Recent advances in multi-object estimation

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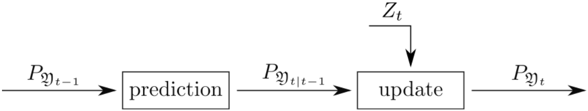
1. Multi-object filtering framework: basics
2. Traditional solutions: strengths and weaknesses
3. Stochastic populations for multi-object filtering

Bayesian filtering

Principle



Z_t

$P_{\mathfrak{Y}_{t-1}}$ → prediction → $P_{\mathfrak{Y}_{t|t-1}}$ → update → $P_{\mathfrak{Y}_t}$

- $P_{\mathfrak{Y}_t}$: "information" known by operator at time t on objects of interest or *targets*
- Z_t: observations produced by the sensor system at time t and collected by the operator

Sensor system for target tracking

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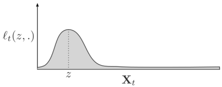
Stochastic description

Likelihood $\ell_t(z, x)$: how likely is obs. z to come from a target with state x?



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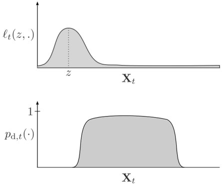

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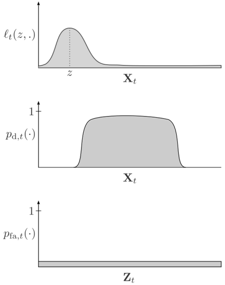
Probability of detection $p_{\mathrm{d},t}(x)$: how likely is a target with state x to be detected?

Probability of false alarm $p_{\mathrm{fa},t}(z)$: how likely is the sensor to produce a false alarm with state z?



Sensor system for target tracking (cont.)

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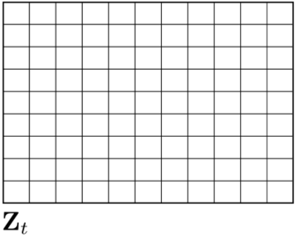
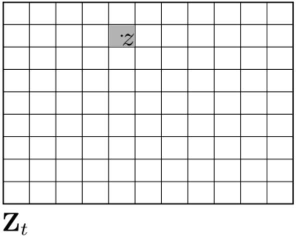


- *Discrete* observation space $\mathbf{Z}_t$

Sensor system for target tracking (cont.)



- *Discrete* observation space $\mathbf{Z}_t$
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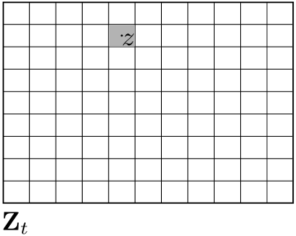

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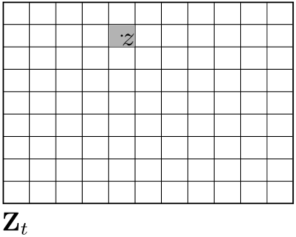

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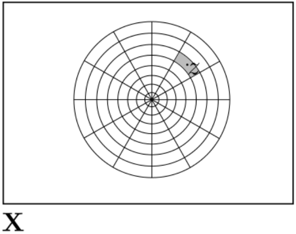

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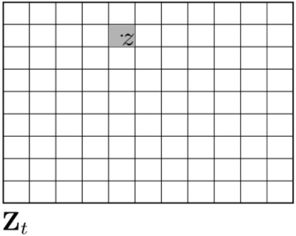

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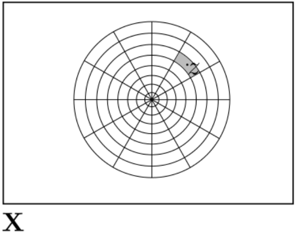
- $\mathbf{Z}_t$ projected onto $\mathbf{X}$ shapes the sensor field of view (FoV)
- Outside of the sensor FoV, $p_{\text{d},t}$ is always zero (i.e. no target detection)



Multi-object filtering: common assumptions

Common assumptions (time t)

1. Targets behave independently
2. Observations are produced independently
3. At most one observation per target (if none, target is *miss-detected*)
4. At most one target per observation (if none, obs. is a *false alarm*)

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The assumptions above...

- 1. ... simplify the estimation problem (notably the data association)
- 2. ... will be used in the context of this presentation
- 3. ... are *not* necessary in the general multi-object estimation framework

1. Multi-object filtering framework: basics
2. Traditional solutions: strengths and weaknesses
    - The track-based approach
    - The RFS-based approach
3. Stochastic populations for multi-object filtering

The track-based approach

General principle

"A potential target = one track."

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Track representation

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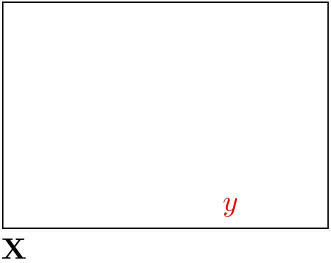

A track y is...

- ... *identified* by its state distribution

The track-based approach

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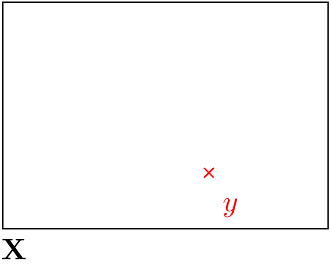

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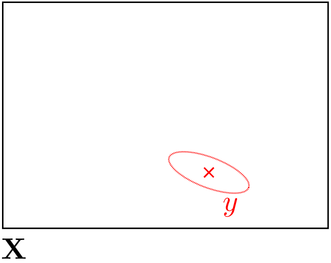

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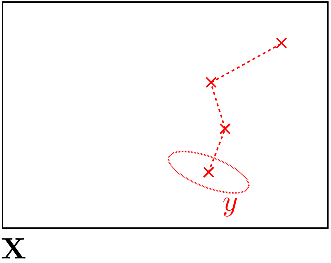

A track y is...

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- ... *described* by its history of past estimates

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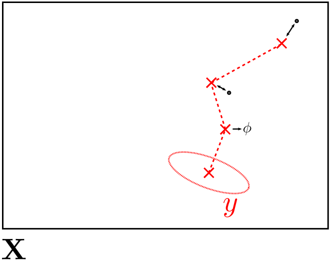
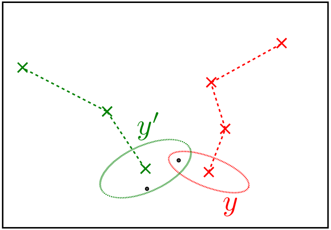


A track y is...

- ... *identified* by its state distribution (e.g. mean + covariance)
- ... *described* by its history of past estimates
- ... *characterised* by its observation path

Track update

Data association



X

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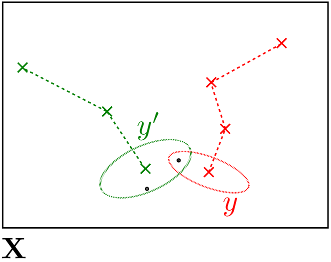

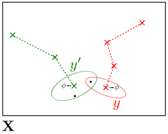

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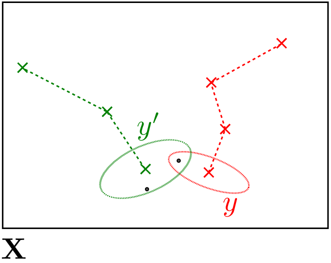

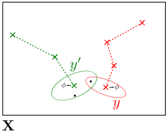

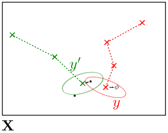

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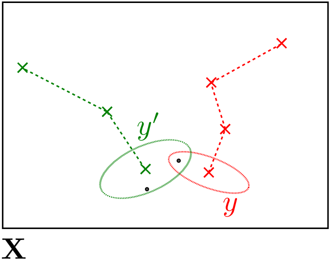

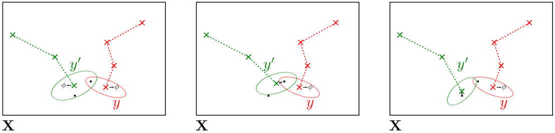

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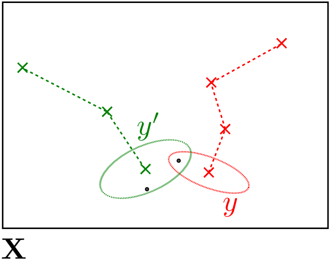

