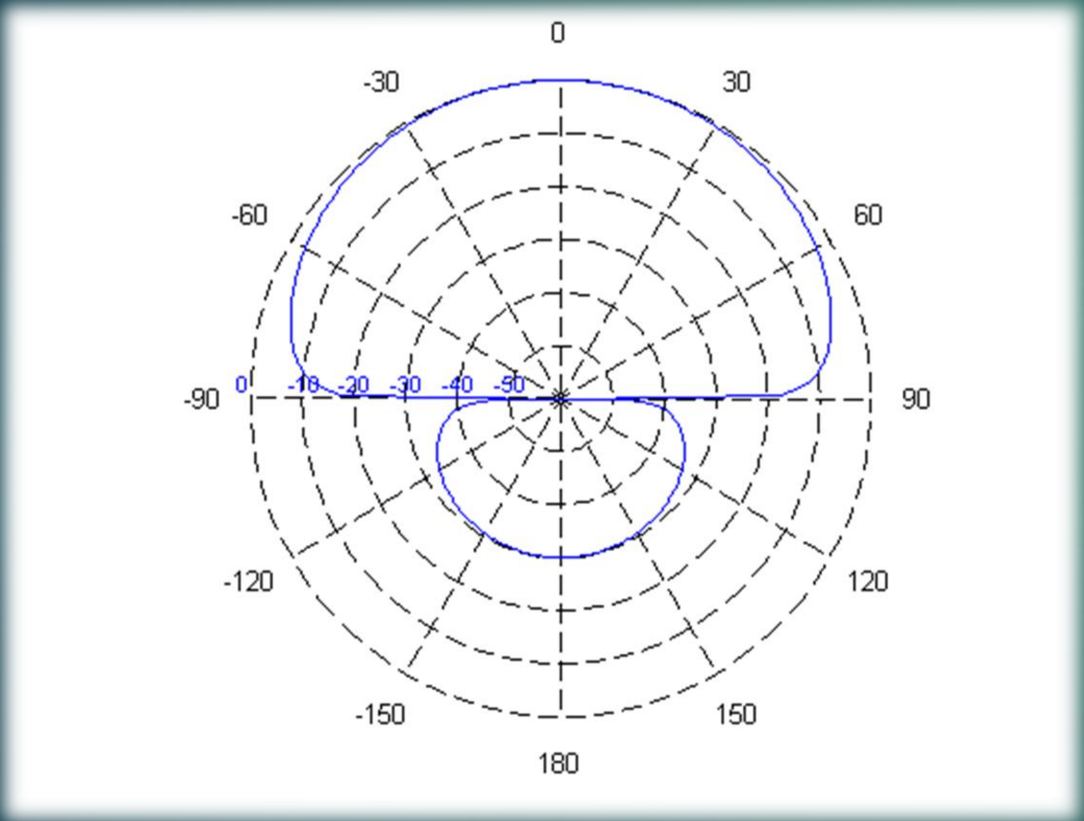
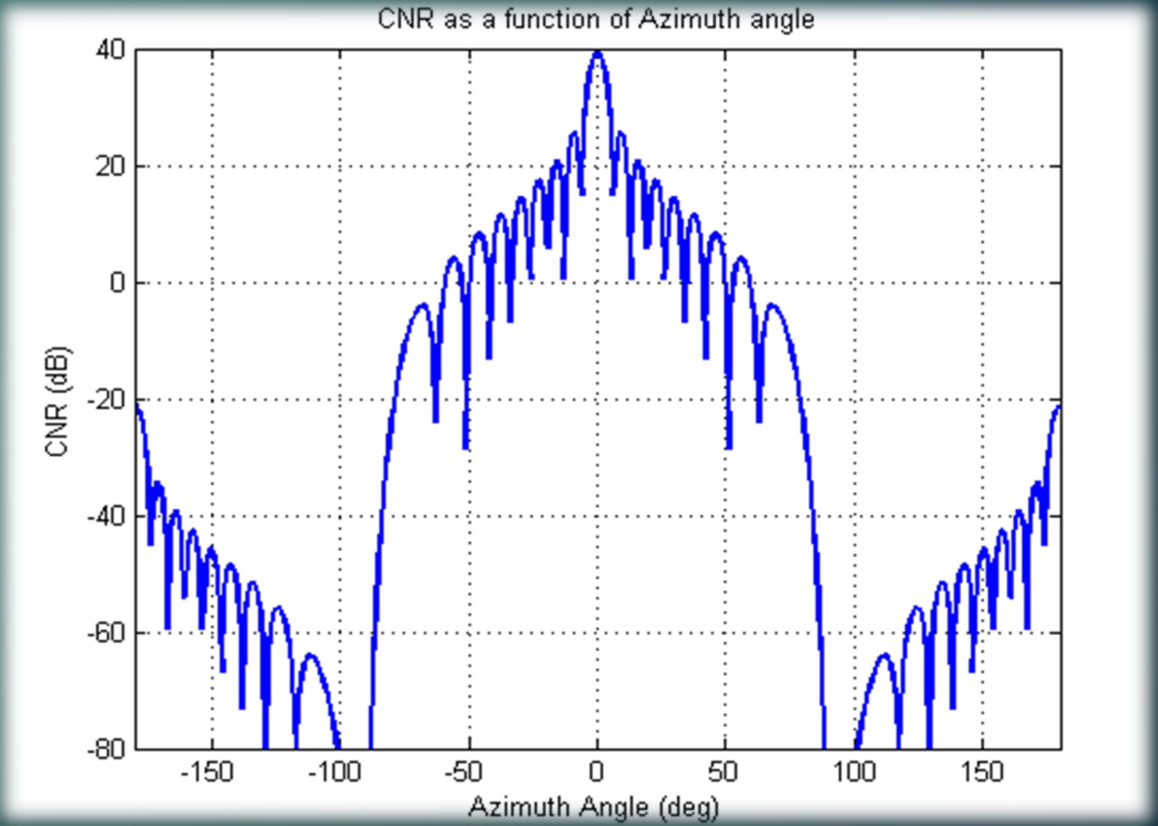
Clutter Signal – Side Looking Airborne Radar (SLAR)



The element pattern with -30dB backlobe level.



Received CNR per element per pulse.

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Clutter Signal – Space-Time Covariance Matrix

- Clutter space-time covariance matrix:

$$\mathbf{R}_c = E\{\chi_c \chi_c^H\} = \sigma^2 \sum_{k=1}^{N_c} \xi_{ik} \mathbf{v}_{ik} \mathbf{v}_{ik}^H$$

Or

$$\mathbf{R}_c = \sigma^2 \sum_{k=1}^{N_c} \xi_{ik} (\mathbf{b}_{ik} \mathbf{b}_{ik}^H) \otimes (\mathbf{a}_{ik} \mathbf{a}_{ik}^H)$$

- Each *ik*-th clutter scatterer contributes a term that is the Kronecker product of a temporal covariance matrix with a spatial covariance matrix.
- This space-time covariance matrix has a Toeplitz-block-Toeplitz structure.
- $\mathbf{R}_c$ is an *M×M block matrix where each block is an N×N* cross-covariance matrix of spatial snapshots from two PRI’s.

