

Energy Efficient Distributed Beamforming with Sensor Selection in Wireless Sensor Networks

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Abstract—We study the energy efficiency for a wireless sensor network, in which multiple sensors having identical information cooperatively transmit signals to a fusion center. To facilitate the cooperation, we propose a transmission scheme consisting of four phases: channel state information acquisition phase, sensor selection phase, beamforming phase, and cooperative transmission phase. Analyses show that there is tradeoff between the energies for sensor selection plus beamforming phases and cooperative transmission phase in deciding the number of sensors to be selected. This observation is captured by numerical and simulation results, which can provide a design guideline for energy saving and prolonging of network lifetime.

I. INTRODUCTION

Distributed beamforming has been studied in wireless sensor networks as a cooperative transmission technique [1]–[5], which can efficiently transmit sensors’ decisions to a fusion center (FC). For coherent combining at the FC, the carrier phases of the signals transmitted by sensors can be aligned through feedback. Due to distributed beamforming, the transmission power of sensors can decrease with the number of sensors. When a certain adaptive algorithm is used for the carrier phase alignment, there are two important issues for distributed beamforming: *i*) the amount of feedback; *ii*) convergence rate. In general, it is desirable to have a fast convergence rate and low feedback rate. In [6], it is shown that binary feedback can be used for the carrier phase alignment.

If time division duplexing (TDD) is used, the phase alignment can be achieved by estimating the channel coefficient from the FC to each sensor based on the channel reciprocity. If the FC broadcasts a pilot signal, each sensor can estimate the channel coefficient. Using this estimated channel coefficient, the phase of the signal transmitted by each sensor can be adjusted for coherent combining at the FC. In this case, adaptive algorithms for the carrier phase alignment are not required. However, there are other issues. Provided that the FC can have a target signal-to-noise ratio (SNR), if all the sensors transmit signals for distributed beamforming, the resulting SNR can be higher than the target SNR. To save the energy consumption or increase the energy efficiency, it would be necessary that the number of active sensor nodes can be minimized, while the target SNR is met in distributed beamforming [7].

In this paper, we propose an approach that allows us to select sensors to maximize the energy efficiency in distributed beamforming. Throughout the paper, it is assumed that TDD is employed and the carrier phase alignment is established.

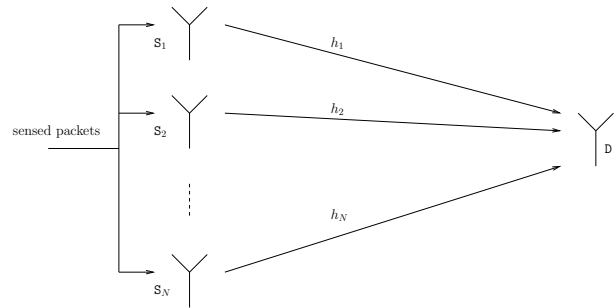


Fig. 1. System model.

For distributed beamforming, we have the following four (4) phases: 1) channel state information (CSI) acquisition phase; 2) sensor selection phase; 3) beamforming phase; 4) cooperative transmission phase. The total energy consumption of the four phases is considered in selecting sensors.

The remainder of this paper is organized as follows. The system model is presented in Section II. A cooperative transmission scheme consisting of four phases is proposed in Section III. Section IV provides analyses for energy consumption. Numerical results are shown in Section V. Section VI concludes the paper with some remarks.

II. SYSTEM MODEL

We consider a wireless sensor network, illustrated in Fig. 1, consisting of N sensors, denoted by S_1, S_2, \dots, S_N , and an FC or destination, denoted by D . All of the sensors and FC are equipped with single antennas. Periodically, the FC collects messages (representing sensors’ decisions over the time) that are commonly available at all the sensors. Furthermore, the sensors use the same coding scheme of L -length codewords. At a certain time, a common packet of L elements, which is denoted by s , will be sent to D .

