Distributed Sensor Registration Based on Random Finite Set Representations

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***Abstract*—In the investigation of multi-sensor fusion problems, it is commonly assumed that all the parameters necessary to transform the information from the sensors to a common frame are known. Imperfect knowledge of these registration parameters induce systematic biases which would inhibit the benefits of multi-sensor fusion. For example, they can result in deterioration of the target localisation accuracy. We consider the problem of sensor registration in a distributed setting in which the sensor platforms propagate random finite set distributions that describe the multi-object scene using Probability Hypothesis Density (PHD) filters, and, transmit these posteriors to their neighbours. To find the respective registration error between two platforms, we propose a recursive method whereby a posterior distribution on the registration error is updated upon receiving the incoming multi-object distribution. The update is performed in a Bayesian fashion using a likelihood that captures the similarity of the multi-object scene as determined by the two sensors. In particular, we utilise the Bhattacharyya distance of multi-object distributions as a measure of similarity. We demonstrate the proposed method through a simulation example.**

I. Introduction

Integration of information from different sensors provides opportunities for improving global situation awareness through the use of diversity provided by, e.g., different sensing modalities and/or measurements from distinct geo-locations. Successful operation of multi-sensor fusion algorithms rely on the accurate knowledge of the sensor registration parameters which are used to transform the information to a common description form. The parameters necessary to characterize such transformations might include sensor positions and relative orientations which are, in practice, often inaccurately known or not known at all.

We consider a distributed multi-sensor setting in which an unknown and varying number of targets are to be tracked. Each sensor receives a set of measurements induced by targets detected with a probability and the surroundings. Together with the set of targets, these measurements are modelled with Random Finite Sets (RFSs). For each platform, the problem of finding the multi-object posterior is addressed through the use of Finite Set Statistics (FISST) [1]. The celebrated PHD [2] and Cardinalised PHD (CPHD) [3] filters are examples of such Bayesian filters with the underlying assumptions that the targets are distributed in accordance with a multi–object Poisson and Independent Identicailly Distributed (iid) cluster distribution respectively.

Each sensor receives its neighbours' multi-object posteriors and fuses them with its local one using a scheme that is appropriate for RFS densities. Such a distributed fusion scheme is described in [4] whereby multi-object posteriors are combined in a pairwise fashion based on Exponential Mixture Densities (EMDs). EMDs are weighted geometric means of the posteriors with a weight parameter ω, which can be selected using an appropriate information metric [5]. One possible choice, for example, is to choose ω so that the incoming and local posteriors have equal Kullback–Leibler divergence with respect to the fused result. This is similar to selecting a middle ground between the two posteriors. This strategy can be realised using Monte Carlo (MC) methods considering Sequential MC (SMC) PHD/CPHD filters [6].

Such fusion methods, however, implicitly assume that the arguments of the posteriors (which are sets of state variables in our case) are in the same coordinate frame. We investigate the registration problem in which the parameters necessary for transforming one of the posterior to the other's reference frame are inaccurate or unknown. We consider two platforms and notice that the local and an incoming posterior admit an observation pair since measurements of neighbours are not accessible in a distributed setting. This view equivalently considers the posteriors as random functions of set variables since the filtering operation is a deterministic operator on the sensor measurements which are random sets themselves. We construct a probabilistic model of the problem based on this treatment and pose the registration problem in a maximum *a-posteriori* estimation setting within this model from a Bayesian perspective.

We assume that the registration parameters evolve according to a Markov model through time. Consequently, computation of the marginal parameter posterior can be carried out using

$$p\left(\theta^n|\{\left(P_i^k,P_j^k\right)\}_{k=1:n}\right) \propto p(P_j^n,P_i^n|P_i^{n-1},P_j^{n-1},\theta^n)\times \int p(\theta^n|\theta^{n-1})p(P_j^{n-1},P_i^{n-1}|P_i^{n-2},P_j^{n-2},\theta^{n-1})\dots\int p(\theta^2|\theta^1)p(P_j^1,P_i^1|\theta^1)p(\theta^1)\mathrm{d}\theta^1\cdots\mathrm{d}\theta^{n-1} \quad (6)$$