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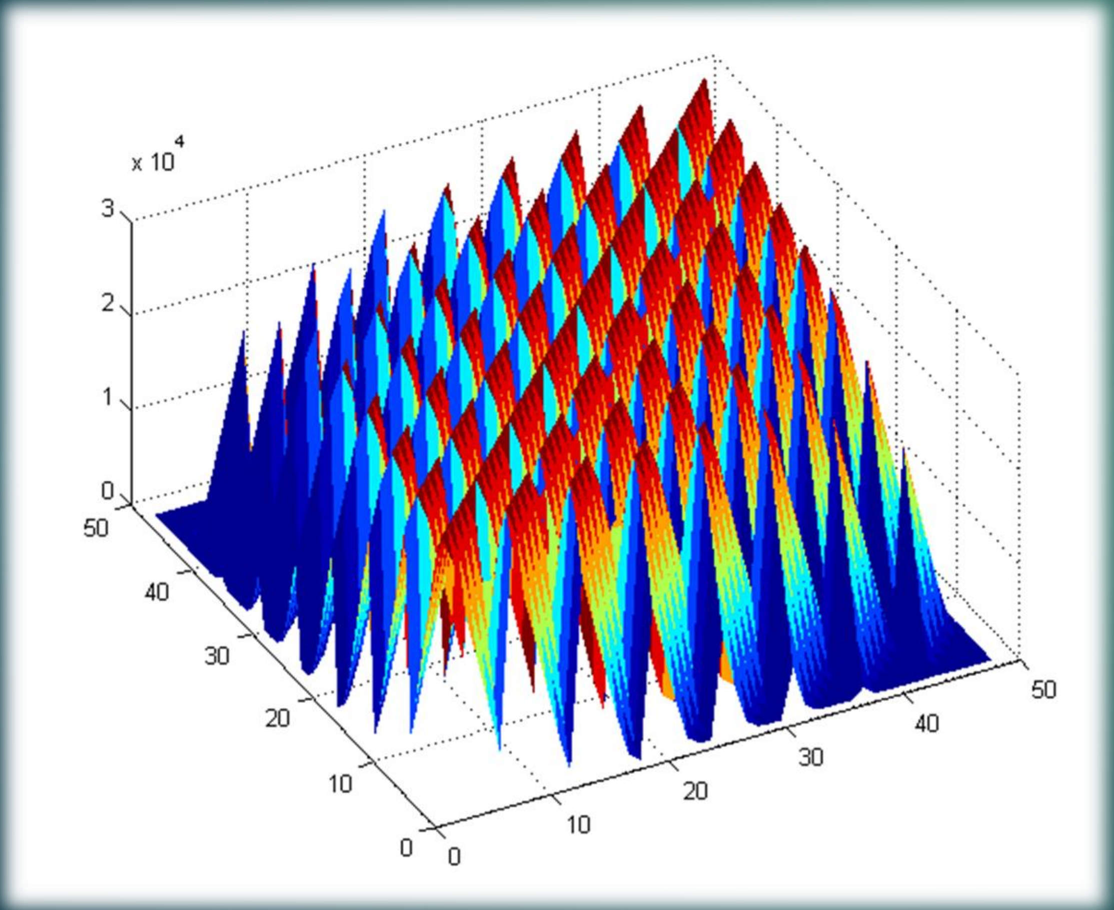
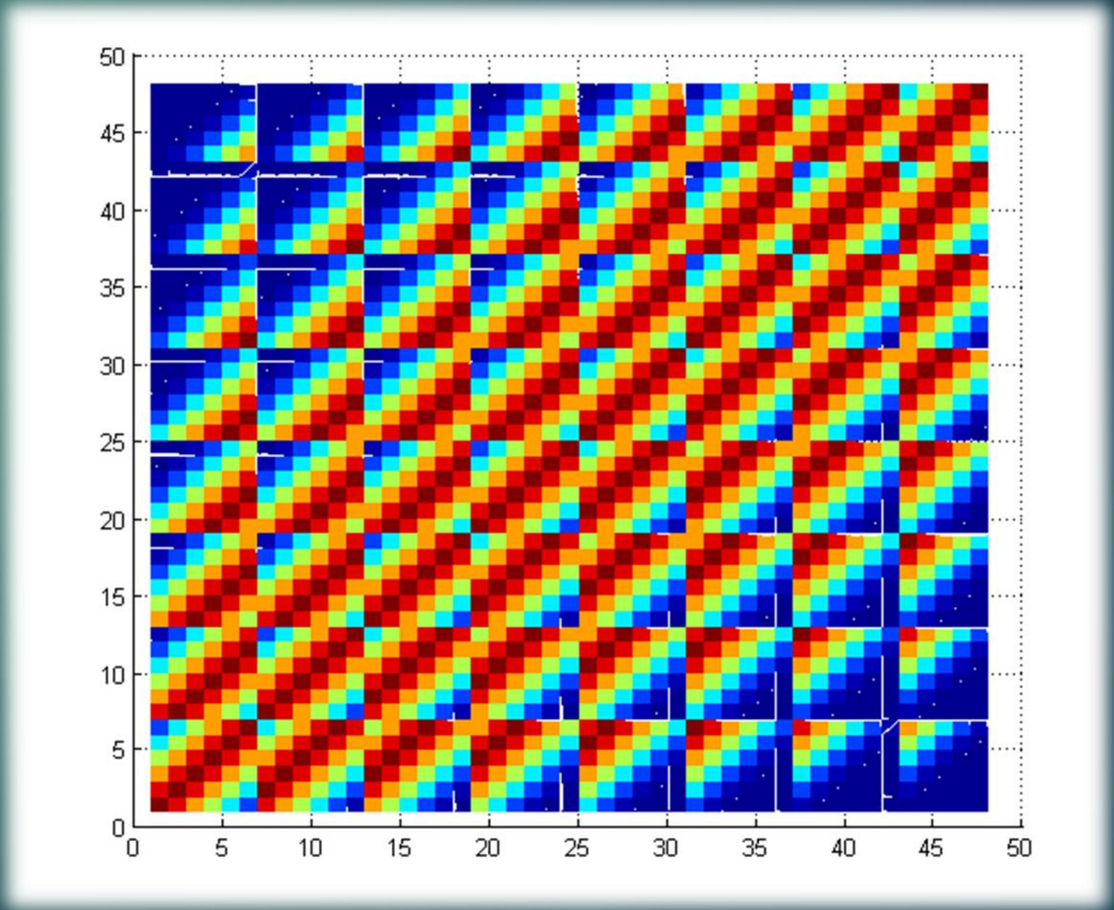
Clutter Signal – Space-Time Covariance Matrix

N = 6, M = 8.

$\mathbf{R}_c$ is 8×8 block matrix where each block is a 6×6 cross-covariance of spatial snapshots.





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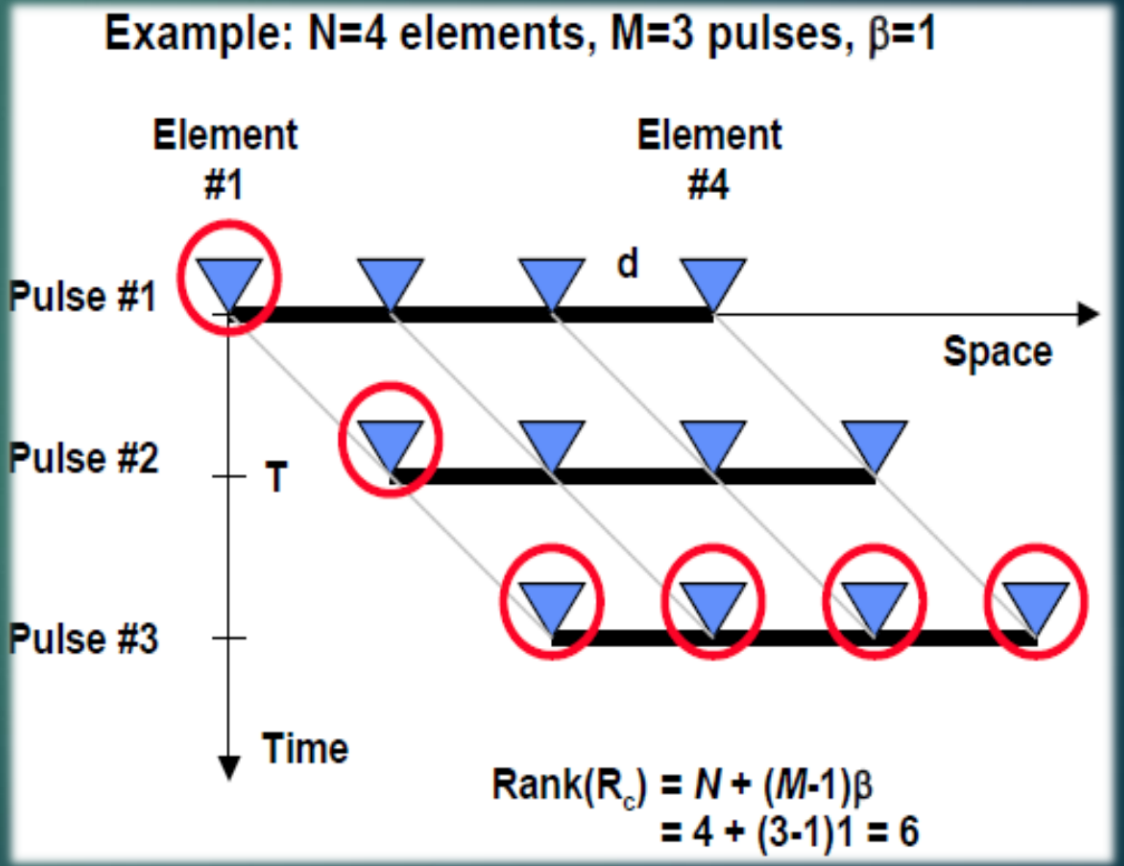
Clutter Signal – Space-Time Covariance Matrix

- The rank of the clutter covariance matrix is important because its an indicator of the severity of the clutter scenario and therefore determines the necessary degrees of freedom required to produce effective clutter cancelation.
- Brennan & Staudaher [4] proved:

$$r_c = \text{rank}(\mathbf{R}_c) = \text{round}[N + (M-1)\beta]$$

- Clutter observations are repeated by different elements on different pulses as the platform moves.
- <u>Only independent observations contribute to the clutter covariance matrix rank.</u>