The channels from the sensors to FC are assumed to be flat fading. Denote by $h_i, i = 1, 2, \dots, N$, the channel gain from S_i to D , which is independent and identically distributed (iid) for different i . This assumption is reasonable under the following scenario: *i*) the sensors are uniformly distributed in approximately equal distance from the FC, *ii*) there are rich scatterers around the FC. As we consider transmissions in TDD mode, this channel gain is identical to that for the link

from D to S_i . Let $\alpha_i = |h_i|^2$. We denote by $f(\alpha)$ the common probability density function (pdf) of the α_i 's.

In this paper, we consider cooperative transmissions based on distributed beamforming. Provided that each sensor knows the channel gains, multiple sensors can cooperatively form distributed beamformers. Let \mathcal{M} , $\mathcal{M} \subset \{1, 2, \dots, N\}$, whose cardinality is denoted by M , be the set of indices of sensors that are activated for distributed beamforming. For the case of using a single sensor S_i , $\mathcal{M} = \{i\}$ and $M = 1$, while $\mathcal{M} = \{1, 2, \dots, N\}$ and $M = N$ if all the available sensors are selected. For given \mathcal{M} , the received signal at the FC is given by

$$\mathbf{y} = \sum_{i \in \mathcal{M}} w_i h_i \mathbf{s} + \mathbf{w}, \quad (1)$$

where w_i is the beamforming weight at S_i and $\mathbf{w} \sim \mathcal{CN}(0, N_0 \mathbf{I})$ is the background noise at D. The transmission power at S_i is therefore given by $P_i = |w_i|^2$. With cooperative transmissions, energy efficiency can be improved because a less total of transmission powers would be required. Let assume SNR_D be the SNR level required by D for successfully decoding the transmitted message. For example, when each message of RL bits is encoded to become a packet of L symbols and the random Gaussian code is employed, $\text{SNR}_D = (2^R - 1)$. If only a single random sensor S_i , who knows its own channel gain h_i , is used to transmit, it is required that $w_i = \sqrt{N_0 \text{SNR}_D} \frac{h_i^*}{\alpha_i}$, resulting a transmission power of $P_i = \frac{N_0 \text{SNR}_D}{\alpha_i}$, to achieve the target SNR. On the other hand, if multiple sensors S_i 's, $i \in \mathcal{M}$, are randomly chosen, the beamforming weights and total transmission power are $w_i = \sqrt{N_0 \text{SNR}_D} \frac{h_i^*}{\sum_{j \in \mathcal{M}} \alpha_j}$ and $P_{\mathcal{M}} = \sum_{i \in \mathcal{M}} |w_i|^2 = \frac{N_0 \text{SNR}_D}{\sum_{i \in \mathcal{M}} \alpha_i}$, respectively. If $f(\alpha)$ is an exponential function (i.e., Rayleigh fading), $\mathbb{E}[P_i] = \infty$ due to a non-zero probability of $|h_i|^2$ being zero, i.e., an infinity energy would be required when a single random sensor is used. However, under this scenario, $\mathbb{E}[P_{\mathcal{M}}]$ is finite when $M > 1$. This case, as an example, shows a great benefit of cooperative transmissions for energy saving.

In the discussion above, the CSI knowledge is assumed to be available at the both sides, the sensors and FC, without any extra cost. Under this assumption, a trivial solution to energy saving is to utilize all the available sensors. However, there must be the costs for CSI acquisition in cooperative schemes. If this cost is taken into account, activating all the available sensors may not necessarily be the best option. In addition, how to obtain CSI at sensors and, if not all of available sensors are selected, how to select good sensors need to be studied. In the next section, we propose a scheme to select M sensors with best channel gain powers among N available sensors. Furthermore, analyses in Section IV can help in deciding a proper number of selected sensors to achieve smallest energy per message for a system with given set of parameters.