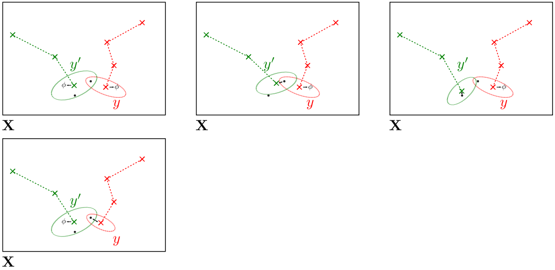

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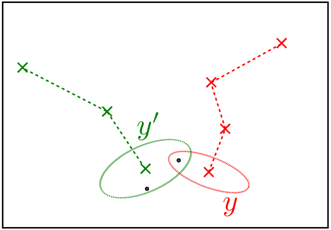

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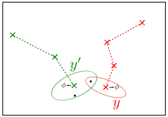

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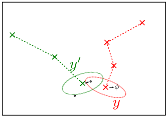

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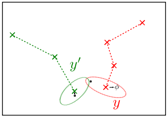

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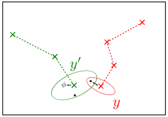

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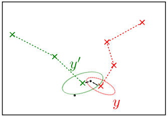

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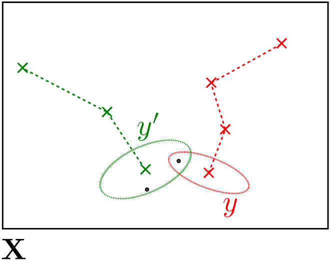

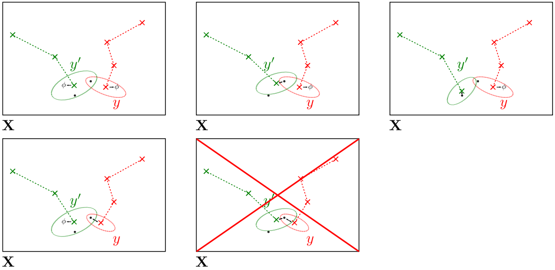

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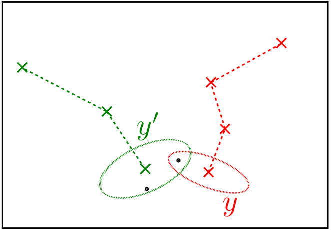

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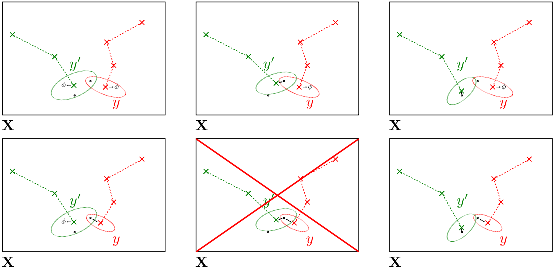

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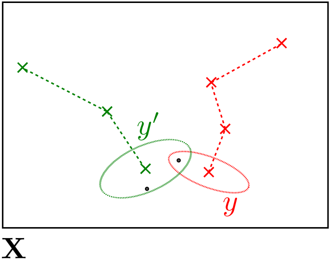

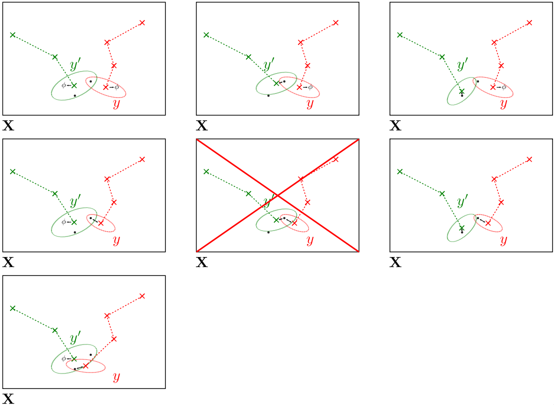

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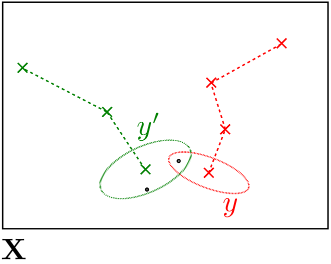

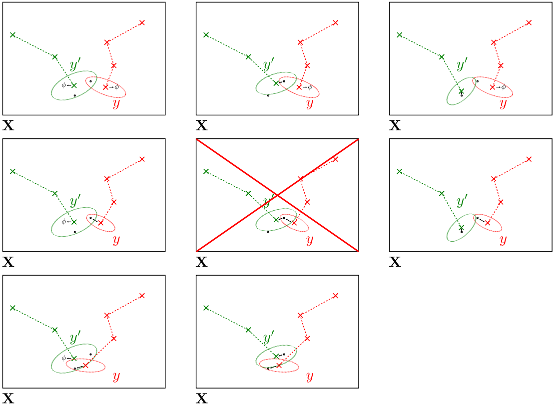

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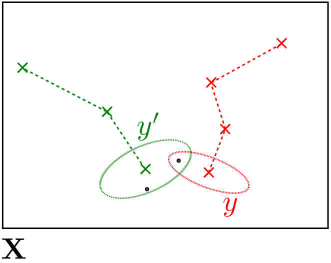

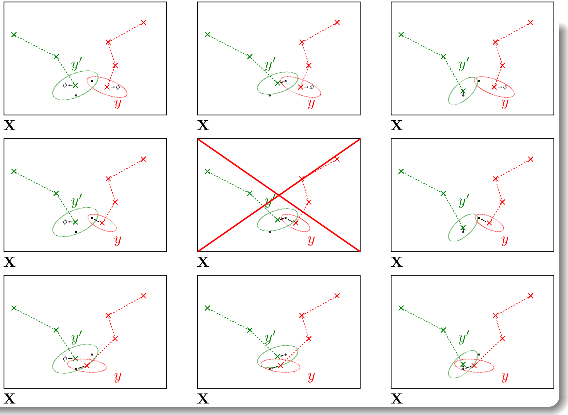

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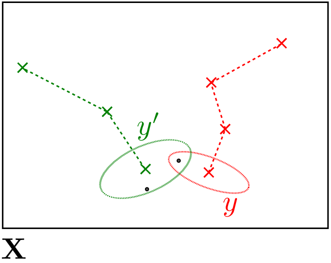

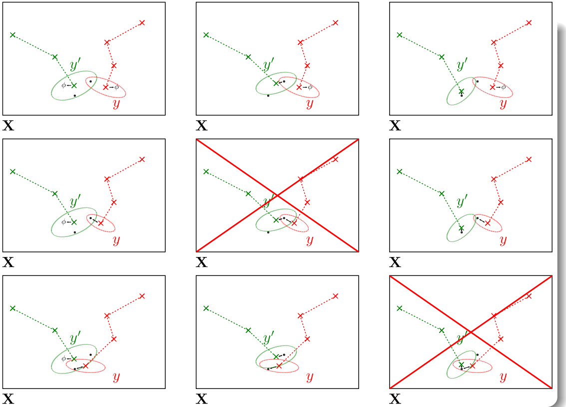

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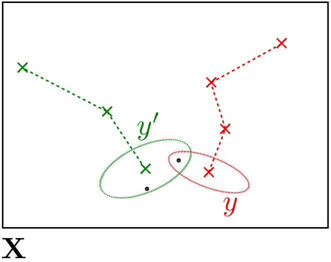
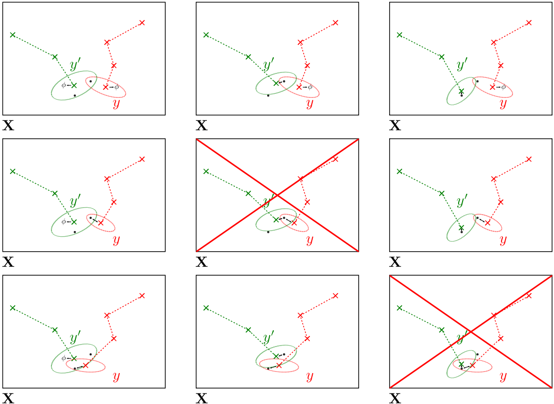
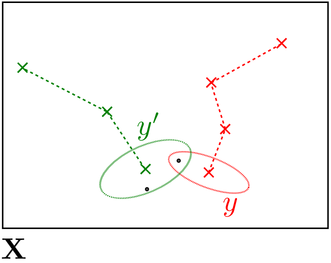


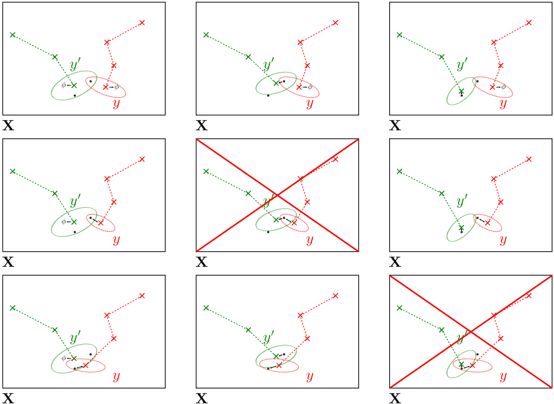


What is propagated to the next step?

