Reliable Sub-Nyquist Wideband Spectrum Sensing Based on Randomised Sampling

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***Abstract*—This paper considers a sub-Nyquist multiband spectrum sensing approach that accomplishes the sensing task using sampling rates significantly lower than those demanded by the classical uniform-sampling-based DSP. It deploys nonuniform randomised sampling in conjunction with an appropriate spectral analysis tool. Reliability guidelines that ensure the credibility of the sensing procedure amid a sought detection performance are presented. They demonstrate the trade-offs between the sampling rate and sensing time. Numerical examples are provided to illustrate the effectiveness of the introduced technique.**

I. Introduction

Spectrum sensing entails scanning parts of the radio spectrum in search of meaningful activity, e.g. presence of a transmission. It has a plethora of application areas such as surveillance, interception and multichannel communication systems. The latter include the emerging cognitive radio paradigm which triggered intensive research into effective spectrum sensing techniques [1, 2]. In the scenario of monitoring a wideband frequency range consisting of a number of non-overlapping spectral subbands and without *a priori* knowledge of the signal characteristics, spectrum sensing methods that rely on nonparametric spectral analysis are regarded as adequate efficient candidates [1-4]. This approach is adopted here where the aforementioned scenario is studied, i.e. multiband spectrum sensing.

Uniform-sampling-based DSP imposes a minimum sampling rate of twice the width of the monitored frequency range despite the subbands activity. Otherwise aliasing causes irresolvable detection problems [5]. If the monitored frequency range(s) is/are considerably wide, uniform-sampling-based spectrum sensing approaches can demand excessively high sampling rates; possibly beyond the capability of the currently available acquisition device(s). In such cases, the sampling rate requirement becomes an impeding factor to the deployment of uniform sampling DSP and consequently alternatives are sought [3].

In this paper, we consider a wideband spectrum sensing approach that utilises randomised sampling in conjunction with an appropriate periodogram-type spectral analysis tool to reliably perform the sensing operation. It uses significantly low sub-Nyquist sampling rates that are notably lower than those demanded by uniform-sampling-based sensing methods. Most importantly, we provide prescriptive guidelines on the required sampling rate and sensing time in a given scenario to meet sought probabilities of detection and false alarm, i.e. to ensure the reliability of the sensing procedure. It is shown that the sampling rate can be arbitrarily low at the expense of infinitely long sensing time. From a large number of possible randomised sampling schemes, here we address the total random sampling (TRS) [6] and stratified sampling with equal partitions (SSEP) [7].

Based on the reliability guidelines, it is shown that the benefits of the adopted spectrum sensing technique become more visible in low spectrum occupancy environments, i.e. for multiband signals with sparse spectrum. Accordingly, the sparsity of the signal spectrum indirectly dictates the savings of the considered randomised-sampling-based sensing approach in terms of the total number of processed samples.

Whilst the adopted methodology, commonly referred to as digital alias-free signal processing (DASP), does not fall under the compressive sensing (CS) framework, it is best suited to sparse signals and deploys nonuniform randomised sampling to ease the sampling rate requirements. However, DASP does not involve computationally demanding operations such as solving certain optimisation problems. We briefly outline in Section IV the differences between the CS and DASP methodologies in the context of spectrum sensing.

II. Multiband Spectrum Sensing

A. System Model and Problem Formulation

Consider a communication system operating over L predefined contiguous disjoint spectral subbands each of width B_C , i.e. the overseen bandwidth is $\mathscr{B}=[f_{in}, f_{in}+B]$ where $B=LB_C$. The maximum number of concurrently active subbands at any particular point in time is L_A. Hence the joint bandwidth of the active subbands never exceeds $B_A=L_AB_C$. Our objective is to devise an approach that is capable of scanning the monitored frequency range $\mathscr{B}$ and identifying the active subbands. Its operational sampling rates should be significantly lower than $f_{US}\geq 2B$ where $2B$ is the minimum rate (not always achievable) that could be used when classical uniform sampling is deployed [5].

