Adaptive Beamforming & Blind Signal Separation Who Needs Higher Order Statistics?

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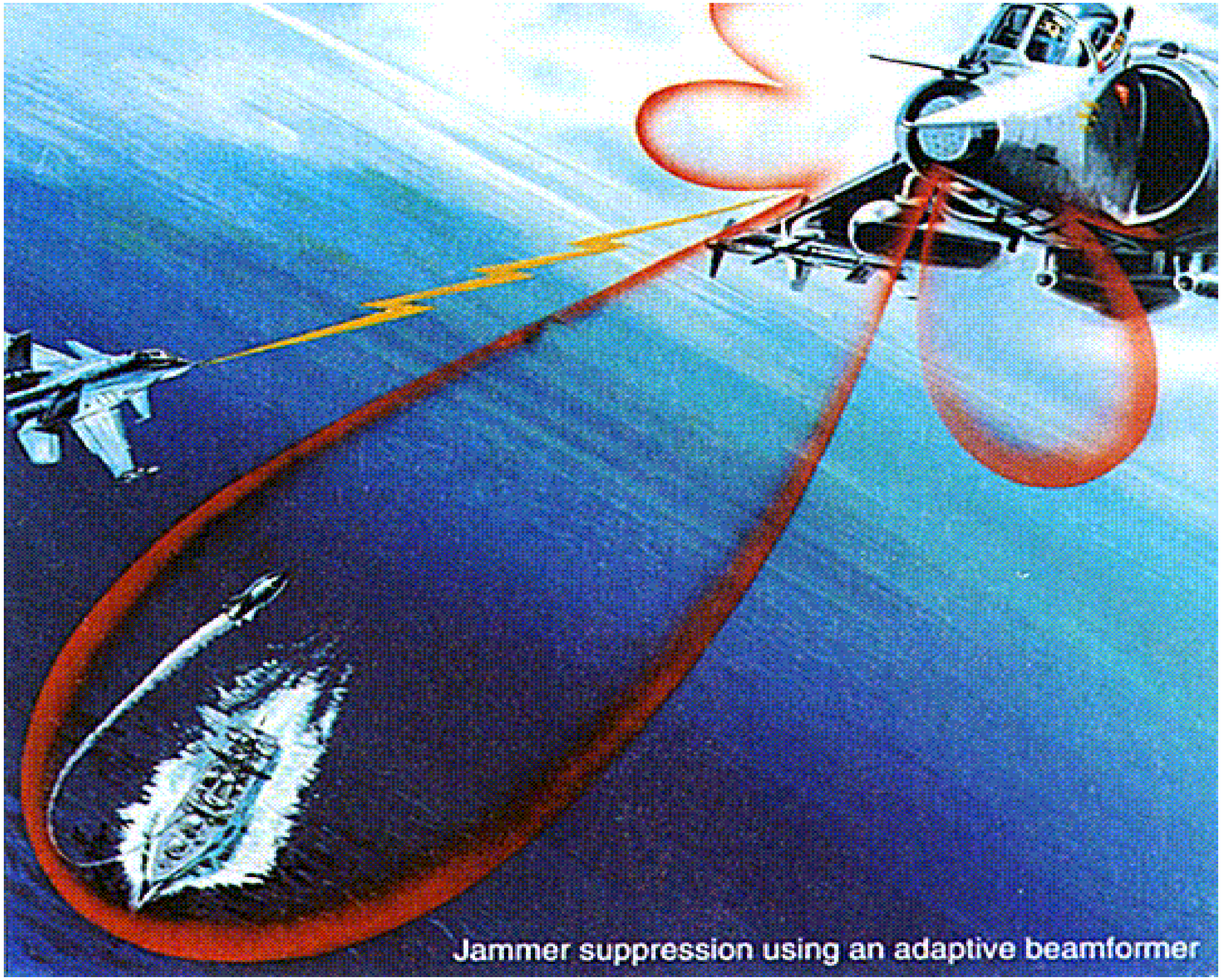
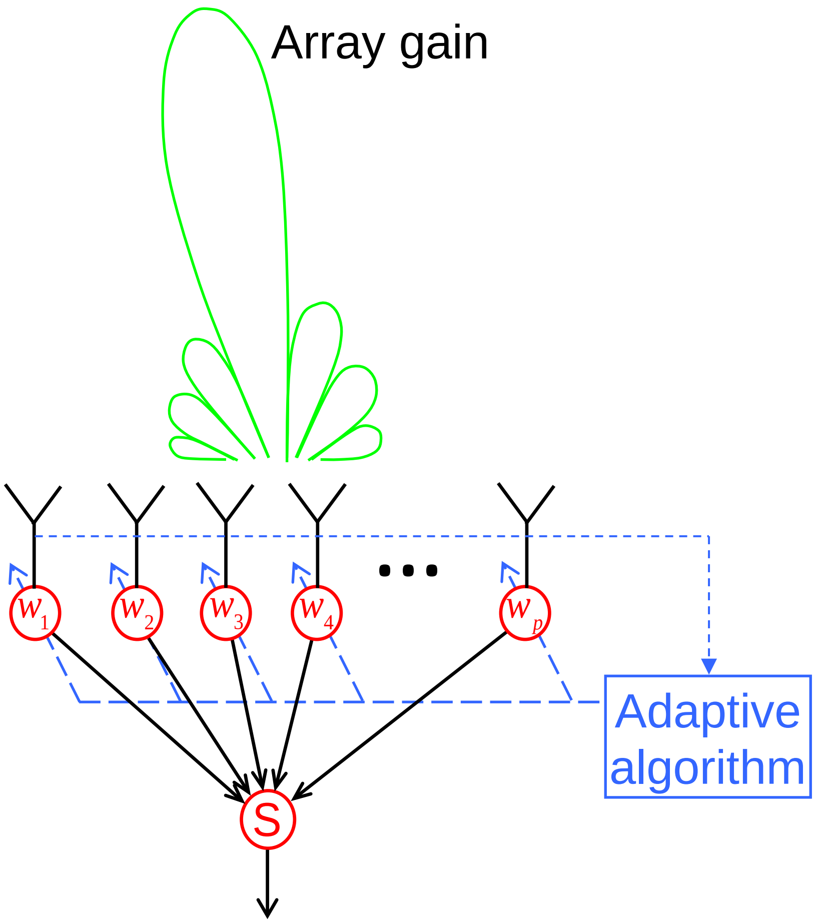
Outline of Talk