Track update

Data association



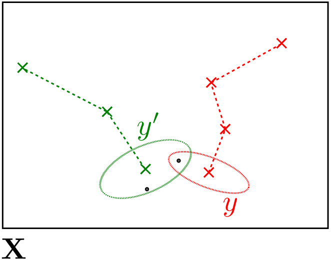


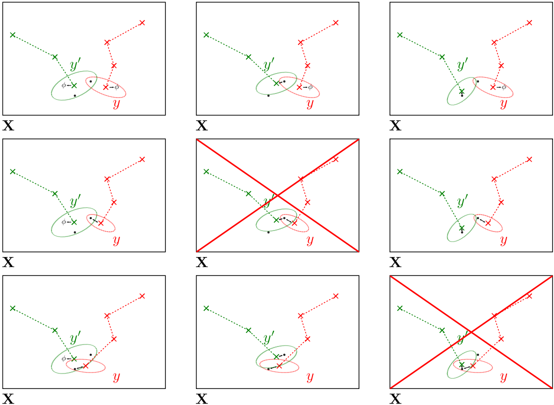
What is propagated to the next step?

All configurations, with associated probabilities → MHT filter

Track update

Data association





What is propagated to the next step?

All configurations, with associated probabilities → MHT filter

A weighted combination of all configurations → JPDA filter

Challenges

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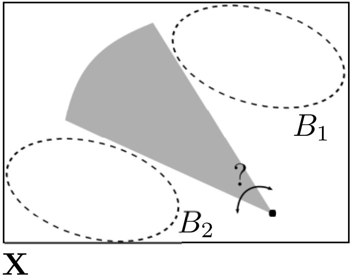

Sensor management problem: explore B_1 or B_2?

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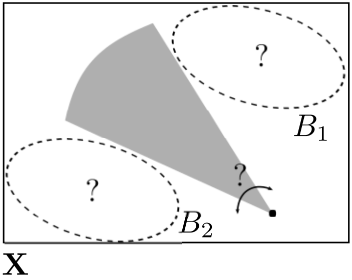

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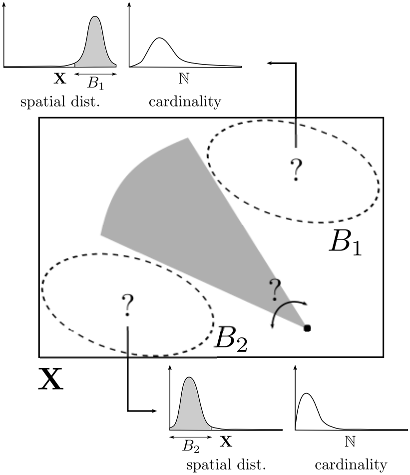

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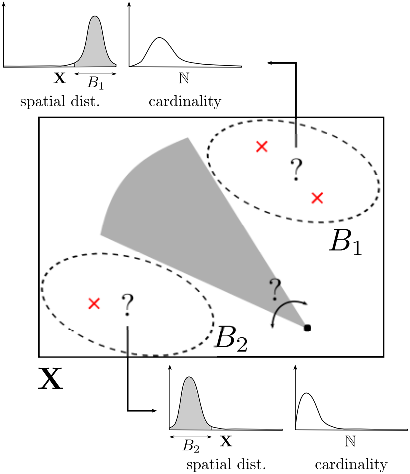

Sensor management problem: explore B_1 or B_2?

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How many tracks to create? Where?

Challenges (cont.)

Track creation/deletion

Challenges (cont.)

Track creation/deletion

- When to create tracks?

Challenges (cont.)

Track creation/deletion

- When to create tracks? For every new observation?

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- When to delete tracks?

Challenges (cont.)

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Challenges (cont.)

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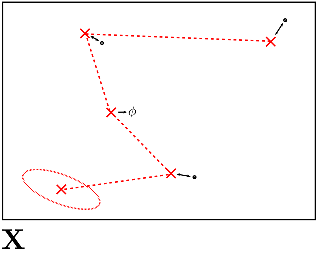

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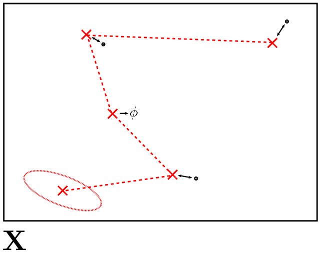

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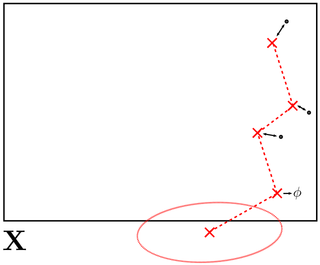

Because the target it represents has "probably" left the scene?

The track-based approach: pros and cons

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Can handle spatial information on tracks “optimally”

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The RFS-based approach

General principle

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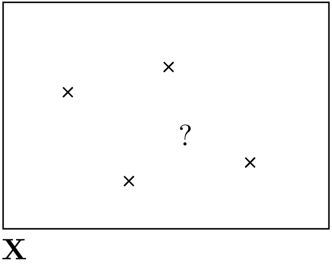

The target RFS Ξ...

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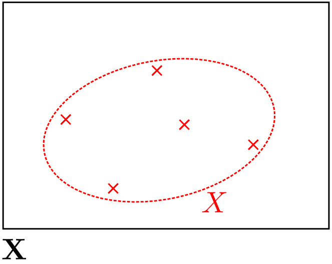

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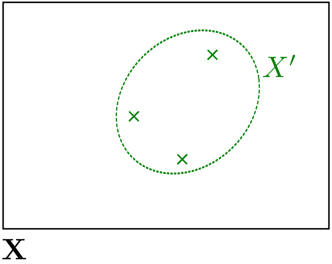

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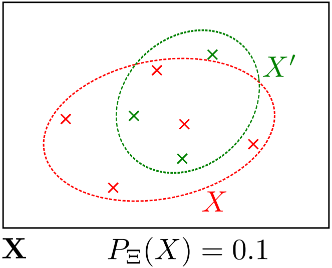

$$P_\Xi(X) = 0.1$$
$$P_\Xi(X') = 0.03$$
$$\ldots$$

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First-moment density for usual RFS-based filters

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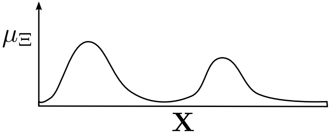
First-moment density μ_Ξ



Approximate description of RFS Ξ

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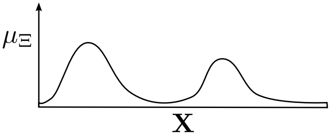

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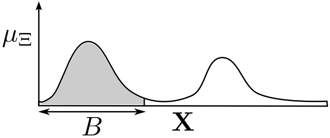

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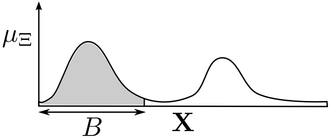

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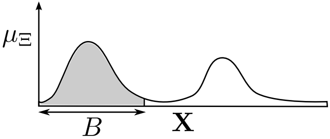