B. Adopted Sensing Technique

The introduced multiband spectrum sensing method utilizes a periodogram-type estimator defined by:

$$X_e^{\mathcal{T}_r}(f) = \frac{c}{\mu\alpha^2}\left|\sum_{n=1}^{N} y(t_n)w(t_n)e^{-j2\pi f t_n}\right|^2 \quad (1)$$

to estimate a detectable frequency representation of the incoming signal from a finite set of its irregularly distributed noisy samples $y(t_n) = x(t_n) + n(t_n)$. Whereas, N is the number of processed samples within the considered time analysis window $\mathcal{T}_r = [t_r, t_r + T_0]$; $c = N/(N-1)$ and $c = 1$ for TRS and SSEP respectively. Windowing function $w(t)$ is introduced to suppress spectral leakage and $\mu = \int_{\mathcal{T}_r} w^2(t)dt$. Evidently, spectrum sensing does not require the detailed spectral shape to be determined within the monitored wide frequency range. This premise is exploited here where a frequency representation that permits detection is sought and estimating the signal's exact power spectral density (PSD) is not our objective.

The standard deviation of a periodogram-type estimator, such as $X_e^{\mathcal{T}_r}(f)$, is known to be of the same order as its expected value. To reduce this uncertainly, we average a K number of the $X_e^{\mathcal{T}_r}(f)$ estimator in (1) calculated over K windows, i.e. $\mathcal{T}_r$ for $r = 1, 2, \ldots K$, thus:

$$\hat{X}_e(f) = \frac{1}{K}\sum_{r=1}^{K} X_e^{\mathcal{T}_r}(f). \quad (2)$$

This evokes shifting $\mathcal{T}_r$ and the repositioning/aligning of $w(t)$. For simplicity, we assume that the active subbands are of similar power levels and the incoming wide sense stationary (WSS) signal $x(t)$ propagates via an additive white Gaussian noise channel where σ_N^2 denotes the noise variance. Transmissions with non-equal power levels and cyclostationay signals are addressed in [6] and [7].

Non-overlapping uncorrelated signal windows are considered here. The adopted sensing procedure for each spectral subband comprises two steps: 1) estimating the magnitude spectrum at selected frequency point(s) and 2) comparing the magnitude(s) with pre-set threshold(s).

We seek inspecting one frequency point per subband to establish its status. This can be achieved by performing spectral analysis within short time windows, i.e. maintaining relatively smooth spectrographs. The examined frequency points are placed at the centre of the system subbands given the windowing effect. The sensing problem can be formulated as a conventional detection binary hypothesis testing problem:

$$\begin{aligned} H_{0,k}: &\quad \hat{X}_e(f_k) < \gamma_k \\ H_{1,k}: &\quad \hat{X}_e(f_k) \geq \gamma_k \qquad k = 1, 2, \ldots L \end{aligned} \quad (3)$$

where γ_k is the threshold, $H_{0,k}$ hypothesis signifies the absence of an activity in subband k and $H_{1,k}$ depicts the presence of an activity. We show below that (3) can deliver reliable spectrum sensing routine provided appropriately selected T_0, average sampling rate $\alpha = N/T_0$ and K.

III. RANDOMISED SAMPLING AND SPECTRAL ANALYSIS

A. *Total Random and Stratified Sampling*

The sampling instants $\{t_n\}_{n=1}^{N}$ of the TRS scheme are independent identically distributed random variables whose probability distribution functions are given by: $p_n(t) = 1/T_0$ for $t \in [t_r, t_r + T_0]$ and zero elsewhere. With stratified sampling $\mathcal{T}_r$ is divided into N strata: $S_1, S_2, \ldots, S_N$ each containing one randomly selected sampling instant. The probability density function of the n-th sampling instant is $p_n(t) = 1/|S_n|$ if $t \in S_n$ and zero elsewhere, $|S_n|$ is the width of S_n. Here, we consider the case where the strata are of equal lengths, i.e. SSEP and $|S_n| = 1/\alpha$. Below, we illustrate that $X_e^{\mathcal{T}_r}(f)$ with TRS and SSEP is a suitable tool for spectrum sensing.

B. *Target Frequency Representations*

Given that the components of the summation in (1) are independent with respect to t_n, the estimator expected value is:

$$C_{TRS}^{\mathcal{T}_r}(f) = E\left[X_e^{\mathcal{T}_r}(f)\right] = \frac{N(P_S + \sigma_N^2)}{(N-1)\alpha} + \Phi_X(f) * \left|W(f)\right|^2 \big/ \mu \quad (4)$$

for TRS where $P_S = E[x^2(t)]$ is the power of the incoming WSS signal, $\Phi_X(f)$ is its PSD and $*$ denotes the convolution operation. Whilst, for the SSEP scheme:

$$C_{SSEP}^{\mathcal{T}_r}(f) \approx \frac{\left[1-\eta(f)\right]P_S + \sigma_N^2}{\alpha} + \Phi_X(f) * \left|W(f)\right|^2 \big/ \mu \quad (5)$$

where $0 \leq \eta(f) \leq 0.5$ assuming $f_{in} >> B$ for simplicity, refer to [7] for further details.