III. PROPOSED COOPERATIVE TRANSMISSION SCHEME

As mentioned, the FC requests information from sensors periodically. In response to a request, the sensors cooperatively transmit until the transmission of all the information in request

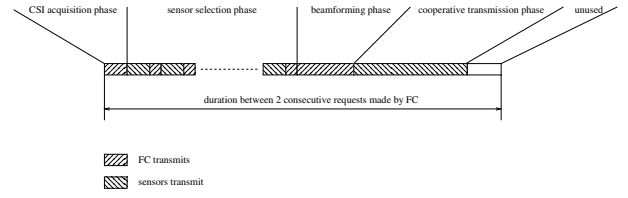


Fig. 2. Proposed Scheme.

is completed. In our proposed scheme of four phases, we assume that the channel gains keep constant in a duration longer than the time required for all the four phases (i.e., slow fading). That is, after formed, distributed beamformers can be used by activated sensors until the last symbol of current packet is sent to the FC without any beamforming change.

The transmission of a packet is performed in four phases, illustrated in Fig. 2, as follows:

- **CSI acquisition phase:** The FC broadcasts a pilot packet of Q_t symbols to the sensors with power P_t . This pilot packet also informs the sensors that the current information is requested by the FC. Based on the received signals of pilot packets, each sensor estimates its channel gain. The energy for this phase is given by

$$E_{bc} = Q_t P_t. \quad (2)$$

In general, there is a tradeoff between the transmission power and the number of pilot symbols in order to achieve a certain estimation quality. If the transmission power is lowered, a larger number of pilot symbols is required. Nevertheless, this required amount of energy is independent of the message to be transmitted and the number of sensors. We consider this energy as a fixed parameter.

- **Sensor selection phase:** The M best sensors are selected by a distributed method based on a binary splitting algorithm [8], [9]. In principle, this phase consists of a sequence of time slots (or rounds), in which the sensors independently polls for selection. In each slot, each sensor compares its channel gain power with a threshold α_{th} to decide whether or not to vote to be selected. Voting decisions are in the form of pilot packets of Q_t symbols that are transmitted with power P_t and that can be used by the FC to detect channel gains in the time slots accessed by single sensor. At the end of the time slot, the FC feeds a ternary indicator back to all the sensors based on the current state: *i*) no sensor has voted, the feedback indicator is an idle (0), *ii*) only one sensor has voted, the feedback indicator is a success (1), *iii*) more than one sensor have voted, the feedback indicator is a collision (e). Define the set of unselected sensors and the set of sensors with the right to vote are backlogged and active sets, respectively. At the beginning of each time slot, α_{th} is recomputed based on the feedback such that the probabilities of that each sensor deciding to vote in the active set is $\frac{P_c}{N_b}$ if the

active set is same as the backlogged set, or p_s otherwise. Here, p_c and p_s are parameters of the algorithm and N_b is the number of sensors in the backlogged set [10]. Algorithm 1 illustrates the behavior of a single sensor S_i in the selection phase. In this algorithm, $get(feedback)$ is the procedure of detecting the feedback indicator sent by FC and $split(a, b, p) \triangleq F^{-1}(pF(a) + (1-p)F(b))$, where $F(\alpha) = \int_0^\alpha f(x)dx$ is the cumulative distribution function of the α_i 's. Note that under the iid channel gain scenario, the threshold values are identical for all the sensors and the sensors with higher channel gain powers are always selected while this algorithm only provides a fairness if the sensors have different channel distributions. Triggered by a single request made by FC, let L_s and L_t be the number of feedback indicators used by FC and the number of transmissions used by all the sensors in the selection phase, respectively. Note that, although closed to each other, $L_s \neq L_t$ in general because there could be more than one or no sensor transmit in a time slot. The energy for selection phase is given by

$$E_{sel} = L_t Q_t P_t + L_s Q_f P_f, \quad (3)$$

where Q_f and P_f are the length of and transmission power for feedback indicators, respectively. From (3), we can see that small L_s and L_t are desirable for saving energy in this phase. Given M and aiming at reducing the average energy in this phase, the parameters p_c and p_s can be optimized with a relatively heavy computation. However, suboptimal choice can be $p_c = 1$ and $p_s = 0.5$ for sufficiently large N with a small loss of performance [8]–[10].

