

Monte Carlo Realisation of A Distributed Multi-Object Fusion Algorithm

Murat Üney ^{#1}, Simon Julier ^{*2}, Daniel Clark ^{†3}, Branko Ristić ^{‡4},

[#] *Department of Electrical, Electronic & Computer Eng., School of Eng. & Physical Sciences, Heriot-Watt University
Riccarton, Edinburgh EH14 4AS, UK*

¹m.oney@hw.ac.uk

^{*} *University College London
Malet Place, London WC1E 6BT, UK*

²s.julier@cs.ucl.ac.uk

[†] *Joint Research Institute in Signal and Image Processing, Heriot-Watt University
Riccarton, Edinburgh EH14 4AS, UK*

³d.e.clark@hw.ac.uk

[‡] *ISR Division, DSTO*

PO Box 1500, Edinburgh SA 51 11, Australia

⁴branko.ristic@dsto.defence.gov.au

Abstract—We consider the problem of distributed target tracking in a multi-object, multi-sensor scenario in which the structure of the joint distribution of the estimate between different nodes is unknown. In this paper we present a preliminary implementation of Generalised Covariance Intersection (GCI) fusion rule for multi-object posteriors through a Monte Carlo realisation. We discuss the subtleties in the case of multi-object distributions and derive a scheme for sampling from Exponential Mixture Densities which are at the heart of the GCI. We demonstrate the improvement in localisation of multiple targets in a simulation scenario.

I. INTRODUCTION

Distributed sensor networks are expected to revolutionise military operations. By distributing information collection and processing throughout a network of heterogeneous, geographically-dispersed information processing nodes, it is anticipated that robust, scalable tracking and estimation algorithms can be developed. However, there are several challenges associated with realising such systems. We focus on two problems: *multi-object tracking*, and *distributed data fusion*.

Many tracking problems inherently involve multiple objects. Objects of military relevance rarely occur in isolation; they are surrounded by other objects either of military or non-military significance. Therefore, the tracking system must be able to account for and track the states of each of these objects. The difficulties are exacerbated by many sensing systems which are noisy, return cluttered measurements, and may fail to detect targets. With careful tuning, approaches such as Multiple Hypothesis Tracking (MHT) can be used in relatively benign environments. However, general experience indicates that MHT can be very brittle. These difficulties are compounded in distributed environments, where individual nodes make track initialisation, track merging and track deletion decisions using local information only. This can cause valid tracks to be pruned, and invalid tracks to be maintained. An alternative approach to multi-object tracking is to use a rigorous multi-object multi-sensor detection and classification algorithm provided through the formalism of Finite Set Statistics (FISST) [1]. The success of FISST for multi-sensor multi-object tracking has been demonstrated by the Probability Hypothesis Density (PHD) filter algorithms [2], [3]. These algorithms replace the exponential complexity of data association techniques

with robust, computationally cheaper [4], e.g., cubic-complexity [5], algorithms that are effective in estimating both the correct number of objects and their state vectors in data with high false alarm rates and missed detections.

Distributed data fusion algorithms combine the state estimates that are generated by a number of fusion centres or nodes. However, the estimates from the different nodes are *not* conditionally independent of one another and, if optimal fusion is to occur, common information has to be “cancelled out” [6]. However, in most networks computing this information is prohibitively expensive. An alternative is to use suboptimal fusion techniques such as Covariance Intersection (CI) [7]. However, CI only utilizes the mean and covariance of the estimates and cannot exploit any additional distribution information such as the number of modes. The generalisation of CI to general probability distributions was first proposed by Mahler [8] and independently derived by Hurley [9]. This generalisation replaces the product form of Bayes Rule with an Exponential Mixture Density (EMD—equivalently a weighted geometric mean). Theoretical [10], [11] and practical analysis [12], [13] has demonstrated that this generalisation has a sound theoretical basis. Although Mahler originally proposed the EMD fusion rule within the context of applying FISST in distributed data fusion problems [8], his discussion included no proofs of the validity of the proposed method, no implementation strategy of how such a method could be realised, and no demonstration of the performance of the method.

In a companion paper [14], we have developed the forms of several widely-used multi-object distributions using the exponential mixture density form. Although these forms are compact and easy to write down, there is no guarantee that these lead to algorithms with good computational performance. Therefore, in this paper we consider the problem of developing initial implementations of these algorithms. We use a Monte Carlo approach within the Probabilistic Hypothesis Density (PHD) framework.