Bayesian recursions. The parameter likelihood function, however, cannot be modeled or evaluated in a straightforward fashion. Therefore, we propose a model for this distribution in the form of the exponential function of a negative distance which, in a sense, reflects the idea that the correct set of registration parameters should lead a significant overlap of the posteriors when one is fixed and the other is transformed accordingly. In particular, we use the Bhattacharyya Distance (BD) of two multi-object posteriors and the likelihood equals to the Bhattacharyya Coefficient (BC) which reflects the affinity of the multi-object scenes as described by two sensors. Consequently, we obtain a recursive Bayesian algorithm which propagates a distribution on the parameters. Doing that, we exploit the information from multiple targets on the unknowns which in turn provides a rapid decrease in error. The update stage of the recursions, being based on the posteriors rather than the observations, has the flexibility that it can be used together with any multi-object filtering scheme including extended-target tracking filters [7] and Mahler and Clark's generalised PHD filters [8].

Previous work on registration includes centralised joint registration and multi-object estimation of a known number of targets using SMC methods [9]. In [10], the joint problem is solved for an unknown number of targets using doubly stochastic point process models in a centralised fashion. A maximum likelihood approach has been applied for the distributed localisation of sensor nodes in a network while tracking a single target [11]. In this work, we consider a distributed setting and utilise the RFS framework which enables us to benefit from multiple target trajectories for updating the registration parameters. In Section II, the problem definition and the Bayesian recursive solution are presented. We introduce our likelihood model for using in the recursions in Section III. The computation of this likelihood using MC methods is the subject of Section IV. We demonstrate our algorithm in an example scenario in Section V and conclude in Section VI.

II. Sensor Registration Recursions

A. Problem Definition

We consider a distributed setting in which a target process generates an RFS X^n at time n that induces measurement sets Z_i^n and Z_j^n on sensors i and j respectively. Sensor j is translated and rotated with respect to sensor i and we aim to find these registration parameters denoted by the vector θ_{ij}^n. Registration parameters affect the generation processes of both ${}^nZ_i$ and Z_j^n which are subsequently filtered using the operators $\mathcal{F}_i$ and $\mathcal{F}_j$ to produce multi-object posterior distributions P_i^n and P_j^n. Being produced by operators acting on random variables, these posteriors can be treated as random variables as well. In Fig. 1, the causal interactions of the variables are illustrated by a Directed Graph. In Fig. 1(a), the variables in the n^{th} time window is given. The posteriors are independent of all the other variables given the previous update and the observations (Fig. 1(b)). As illutrated in Fig. 1(c) and Fig. 1(d), the target and registration parameter processes are Markov over time.

In a distributed setting, the multi-target posteriors (P_i^n and P_j^n) rather than the raw observations (Z_i^n and Z_j^n) are communicated. This treatment enables us to pose the sensor registration problem as a maximum *a-posteriori* (MAP) problem. At time step n, the registration parameters are given by the maximum of their distribution given the history of target posteriors:

$$\hat{\theta}_{ij}^n = \arg\max_{\theta^n\in\Theta} p\left(\theta_{ij}^n|\{\left(P_i^k,P_j^k\right)\}_{k=1:n}\right) \quad (1)$$

where $\{\left(P_i^k,P_j^k\right)\}_{k=1,\dots,n}$ denotes the history of the posteriors and the registration parameters up to time n. For notational simplicity, we drop the subscript in the registration parameters for the rest of the paper and so $\theta^n = \theta_{ij}^n$.

B. Recursive Bayesian Solution

The posterior distribution we seek in (1) is the marginal of the joint posterior, i.e.,

$$p\left(\theta^n|\{\left(P_i^k,P_j^k\right)\}_{k=1:n}\right) = \int\cdots\int p\left(\theta^1,\dots,\theta^n|\{\left(P_i^k,P_j^k\right)\}_{k=1:n}\right)\mathrm{d}\theta^1\cdots\mathrm{d}\theta^{n-1} \quad (2)$$

which satisfies the proportionality relation

$$p\left(\theta^1,\dots,\theta^n|\{\left(P_i^k,P_j^k\right)\}_{k=1:n}\right) \propto p(\theta^1,\dots,\theta^n)p\left(\{\left(P_i^k,P_j^k\right)\}_{k=1:n}|\theta^1,\dots,\theta^n\right). \quad (3)$$

The first term on the right hand side of (3) is the joint registration prior which factorises according to the Markov model (Fig. 1(d)) as

$$p(\theta^1,\dots,\theta^n) = p(\theta^1)\prod_{k=2}^{n} p(\theta^k|\theta^{k-1}). \quad (4)$$