- **Beamforming phase:** Cooperative beamformers are efficiently constructed if each of selected sensors knows the sum of channel power gains of all the selected sensors. Let $S_{[1]}, S_{[2]}, \dots, S_{[M]}$ denote the M selected sensors in the descending order. Since $h_{[i]}, i = 1, 2, \dots, M$, are known to D, it transmits with power $\frac{N_0 \text{SNR}_s}{\alpha_{[M]}}$, where SNR_s is an identical required SNR at all the sensors, in Q_b symbols to inform all the selected sensors the parameter $\sum_{i=1}^M \alpha_{[i]}$. The energy for this phase is given by

$$E_{bf} = \frac{Q_b N_0 \text{SNR}_s}{\alpha_{[M]}}. \quad (4)$$

As can be seen from (4), a larger M is used, a smaller $\alpha_{[M]}$ can be, which leads to a larger amount of energy to be required.

- **Cooperative transmission phase:** M selected sensors cooperatively send the packet to the FC in a L -symbol duration. The i th best sensor, $S_{[i]}$, transmits with power $\frac{N_0 \text{SNR}_d \alpha_{[i]}}{(\sum_{j=1}^M \alpha_{[j]})^2}$. The energy for this phase is given by

$$E_{co} = \frac{LN_0 \text{SNR}_d}{\sum_{i=1}^M \alpha_{[i]}}. \quad (5)$$

As can be seen from (5), more sensors are selected, less energy would be required.

Algorithm 1 Algorithm performed in the selection phase.

Input: $N, M, p_c, p_s, F(\cdot)$

Output: *selected*

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1: selected  $\leftarrow 0$ ;
2: feedback  $\leftarrow 0$ ; prevfeedback  $\leftarrow 0$ ;
3:  $N_{sel} \leftarrow 0$ ;  $N_{bl} \leftarrow N$ ;
4: voted  $\leftarrow 0$ ; level  $\leftarrow 0$ ;  $\alpha_{low}(level) \leftarrow \infty$ ;
5: while  $N_{sel} < M$  do
6:   if feedback = 1 then
7:      $N_{sel} \leftarrow N_{sel} + 1$ ;  $N_{bl} \leftarrow N_{bl} - 1$ ;
8:     if voted then
9:       selected  $\leftarrow 1$ ;
10:    end if
11:  end if
12:  if !selected then
13:    if feedback =  $e$  then
14:      level  $\leftarrow level + 1$ ;
15:       $\alpha_{up}(level) \leftarrow \alpha_{up}(level - 1)$ ;
16:       $\alpha_{low}(level) \leftarrow$ 
17:         $split(\alpha_{low}(level - 1), \alpha_{up}(level - 1), p_s)$ ;
18:    else
19:      if feedback = 0 and prevfeedback =  $e$  then
20:         $\alpha_{up}(level) \leftarrow \alpha_{low}(level)$ ;
21:         $\alpha_{up}(level - 1) \leftarrow \alpha_{low}(level)$ ;
22:         $\alpha_{low}(level) \leftarrow$ 
23:           $split(\alpha_{low}(level - 1), \alpha_{up}(level - 1), p_s)$ ;
24:      else
25:        if level > 0 then
26:          level  $\leftarrow level - 1$ ;
27:        else
28:           $\alpha_{up}(level) \leftarrow \alpha_{low}(level)$ ;
29:           $\alpha_{low}(level) \leftarrow split(0, \alpha_{up}(level), \frac{p_c}{N_{bl}})$ ;
30:        end if
31:      end if
32:    end if
33:     $\alpha_{th} \leftarrow \alpha_{low}(level)$ ; voted  $\leftarrow (|h_i|^2 > \alpha_{th})$ ;
34:    prevfeedback  $\leftarrow feedback$ ;
35:    get(feedback);
36:  end if
37: end while

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The total energy required to transmit a packet is given by

$$E_{tot} = E_{bc} + E_{se} + E_{bf} + E_{co}. \quad (6)$$

The next section analyzes the expectation of this total energy.