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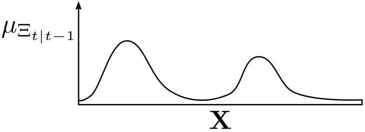

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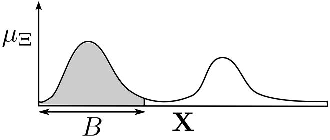

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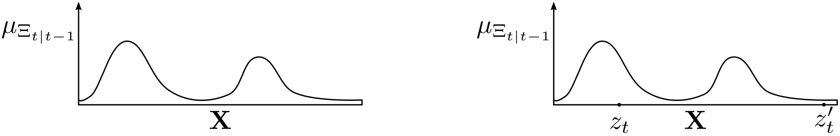

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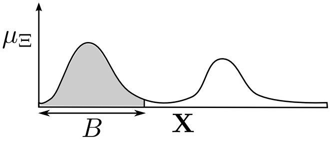

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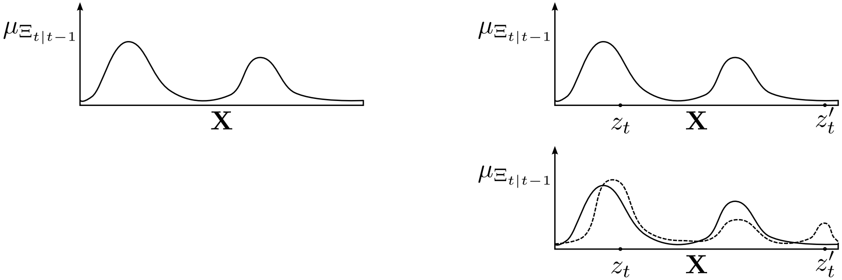

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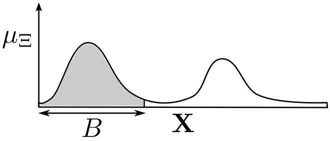

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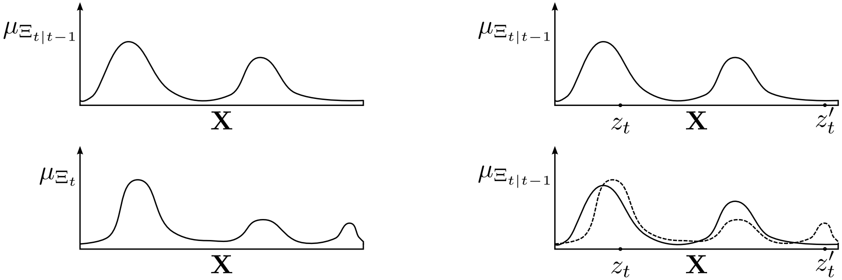

Challenges

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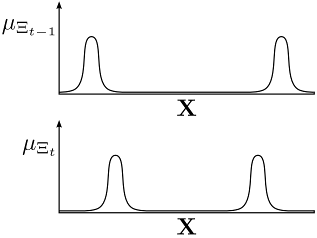
No equivalent of track history: what are the consequences?



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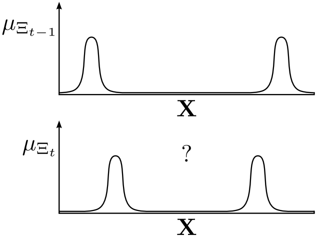

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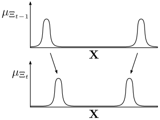

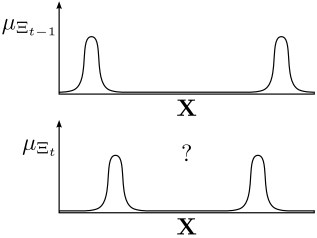

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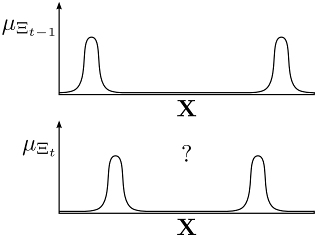

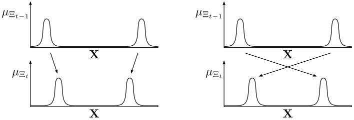

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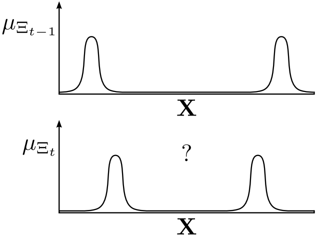

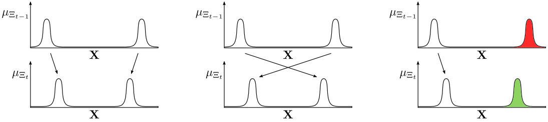

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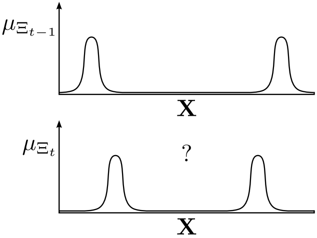

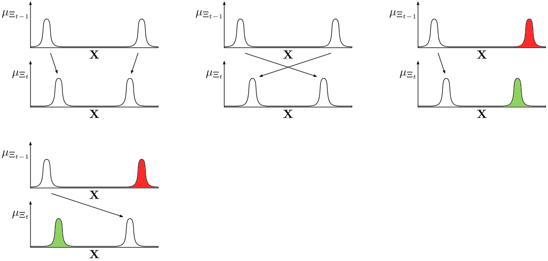

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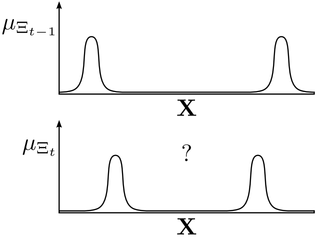

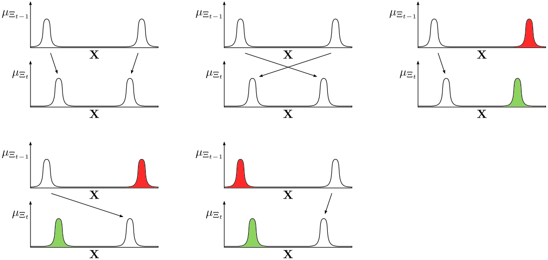

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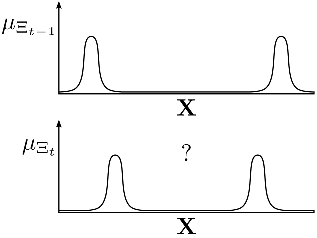

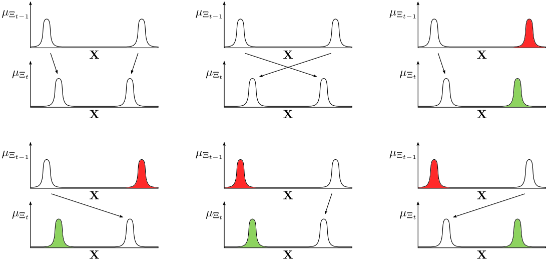

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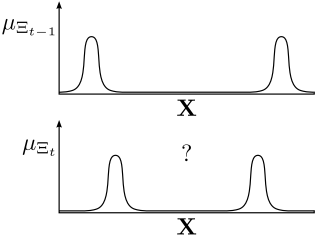

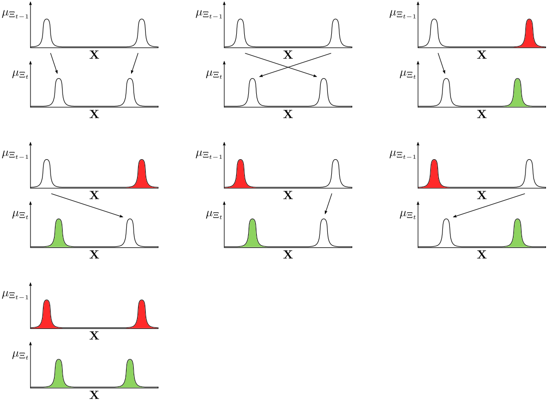

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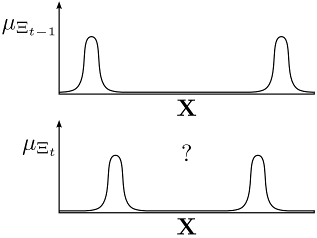

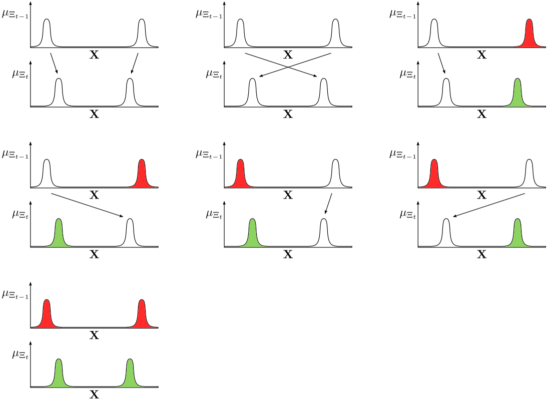

No inherent solution to link individuals from successive populations

Introducing labelling on top of RFS framework recently explored (Labeled Multi-Bernoulli filter, Vo et al.)