- Classic Adaptive Beamforming (ABF)
  - Least squares formulation
  - Second order statistics (SOS)
  - Weight jitter, beam misalignment etc.
- Blind Signal Separation (BSS)
  - Need for Higher Order Statistics (HOS)
  - Case of disparate signal powers
- Direction-Weighted PCA (DWPCA)
  - Paradigm shift: ABF to semi-BSS (SBSS)
  - Ready extension to broadband ABF/SBSS

Adaptive Beamforming



Adaptive Beamforming



Output Signal $e(t) = \mathbf{w}^H \mathbf{x}(t)$

- Complex weights (phase and amplitude) - narrowband
- Minimise output power subject to look-direction constraint

$$\mathbf{w}^H \mathbf{c}(\theta) = \mu$$

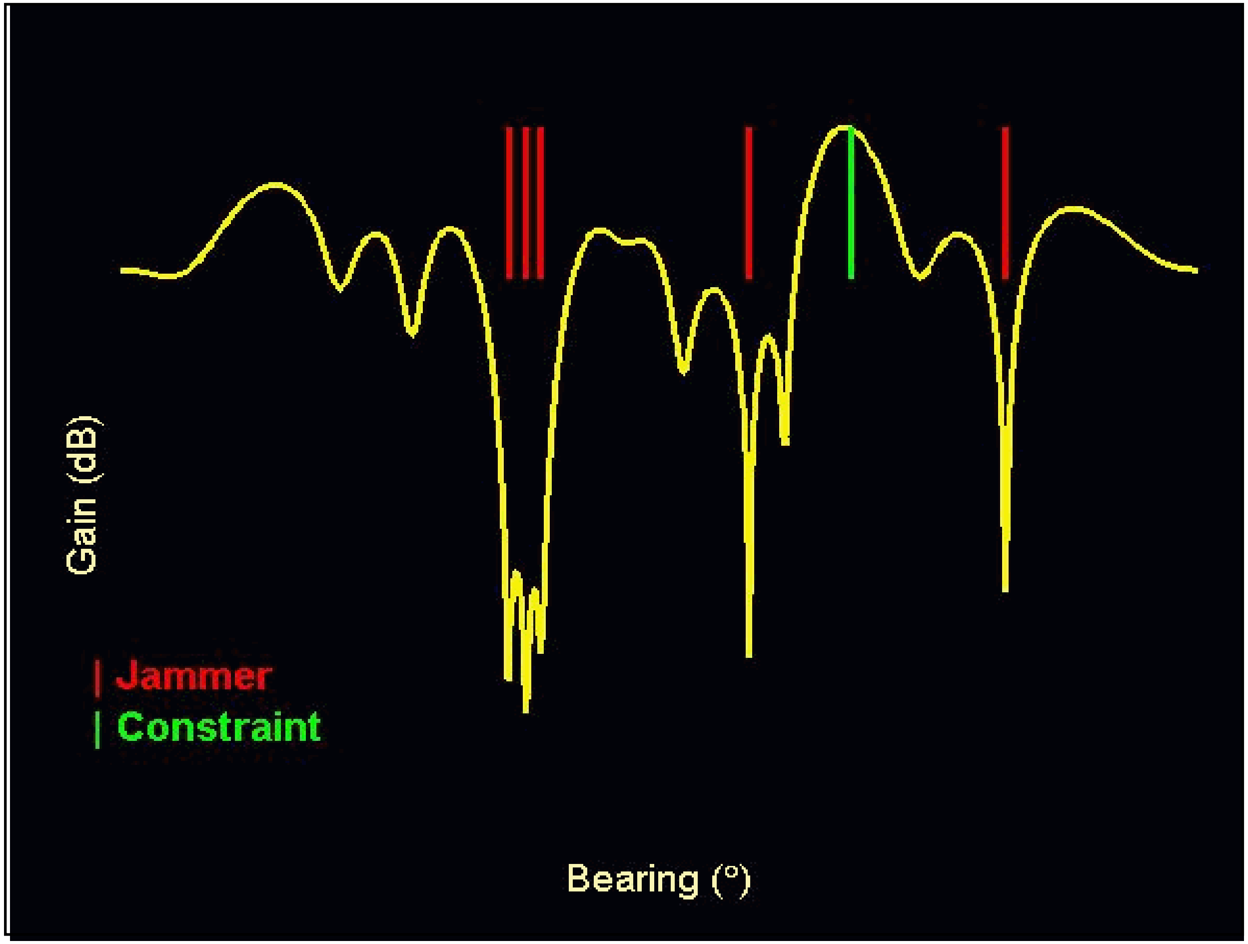
- Least squares weight vector

$$\mathbf{M}\mathbf{w} = \lambda \mathbf{c}$$

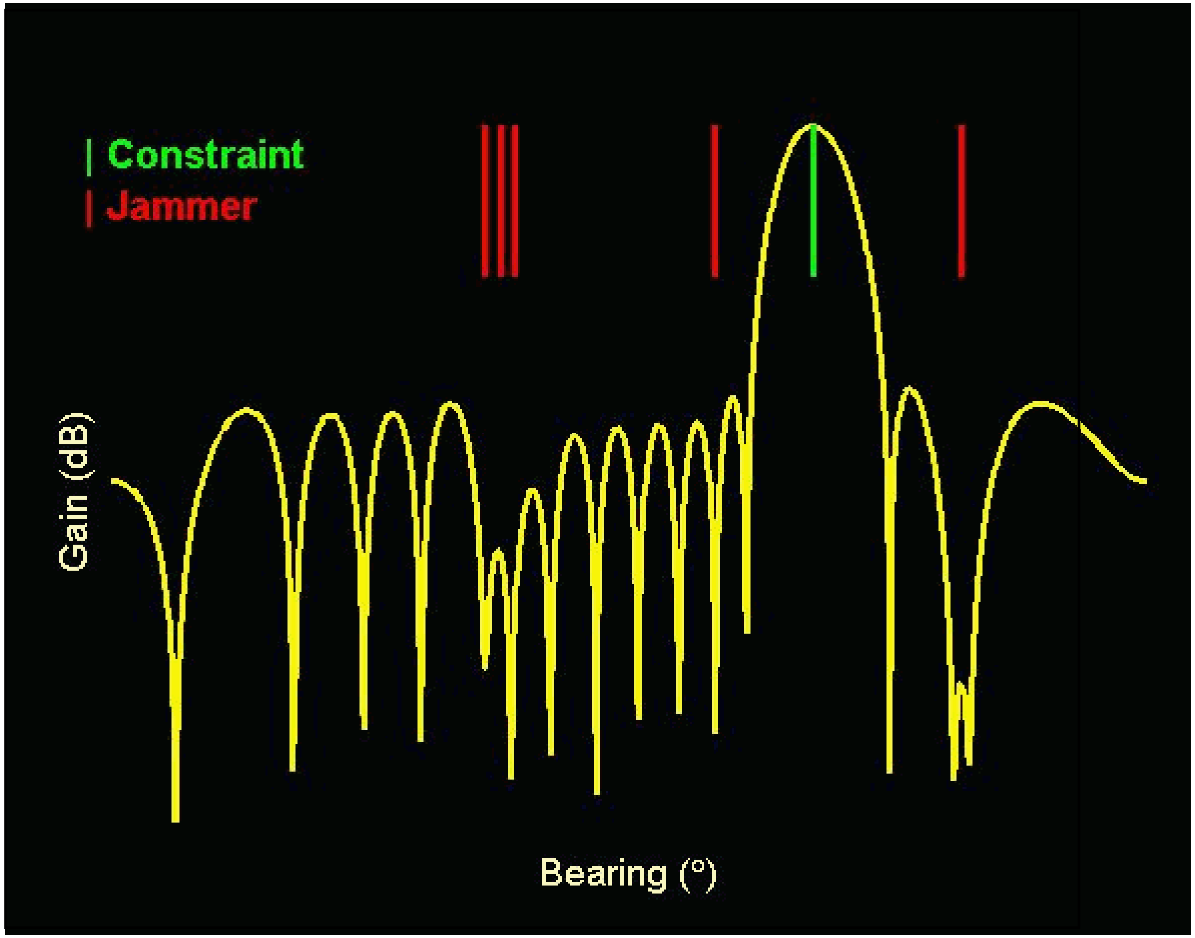