The second term is the joint likelihood and upon realisation of the Directed Graph in Fig. 1 admits the equality

$$p(\left(P_i^k,P_j^k\right)|\{\left(P_i^l,P_j^l\right)\}_{l=1:k-1},\theta^k,\dots,\theta^1) = p(\left(P_i^k,P_j^k\right)|\left(P_i^{k-1},P_j^{k-1}\right),\theta^k)$$

for all $k=1,\dots,n$, we can express this likelihood with the following factorisation:

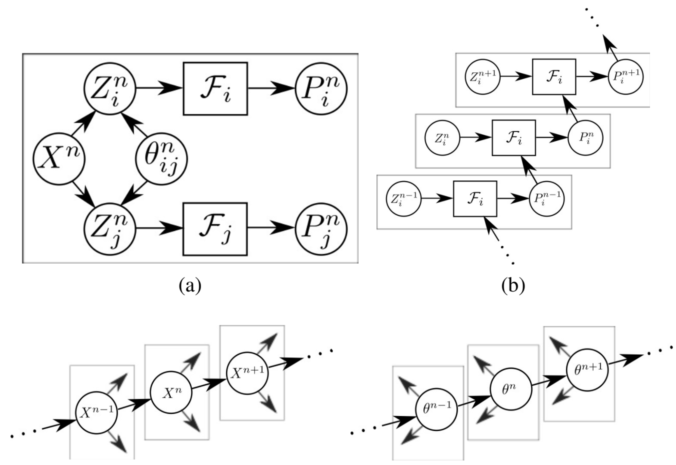

Fig. 1. Directed Acyclic Graph (DAG) representation for finding the respective registration parameters between sensor i and j: (a) Conceptual relationship between set variables, filtering operators, alignment parameters and posteriors. (b) Temporal evolution of the i^{th} RFS posterior. (c) Temporal evolution of the multi-object state. (d) Temporal evolution of registration parameters.

$$p\left(\{(P_i^k, P_j^k)\}_{k=1:n}|\theta^1, \ldots, \theta^n\right) = \prod_{k=1}^{n} p\left((P_i^k, P_j^k) \mid (P_i^{k-1}, P_j^{k-1}), \theta^k\right). \quad (5)$$

Substituting (4) and (5) into (3) leads to (6). Exploiting the Markov assumptions, this can be simplified into a two-step recursive equation consisting of the prediction

$$p\left(\theta^n|\{(P_i^k, P_j^k)\}_{k=1:n-1}\right) = \int p(\theta^n|\theta^{n-1})p\left(\theta^{n-1}|\{(P_i^k, P_j^k)\}_{k=1:n-1}\right) \mathrm{d}\theta^{n-1}, \quad (7)$$

followed by the update

$$p\left(\theta^n|\{(P_i^k, P_j^k)\}_{k=1:n}\right) = p\left(P_i^n, P_j^n|P_i^{n-1}, P_j^{n-1}, \theta^n\right) p\left(\theta^n|\{(P_i^k, P_j^k)\}_{k=1:n-1}\right). \quad (8)$$

Here $p\left(P_i^n, P_j^n|P_i^{n-1}, P_j^{n-1}, \theta^n\right)$ is the registration parameter likelihood function. We now discuss how this can be developed.

III. Modelling the Registration Parameter Likelihood

The likelihood term $p(P_j^n, P_i^n|P_i^{n-1}, P_j^{n-1}, \theta^n)$ in the update stage (8) captures the dependency of the posteriors from sensors i and j with the previous updates and with the registration parameters. It is not straigthforward to find a tractable form for this term and evaluate it. Instead, we model this distribution with an exponential function of a negative distance metric for the following reasons.

The first issue is that P_i^n and P_j^n are distributions which describe the same multi-target scene. Had both sensors been perfectly aligned (and the true value of the respective registration parameters were zero), they would have converged towards the same distributions, subject to the multi-target and observation dynamics. Let $\tilde{P}_j^n$ denote the random posterior under the condition that the true value of the respective registration θ^n is the zero vector (as illustrated in Fig. 2). For this perfect alignment case, we expect P_i^n and $\tilde{P}_j^n$ to be closely located with respect to a distance measure. An example of such a measure is the Bhattacharyya Distance (BD) of two RFS distributions $p(X)$ and $q(X)$ given by

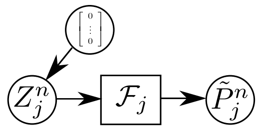

Fig. 2. j^{th} multi-object posterior for the hypothetical case that sensor j is perfectly aligned with sensor i (denoted by $\tilde{P}_j^n$).