The RFS-based approach: pros and cons

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Provides probability framework incorporating all system uncertainties

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Lacks intuitive interpretation (on some points)

- Poisson (resp. i.i.d.) approximation in PHD (resp. CPHD) filter

1. Multi-object filtering framework: basics

2. Traditional solutions: strengths and weaknesses

3. Stochastic populations for multi-object filtering
    - Multi-object estimation framework
    - Bayesian filtering
    - Closed-loop sensor management

Estimation framework for stochastic populations

General principle

> “A potential target is represented by a specific amount of information: not too little, but *not too much either*.”

Estimation framework for stochastic populations

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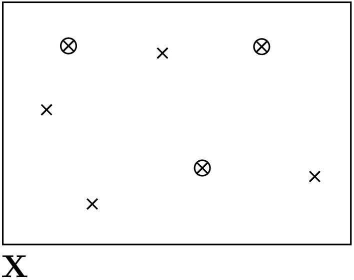
Outline

- Well-defined probabilistic framework, developed by J. Houssineau (supervisor: D. Clark)
- Level of description depends whether individual is *distinguishable* or *indistinguishable*
- Ongoing developments beyond tracking (e.g. sensor management, sensor calibration, performance assessment, ...)

Target distinguishability



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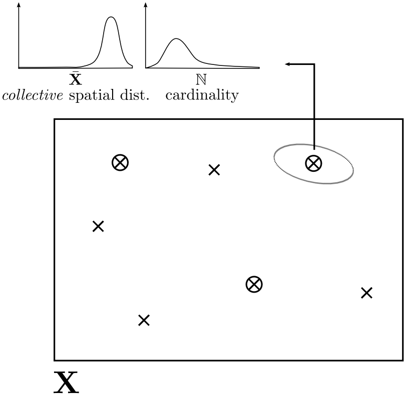

Indistinguishable target:

- *unidentified* member of larger population
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- e.g., "*one of the* potential individuals that entered 10 time steps ago and has not been detected yet"

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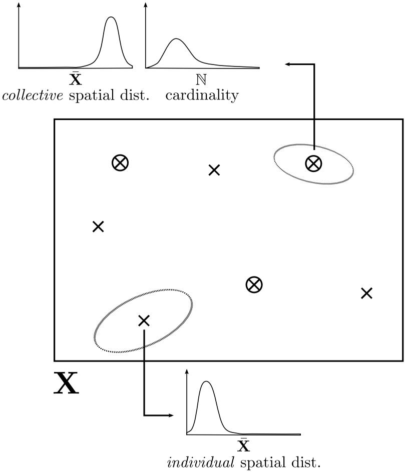


Indistinguishable target:

- *unidentified* member of larger population
- no specific information available
- e.g., "*one of the* potential individuals that entered 10 time steps ago and has not been detected yet"

Distinguishable target:

- individual *characterised* by specific information
- e.g., "*the* potential individual that entered 10 time steps ago and produced observations z_{21}^3, z_{25}^1"

Individual management: where are the targets?

Spatial distribution and "empty state" ψ

Individual management: where are the targets?

Spatial distribution and “empty state” ψ

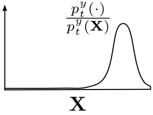
p_t^y

What is the target state?

Individual management: where are the targets?

Spatial distribution and “empty state” ψ

p_t^y =



What is the target state?

If the target is the scene,
what is its state?

Individual management: where are the targets?

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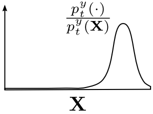
p_t^y =  + $p_t^y(\{\psi\})$

What is the target state?	If the target is the scene, what is its state?	Is the target in the scene?

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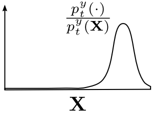

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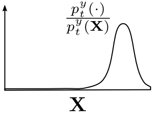
p_t^y	=		+	$p_t^y(\{\psi\})$
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Distinguishable target and associated track

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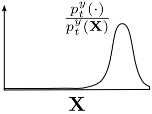

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What is the target state?

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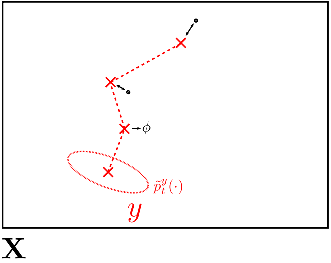
Is the target in the scene?

Distinguishable target and associated track

- *First detection* triggers distinguishability
- Associated track y describes observation path and current info on target state



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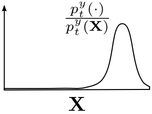
Spatial distribution and "empty state" ψ

p_t^y =



\+ $p_t^y(\{\psi\})$

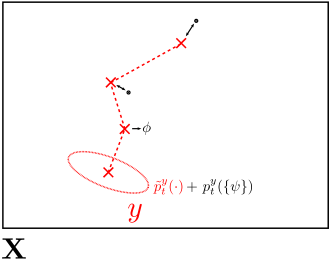
What is the target state?

If the target is the scene, what is its state?

Is the target in the scene?

Distinguishable target and associated track

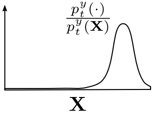
- *First detection* triggers distinguishability
- Associated track y describes observation path and current info on target state
- Usual notion of track, except probability of absence $p_t^y(\{\psi\})$



Individual management: where are the targets?

Spatial distribution and "empty state" ψ

p_t^y =



\+ $p_t^y(\{\psi\})$

What is the target state?

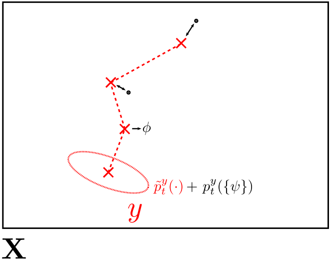
If the target is the scene, what is its state?

Is the target in the scene?

Distinguishable target and associated track

- *First detection* triggers distinguishability
- Associated track y describes observation path and current info on target state
- Usual notion of track, except probability of absence $p_t^y(\{\psi\})$

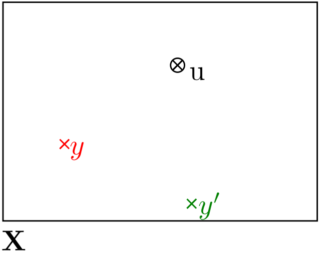
  Note: $1 - p_t^y(\{\psi\})$ does *not* assess track credibility, but its presence in the scene



Population management: which are the true targets?

Joint probability of existence

Assesses *joint* existence of targets:



Population management: which are the true targets?

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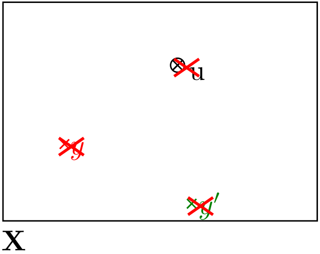
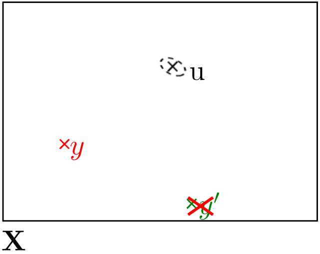


Assesses *joint* existence of targets:

- $P(\emptyset, 0)$

Population management: which are the true targets?