The structure of this paper is as follows. In Section II, we describe multi object PHDs and the EMD fusion rule. Section III formalises the problem statement and describes our Monte Carlo-based realisation of the EMD fusion rule. An example illustrating the algorithm is provided in Section IV and conclusions are given in Section V.

II. FINITE SET STATISTICS

In this section we provide an overview of multi-object density functions, their Probability Hypothesis Densities (PHDs) and fusion of multi-object posteriors together with the implications on the corresponding PHD.

A. Multi-object density functions and PHDs

The problem of tracking multiple targets in clutter has traditionally been performed using MHT. However, many years of experience has shown that MHT can be both brittle and computationally expensive. Recent developments based on random finite sets have developed classes of multitarget tracking algorithms which are both robust and operate in linear time.

Within the framework of random finite sets, the state is represented by the random set variable $X = \{x_1, \dots, x_N\}$. Both the cardinality of the set, N , and the values of the individual random variables x_i are random and x_i takes values from the space \mathcal{X} . X is characterized by the *multi-object density* $f(X)$, which leads to the following definition of a set integral:

$$\int f(X)\delta X := f(\emptyset) + \sum_{n=1}^{\infty} \frac{1}{n!} \int f(\{x_1, \dots, x_n\}) dx_1 \dots dx_n. \quad (1)$$

The multi-object density $f(\{x_1, \dots, x_n\})$ is constructed from the joint density $f(x_1, \dots, x_n)$ via the expression

$$f(\{x_1, \dots, x_n\}) := n!p(n)f(x_1, \dots, x_n),$$

where $p(n)$ is the distribution of the cardinality of X and the factorial term is required to take all possible permutations of the unordered elements of X into account¹.

The combinatorial nature of the multi-object densities renders a multi-object extension of the conventional recursive Bayesian filtering in which the posterior is propagated intractable. However, approximation strategies for propagating the first-order moment, known as the Probability Hypothesis Density (PHD), have proven to be very successful [16]². The PHD $D(x)$ of X is a function over the single element state space \mathcal{X} and

$$\int_{\mathcal{S}} D(x)dx = E\{|X \cap \mathcal{S}|\}. \quad (2)$$

where $|\cdot|$ is the set cardinality. In other words, the integral of the PHD over a region \mathcal{S} is the *expected number of targets in \mathcal{S}* [15]. In general, it can be written as some single-object probability distribution $s(x)$ multiplied by the expected number of targets (the expectation of the cardinality distribution, i.e. $E\{n\} = \sum_{n=1}^{\infty} np(x)$) [15].

For any given multi-object distribution, the resulting PHD can be written in the form of some single-object probability distribution $s(x)$ multiplied by the expected number of targets [15], i.e.

$$D(x) = s(x) \cdot \sum_{n=1}^{\infty} n \cdot p(n). \quad (3)$$

B. Exponential Mixture Densities for Multi-Object Posteriors

The proposal that the generalisation of Covariance Intersection is the Exponential Mixture Density was proposed by Mahler specifically to extend FISST to suboptimal distributed environments [8].

¹To be precise, $f(x_1, \dots, x_n)$ is a symmetric function of its arguments (see, e.g., pg.364 of [15], for further details).

²It might be noted that the computational costs of FISST are much *worse* than those for MHT. However, the PHD approximation derived from FISST is both computationally cheaper and better-behaved than MHT.

This generalisation has proved to be extremely valuable for distributed estimation in the single-target case [10]–[13]. Consider two multi-object posteriors, $f_0(X|Z_0^{1:k})$ and $f_1(X|Z_1^{1:k})$, conditioned on measurement set sequences $Z_0^{1:k}$ and $Z_1^{1:k}$ respectively. The measurement sets come from two different sensor suites. The goal is to construct the fused estimate $f(X|Z_0^{1:k} \cup Z_1^{1:k})$. However, Bayesian solution requires the maintenance of the distribution over common information, $f(X|Z_0^{1:k} \cap Z_1^{1:k})$ which is not tractable. Instead, the EMD fusion rule approximates the joint distribution $f(X|Z_0^{1:k} \cup Z_1^{1:k}) \approx f_{\omega}(X|Z_0^{1:k}, Z_1^{1:k})$, where [8]

$$f_{\omega}(X|Z_0^{1:k}, Z_1^{1:k}) = \frac{f_0(X|Z_0^{1:k})^{(1-\omega)} f_1(X|Z_1^{1:k})^{\omega}}{\int f_0(X|Z_0^{1:k})^{(1-\omega)} f_1(X|Z_1^{1:k})^{\omega} \delta X}, \quad (4)$$

where $\omega \in [0, 1]$ is a free variable and can be selected using an information criterion such as minimising the entropy of $f_{\omega}(X|Z_0^{1:k}, Z_1^{1:k})$.