- Sample covariance matrix

$$\mathbf{M} = \mathbf{X}^H \mathbf{X}$$

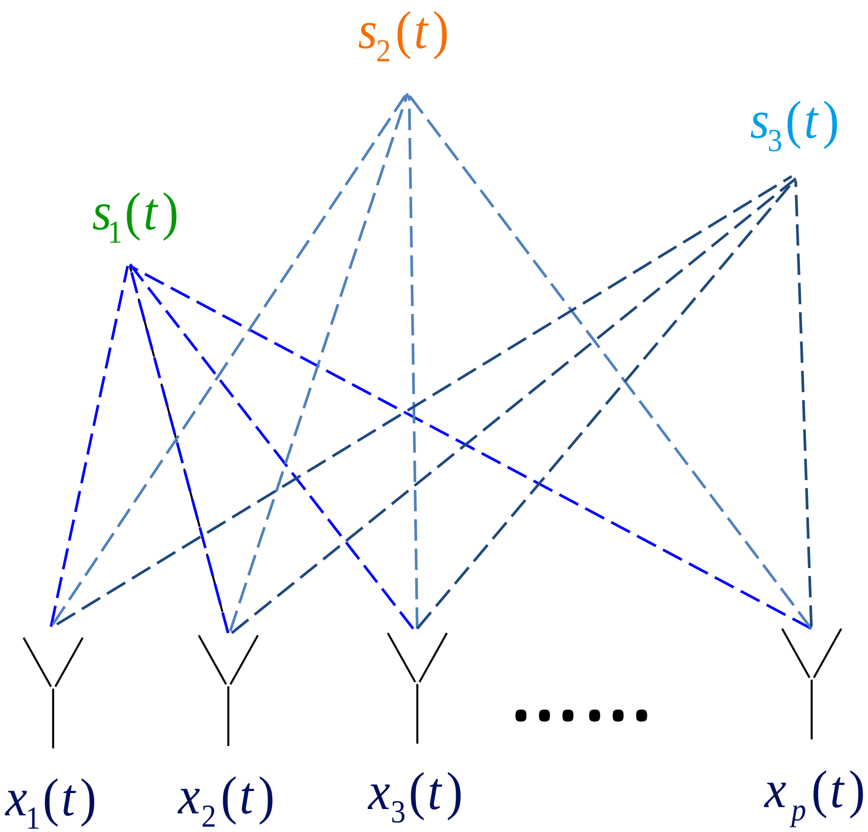
Unstabilised Beam Pattern



Stabilised Beam Pattern



Blind Signal Separation



- Signal model

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)$$

- Data matrix

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N}$$

- Unknown mixture matrix A
- Unknown signals S
- Input signals non-Gaussian and statistically independent

Blind Signal Separation

- Independent component analysis (ICA)
- Avoids need for array calibration
  - Foetal heartbeat monitor
  - HF communications
- Involves use of higher order statistics (HOS)
- Requires signals to be non-Gaussian
  - Typical of man-made signals
  - Digital communication signals