Joint probability of existence



Assesses *joint* existence of targets:

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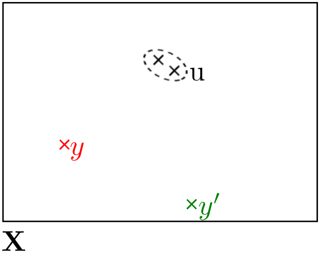
- $P(\{y\}, 1)$

Population management: which are the true targets?

Joint probability of existence



Assesses *joint* existence of targets:

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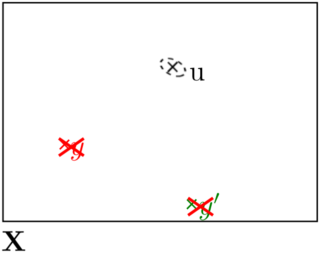
Population management: which are the true targets?

Joint probability of existence



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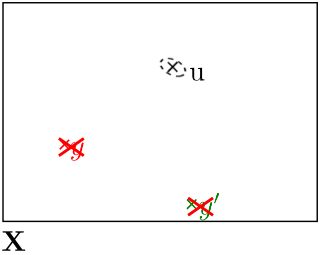
Population management: which are the true targets?

Joint probability of existence



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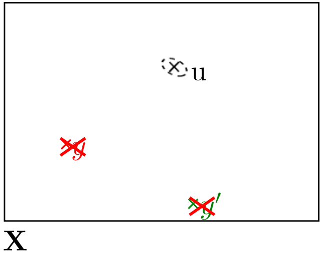
At time t:

- Set of tracks Y_t, m_t populations of indistinguishable targets

Population management: which are the true targets?

Joint probability of existence



Assesses *joint* existence of targets:

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- $P(\{y, y'\}, 2)$
- $P(\emptyset, 1)$
- $\dots$

At time t:

- Set of tracks Y_t, m_t populations of indistinguishable targets
- $\sum_{Y \subseteq Y_t} \sum_{n_1 \geq 0} \cdots \sum_{n_{m_t} \geq 0} P_t(Y, n_1, \dots, n_{m_t}) = 1$

Individual+population management: what is the true multi-target configuration?

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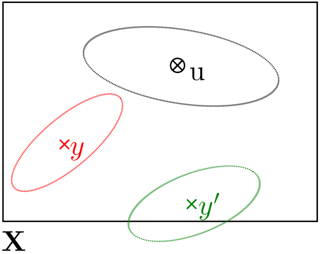
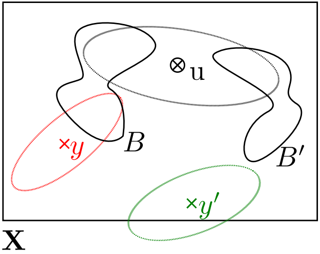
Elementary events



Individual+population management: what is the true multi-target configuration?

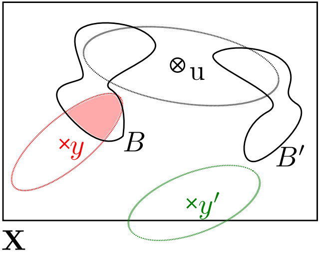
Elementary events



Probability that:

Individual+population management: what is the true multi-target configuration?

Elementary events

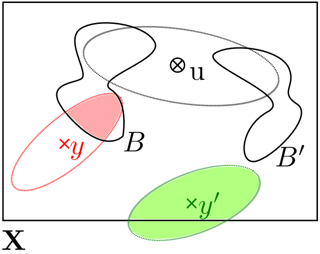


Probability that:

- y exists and lies within B, *and*

Individual+population management: what is the true multi-target configuration?

Elementary events

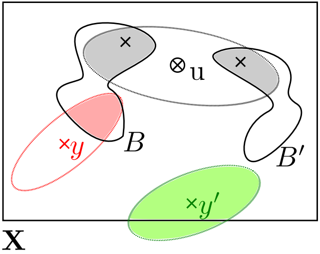


Probability that:

- y exists and lies within B, *and*
- y' exists, *and*

Individual+population management: what is the true multi-target configuration?

Elementary events

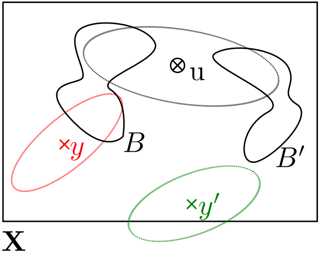


Probability that:

- y exists and lies within B, *and*
- y' exists, *and*
- one undetected targets in B, one in B', none elsewhere.

Individual+population management: what is the true multi-target configuration?

Elementary events



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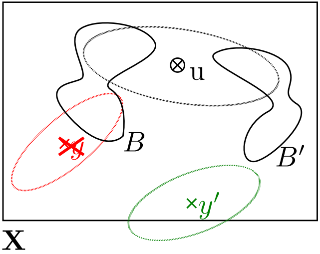
Individual+population management: what is the true multi-target configuration?

Elementary events

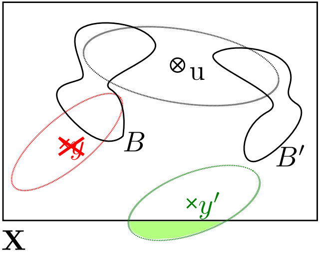


Probability that:

- y does not exist, *and*

Individual+population management: what is the true multi-target configuration?

Elementary events

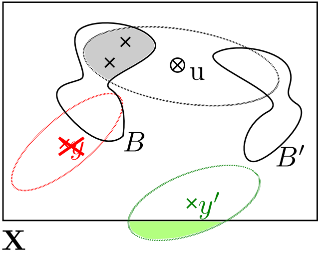


Probability that:

- y does not exist, *and*
- y' exists, but has left the scene, *and*

Individual+population management: what is the true multi-target configuration?

Elementary events

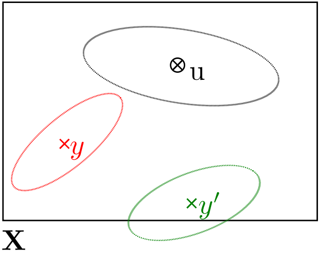


Probability that:

- y does not exist, *and*
- y' exists, but has left the scene, *and*
- two undetected targets in B, none elsewhere.

Individual+population management: what is the true multi-target configuration?

Elementary events



Probability that:

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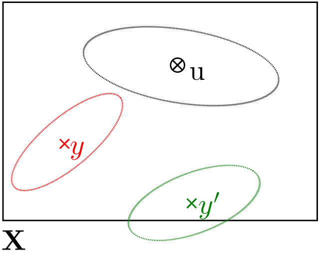
Compound events

Individual+population management: what is the true multi-target configuration?

Elementary events



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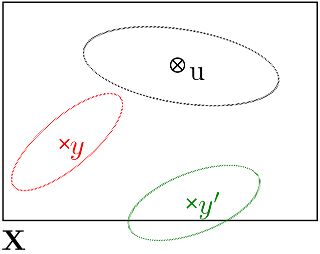
- Probability that there are *at least* two undetected targets in B

Individual+population management: what is the true multi-target configuration?

Elementary events



Probability that:

- y does not exist, *and*
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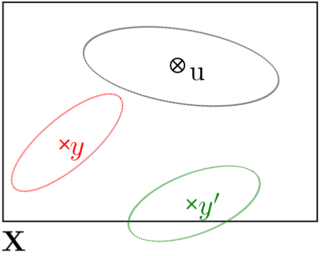
- two undetected targets in B, none elsewhere.

Compound events

- Probability that there are *at least* two undetected targets in B
- Probability that *either* y exists *or* y' has left the scene, *and* there are no undetected targets in the scene

Individual+population management: what is the true multi-target configuration?

Elementary events



Probability that:

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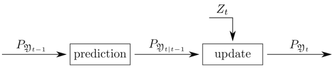
- Probability that there are *at least* two undetected targets in B
- Probability that *either* y exists *or* y' has left the scene, *and* there are no undetected targets in the scene
- ....