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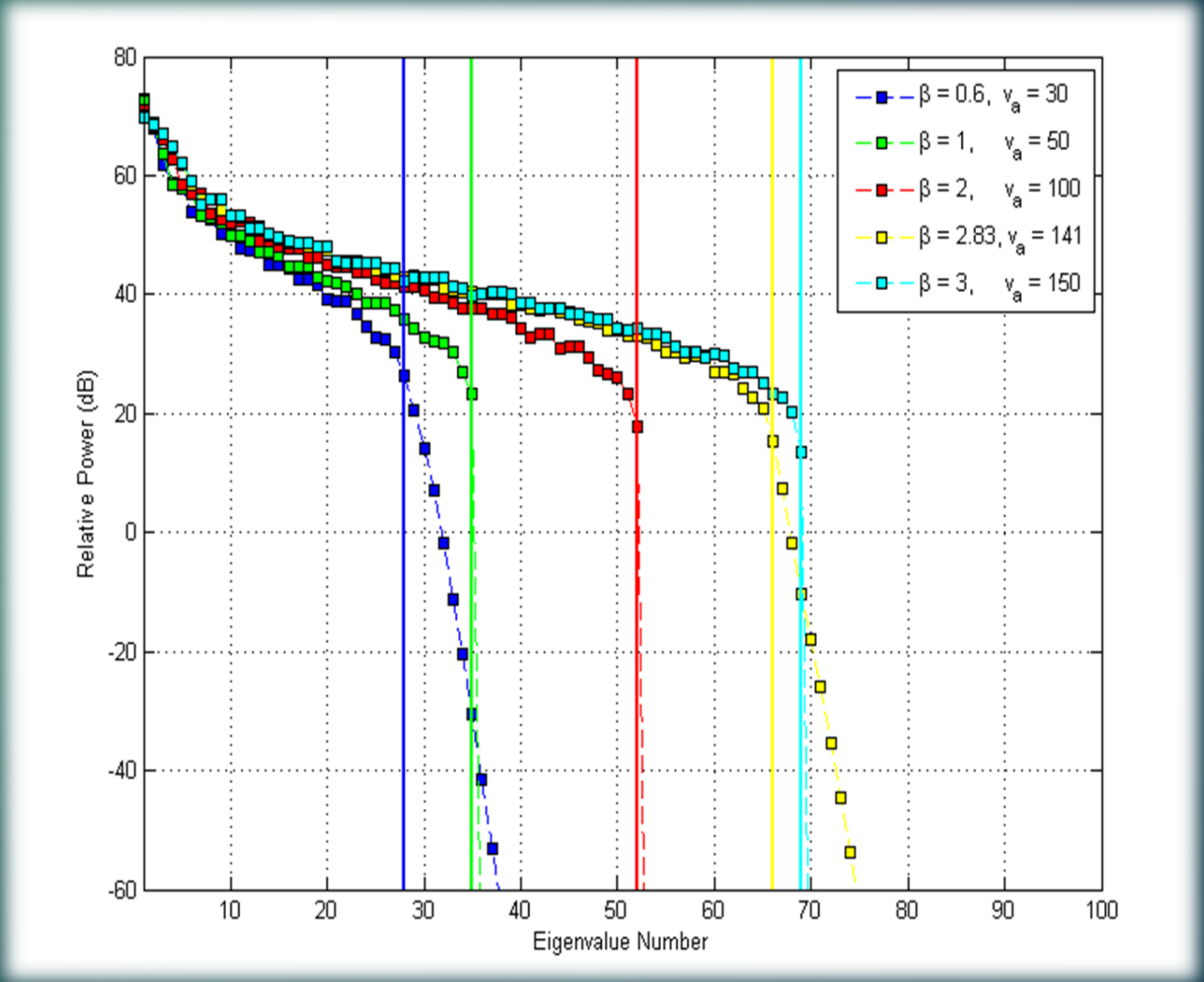
Clutter Signal – Space-Time Covariance Matrix

- Equivalently, by eigenvalue decomposition we may write:

$$\mathbf{R}_c = \mathbf{E}_c \boldsymbol{\Lambda}_c \mathbf{E}_c^H = \sum_{i=1}^{NM} \lambda_i \mathbf{e}_i \mathbf{e}_i^H$$

- $\boldsymbol{\Lambda}_c = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{MN})$ is a diagonal matrix of the eigenvalues $\{\lambda_i\}$. According to Brennan's rule only the first r_c eigenvalues are nonzero.
- When β is an integer, the eigenspectrum exhibits a sharp cutoff, as the covariance matrix is singular.
- $r_c = \mathrm{rank}(\mathbf{R}_c) = \mathrm{round}[N + (M-1)\beta]$
- The eigenspectrum shows how large the clutter subspace is [5].



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Clutter Signal – Velocity Misalignment

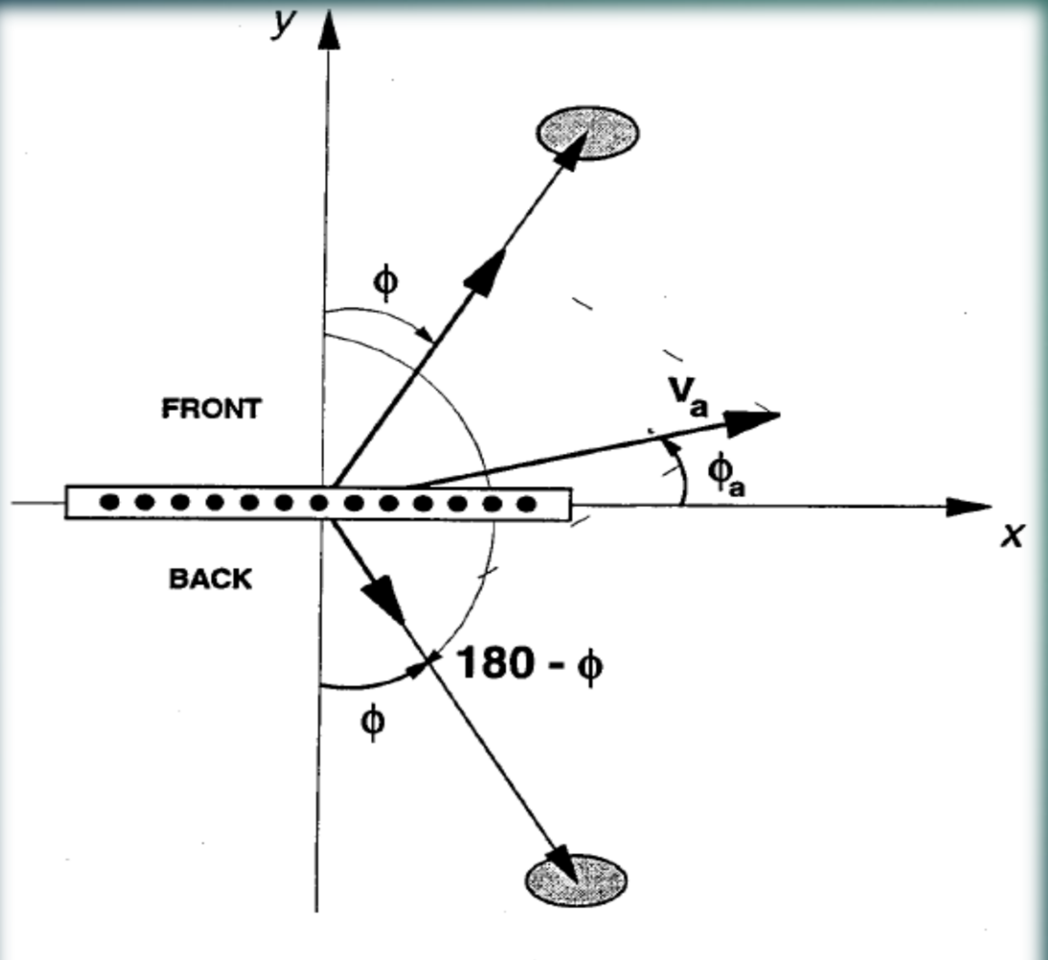
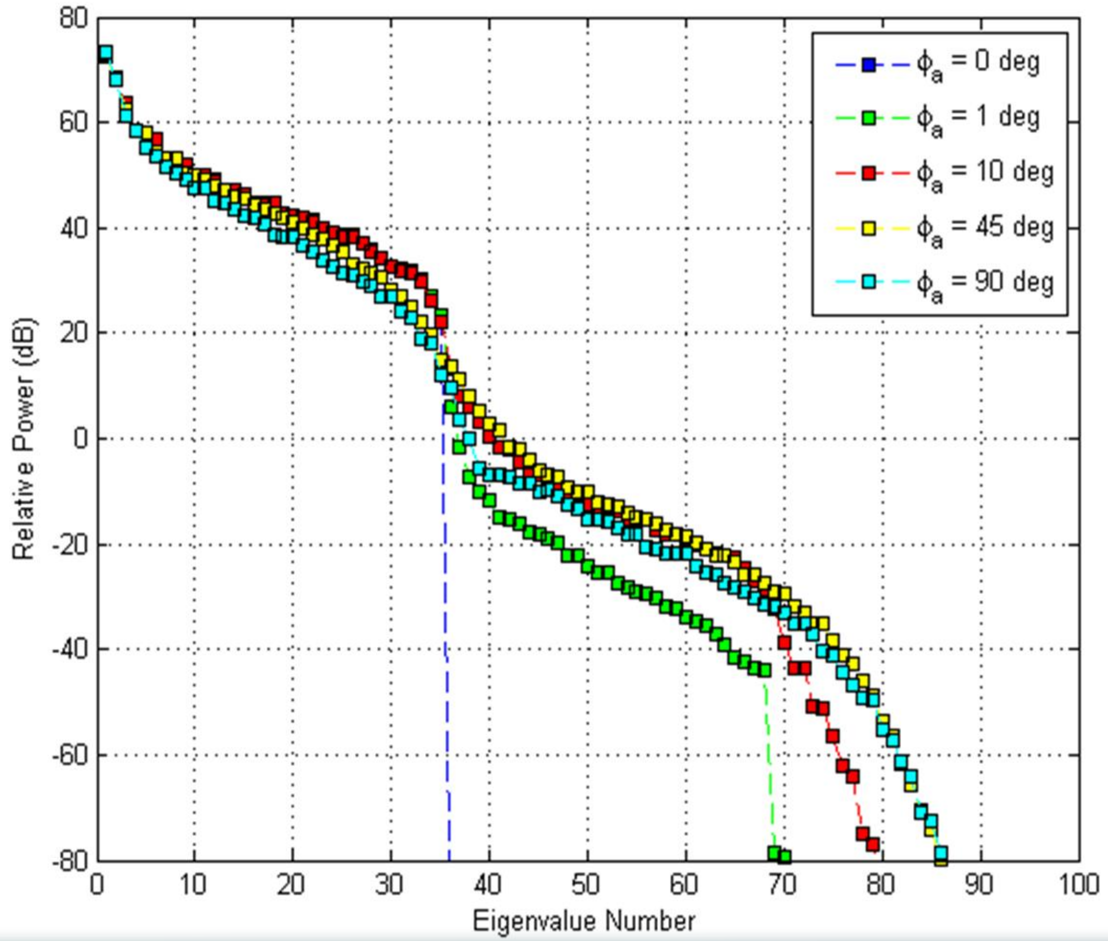


Figure 12. Array geometry with velocity misalignment.