The EMD fusion rule is not practical for more than a few targets and so utilisation of approximation strategies, such as the PHD filter [2], are required. In our accompanying paper on robust fusion of multi-object densities [14], we derived explicit formulae for specific tractable types of multi-object posteriors to fuse. The most general of the cases that was considered was that of independently, identically distributed cluster (i.i.d. cluster) processes. We summarise the form of the PHD of the updated distribution in this scenario.

C. EMD Fusion of IID Cluster Processes

Suppose that we have two posteriors, f_0 and f_1 , that we wish to fuse that are of the form of an i.i.d. cluster process, i.e.,

$$f_0(X) = n! \cdot p_0(n) \prod_{x \in X} s_0(x),$$

$$f_1(X) = n! \cdot p_1(n) \prod_{x \in X} s_1(x),$$

where $s_0(x)$ and $s_1(x)$ are densities on the single-object state-space and $p_0(n)$ and $p_1(n)$ are cardinality distributions in the number of targets. Then the fused first-moment, or PHD, is found with

$$D_{\omega}(x) = s_{\omega}(x) \cdot \sum_{n=1}^{\infty} n \cdot p_{\omega}(n), \quad (5)$$

where the updated i.i.d. location density and cardinality distribution are

$$s_{\omega}(x) = \frac{s_0^{(1-\omega)}(x) s_1^{\omega}(x)}{\int s_0^{(1-\omega)}(y) s_1^{\omega}(y) dy} \quad (6)$$

$$p_{\omega}(n) = \frac{p_0^{(1-\omega)}(n) p_1^{\omega}(n) \left(\int s_0^{(1-\omega)}(x') s_1^{\omega}(x') dx' \right)^n}{\sum_{m=0}^{\infty} p_0^{(1-\omega)}(m) p_1^{\omega}(m) \left(\int s_0^{(1-\omega)}(y) s_1^{\omega}(y) dy \right)^m}. \quad (7)$$

In the next section, we investigate a practical implementation of the Generalised Covariance Intersection using Monte Carlo representations of the intensity functions.

III. MONTE CARLO FUSION OF IID CLUSTER PHDs

A. Problem Definition

Suppose that there are 2 sensors observing a common region and at time k collect noisy measurements Z_0^k and Z_1^k respectively due to multiple targets and clutter with non-zero probability of missing either of the targets. The sensors run local CPHD filters utilizing Sequential MC realizations and produce a particle representation

of the PHD for the multi-object scene together with a cardinality distribution. The posterior PHD from the j th sensor is given by

$$D_j^{k|k}(x|Z_j^{1:k}) \approx \tilde{\mu}_j^{k|k} \tilde{s}_j^{k|k}(x), \quad (8)$$

where $\tilde{\mu}_j^{k|k}$ and $\tilde{s}_j^{k|k}(x)$ are the estimates of the expected number of targets and the localisation density respectively³. The localisation density estimate is represented by using a set of N_j particles of the form

$$\tilde{s}_j(x) = \sum_{i=1}^{N_j} \zeta_j^{(i)} \delta(x - x_j^{(i)}) \quad (9)$$

where $\zeta_j^{(i)}$ is the weight of the particle $x_j^{(i)}$. Note that if the points $x_j^{(i)}$ for $i = 1, \dots, N_j$ are sampled from a proposal distribution $q(x)$, i.e. $x_j^{(i)} \sim q(x)$, then

$$\zeta_j^{(i)} \propto \frac{s_j(x_j^{(i)})}{q(x_j^{(i)})}. \quad (10)$$

Our problem is, having received $\{\zeta_j^{(i)}, x_j^{(i)}\}$ for $j = 0, 1$, to find a particle representation for the EMD given by (6), which is the localisation density component of the fused PHD under the assumption that the multi-object scene admits an i.i.d. cluster model.