IV. AVERAGE ENERGY PER MESSAGE ANALYSES

In this section, we analyze the expected energy required to transmit a message for a given set of parameters $N, M, \text{SNR}_d, \text{SNR}_s, P_t, Q_t, P_f, Q_f, Q_b$, and L . Throughout this section, we assume that $f(\alpha) = \exp(-\alpha)$, $\alpha \geq 0$ ¹.

In the selection phase, it is not easy to derive the expectations of L_t and L_s for different M and N . However,

¹This assumption, used for analyses, is not necessary for the operation of scheme proposed in the previous section.

in the literature [9], [10], the asymptotic expressions (i.e., $N = \infty$) can be used as good approximates for large networks. Therefore, we derive lower bounds for $\mathbb{E}[L_t]$ and $\mathbb{E}[L_s]$ for $N = \infty$ and use them as approximations for designing purposes.

Let $l_t(M)$ and $l_s(M)$ respectively denote the expectations of transmissions and rounds for selecting M sensors among $N = \infty$ available sensors with $p_c = 1$ and $p_s = 0.5$. They follows the recursive equations:

$$l_s(M) = \frac{1}{e-1} \left[e + \lambda(M) + \sum_{m=1}^{M-1} \frac{l_s(M-m)}{m!} \right]; \quad (7)$$

$$l_t(M) = \frac{1}{e-1} \left[e + \mu(M) + \sum_{m=1}^{M-1} \frac{l_t(M-m)}{m!} \right], \quad (8)$$

where

$$\lambda(M) = \sum_{n=2}^{\infty} \frac{a(M,n)}{n!} \quad \text{and} \quad \mu(M) = \sum_{n=2}^{\infty} \frac{b(M,n)}{n!}. \quad (9)$$

The physical meaning of $a(M,n)$ (resp. $b(M,n)$) is that it represents the expected number of rounds (resp. transmissions) required for selecting M sensors after a collision of size n with splitting parameter $p_s = 0.5$. Note that $a(M,n) = a(n,n)$ and $b(M,n) = b(n,n)$ if $M > n$. It can be proved that $a(1,n) > \log_2(n)$ [8]. Furthermore, $a(M,n) > a(M-1,n) + 1$ for $M \leq n$. Analogously, we can also prove that $b(1,n) > n-1$ and $b(M,n) > b(M-1,n) + 1$ for $M \leq n$. Thus, for $M \leq n$ $a(M,n) > \log_2(n) + M - 1$ and $b(M,n) > n + M - 2$ while $a(M,n) > \log_2(n) + n - 1$ and $b(M,n) > 2n - 2$ for $M > n$. Hence,

$$\lambda(M) > \sum_{n=2}^M \frac{\log_2(n) + n - 1}{n!} + \sum_{n=M+1}^{\infty} \frac{\log_2(n) + M - 1}{n!}; \quad (10)$$

$$\mu(M) > \sum_{n=2}^M \frac{2n-2}{n!} + \sum_{n=M+1}^{\infty} \frac{n+M-2}{n!}. \quad (11)$$

Using the ratio test, we can show that the lower bounds for $\lambda(M)$ and $\mu(M)$ are convergent. From (7), (8), (9), (10), and (11), we can find lower bounds for $l_s(M)$ and $l_t(M)$, which are denoted by $\tilde{l}_s(M)$ and $\tilde{l}_t(M)$. We rely on these bounds to find approximates of average energy for sensor selection phase as

$$\mathbb{E}[E_{\text{sel}}] \approx \tilde{l}_t(M) Q_t P_t + \tilde{l}_s(M) Q_f P_f. \quad (12)$$

When M increases, $\mathbb{E}[E_{\text{sel}}]$ is expected to be increased since $\tilde{l}_t(M)$ and $\tilde{l}_s(M)$ increase with M .