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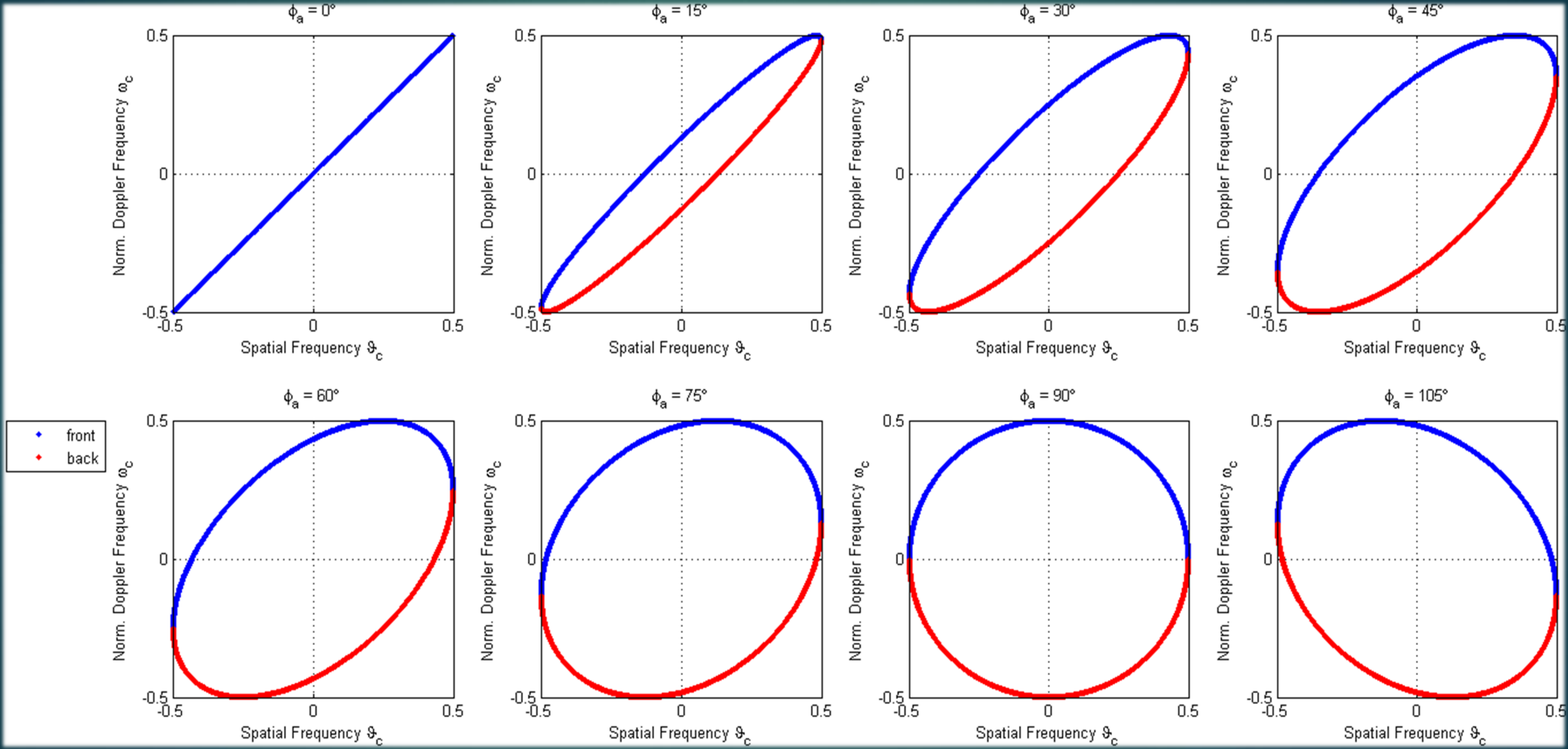
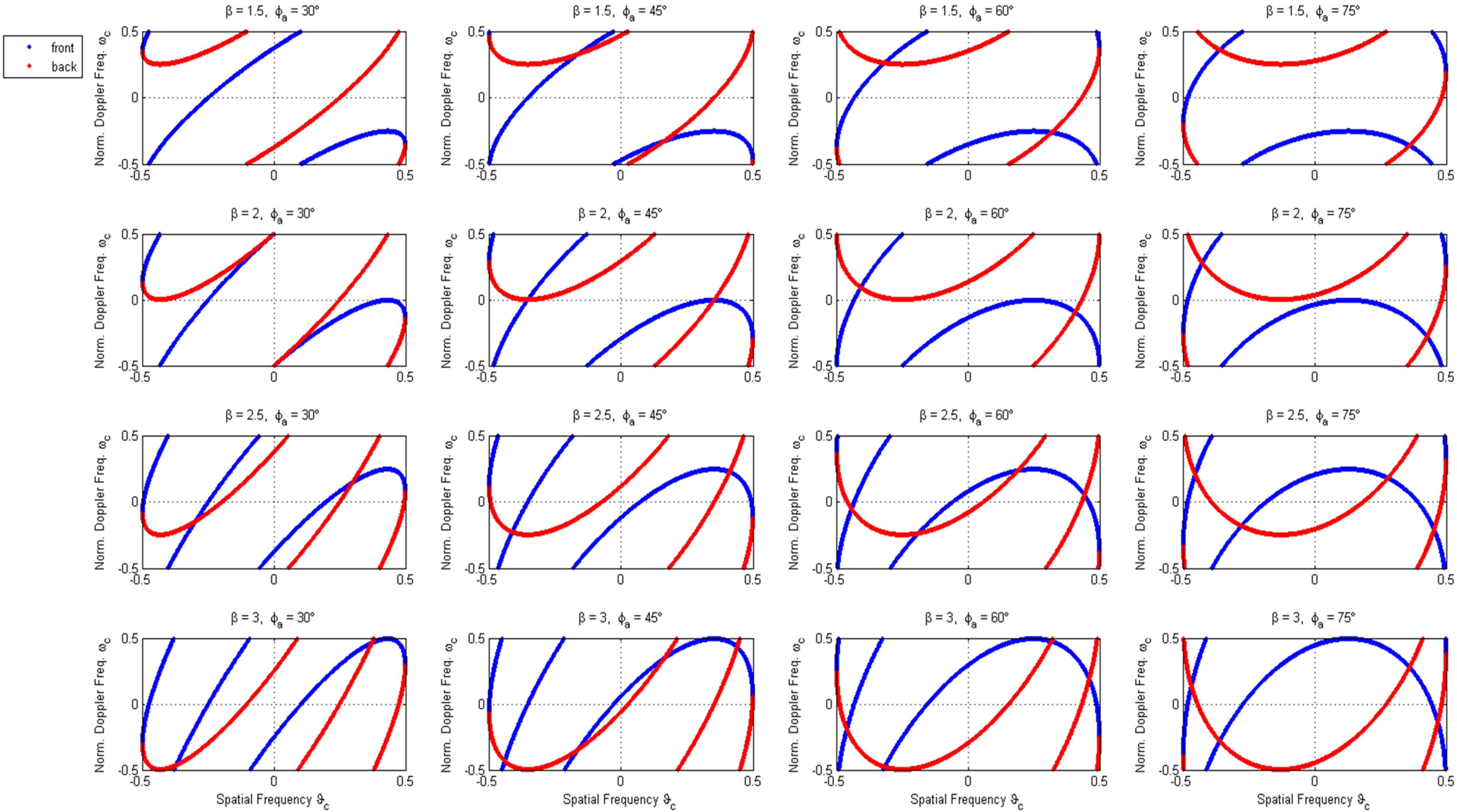
Clutter Signal – Velocity Misalignment – β=1Case.