$$BD(p(X), q(X)) = -\ln \int \sqrt{p(X), q(X)}\delta X \quad (9)$$

where the integral term is known as the Bhattacharyya Coefficient (BC) and the set integral is given by

$$\int f(X)\delta X := f(\emptyset) + \sum_{n=1}^{\infty} \frac{1}{n!} \int f(\{x_1, ..., x_n\})\mathrm{d}x_1...\mathrm{d}x_n. \quad (10)$$

It is not easy to achieve a tractable upper bound for the expected value of $BD(P_i^n(X), \tilde{P}_j^n(X))$ over all possible sets of sensor measurements, nevertheless, we can empirically show that the greater the misalignment between the sensors is, the greater the BD between the multi-object posteriors is on the average. As an example, in Fig. 3, we present the BD of the posteriors of two sensors in a 2D scenario in which the registration parameter was the displacement of the two sensors in the x direction. Each sensor computed its own distribution from its own sensors, and the BD was computed as a function of the displacement in the x direction by both sensors. As can be seen, the BD tends to monotonically increase with the displacement.

The second issue is that, in a distributed setting, platform i receives the posterior of platform j, i.e., P_j^n whereas the similarity of P_i^n is with $\tilde{P}_j^n$. Suppose that there exists a transformation $\mathcal{T}$ that reverses the effects of translation and rotation on P_j^n so that we can compute the j^{th} multi-object posterior for the hypothetical case that sensor j is perfectly aligned with sensor i. In other words, suppose that

$$\tilde{P}_j^n = \mathcal{T}(P_j^n; \theta^n)$$

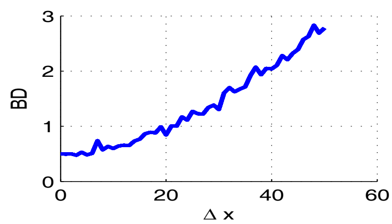

Fig. 3. Displacement of two sensor platforms along x-axis vs. the Bhattacharyya distance between their posteriors (computed using MC methods and averaged over 15 simulations in an example scenario).

almost surely. We model the likelihood using the negative exponential of the BD between the transformed posterior and P_i^n, i.e.,

$$p(P_j^n, P_i^n | P_i^{n-1}, P_j^{n-1}, \theta^n) \propto \exp(-BD(P_i^n, \tilde{P}_j^n)) \propto \int \sqrt{P_i^n(X), \mathcal{T}(P_j^n(X); \theta^n)} \delta X \quad (11)$$

One way of finding $\tilde{P}_j^n$ given the realisation of the registration parameters $\theta^1, \ldots, \theta^n$ is to transform the observation sequence $Z_j^1, ..., Z_j^n$ to the coordinate frame of sensor i and then filter them. Since platform i does not have access to these observations, it might not be possible to compute $\tilde{P}_j^n$ only by using the posterior sensor j outputs (P_j^n) using its observations. Nevertheless, an approximate computation of $\tilde{P}_j^n$ can be carried out through inverse rotation and translation of the arguments of P_j^n which can be assumed to have removed the effects of θ^n approximately. Suppose that θ^n is a tuple of a spatial translation vector θ_T^n and Euler angles $\theta_E^n = (\psi, \theta, \phi)$ (or, rotation angles in the $\mathrm{x-y}, \mathrm{y-z}$ and $\mathrm{x-z}$ planes respectively), i.e., $\theta^n = (\theta_T^n, \theta_E^n)$. Then, we approximate the transformed posterior with

$$\tilde{P}_j^n(\{x_1, \ldots, x_N\}) \approx |\mathbf{R}_{\theta_E^n}| \quad (12)$$
$$P_j^n(\{\mathbf{R}_{\theta_E^n}^{-1}(x_1 - \theta_T^n), \ldots, \mathbf{R}_{\theta_E^n}^{-1}(x_N - \theta_T^n)\})$$

where $\mathbf{R}_{\theta_E^n}$ is the Directional Cosine Matrix for θ_E^n and $|\mathbf{R}_{\theta_E^n}| = 1$ is its determinant.