In the beamforming phase, since the α_i 's are independent and exponentially distributed, $\alpha_{[M]}$, the $(N-M+1)$ th element in the ascending order list of $\{\alpha_i\}_{i=1}^N$, can be written as [11]

$$\alpha_{[M]} = \sum_{i=1}^{N-M+1} \frac{1}{N-i+1} \beta_i, \quad (13)$$

where $\beta_i = i(\alpha_{[N-i+1]} - \alpha_{[N-i]})$ and the β_i 's are independent and distributed as same as the α_i 's. Let $f_{[M]}(\alpha)$ denote the pdf

of $\alpha_{[M]}$. From (13), the closed-form expression of $f_{[M]}(\alpha)$ can be found, which is in a form of a weighted sum of exponential functions with different weights. The average energy for the beamforming phase is given by

$$\mathbb{E}[E_{\text{bf}}] = Q_b N_0 \text{SNR}_s \int_0^{\infty} \frac{1}{\alpha} f_{[M]}(\alpha) d\alpha. \quad (14)$$

In computing $\mathbb{E}[E_{\text{bf}}]$, a numerical method in [12] can be used. Note that $f_{[M]}(0) = 0$ if $M < N$. Therefore, $\mathbb{E}[E_{\text{bf}}]$ is finite and it increases with M if not all of available sensors are selected. The case of selecting all the available sensors (i.e., $M = N$) is not practical for Rayleigh fading as an infinite energy for beamforming phase would be required.

In considering the cooperative transmission phase, we define $\alpha_{[1:M]} = \sum_{i=1}^M \alpha_{[i]}$ and denote its pdf by $f_{[1:M]}(\alpha)$. From (14), we have

$$\alpha_{[1:M]} = \sum_{i=1}^{N-M+1} \frac{M}{N-i+1} \beta_i + \sum_{i=N-M}^N \beta_i \quad (15)$$

Similar to $f_{[M]}(\alpha)$, the closed-form expression of $f_{[1:M]}(\alpha)$ can be found. The average energy for the cooperative transmission phase is given by

$$\mathbb{E}[E_{\text{co}}] = L N_0 \text{SNR}_d \int_0^{\infty} \frac{1}{\alpha} f_{[1:M]}(\alpha) d\alpha. \quad (16)$$

Opposed to the case of using a single random sensor, $\mathbb{E}[E_{\text{co}}]$ is finite even that $M = 1$ (i.e., only the best sensor is activated). Therefore, energy saving can be exploited by the sensor selection. Since $\alpha_{[1:M]}$ is the weighted sum of N iid random variables as can be seen from (15), a diversity of order N (selection diversity) can be achieved for any number of selected sensors with better channel gain powers provided that N sensors are available. In addition, the weights in (15) can be seen as a form of a cooperation (or array) gain. Note that the weights increase with M until 1 is reached. Therefore, more energy is saved in this phase when more sensors are selected.

In the design point of view, a proper choice of M , the number of activated sensors, is dependent on the system parameters. For example, assume that L , the packet length, varies while the others parameters are fixed. If L increases, i.e., more information per request is transmitted, $[E_{\text{co}}]$ increases and becomes more dominant in the total energy. Therefore, M should be increased to exploit more benefit of cooperative transmissions. On the other hand, if less information is sent per requested, M should be decreased to reduce the overhead including sensor selection and beamforming.