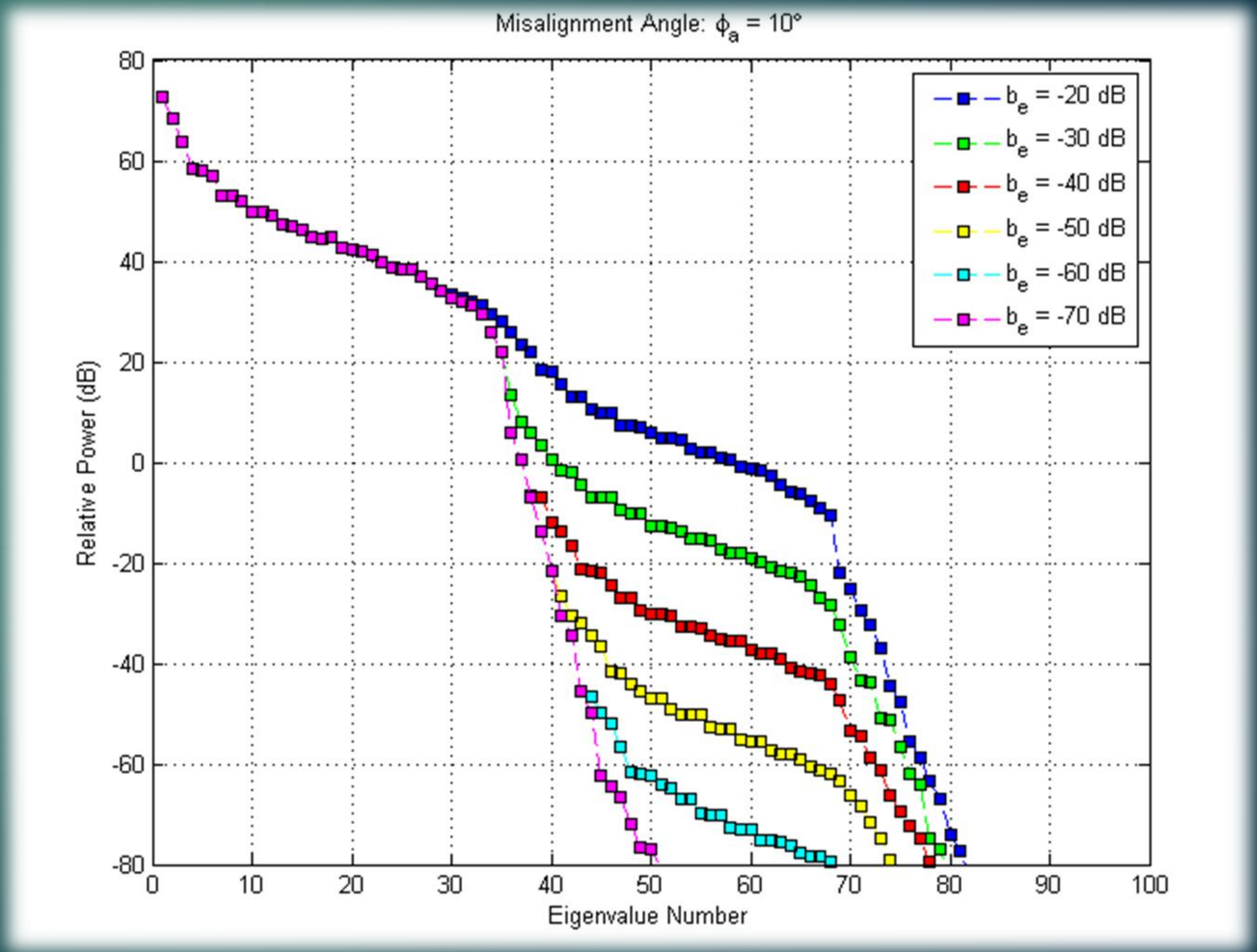




front
back
β = 1.5, ϕa = 30°
β = 1.5, ϕa = 45°
β = 1.5, ϕa = 60°
β = 1.5, ϕa = 75°
β = 2, ϕa = 30°
β = 2, ϕa = 45°
β = 2, ϕa = 60°
β = 2, ϕa = 75°
β = 2.5, ϕa = 30°
β = 2.5, ϕa = 45°
β = 2.5, ϕa = 60°
β = 2.5, ϕa = 75°
β = 3, ϕa = 30°
β = 3, ϕa = 45°
β = 3, ϕa = 60°
β = 3, ϕa = 75°
Norm. Doppler Freq. ωc
Spatial Frequency ϑc
-0.5
0
0.5

Array Radar Signal Environment
Clutter Signal – Backlobe Power Level

Clutter Eigenspectra with misalignment angle $\varphi_{\mathrm{a}} = 10°$ for different array element backlobe power levels.



Array Radar Signal Environment
Clutter Signal – Intrinsic Clutter Motion (ICM)

- Natural variation in clutter reflectivity may occur with land clutter due to wind and with sea clutter due to motion of the ocean waves.

- Any source of fluctuation causes a broadening of the Doppler spectrum of a single clutter echo.

- The pulse-to-pulse fluctuations due to any of these sources is termed as Intrinsic Clutter Motion (ICM).

- The presence of ICM requires a wider clutter cancellation notch or equivalently more adaptive degrees of freedom for effective cancellation.

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Clutter Signal – Intrinsic Clutter Motion (ICM)

- We already have modeled the echo from the *ik*-th clutter patch:
- $\boldsymbol{\chi}_{c_{ik}} = \alpha_{ik}\mathbf{v}_{ik} = \alpha_{ik}\mathbf{b}(\varpi_{ik})\otimes\mathbf{a}(\vartheta_{ik})$
- Temporal fluctuation is represented by replacing the single common amplitude α_{ik} by the *M*×1 vector:
- $\boldsymbol{\alpha}_{ik} = [\alpha_{ik,0}, \alpha_{ik,1}, \ldots \alpha_{ik,M-1}]^T$
- $\alpha_{ik,m}$ is the random amplitude of the *ik*-th clutter patch at the *m*-th pulse.

- As a result, the space-time clutter snapshot including ICM is:

$$\boldsymbol{\chi}_{c_{ik}} = (\boldsymbol{\alpha}_{ik}\odot\mathbf{b}(\varpi_{ik}))\otimes\mathbf{a}(\vartheta_{ik})$$

- ICM is modeled as a WSS random process with a Gaussian Doppler spectrum.
- The temporal autocorrelation of the fluctuation is also Gaussian:

$$\gamma(m) = E\{\alpha_{ik,l+m}\alpha_{ik,l}^{*}\} = \exp\left\{-\frac{\kappa_c^2 T_r^2}{2} m^2\right\}$$

- The spectral standard deviation κ_c is expressed in terms of a velocity standard deviation σ_v :

$$\kappa_c = \frac{4\pi}{\lambda_0}\sigma_v$$

which is an experimentally measured quantity.

Array Radar Signal Environment
Clutter Signal with ICM – Space-Time Covariance Matrix

- The space-time covariance matrix due to the *ik-th* clutter patch including ICM is:

$$\mathbf{R}_{c,ik} = \sigma^2 \xi_{ik} (\boldsymbol{\Gamma}_{ik} \odot \mathbf{b}_{ik} \mathbf{b}_{ik}^H) \otimes \mathbf{a}_{ik} \mathbf{a}_{ik}^H$$

where

$$\boldsymbol{\Gamma}_{ik} = E\left\{\boldsymbol{\alpha}_{ik} \boldsymbol{\alpha}_{ik}^H\right\} = \mathrm{Toeplitz}\{\gamma(0), \gamma(1), \dots, \gamma(M-1)\}$$

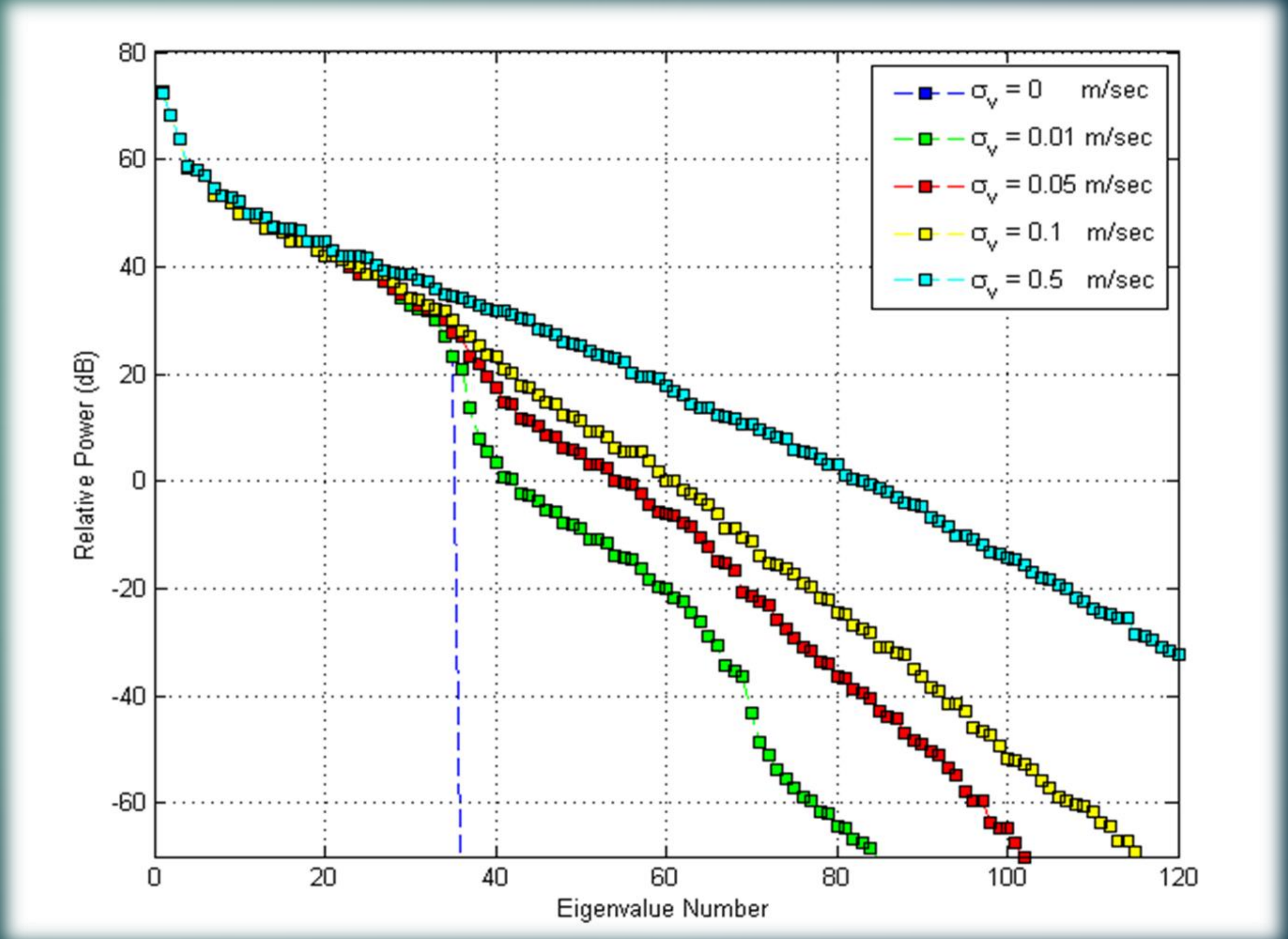
Is the M×M covariance matrix of temporal fluctuation for the *ik*-th clutter patch.