Principal Components Analysis (PCA)

- Signal model $\mathbf{X} = \mathbf{AS} + \mathbf{N}$
- Singular value decomposition (SVD)

$$\mathbf{X} = \begin{bmatrix} \mathbf{U}_s & \mathbf{U}_n \end{bmatrix} \begin{bmatrix} \mathbf{D}_s & 0 \\ 0 & \sigma \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{V}_s \\ \mathbf{V}_n \end{bmatrix}$$

$$= \mathbf{U}_s \mathbf{D}_s \mathbf{V}_s + \sigma \mathbf{U}_n \mathbf{V}_n$$

- Signal subspace $\mathbf{V}_s = \mathbf{D}_s{}^{-1} \mathbf{U}_s{}^{H} \mathbf{X}$

$$\mathbf{V}_s \mathbf{V}_s{}^{H} = \mathbf{I}_s$$

Hidden Rotation Matrix

- Signal subspace

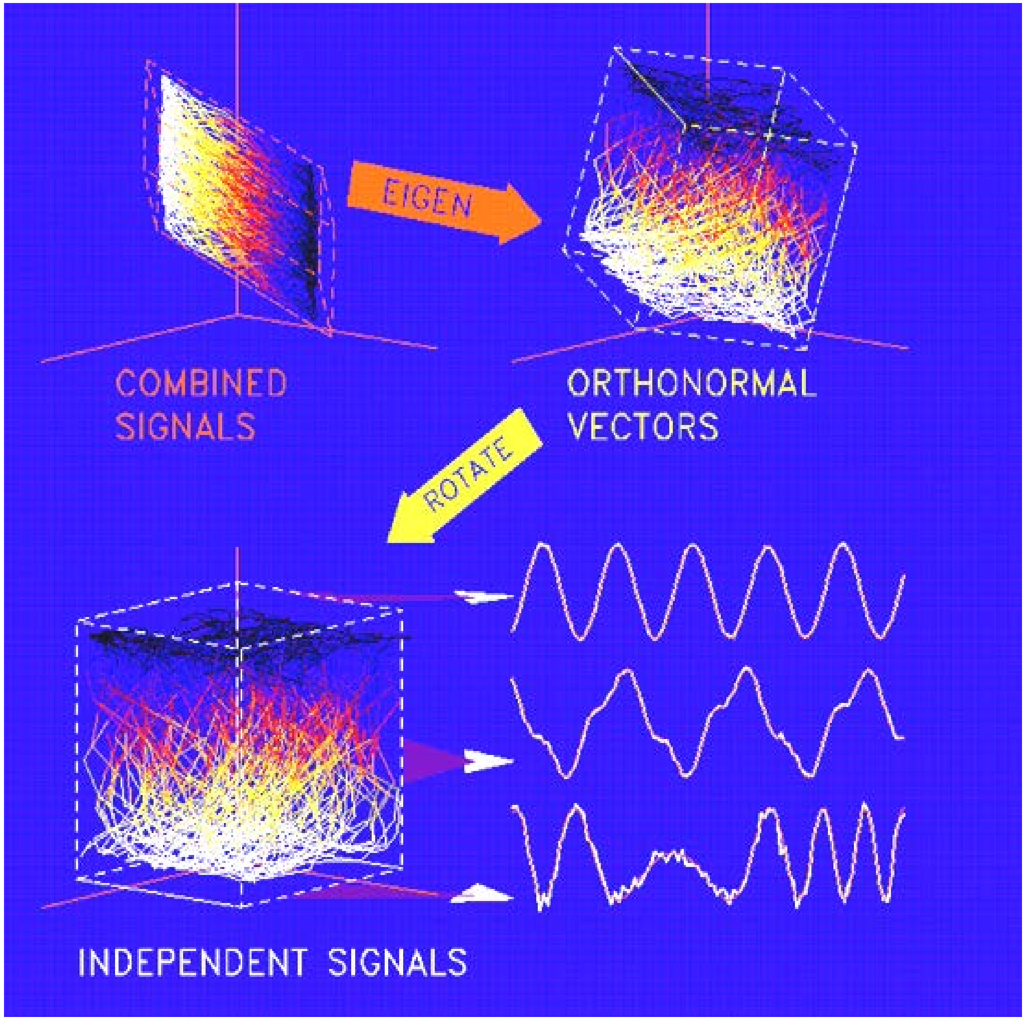
$$\mathbf{V}_s \approx \mathbf{D}_s^{-1}\mathbf{U}_s^{H}\mathbf{A}\mathbf{S}$$

- In terms of second order statistics

$$\mathbf{Q}\mathbf{V}_s = \mathbf{S}$$

  – where $\mathbf{Q}$ is an arbitrary unitary matrix
- Knowledge of $\mathbf{Q}$ $\Rightarrow$ knowledge of mixture matrix
  – cannot be determined from 2nd order statistics
  – exploit HOS if signals are non-Gaussian
  – or some form of prior knowledge (SBSS)