After substituting from (12) into (11), the registration parameter likelihood model is obtained as

$$p(P_j^n, P_i^n | P_i^{n-1}, P_j^{n-1}, \theta^n) \propto \int \sqrt{P_i^n(X) P_j^n(\{\mathbf{R}_{\theta_E^n}^{-1}(x_1 - \theta_T^n), \ldots, \mathbf{R}_{\theta_E^n}^{-1}(x_N - \theta_T^n)\})} \delta X \quad (13)$$

which will be used within the Bayesian registration recursions given by (7) and (8).

IV. Realisation via Monte Carlo Methods

The realisation of the registration recursions given in (7) and (8) with the registration parameter likelihood model given in (13) will be carried out using sequential Monte Carlo (SMC) methods. In particular, we use a Sampling/Importance Sampling scheme similar to the Bootstrap filter [12]. We begin with M samples from a prior distribution on the parameters $p(\theta^1)$, i.e., $\{\theta^{1,(1)}, ..., \theta^{1,(M)}\}$ such that $\theta^{1,(i)} \sim p(\theta^1)$. At time step k, the likelihood model (13) is evaluated for each sample $\theta^{k,(i)}$ generated from the prediction density given by (7). These quantities are denoted by $\zeta^{k,(i)}$ and given two posteriors P_i^k and P_j^k, they are found using Monte Carlo methods details of which are discussed later in this section. After finding $\zeta^{k,(i)}$ for each parameter point $\theta^{k,(i)}$, Importance Sampling $\{(\zeta^{k,(i)}, \theta^{k,(i)})\}_{i=1}^M$ yields samples generated approximately from the required posterior given by (8).

For the rest of this section, we discuss approximate computation of the likelihood model using MC methods. We consider multi-object posteriors outputby adaptive birth process driven CPHD filters [13]. At time k, the RFS distribution of platform i is a cluster processes given by

$$p_i^k(X|Z^{1:k}) = \kappa_i^k(N|Z^{1:k}) N! \prod_{x \in X} s_i^k(x|Z^{1:k}) \quad (14)$$

Here, $N = |X|$ is the cardinality of the random set variable X. κ_i is a distribution over non-negative integers enumerating the probability of number of targets and known as the posterior cardinality distribution. s_i is the posterior intensity function localising the targets. The SMC realisation of the CPHD filter propagates particles generated from s_i for representing the localisation density:

$$S_i^k \triangleq \{x_k^{(p)}\}_{p=1}^P \text{ where } x_k^{(p)} \sim s_i$$

The localisation density κ_i is stored in an array of chosen length.

The coordinate transformation of the cluster posterior from sensor j then involves only the transformation of its intensity function, or the localisation density, i.e.,

$$\tilde{s}_j^k(x) = s_j^k(\mathbf{R}_{\theta_E^n}^{-1}(x - \theta_T^n)).$$

Equivalently, the representation of $\tilde{s}_j^k(x)$ using the particle set S_j^k is obtained by applying the transformation to each particle separately. In other words

$$\tilde{S}_i^k \triangleq \{\tilde{x}_k^{(p)}\}_{p=1}^P \text{ where } \tilde{x}_k^{(p)} = \mathbf{R}_{\theta_E^n}^{-1}(x_k^{(p)} - \theta_T^n)$$

The BC modeling the likelihood of concern for two cluster processes can be found by substituting from (14) and the RHS of (13) into (10) as

$$BC(p_i(X), p_j(X)) = \sum_{N=0}^{\infty} \sqrt{\kappa_i(N)\kappa_j(N)} Z^N \quad (15)$$
$$Z = \int_{\mathcal{X}} \sqrt{s_i(x) s_j(x)} \, \mathrm{d}x. \quad (16)$$

We consider IS evaluation of Z for which we need a distribution with heavier tails compared to $s_i^{0.5}(x) s_j^{0.5}(x)/Z$ [14]. Non-degenerate mixtures of $s_i(x)$ and $s_j(x)$ are such proposal densities and the particle set

$$S_U \triangleq S_i \cup S_j \quad (17)$$

is constituted of $P_U = P_i + P_j$ samples from such a mixture density given by

$$s_U(x) = \frac{P_i s_i(x) + P_j s_j(x)}{P_i + P_j} \quad (18)$$

Using S_U, the IS estimate [14] of Z is given by

$$\hat{Z}_{IS} \triangleq \sum_{x \in S_U} \frac{\sqrt{s_i(x) s_j(x)}}{P_i s_j(x) + P_j s_j(x)} \quad (19)$$