- Finally, by use of superposition:

$$\mathbf{R}_c = \sigma^2 \sum_{k=1}^{N_c} \xi_k (\boldsymbol{\Gamma}_k \odot \mathbf{b}_k \mathbf{b}_k^H) \otimes \mathbf{a}_k \mathbf{a}_k^H$$

Array Radar Signal Environment
Clutter Signal with ICM – Space-Time Covariance Matrix

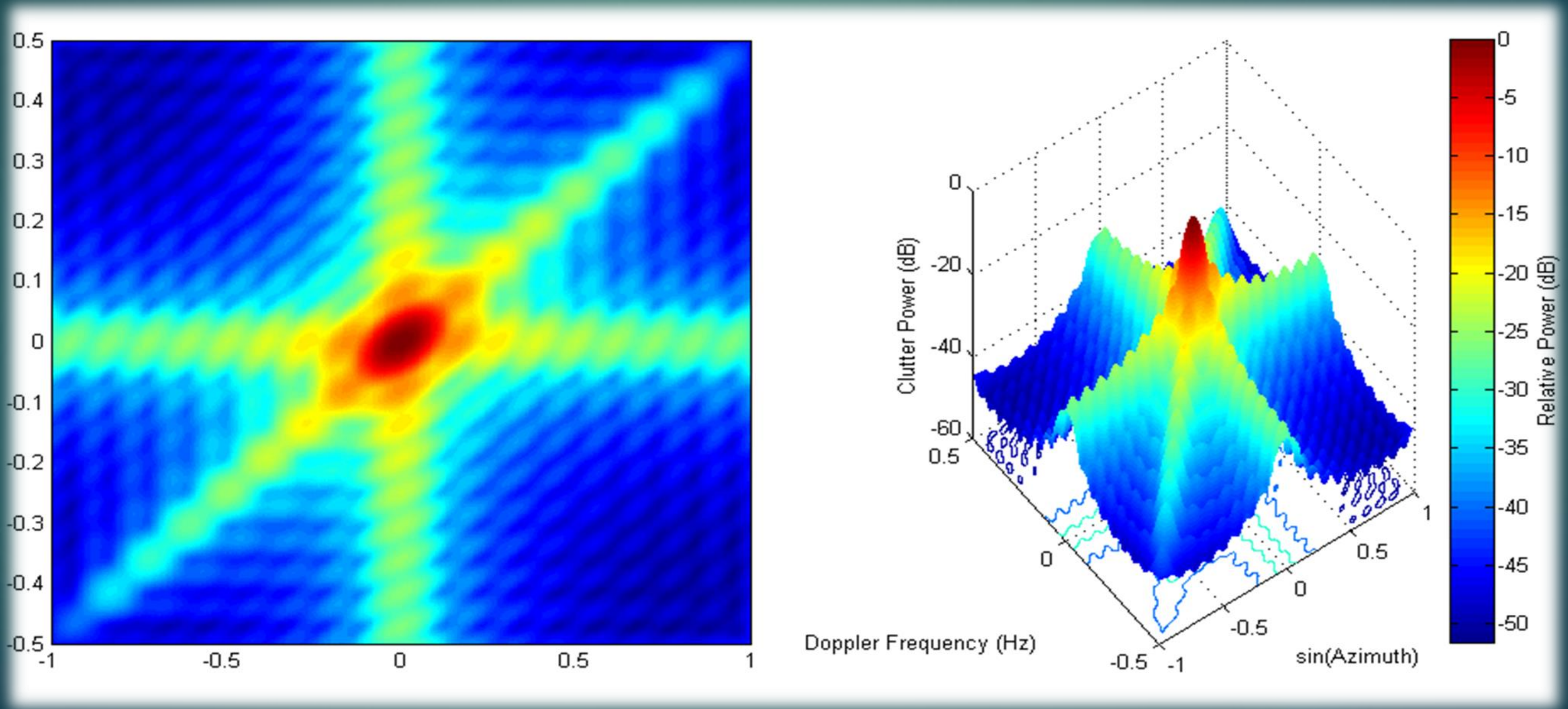
- Effect of ICM on the clutter eigen-spectrum. Zero misalignment and β=1.
- As σ_v increases, the tails of the eigen-spectrum become larger as the rank of the covariance matrix increases.



Array Radar Signal Environment

Clutter Signal – Fourier Power Spectrum - SLAR

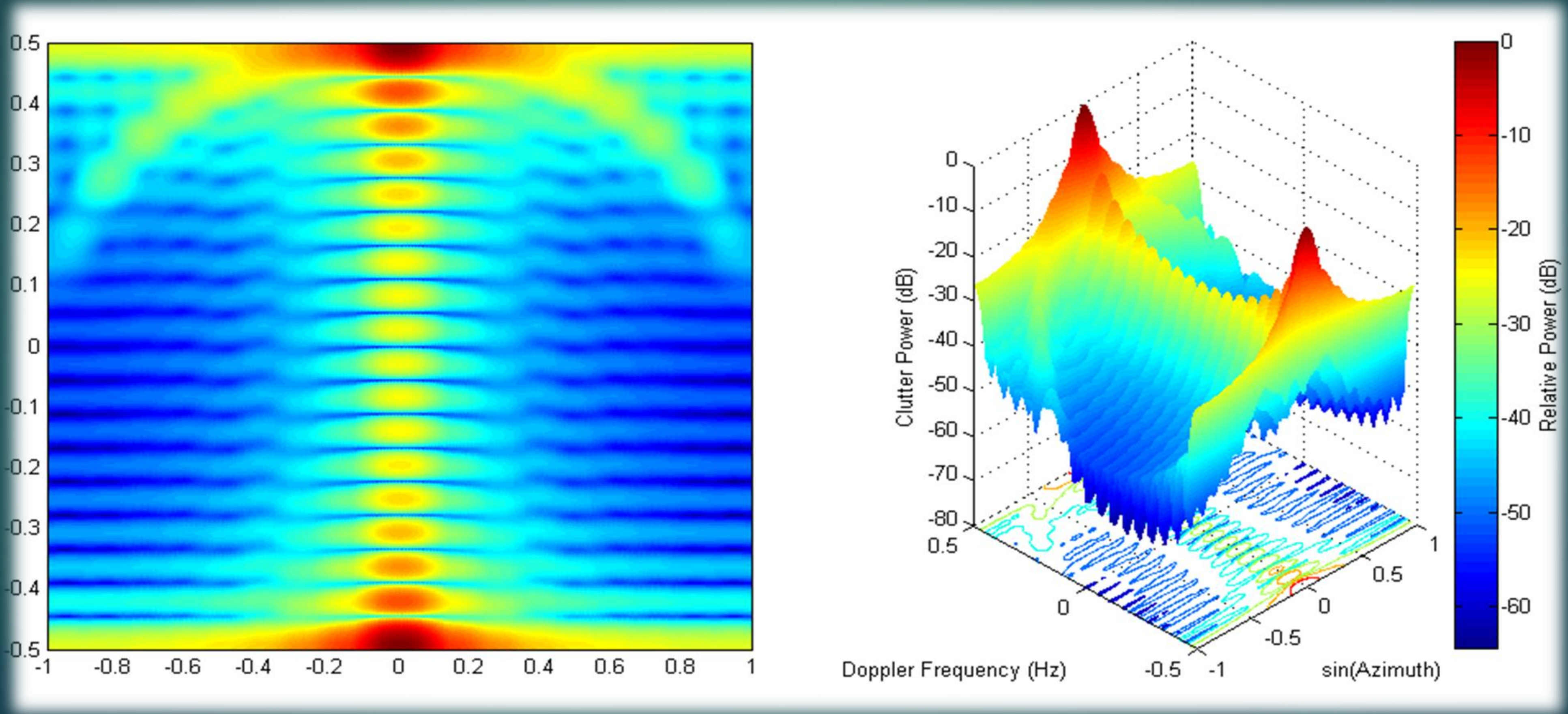
$$P_{MF}(\mathbf{v}) = \frac{\mathbf{v}^H(\varpi,\vartheta)\mathbf{R}_c\mathbf{v}(\varpi,\vartheta)}{\mathbf{v}^H(\varpi,\vartheta)\mathbf{v}(\varpi,\vartheta)}$$