B. Particle Representation of an Exponential Mixture Density

The evaluation of (6) is not trivial for a number of reasons. First, given a particle representation of $s_j(x)$, only an approximate evaluation, i.e. $\tilde{s}_j(x)$, is possible for which Kernel Density Estimation (KDE) methods are employed [17]. A KDE approximation is given by

$$\tilde{s}_j(x) = \frac{1}{V_h S(N_j)} \sum_{i=1}^{N_j} \zeta_j^{(i)} K\left(\frac{x - x_j^{(i)}}{h}\right) \quad (11)$$

where $K(\cdot)$ is a Kernel function (often selected from the class of radial basis functions), h is the scale factor or bandwidth (BW) and V_h is the volume of K which depends also on h . $S(N_j)$ is a normalisation constant.

Given $\tilde{s}_j(x)$, another difficulty is that it is computationally infeasible to evaluate $\tilde{s}_\omega(x)$ on a conventional grid due to the curse of dimensionality. In addition, for the case in which further processing based on the EMD is needed, e.g., in order to proceed fusion by introducing PHDs output by additional sensors, a particle representation $\{\zeta_\omega^{(i)}, x_\omega^{(i)}\}$ is preferable to approximate evaluations of $s_\omega(x)$.

C. Particle Representation of the EMD Fusion Rule

For the particle set to represent $s_\omega(x)$, a reasonable choice is the *union* of the already received sets of particles, i.e.

$$P_\omega = P_0 \cup P_1 \quad (12)$$

where

$$\begin{aligned} P_0 &= \{x_0^{(1)}, x_0^{(2)}, \dots, x_0^{(N_0)}\} \\ P_1 &= \{x_1^{(1)}, x_1^{(2)}, \dots, x_1^{(N_1)}\} \end{aligned}$$

Therefore we seek $\zeta_\omega^{(k)}$ for $x^{(k)} \in P_\omega$ such that for an arbitrary density $q(x)$, if $x^{(k)} \sim q(x)$, then $\zeta_\omega^{(k)}$ satisfies the relation given by (10), which specialises to

$$\zeta_\omega^{(k)} \propto \frac{s_0^{1-\omega}(x^{(k)}) s_1^\omega(x^{(k)})}{q(x^{(k)})}. \quad (13)$$

³For the rest of the paper, we drop the superscripts $k|k$ and omit the conditioning notation for posteriors where they can be identified from the context.

Consider all samples $x^{(i)} \in P_0$ which are generated from some distribution $q(x)$. Then, considering (10), it holds that $q(x^{(i)}) \propto s_0(x^{(i)})/\zeta_0^{(i)}$ for $i = 1, \dots, N_0$ which after substituting in the right hand side (RHS) of (13) yields

$$\zeta_\omega^{(i)} \propto \zeta_0^{(i)} s_0^{-\omega}(x^{(i)}) s_1^\omega(x^{(i)}) \quad (14)$$

enabling us to utilize the RHS to compute weights for representing $s_\omega(x)$ with the particle set P_0 .

Similarly, for $x^{(i)} \in P_1$, the weight $\zeta_\omega^{(i)}$ is obtained after substituting the relation $q(x^{(i)}) \propto s_0(x^{(i)})/\zeta_0^{(i)}$ in (13) as

$$\zeta_\omega^{(i)} \propto \zeta_1^{(i)} s_0^{1-\omega}(x^{(i)}) s_1^{\omega-1}(x^{(i)}) \quad (15)$$

Hence, for the particles in P_0 and P_1 the RHSs of (14) and (15) respectively provide weights for the particle representation of the EMD $s_\omega(x)$. For our case, since the evaluation of $s_j(x)$ is possible only through its KDE given by (11), we substitute $\tilde{s}_j(x)$ in place of $s_j(x)$ in (14) and (15).

After obtaining $\{\zeta_\omega^{(k)}, x^{(k)} | x^{(k)} \in P_\omega\}$ it is also possible to obtain an equal weighted representation after resampling. Let us denote these set of particles, i.e. the elements of P_ω selected through resampling based on $\zeta_\omega^{(k)}$ s, by $P_\omega^\#$. Note that, $P_\omega^\# \subseteq P_\omega$.