Bayesian filtering with stochastic populations (1/2)

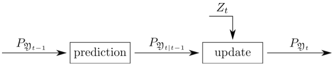
Principle



Maintain and update a dynamical stochastic population reflecting the true multi-target configuration

Bayesian filtering with stochastic populations (1/2)

Principle

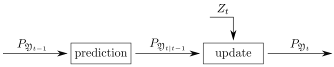


Maintain and update a dynamical stochastic population reflecting the true multi-target configuration

Filtering design: a few modelling choices

Bayesian filtering with stochastic populations (1/2)

Principle



$P_{\mathfrak{Y}_{t-1}}$

prediction

$P_{\mathfrak{Y}_{t|t-1}}$

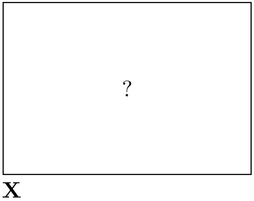
Z_t

update

$P_{\mathfrak{Y}_t}$

Maintain and update a dynamical stochastic population reflecting the true multi-target configuration

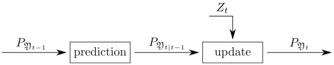
Filtering design: a few modelling choices



- How to represent incoming targets at time t?

Bayesian filtering with stochastic populations (1/2)

Principle



Maintain and update a dynamical stochastic population reflecting the true multi-target configuration

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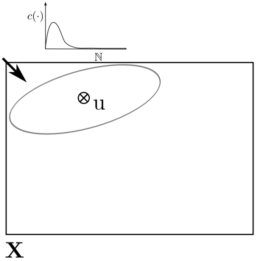
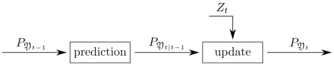


- How to represent incoming targets at time t? One ind. population?

Bayesian filtering with stochastic populations (1/2)

Principle



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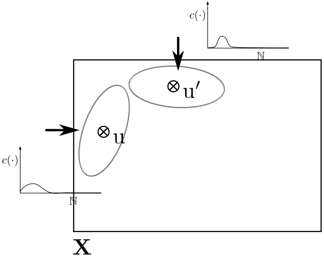
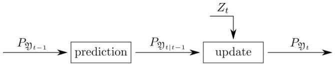


- How to represent incoming targets at time t? One ind. population? Two?

Bayesian filtering with stochastic populations (1/2)

Principle



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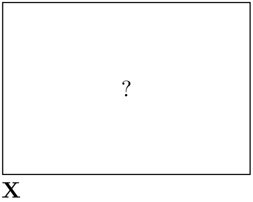
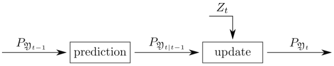


- How to represent incoming targets at time t? One ind. population? Two?
- How to mitigate information loss?

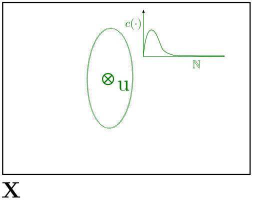
Bayesian filtering with stochastic populations (1/2)

Principle



Maintain and update a dynamical stochastic population reflecting the true multi-target configuration

Filtering design: a few modelling choices



- How to represent incoming targets at time t? One ind. population? Two?
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  Merge "close" ind. populations?

Bayesian filtering with stochastic populations (1/2)

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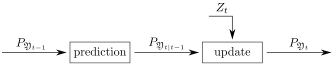

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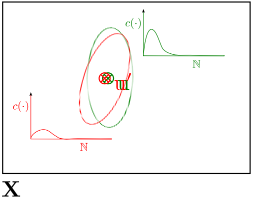
Filtering design: a few modelling choices



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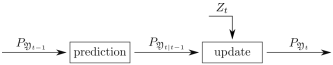

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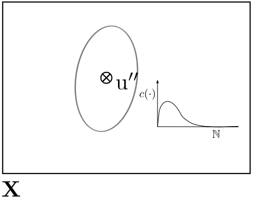

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Bayesian filtering with stochastic populations (1/2)

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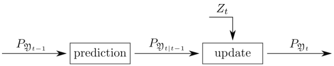
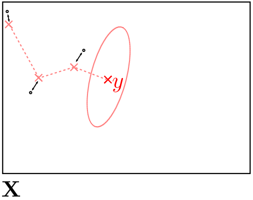


Maintain and update a dynamical stochastic population reflecting the true multi-target configuration

Filtering design: a few modelling choices



- How to represent incoming targets at time t? One ind. population? Two?
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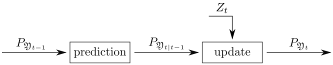
  Merge “close” ind. populations? Merge “close” tracks?

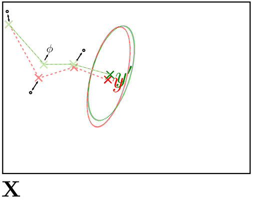
Bayesian filtering with stochastic populations (1/2)

Principle



Maintain and update a dynamical stochastic population reflecting the true multi-target configuration

Filtering design: a few modelling choices



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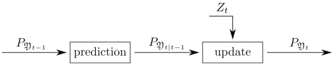
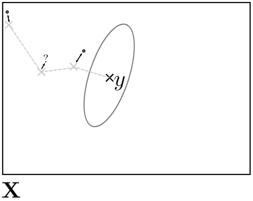
Bayesian filtering with stochastic populations (1/2)

Principle



Maintain and update a dynamical stochastic population reflecting the true multi-target configuration

Filtering design: a few modelling choices



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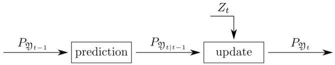
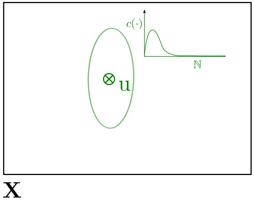
Bayesian filtering with stochastic populations (1/2)

Principle



Maintain and update a dynamical stochastic population reflecting the true multi-target configuration

Filtering design: a few modelling choices



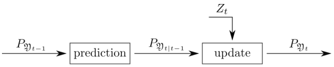
- How to represent incoming targets at time t? One ind. population? Two?
- How to mitigate information loss?

  Merge "close" ind. populations? Merge "close" tracks?

  "Blend" a track *into* a "close" ind. population?

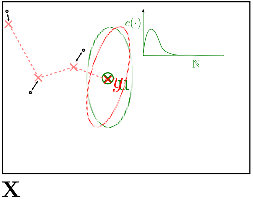
Bayesian filtering with stochastic populations (1/2)

Principle



Maintain and update a dynamical stochastic population reflecting the true multi-target configuration

Filtering design: a few modelling choices



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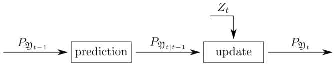
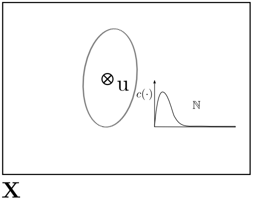
Bayesian filtering with stochastic populations (1/2)

Principle



Maintain and update a dynamical stochastic population reflecting the true multi-target configuration

Filtering design: a few modelling choices



- How to represent incoming targets at time t? One ind. population? Two?
- How to mitigate information loss?

  Merge “close” ind. populations? Merge “close” tracks?

  “Blend” a track *into* a “close” ind. population?

Bayesian filtering with stochastic populations (2/2)

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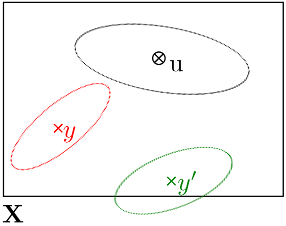
Prediction step



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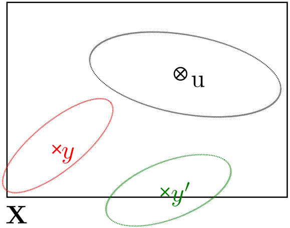

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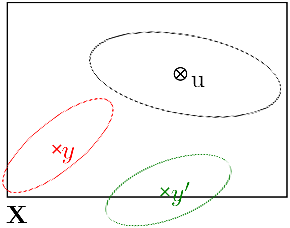

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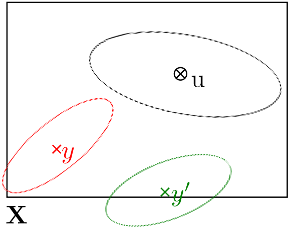

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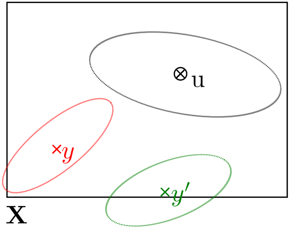

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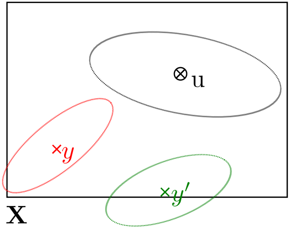

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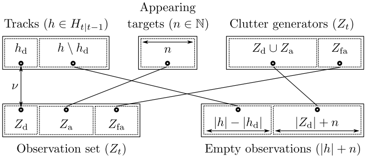

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  Update of joint probabilities of existence

  Pop. management (indist. → tracks, etc.)