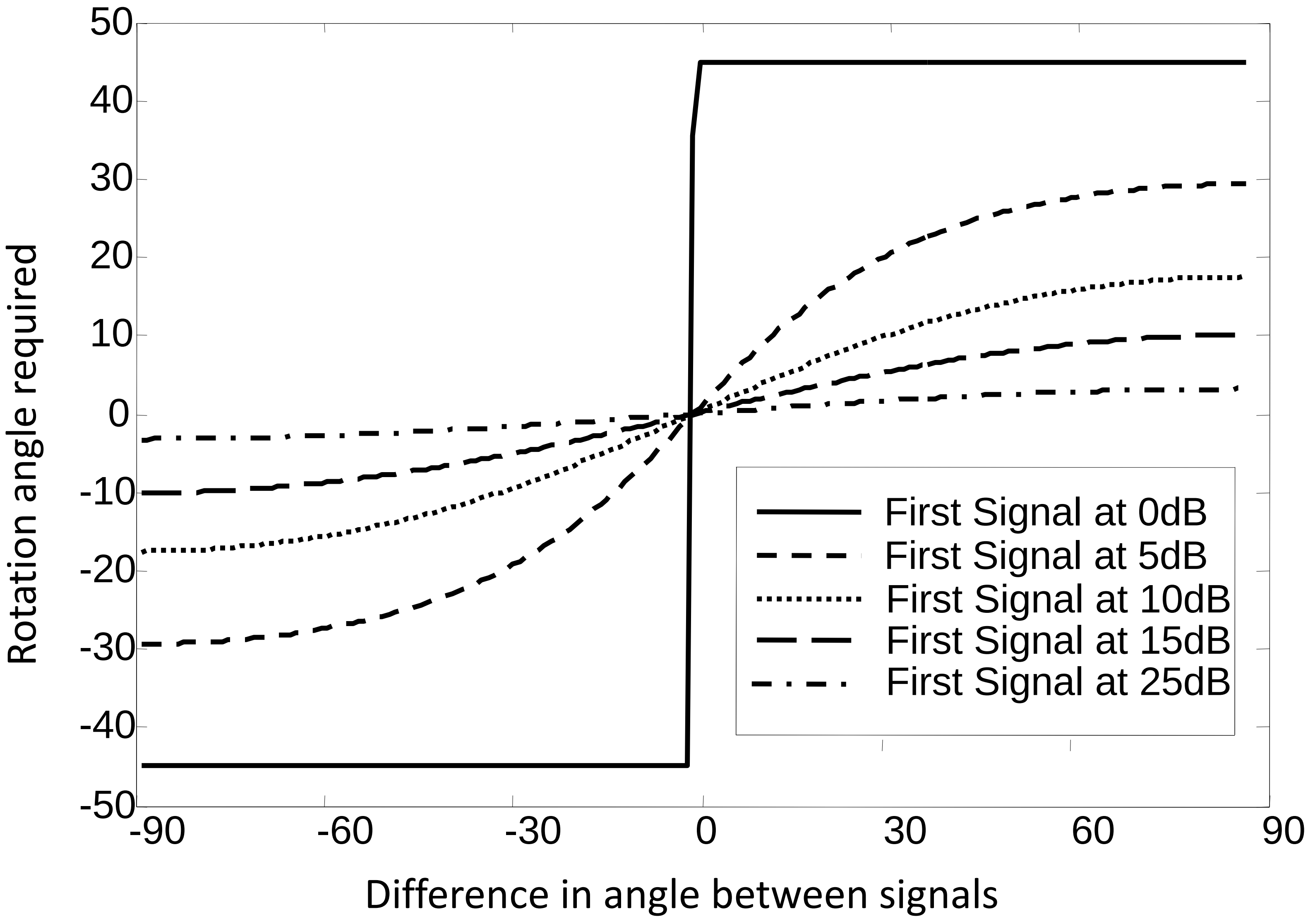
Independent Component Analysis



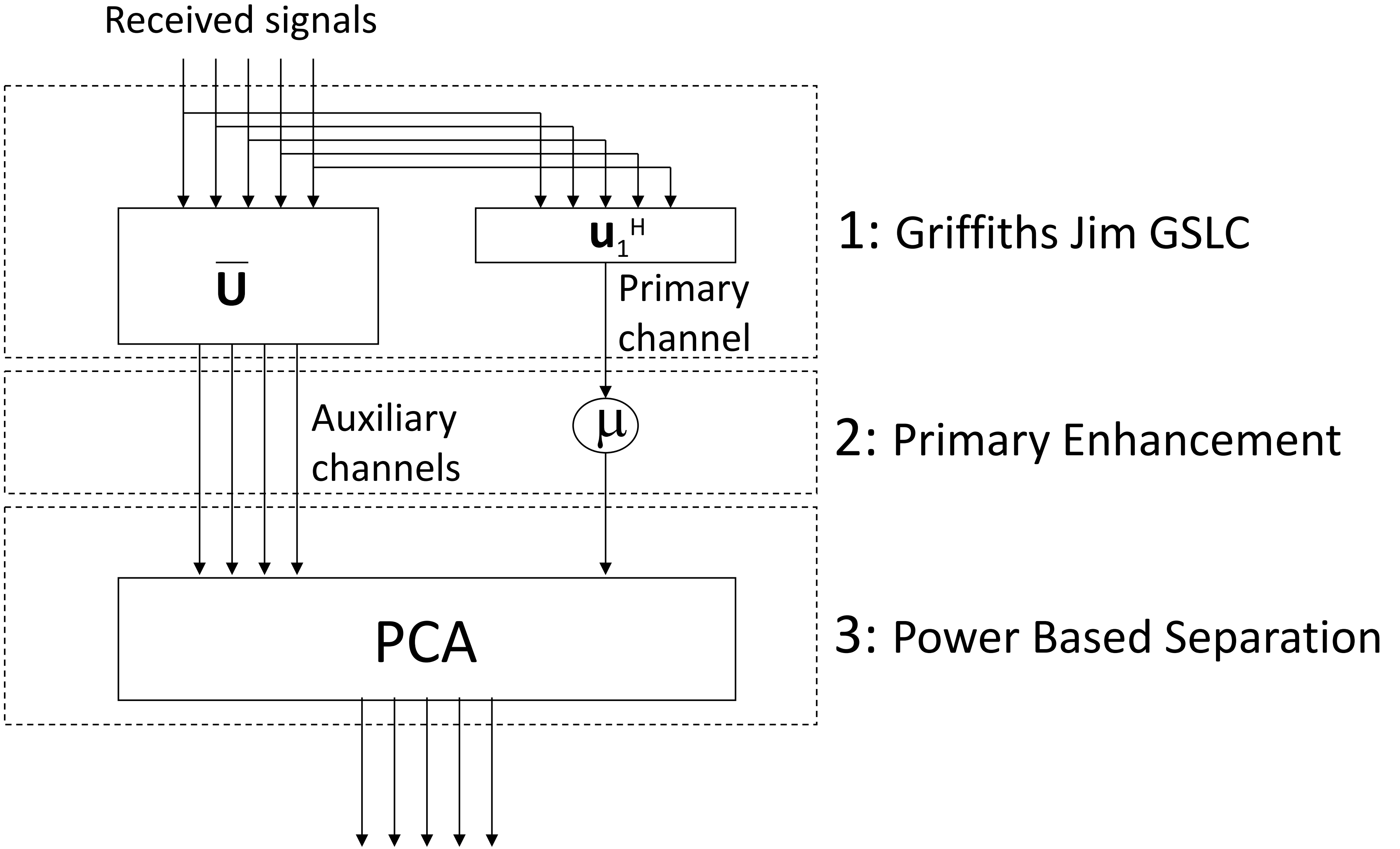
ICA – Practical Considerations

- High computational load (HOS)
- Normalising component powers is dangerous
- Assumes that weak components are noise
  - One man's noise is another man's signal
- Signal powers provide important prior information
- Different power levels reduce need for HOS
  - No hidden rotation matrix
- SOS may be sufficient for signal separation
  - power inversion effect