V. NUMERICAL AND SIMULATION RESULTS

In our simulations, channel gain powers are assumed to be independent and exponentially distributed with unit mean (i.e., Rayleigh fading). The noise powers are identically unit at all the nodes. Both the SNRs required at FC and sensors are assumed to be $\text{SNR}_d = \text{SNR}_s = 1$. In addition, the energy for CSI acquisition is assumed to be fixed and constant, which is $E_{\text{bc}} = Q_t P_t = 5$. In addition, in the sensor selection phase, we assume that the transmission power for (ternary) feedback

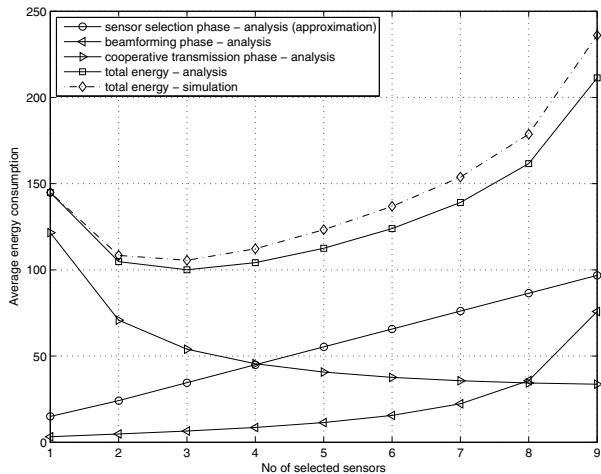


Fig. 3. Energy consumption for different number of selected sensors

$P_f = 1$ and the number of symbols in a feedback packet $Q_f = 1$. In the beamforming phase, the FC informs the sum of channel gain powers of the channels from selected sensors in an encoded packet of $Q_b = 8$ symbols.

For a packet length of $L = 300$ and $N = 10$ available sensors, Fig. 3 presents the average energy consumed per packet for different numbers of selected sensors (M varies from 1 to 9). It can be seen that the energy for cooperative transmissions (i.e., E_{co}) is always improved when more BSs are selected. However, the rate of improvement is reduced with M since the contributions of the worst sensors become less significant in comparison with that of the better sensors. On the other hand, since the expected number of rounds and transmissions for distributed sensor selection increase with M , the energy for the selection phase (i.e., E_{sel}) is shown to increase with M . Furthermore, because a higher power is required to inform the sum of channel gain powers (inversely proportional to the M th largest channel gain powers) when M increases, the energy for beamforming phase (i.e., E_{bf}) dramatically increases as M becomes larger. As such, the total energy would be a U-shape function of M as shown in Fig. 3. Simulation results clearly confirm this observation.

As discussed, the best number of selected sensors is dependent on the system parameters. Fig. 4 shows the best numbers of selected sensors (which requires the least energy) for different values of the packet length. As expected, the best number of selected sensors increases with the packet length in order to further exploit the energy efficient benefit in cooperative transmission phase.

VI. CONCLUDING REMARKS

In this paper, we have discussed the energy utilization for a proposed scheme of cooperative transmission. In our proposed scheme, four phases: CSI acquisition, sensor selection, beamforming, and cooperative transmission, are required. Our analyses showed that when more sensors are selected to send information to FC, energy efficiency in cooperative

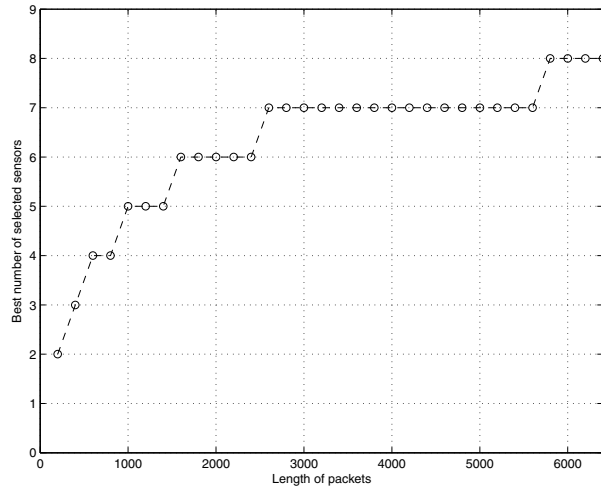


Fig. 4. Best numbers of sensors for different length of packets

transmission phase is improved but it requires more energy for sensor selection and beamforming. The analyses can be used as a guideline for deciding the number of selected sensors as illustrated by numerical results.

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