For $\omega = 0$, it is expected that most of the elements of $P_\omega^\#$, would be from P_0 . In other words,

$$|P_\omega^\# \cap P_0| \gg |P_\omega^\# \cap P_1| \quad (16)$$

Symmetrically, for $\omega = 1$, most of the elements of $P_\omega^\#$ are expected to be from P_1 , i.e.

$$|P_\omega^\# \cap P_1| \gg |P_\omega^\# \cap P_0|$$

This is supported through experiments with EMDs of Gaussian mixtures which are not presented in this paper due to space limitations.

However, this has not been our experience, in the case of the localisation densities of PHDs. It might be argued that the problem is due to the inaccuracy of the KDE approximation possibly related to the selection of the BW h , which has a major influence on the behaviour of the approximation [17]. Given a particle representation, there is a number of methods for finding h including the rules of thumb (RUT) schemes which yield straightforward computations and least square cross validation (LCV) which is an iterative scheme requiring more resources [18]. In Fig. 1(a), equal weighted particles (particles after resampling) representing the localisation component of posterior PHDs output by two CPHD filters (i.e., $s_0(x)$ and $s_1(x)$ respectively) are presented. For $\omega = 0$, after calculating the weights given by (14) and (15) based on the KDEs with RUT, the particle set P_ω is resampled and $P_\omega^\#$ is obtained (magenta particles in Fig. 1(b)).

Note that, for the case, $P_\omega^\#$ should represent $s_0(x)$, and have a significantly higher number of common elements with P_0 compared to P_1 . However, besides the particles from P_0 , a large number of particles from P_1 are *leaked* which is best seen in Fig. 2. We zoom in the left-mid cluster in Fig. 1(a) and present the elements of P_0 (blue circles), P_1 (black triangles) and $P_\omega^\#$ (boxed particles) over that region. Due to inaccurate estimating the BW, there is a significant number of particles in P_1 which do not *intersect* with P_0 yet have non-trivial weights (i.e., $\zeta_\omega^{(i)}$ are not negligible), and appear in $P_\omega^\#$ after resampling (cyan boxed triangles).

This phenomenon is apparent also for $\omega = 1$ (Fig. 1(c)) implying an inaccurate particle representation of $s_\omega(x)$ for $0 < \omega < 1$. KDEs based on LCV pose the same problem (Fig. 1(d),(e)).

We utilize a similar approach to adaptive KDE, and first, treat $s_j(x)$ as a mixture density of G components, i.e.,

$$s_j(x) = \frac{1}{G} \sum_{i=1}^G g_j^i(x)$$

which is a convenient assumption in a multi-object tracking scenario with G as the number of targets. Given a particle representation for a mixture density, it is possible to identify the components through clustering [19]. Clustering of particles representing a PHD is also