It is noticed from (4) and (5) that $E = [X_e^{\mathcal{T}_r}(f)]$ consists of a detectable feature given by the signal windowed PSD $\Phi_X(f) * |W(f)|^2 / \mu$ plus components that merely act as amplitude offsets. They do not undermine the detectability of the spectral components in $C_{TRS}^{\mathcal{T}_r}(f)$ and $C_{SSEP}^{\mathcal{T}_r}(f)$ pertaining to an active subband. The aforementioned additional components are commonly referred to by smeared-aliasing; a phenomenon associated with randomised sampling and depends on the characteristics of the used scheme. Thus, the adopted estimator with TRS and SSEP poses as a legitimate tool to sense the activity of the overseen system subbands.

To save on computations, one frequency point per subband is examined in (3). Achieving the minimum possible sensing time $\tilde{T} = KT_0$ is highly desirable for any detection technique. Accordingly, appropriately short $\mathcal{T}_r$ is employed in (2) to minimize $\tilde{T}$ and aid maintaining low resolution spectrographs without overshadowing the distinguishable features in $E = [X_e^{\mathcal{T}_r}(f)]$, i.e. $\Phi_X(f) * |W(f)|^2 / \mu$. It was noted in [6] and [7] that $T_0 \geq n/B_C$, $n \geq 1$, serves as a practical guideline .

IV. RELIABLE SPECTRUM SENSING

The reliability of a sensing approach is reflected by its ability to meet a sought system behaviour that is commonly expressed by the receiver's operating characteristics (ROC). In this section we deploy the ROC to derive the pursued dependability conditions.

A. Reliability Guidelines

According to central limit theorem and for a large number of averaged windows, $\hat{X}_e(f)$ becomes normally distributed, moderate value of K suffices in practice. Hence we have: $\hat{X}_e(f_k) \sim \mathcal{N}\left(m_0(f_k), \sigma_0^2(f_k)\right)$ and $\hat{X}_e(f_k) \sim \mathcal{N}\left(m_1(f_k), \sigma_1^2(f_k)\right)$ for $H_{0,k}$ and $H_{1,k}$ respectively. We note: $m(f_k) = E[\hat{X}_e(f_k)]$ is the estimator mean and equal to that in (4) or (5) whereas $\sigma^2(f_k) = \mathrm{Var}\left\{\hat{X}_e(f_k)\right\}$ is the estimator variance. The formulas for the latter were omitted due to space limitations (refer to [6] and [7] for more details). Using the detection decision described by (3), the probability of a false alarm in a particular subband is given by :

$$P_{f,k}(\gamma_k) = \Pr\left\{H_{1,k} \middle| H_{0,k}\right\} = Q\left[\left(\gamma_k - m_0(f_k)\right)/\sigma_0(f_k)\right] \quad (6)$$

and the probability of correct detection is:

$$P_{d,k}(\gamma_k) = \Pr\left\{H_{1,k} \middle| H_{1,k}\right\} = Q\left[\left(\gamma_k - m_1(f_k)\right)/\sigma_1(f_k)\right] \quad (7)$$

where $Q(z)$ is the tail probability of a zero mean and unit variance normal distribution. Due to nonuniform sampling, the false alarm can be triggered not only by the present noise but also by the present smeared-aliasing at all frequencies.

In practice, the user describes the desired performance of the detector by the two probabilities:

$$P_{f,k} \le \Delta_k \quad \text{and} \quad P_{d,k} \ge \ell_k \quad (8)$$

for one or more of the system subbands. Given (6) and (7), we can write:

$$m_1(f_k) - m_0(f_k) \ge Q^{-1}(\Delta_k)\sigma_0(f_k) - Q^{-1}(\ell_k)\sigma_1(f_k) \quad (9)$$

which defines the reliability condition of the sensing procedure. By substituting the mean and variance values and taking a conservative approach, it can be shown that (9) leads to:

$$\tilde{T}_{TRS} \ge \left\{ \frac{2B_A N T_0\left(1 + SNR^{-1}\right)}{(N-1)\alpha}\left[Q^{-1}(\Delta_k) - Q^{-1}(\ell_k)\right] - T_0 Q^{-1}(\ell_k) \right\}^2 \quad (10)$$