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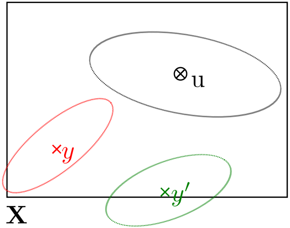

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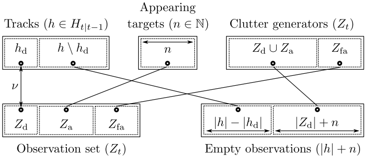

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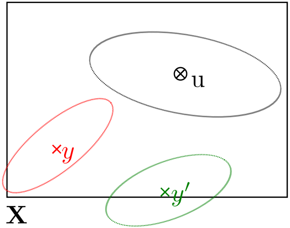

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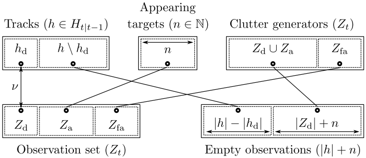

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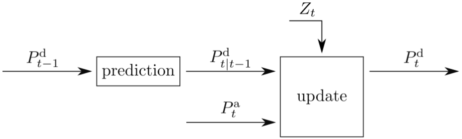

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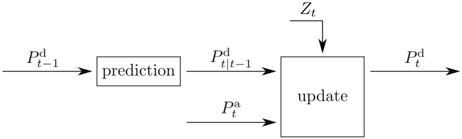

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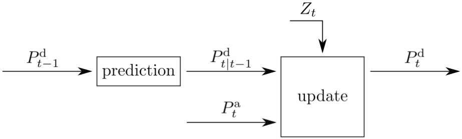

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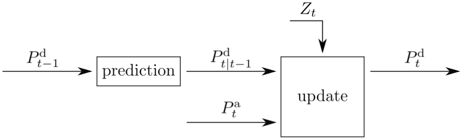

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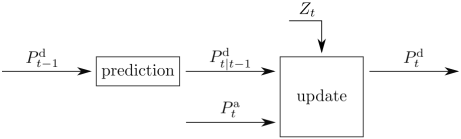

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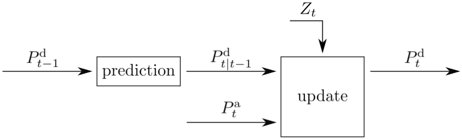

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- Approximate DISP: HISP, similar cost to PHD but better tracking performances (early analysis)

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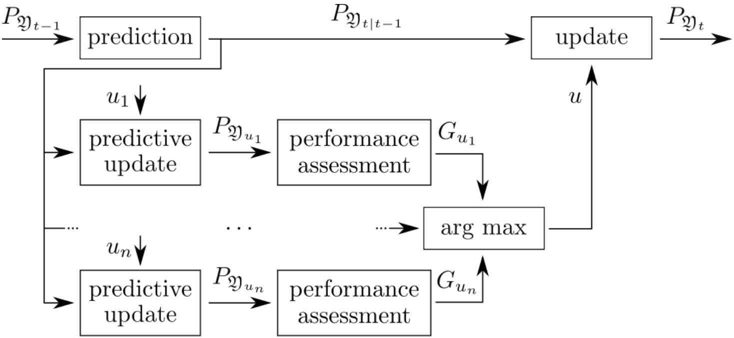

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The operator has access to the predicted information $P_{\mathfrak{Y}_{t|t-1}}$ and considers some action $u \in U_t$ for the next observation. Can we quantify the expected information gain G_u of action u?

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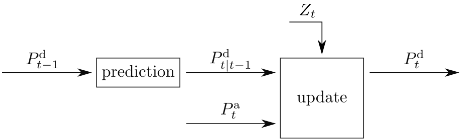

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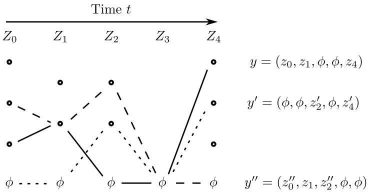

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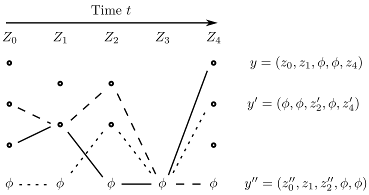

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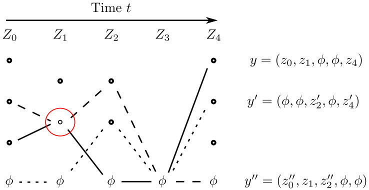

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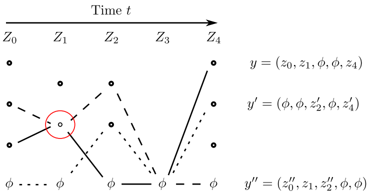

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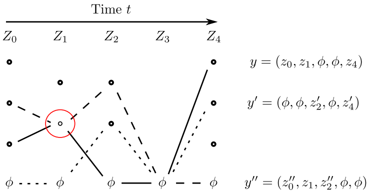

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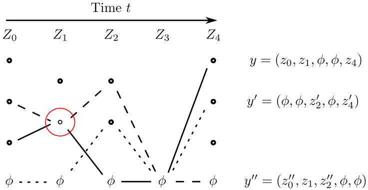

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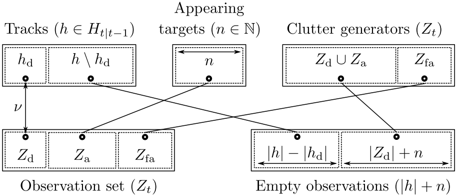

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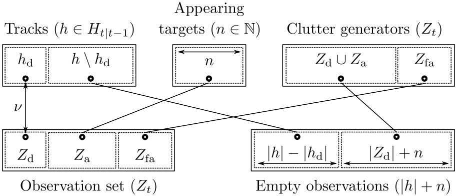

Tracks ($h \in H_{t|t-1}$)

Appearing targets ($n \in \mathbb{N}$)

Clutter generators (Z_t)

h_{d} $h \setminus h_{\mathrm{d}}$

n

$Z_{\mathrm{d}} \cup Z_{\mathrm{a}}$ Z_{fa}

ν

Z_{d} Z_{a} Z_{fa}

$|h| - |h_{\mathrm{d}}|$ $|Z_{\mathrm{d}}| + n$

Observation set (Z_t)

Empty observations ($|h| + n$)

Each association $\mathbf{a} = (h, n, \mathbf{h} \in \mathrm{Adm}_{Z_t}(h, n))$ leads to a unique hyp. $\hat{h} \in H_t$:

- Assessed by prob. $P_u^{\mathbf{a}}$ (i.e. how likely is the association producing $\hat{h}$?)
- Composed of tracks $\hat{h} = \bigcup_{y \in h_{\mathrm{d}}} \{y : \nu(y)\} \cup \bigcup_{y \in h \setminus h_{\mathrm{d}}} \{y : \phi\} \cup \bigcup_{z \in Z_{\mathrm{a}}} \{\mathrm{a} : z\}$

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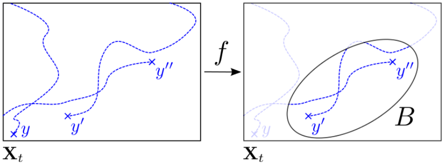

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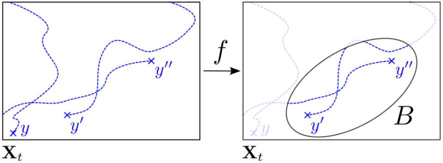

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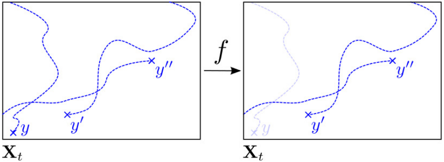

Thank you for your attention!