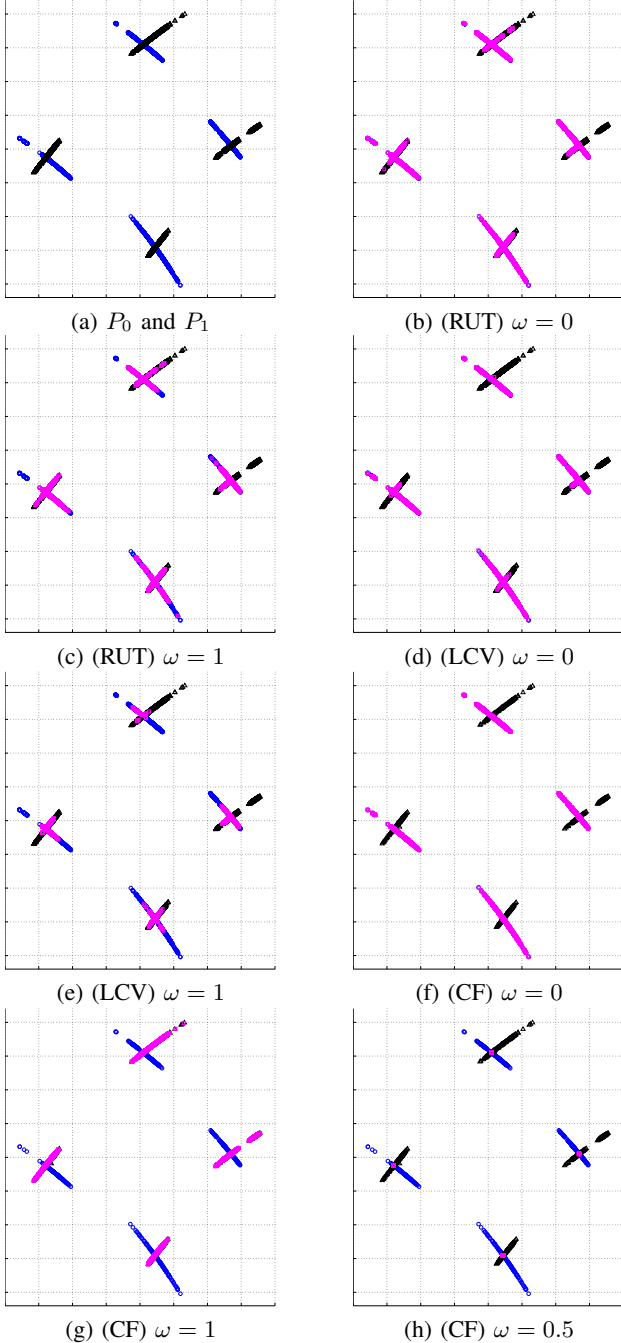


Fig. 1. Equal weighted particles at time step k of a multi-object tracking scenario representing; (a) localisation densities associated with the posterior PHD of CPHD filter 0 (blue circles) and filter 1 (black triangles), (b) EMD for $\omega = 0$ based on RUT KDE (magenta) overlaid on (a), (c) similar for $\omega = 1$, (d) EMD for $\omega = 0$ based on LCV KDE, (e) similar for $\omega = 1$ (LCV), (f) EMD for $\omega = 0$ based on clustered fusion utilizing RUT (CF), (g) similar for $\omega = 1$, (h) similar for $\omega = 0.5$.

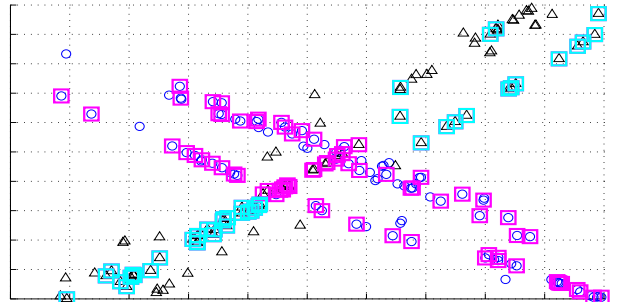


Fig. 2. Leakage of spurious particles to the EMD representation for $\omega = 0$: The particles representing $s_0(x)$, i.e., P_0 (blue circles), and $s_1(x)$, i.e., P_1 (black triangles), respectively, are seen (zoomed in the mid-left clusters in Fig. 1(a)). For the case, most of the elements of P_ω^\sharp (boxed particles) should be from particle set P_0 . However, due to inaccurate BW estimation, among the particles in P_1 , there is a significant number of those which do not lie in the intersection with P_0 yet have non-trivial weights and appear in P_ω^\sharp after resampling (cyan boxed triangles).

of concern for state estimation in the context of SMC realisation of CPHD filters [20].

We consider the CPHD update equation (Theorem 2 in [21]) in which the weight of each particle is updated by a sum of contributions from each observation as well as a term due to a possible missed detection. We label each particle based on the maximum contribution, either with one of the observations or as missed detection. Such a scheme is introduced in [20] and it is shown that the mean of clusters that are persistently associated with an observation in consecutive time steps yield a principled multi-object state estimate. Hence, after clustering both P_0 and P_1 , we employ an association procedure to determine overlapping clusters and form individual EMDs accordingly. For the case, the association can be performed with polynomial complexity using, e.g., the Hungarian algorithm⁴. An example for this approach is provided in Fig. 1(f)–(h) in which the selection of P_0 or P_1 is apparent through selection of $\omega = 0$ or $\omega = 1$ respectively, as well as the improvement in localisation for $\omega = 0.5$. An automatic selection mechanism for ω as described in Section II-B is left out of the scope of this work.

IV. EXAMPLE

As an example scenario we consider 4 targets moving with constant velocity in the common region of interest of two sensors located at $[-100, -100]^T$ and $[1100, -100]^T$ respectively. All targets are born at step 1 and remain in the scene for an observation interval of 150 steps (Figure 3). Each sensor has a detection probability of $p_D = 0.95$ and make AWGN measurements in the polar coordinates with $\sigma_\phi = 0.035$ and $\sigma_R = 0.1$ in the azimuth and range respectively. The clutter is Poisson over the region of interest with rate $\lambda = 10$.