Hidden Rotation



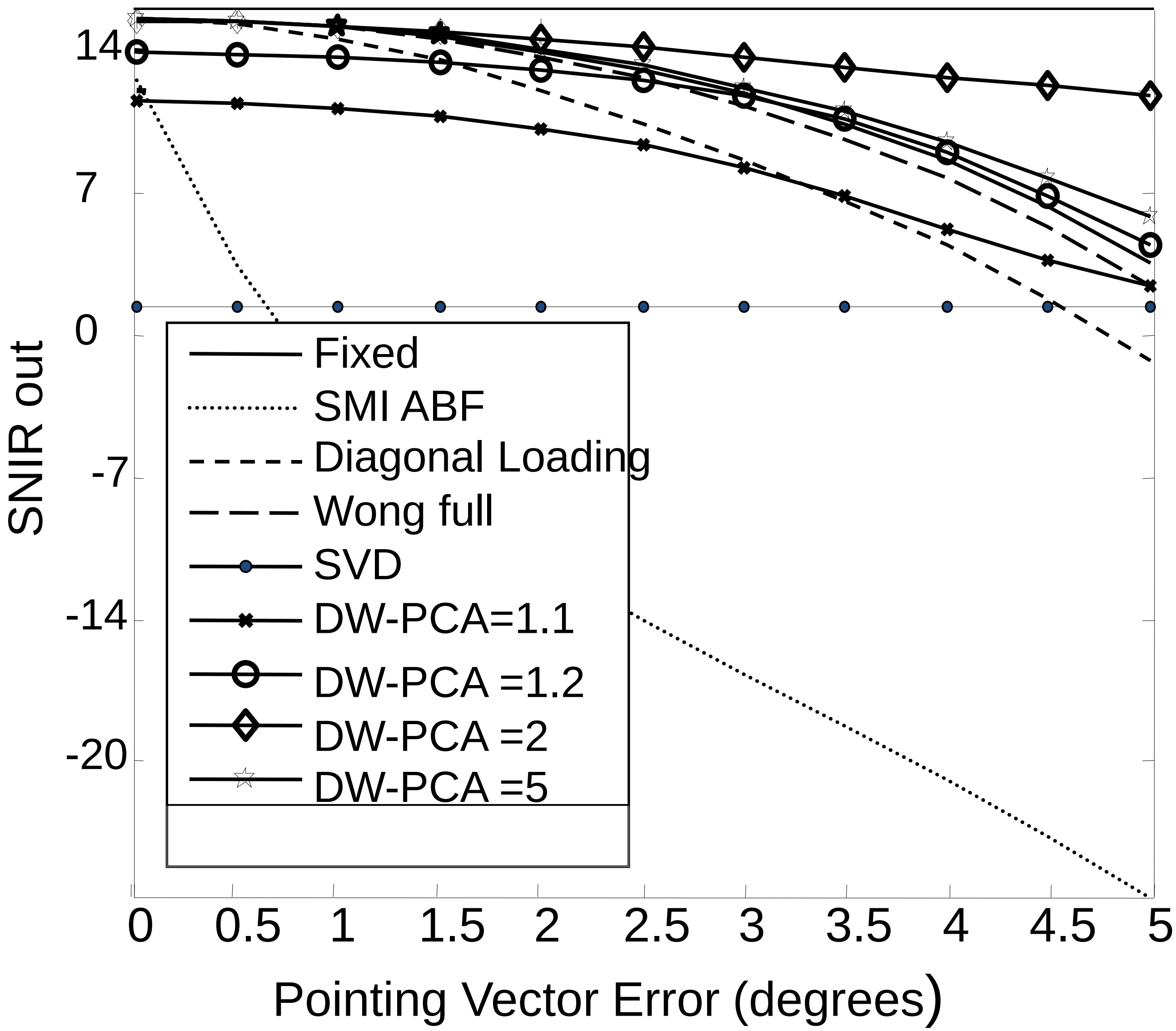
Direction-Weighted PCA



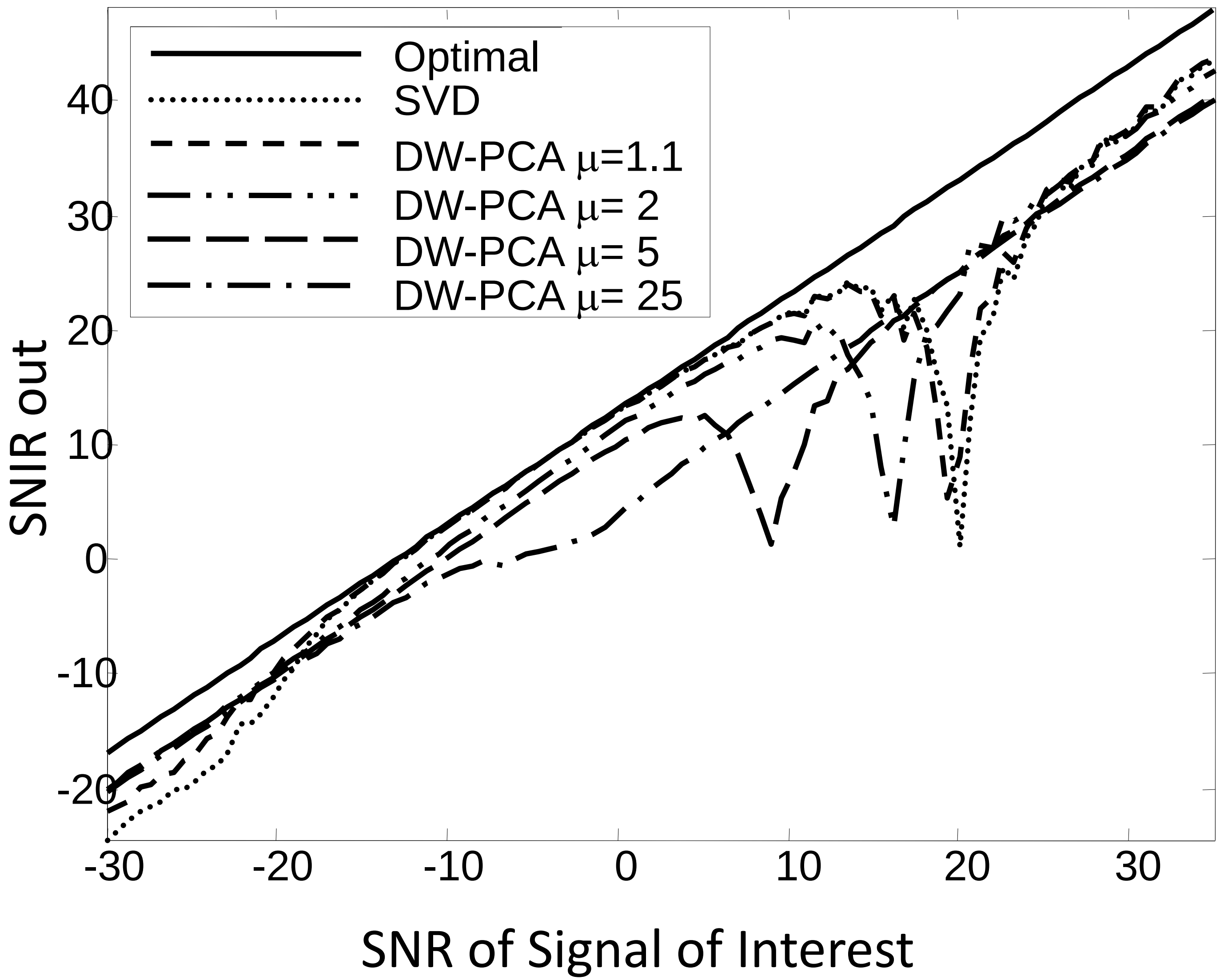
DWPCA – Points to Note

- PCA constrains weight vectors to unit norm resulting in stable operation
- Setting $\mu = 1$ corresponds to pure PCA
- Tends to fixed beamformer as $\mu \to \infty$
- Setting $\mu = 0$ nulls out signal from look direction