Array Radar Signal Environment

Clutter Signal – Fourier Power Spectrum - FLAR

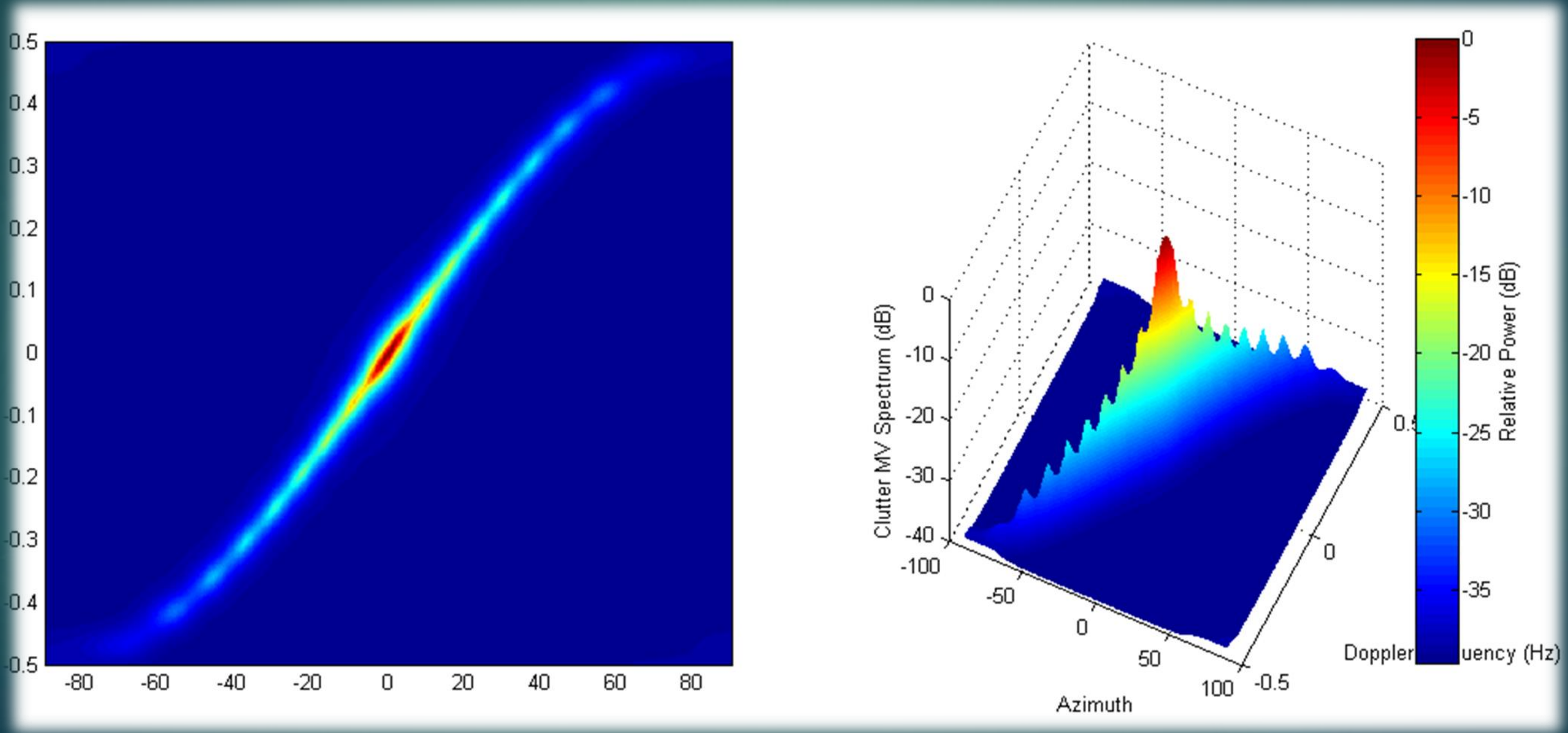
$$P_{MF}(\mathbf{v}) = \frac{\mathbf{v}^H(\varpi,\vartheta)\mathbf{R_c}\mathbf{v}(\varpi,\vartheta)}{\mathbf{v}^H(\varpi,\vartheta)\mathbf{v}(\varpi,\vartheta)}$$



Array Radar Signal Environment

Clutter Signal – Minimum Variance Spectrum - SLAR

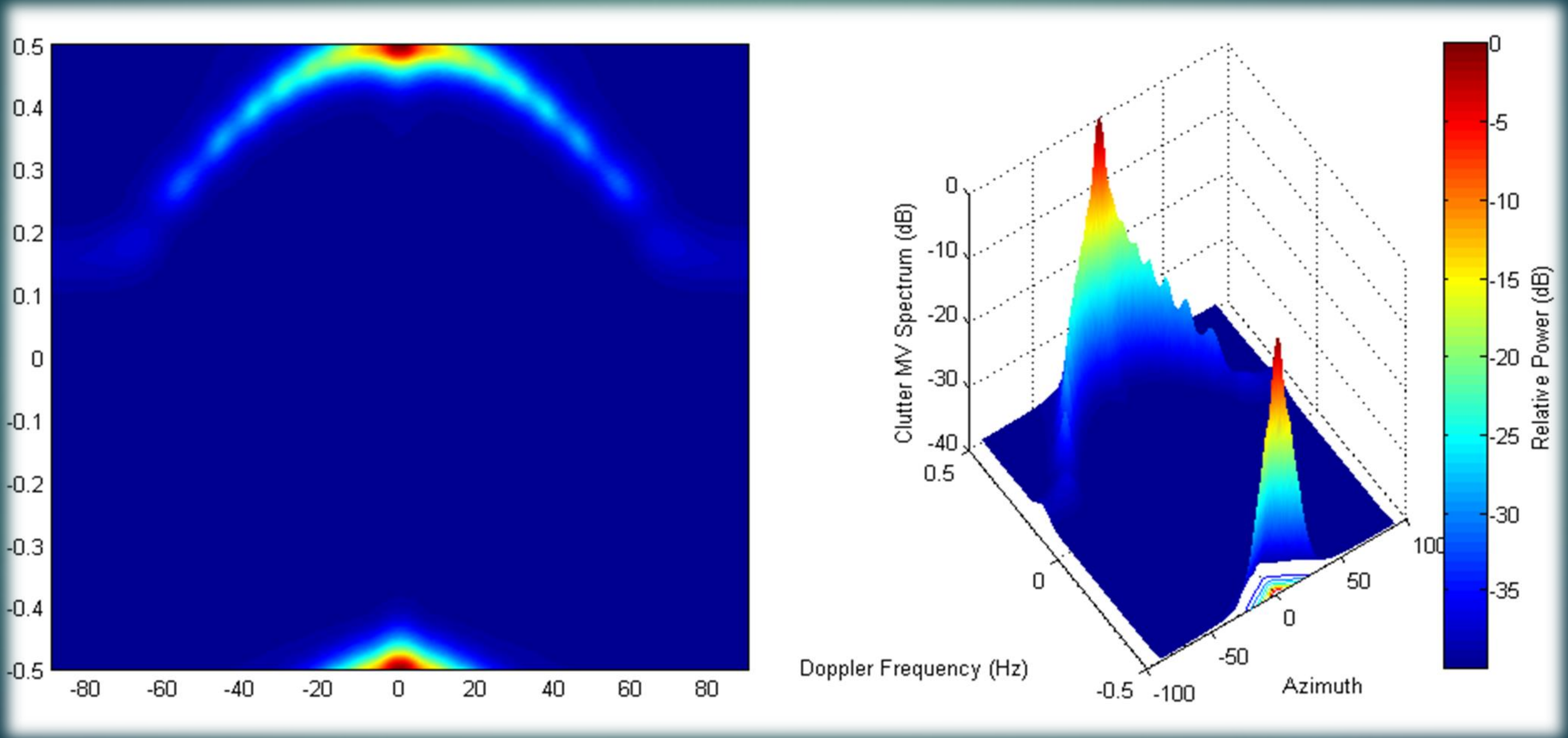
$$P_{MV}(\mathbf{v}) = \left(\mathbf{v}^{\boldsymbol{H}}(\varpi,\vartheta)\mathbf{R}_{c}^{-1}\mathbf{v}(\varpi,\vartheta)\right)^{-1}$$