where $SNR = P_S/\sigma_N^2$. This is the combined lower limit on the sensing time $\tilde{T}_{TRS}$ and the average sampling rate α of (2) to satisfy (8) for TRS where $K = \tilde{T}_{TRS}/T_0$. For SSEP it is:

$$\tilde{T}_{SSEP} \ge \left\{ \frac{2B_A Q^{-1}(\Delta_k)\left(1 + SNR^{-1}\right) - Q^{-1}(\ell_k)\left[2B_A\left(0.5 + SNR^{-1}\right) + \alpha\right]}{(\alpha - B_A)/T_0} \right\}^2 \quad (11)$$

It is noted that (10) and (11) assume worst expected system conditions where L_A subbands are active (i.e. maximum spectrum occupancy) and smeared-aliasing has the most harmful effect on the detection process. The latter is more of an issue with SSEP due to its smeared-aliasing variations across $\mathcal{B}$. Formulas (10) and (11) give a combined conservative guideline on the required sensing time and average sampling rate for predefined detection probabilities, maximum expected spectrum occupancy B_A and signal to noise ratio. It is a clear illustration of the trade-offs between the sampling rate and the sensing time in relation to achieving dependable sensing. According to (10) and (11), we can use remarkable low sub-Nyquist sampling rates for the sensing operation at the expense of longer sensing time. In fact the average sampling rate can be arbitrary low for TRS and SSEP. A closer look at (10) and (11) will show that SSEP demands longer sensing time compared to TRS. However, SSEP lends itself to more practical implementations in hardware compared to TRS and other randomised schemes (see [7] for more details). Overlapping and correlated signal windows can be easily incorporated into the introduced approach using existing results in literature on variance reductions, e.g. Welch periodograms.

Equations (10) and (11) clearly show that the required sensing time and/or the sampling rate are proportional to spectrum occupancy B_A, i.e. the sparsity level of the multiband signal in the frequency domain. In fact, it can be shown that notable saving can be made on the total number of processed samples (function of sampling rate and sensing time) only for very sparse signals where spectrum occupancy is low $B_A \ll B$.

B. Existing Sensing Techniques

Table 1 depicts a list of spectrum sensing techniques highlighting their abilities to conduct multiband detection, sampling rate requirements and computational complexities [1-4]. The latter is an indicative measure where the energy detector is used as a benchmark. Energy detector involves taking the FFT of the signal and averaging. A technique which entails computationally demanding operations, e.g. solving an optimisation problem, is considered high complexity.

TABLE 1. SPECTRUM SENSING APPROACHES

Approach	**Minimum Sampling rate**	**Multiband**	**Computational Complexity**
Energy detector	Nyquist	✓	Low
Multitaper estimator	Nyquist	✓	High
Wavelet-based	Nyquist	✓	Moderate
CS-based	Sub-Nyquist	✓	High
Adopted Method	Sub-Nyquist	✓	Low
Matched filtering	Nyquist	✗	Low
Feature detector	Nyquist	✗	Moderate
Covariance detector	Nyquist	✗	Moderate

From Table 1, it is noticed that only the introduced approach and compressive-sensing-based ones permit sub-Nyquist sampling rates. They furnish considerable savings on the digital data acquisition and alleviate the sampling rate limitation of DSP for wideband signals. The simplicity and the low computational complexity of the adopted approach are its main advantages over CS techniques. The latter methods involve solving underdetermined sets of linear equations. CS methods similar to that in [4] impose a minimum sampling rate of $4B_A$ (still sub-Nyquist in low spectrum occupancy environments). Other CS approaches , e.g. [3], aim to estimate the signal autocorrelation function from its compressed samples. They presume that the reconstructed empirical autocorrelation function from one signal realization is identical to the WSS signal exact autocorrelation function assuming infinitely long sensing time. On the other hand, the introduced technique can use arbitrary low sampling rates and explicitly provide the required sensing time to achieve certain desired probabilities of detection using (10) and (11). Due to space limitations, a detailed comparison with numerical examples is outside the scope of this paper.