- Behaves like soft constraint method for general values of μ

Performance of ABF and DWPCA



Performance of ABF and DWPCA



Polynomial Matrix EVD (PEVD)

- Definition of PEVD

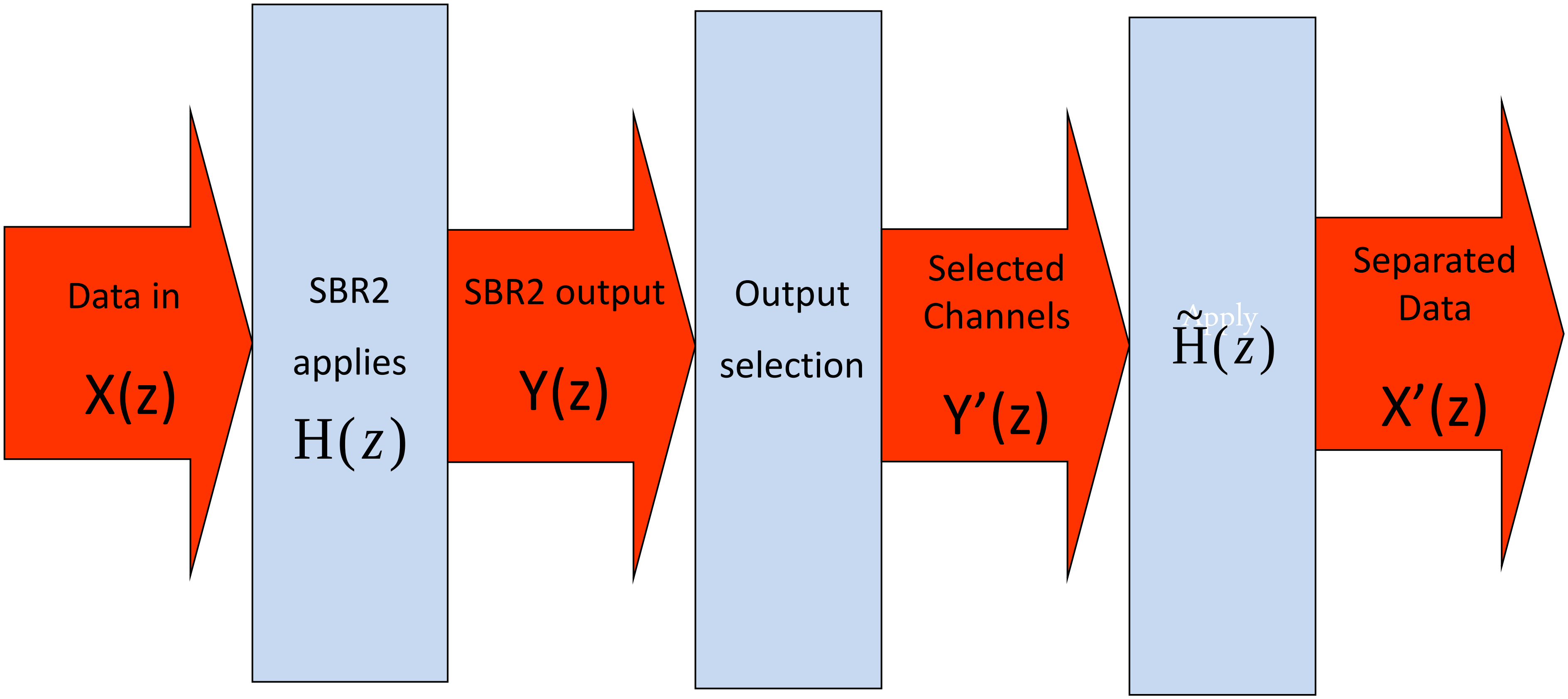
$$\underline{\mathbf{R}}_{vv}(z) = \underline{\mathbf{H}}(z)\underline{\mathbf{R}}_{xx}(z)\underline{\tilde{\mathbf{H}}}(z) = \begin{bmatrix} \underline{d}_1(z) & 0 \\ 0 & \underline{d}_p(z) \end{bmatrix}$$