Here, we substitute Kernel Density Estimates $\hat{s}_i(x)$ and $\hat{s}_j(x)$ in (19) which we obtain through a regularised form of the SMC CPHD filter and obtain a computationally feasible estimate of Z as

$$\hat{Z} \triangleq \sum_{x \in S_U} \frac{\sqrt{\hat{s}_i(x) \hat{s}_j(x))}}{P_i \hat{s}_i(x) + P_j \hat{s}_j(x)}. \quad (20)$$

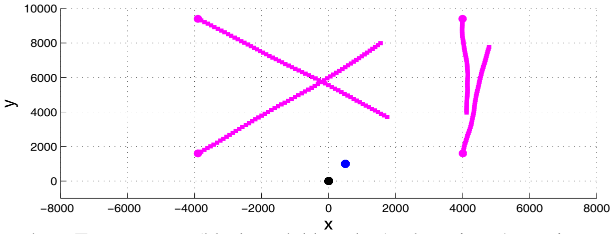

Fig. 4. Two sensors (black and blue dots) observing 4 moving targets (magenta lines) initially located at the circled positions.

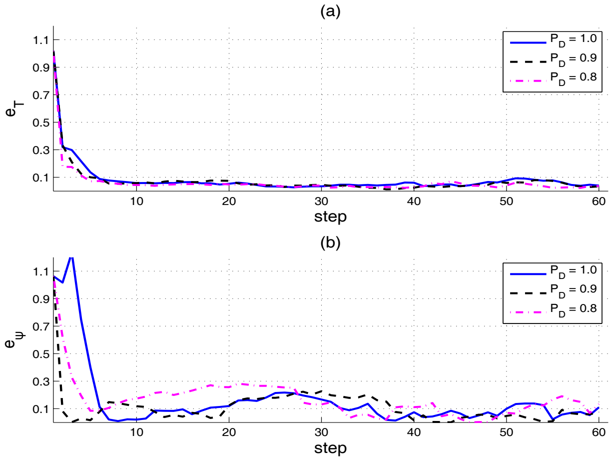

Fig. 5. Performance of the recursive registration algorithm in the example scenario: Normalised error in (a) translation, and, (b) azimuth angle, for $P_D = 1.0, 0.9$ and 0.8 (blue solid, black dashed and magenta dash-dot lines respectively).

For estimating the BC which is the proposed likelihood to use in the update equation (8), we substitute from (20) into (15) together with the finite arrays storing $\kappa_i(N)$ and $\kappa_j(N)$.

V. Example

In this section, we demonstrate the proposed sensor registration method in an example multi-target scenario in which two range-bearing sensors observe 4 targets moving with constant velocity (and slight process noise) for 50 time steps (Fig. 4) in 2-D. The standard deviations in range and bearing are 3 m. and 1° respectively. The clutter is Poisson with rate $\lambda = 10$. One of the sensors (the black sensor) is located at the origin with a bearing aligned with the y-axis. The second sensor (the blue sensor) is located at $[500,\ 1000]^T$ on the x-y plane with a bearing angle of 3° with respect to the first one.

The first sensor receives the second sensor's multi-object posterior output by SMC CPHD filtering and estimates the position and the bearing using the proposed approach together with the Mean Squared Error estimator[1]. In Fig. 5, normalised error in translation given by $e_T \triangleq \left\| \theta_T - \hat{\theta}_T \right\| / \|\theta_T\|$ and in azimuth angle given by $e_E \triangleq \left\| \theta_E - \hat{\theta}_E \right\| / \|\theta_E\|$ where $\theta_E = \psi$ and for probability of detection $P_D = 1.0, 0.9$ and 0.8. It is seen that the recursions lead to a rapid decrease in the error consistently for different qualities of sensor measurements.

[1]We assume that the MSE estimate approximately equals to the MAP estimate which, in our case, holds as after the convergence the parameter posterior tends to be convex.

VI. Conclusion and Future Work

In this paper, we presented a recursive Bayesian solution to the sensor registration problem in a distributed fusion setting. We introduced a model for the parameter likelihood given multi-object posteriors from platforms running PHD filters. This approach provides us the advantages of exploiting from multiple target constellations which lead to a rapid decrease in error, and, making it possible to use various types of PHD filters under registration uncertainties. Possible future directions include empirical investigation of the convergence properties. Another direction involves improving the accuracy of the algorithm as well as the convergence properties by deploying a stochastic tempering stage and a Markov Chain MC move step in the update stage similar to that used in [13].

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