Array Radar Signal Environment

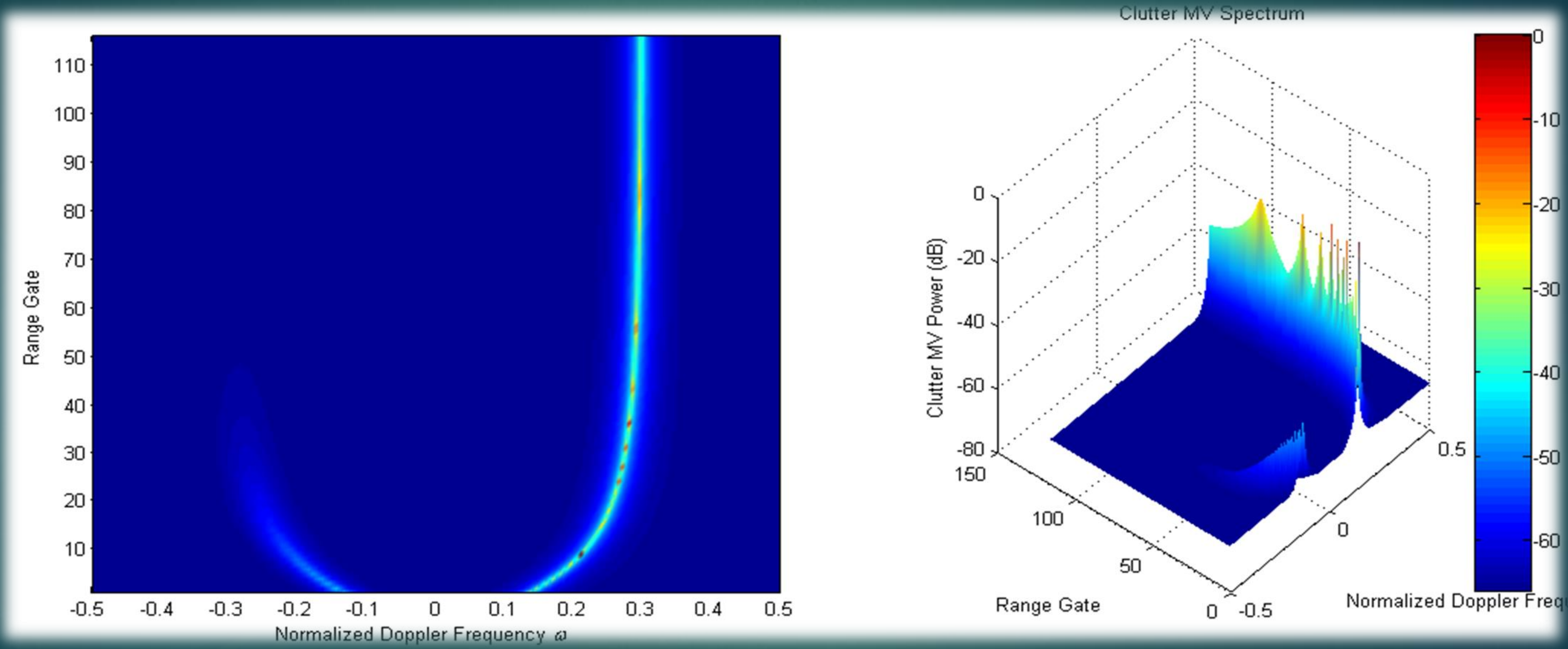
Clutter Signal – Minimum Variance Spectrum - FLAR

$$P_{MV}(\mathbf{v}) = \left(\mathbf{v}^H(\varpi,\vartheta)\mathbf{R}_c^{-1}\mathbf{v}(\varpi,\vartheta)\right)^{-1}$$



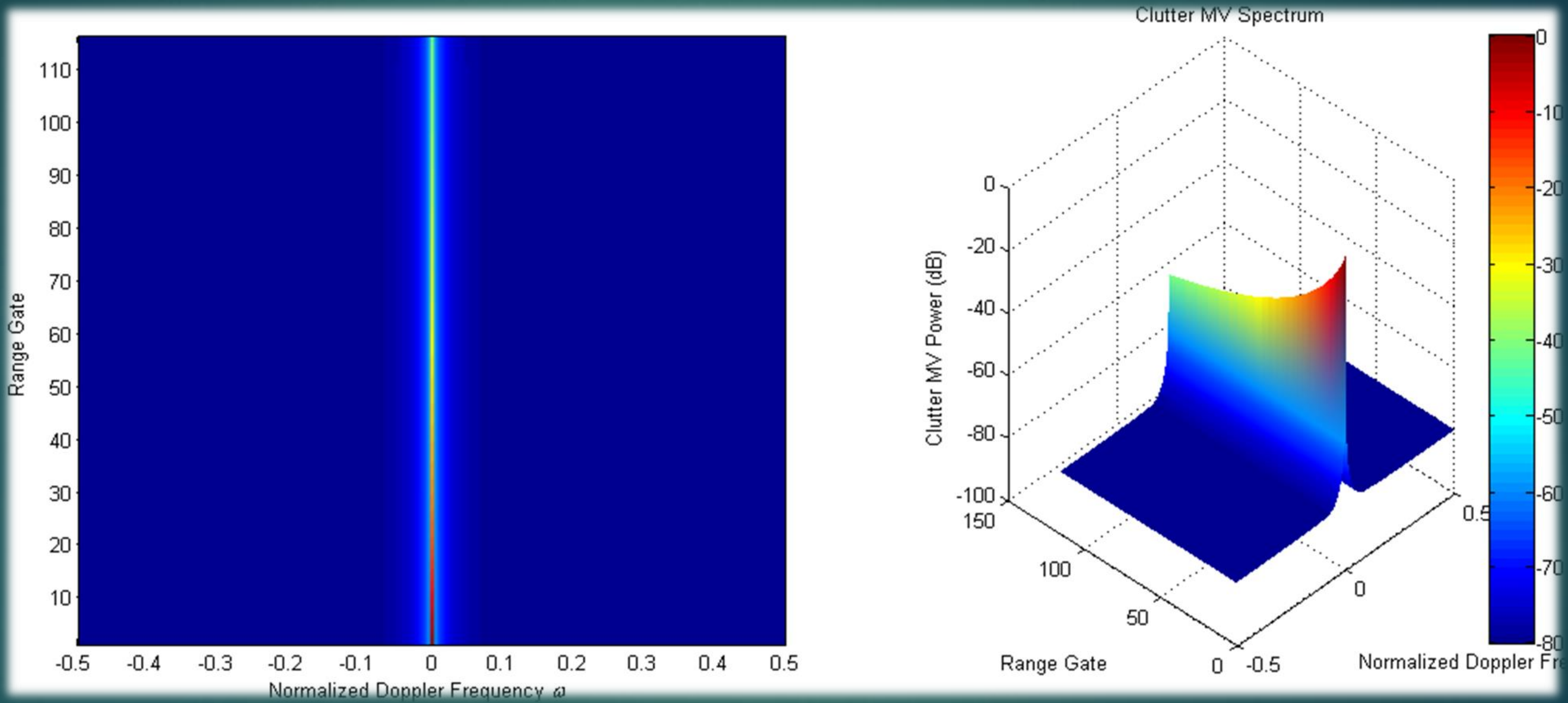
Array Radar Signal Environment

Clutter Signal – Range Dependence - FLAR



Plotting clutter Power versus Doppler and Range results in the Range-Doppler Matrix. For FLAR we notice a certain dependency of the clutter Doppler with range, especially at short range [5].

Array Radar Signal Environment
Clutter Signal – Range Dependence - SLAR



For SLAR the clutter trajectory is a straight vertical line which corresponds to the fact that the clutter Doppler is range-independent [5].

Signal Modeling Summary

- Space-Time Data Snapshot:

$$\boldsymbol{\chi} = \alpha_t \mathbf{v}_t + \boldsymbol{\chi}_u$$

- Undesired Signal Component:

$$\boldsymbol{\chi}_u = \boldsymbol{\chi}_n + \boldsymbol{\chi}_j + \boldsymbol{\chi}_c$$

- Interference-plus-Noise Covariance Matrix:

$$\mathbf{R}_u = E\{\boldsymbol{\chi}_u \boldsymbol{\chi}_u^H\} = \mathbf{R}_c + \mathbf{R}_j + \mathbf{R}_n$$

Where:

$$\mathbf{R}_c = \sigma^2 \sum_{k=1}^{N_c} \xi_k (\boldsymbol{\Gamma}_k \odot \mathbf{b}_k \mathbf{b}_k^H) \otimes \mathbf{a}_k \mathbf{a}_k^H$$

$$\mathbf{R}_j = \mathbf{I}_M \otimes \left(\mathbf{A}_j \boldsymbol{\Xi}_j \mathbf{A}_j^H\right)$$

$$\mathbf{R}_n = \sigma^2 \mathbf{I}_{MN}$$

Array Radar Signal Environment

Clutter Signal – Doppler Bandwidth Widening Factors

- Finite System Bandwidth – Clutter Signal Decorrelation across the array – Apply broadband STAP.
- Range Gate Migration or "Range Walk" causes temporal decorrelation of clutter returns due to platform motion.
- Doppler Spread within a Range Gate for low range resolution systems.
- Doppler and Range ambiguities.
- Due to the finite width of the radar main beam, the Doppler frequency of the main beam clutter extends over an interval of frequencies.

References

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[3] Klemm R., “Principles of Space-Time Adaptive Processing”, 3rd Ed., IET Radar Sonar and Navigation Series 21, London, UK, 2006.

[4] Brennan L.E. and Staudaher F.M., "Subclutter Visibility Demonstration," Technical Report RL-TR-92-21, Adaptive Sensors Incorporated, March, 1992.

[5] Klemm R., “Doppler Properties of Airborne Clutter”, RTO-EN-027, RTO LECTURE SERIES 228, Military Application of Space-Time Adaptive Processing, 19-20 Sept. 2002, Wachtberg, Germany.