- $\underline{\mathbf{R}}_{xx}(z)$ is para-Hermitian (cross-spectral density);

$$[\mathbf{R}_{xx}(\tau)]_{ij} = \mathrm{E}\{x_i(t)x_j^*(t-\tau)\} \quad \underline{\mathbf{R}}_{xx}(z) = \sum_{\tau} \mathbf{R}_{xx}(\tau)z^{-\tau}$$

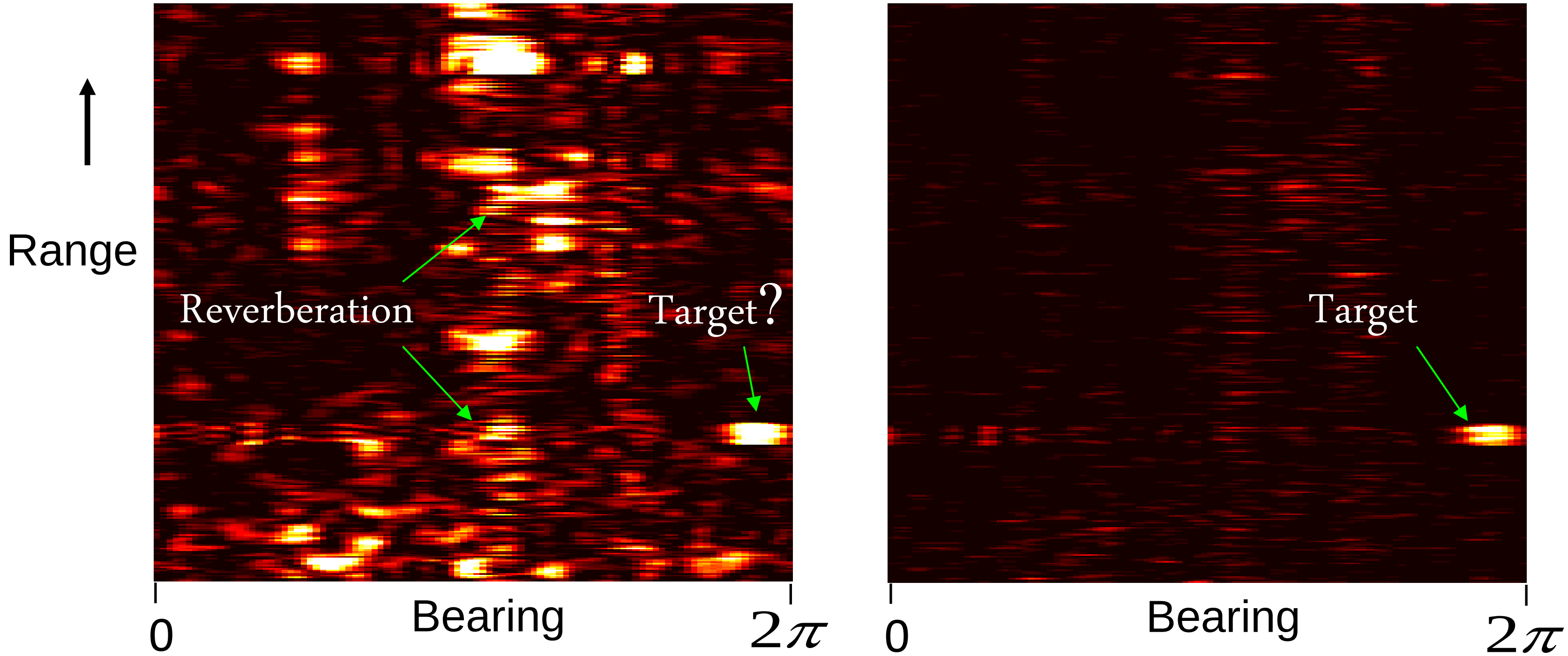
- $\underline{\mathbf{H}}(z)$ is paraunitary i.e $\mathbf{H}(z)\tilde{\mathbf{H}}(z) = \tilde{\mathbf{H}}(z)\mathbf{H}(z) = \mathbf{I}$

Signal separation using SBR2



Sonobuoy Array

Sonar reverberation suppression



- SBR2 algorithm used to reduce reverberation
- Only SOS required for PEVD – difficult to hide PU matrix
- Can be enhanced using broadband DWPCA

Potential Military Applications

- MIMO communication networks
- Underwater acoustic communications
- Sonobuoy array signal processing
- Sonar towed / flank array processing
- Monitoring seismic events (test ban treaty)
- Acoustic monitoring for asset / harbour protection

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- References

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