V. SIMULATIONS

Consider a multiband system comprising $L=20$ subbands where $B_C=5$ MHz. The system subbands are located in $f\in\left[1.35,1.45\right]$ GHz frequency range. A Blackman window is employed where $T_0=0.8\,\mu s$. 16QAM signals with maximum bandwidths and similar power levels are transmitted over the active subbands. A spectrum occupancy of 10% is assumed, i.e. $L_A=2$ and $B_A=10$ MHz. A sampling rate $\alpha=70$ MHz is used and the SNR is -0.2 dB. For the specified probabilities: $P_d\geq 0.95$ and $P_f\leq 0.08$ for all the system subbands, the required sensing time is $\tilde{T}_{TRS}\geq 9.6\,\mu s$ and $K\geq 12$ in (2) for TRS whereas for SEEP $\tilde{T}_{SSEP}\geq 11.2\,\mu s$ and $K\geq 14$ according to (10) and (11). Figures 1 and 2 show the simulated ROC of the adopted method for various sensing times and a threshold sweep (10000 independent experiments are used per plot).

Figures 1 and 2 confirm the moderate conservative nature of the given reliability conditions where the desired performance is achieved for $\tilde{T}_{TRS}\geq 9.6\,\mu s$ and $\tilde{T}_{SSEP}\geq 11.2\,\mu s$, i.e. as recommended by (10) and (11). This affirms the effectiveness of the derived reliability guidelines. If uniform sampling (US) is deployed the minimum valid bandpass sampling rate that would avoid aliasing within $\mathscr{B}$ is 224 MHz. Thus, by adopting the introduced randomised-sampling-based method around 67% saving on the sampling rate is achieved in comparison to uniform sampling. It is unambiguously clear that spectrum sensing with randomised sampling offers tangible benefits in terms of the sampling rates rendering notable reductions in the data acquisition requirements.

On the other hand, the total number of processed signal samples N_T is dependent not only on the sampling rate but also on the sensing time. Figure 3 depicts N_T versus changing spectrum occupancies for TRS, SSEP and uniform sampling. Cyclostationary BPSK transmissions are assumed to be present, whereas the rest of the parameters are similar to the above example. It can be noticed that the savings in terms of the total number of processed samples of the randomised-sampling-based approach becomes more visible as the spectrum occupancy declines, i.e. the signal sparsity level increases. This agrees with the general framework of compressive sensing which capitalises on the sparsity premise.

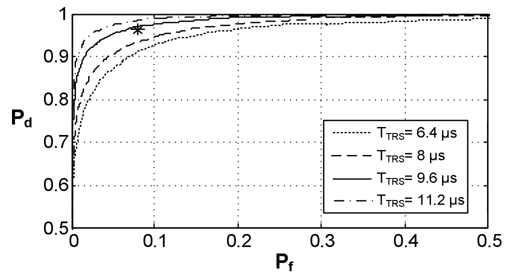

Figure 1. ROC of the adopted technique with TRS for various $\tilde{T}_{TRS}$ and a threshold sweep. Asterisk is $(0.08, 0.95)$; minimum sought performance.

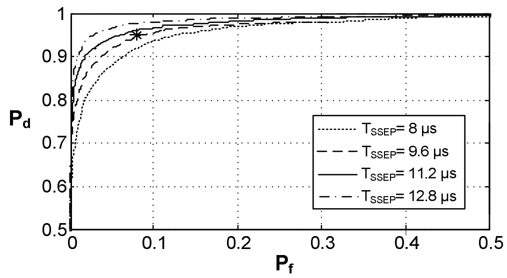

Figure 2. ROC of the adopted technique with SSEP for various $\tilde{T}_{SSEP}$ and a threshold sweep. Asterisk is $(0.08, 0.95)$; minimum sought performance.

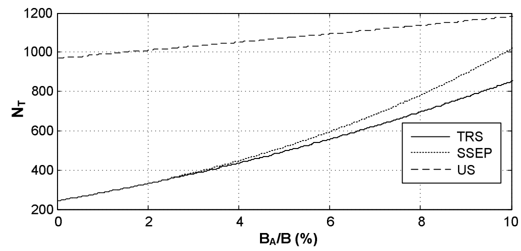

Figure 3. Total number of processed samples for varying spectrum occupancies. TRS, SSEP and uniform sampling are shown.

VI. CONCLUSION

A sub-Nyquist spectrum sensing approach was reviewed. Its simplicity and low computational complexity are among its key merits. This paper serves as an impetus to further research on randomised sampling based techniques. This includes looking at their applicability to areas such as radar signal processing where reducing the sampling rate is highly desirable especially that the treated signals are typically very sparse when expressed in an appropriate basis/frame.

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