The sensors utilise SMC CPHD filters and at each time step, the localisation densities of the posterior PHDs are fused using the method described in Section III-B for $\omega = 0, 0.09, \dots, 1$. For assessment of the localisation performance, we compute the OSPA localisation metric [22] at each time step after producing state estimates simply by taking the mean of the clusters.

In Figure 4, averaged results for 100 Monte Carlo trials are presented. At each time step, we select the fusion result as that for the ω which achieves the minimum OSPA. It is seen that the fusion rule exploits the supplementary information provided by different sensors and improves the localisation accuracy significantly. Another

⁴Note that, in arbitrary networks, this scheme extends in a similar fashion with the utilisation of CI. Note, also that, the multi-sensor CPHD/PHD filters, i.e., the centralised solutions, are intractable, in general [16].

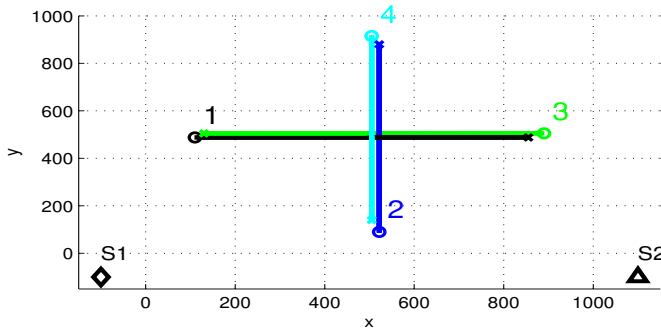


Fig. 3. Example scenario; 4 targets start to move from locations indicated by circles and are observed by two sensors shown by the diamond and the triangle.

observation is that, for this scenario, for a wide range of values of ω for $0 < \omega < 1$, the improvement in localisation is similar. This can be seen in Figure 5 where we present the mean value of minimum OSPA achieving ω at each time step together with ± 1 standard deviation bounds. Hence, fixing ω at 0.5 would produce near optimal results on the average.

V. CONCLUSIONS

In this work, we considered a multi-sensor multi-target tracking scenario and presented our preliminary results on distributed fusion of multi-object posteriors. Specifically, we exploit the merits of Covariance Intersection, which is well established for single object posteriors, in a multi-object setting. Under the i.i.d. cluster assumption for the multi-object scene, principled fusion is possible through PHDs which are produced by Bayesian multi-object filters. We have developed a Monte Carlo realisation for generalisation of the Covariance Intersection through Exponential Mixture Densities and demonstrated the improvement provided in localisation of multiple targets.

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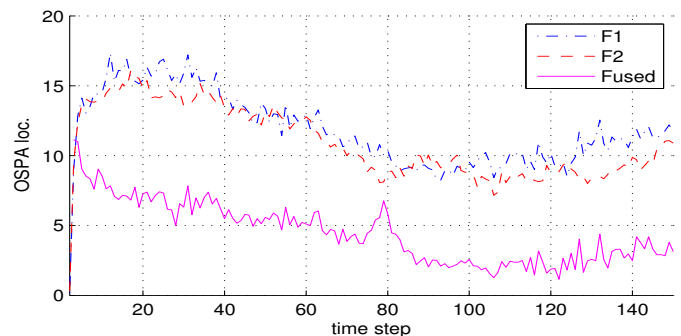


Fig. 4. OSPA localisation distances for CPHD filter 1 (dashed blue line), CPHD filter 2 (dotted red line) and the fusion output (solid magenta line) averaged over 100 MC runs vs. time step.

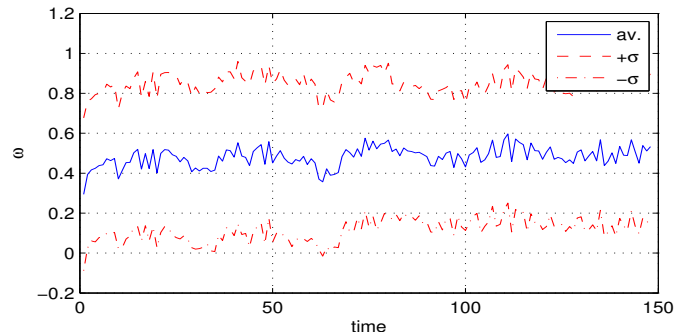


Fig. 5. Mean of the min. OSPA achieving ω (solid line) vs. time steps together with $\pm 1\sigma$ bounds.