RF Interference Mitigation for UWB SAR using Image Sparsity

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RFI Suppression and Sparse Image Formation for UWB SAR: Objectives/Motivations

- Propose an image formation algorithm for ultra wideband (UWB) synthetic aperture radar (SAR) (VHF/UHF spectrum).
- Suppress radio frequency interference (RFI) and reconstruct a SAR image.
- RFI suppression shouldn't introduce artefacts.
- Achieved by leveraging image sparsity model.

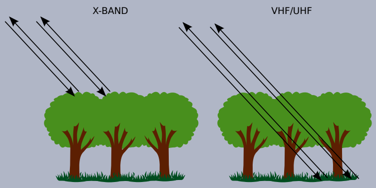
Outline

Outline

Background

Why use VHF/UHF Spectrum?

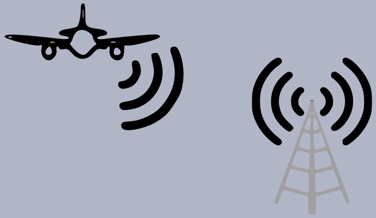
- Foliage Penetration (FoPEN) Radar
- Ground Penetration Radar (GPR)
- Scattering is dependent on wavelength.



Background

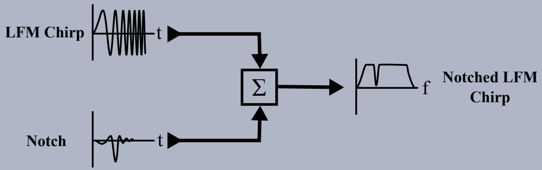
Issues which effect the VHF/UHF spectrum

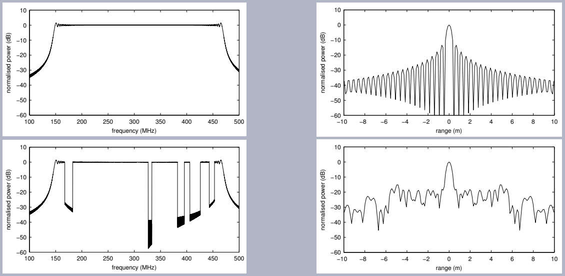
- Interference between SAR systems and radio, television and communications systems.
- Radio frequency interference (RFI)
- Interference Types:
  1. SAR systems can interfere with other spectrum users.
  2. Other users in the spectrum can interfere with SAR system.



Background

Notched LFM on Transmit

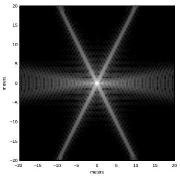




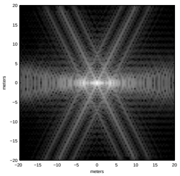
Background

RFI suppression

- Strong interference from AM/FM transmitters.
- RFI pre-processing suppression methods: estimate-and-subtract or linear filter.
- Estimate-and-subtract: estimate the frequencies and phases of the RFI and then abstract.
  Can be computationally expensive and approximation dependent.
- Linear filter: minimise RFI using linear filter, e.g. LMS filter and Wiener filter.
  Can produce large side lobes.



example PSF



filtered PSF

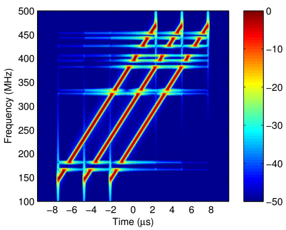
Outline

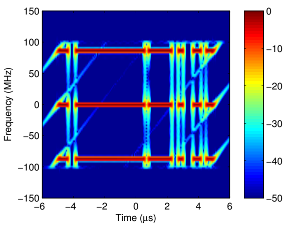
UWB SAR Model

Dechirping of Notched LFM

- Received signal is weighted by the spectrum of the transmitted notched LFM chirp.

$$\tilde{y}(t) \approx |S(\omega_0 + 2\alpha t)|\, y(t)$$





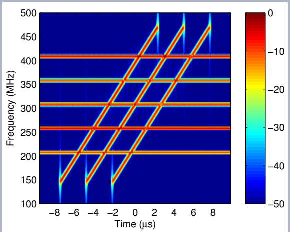
UWB SAR Model

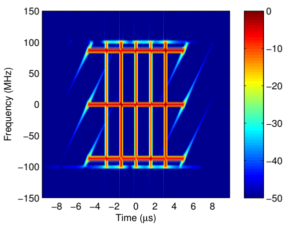
Dechirping of RFI

- The RFI signal after dechirping is approximately given by:

$$\tilde{s_{\text{rfi}}}(t) \approx \left[\frac{1}{2\pi} S_{\text{rfi}}(\omega_0 + 2\alpha t) \exp\left(j\alpha t^2\right)\right] * \left[\sqrt{\frac{\pi}{j\alpha}} \frac{\omega_{\text{if}}}{2\pi} \operatorname{sinc}\left(\frac{\omega_{\text{if}} t}{2}\right)\right]$$

- If $s_{\text{rfi}}(t)$ is a random signal then the $\mathbb{E}\left(|\tilde{s_{\text{rfi}}}(t)|^2\right)$ will be the PSD of $s_{\text{rfi}}(t)$ convolved with a squared sinc function.





UWB SAR Model

Notation

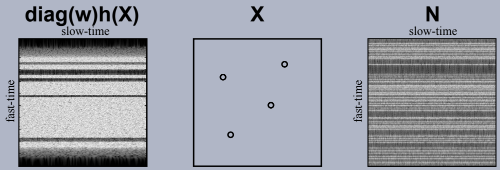
Spotlight-mode Dechirped SAR with RFI System Model:

$$\mathbf{Y} = \text{diag}\,(\mathbf{w})h(\mathbf{X}) + \mathbf{N}$$

$$\mathbf{Y} \in \mathbb{C}^{M' \times N'},\ \mathbf{N} \in \mathbb{C}^{M \times N},\ \mathbf{w} \in \mathbb{R}^{M}$$

Linear Reconstruction:

$$\hat{\mathbf{X}} = g(\mathbf{Y}),\ \text{where},\ g(h(\mathbf{X})) = \mathbf{X}$$



Outline

RFI Suppression

Filter-based RFI suppression

Linear RFI Filtered Reconstruction:

$$\hat{\mathbf{X}} = g(\mathbf{H}\,\mathrm{vec}(\mathbf{Y}))$$

$$\mathbf{H} = \mathrm{diag}([\mathbf{H}_1, \cdots, \mathbf{H}_{N'}])$$

Wiener Filter:

$$\mathbf{H}_{n'} = \mathbf{I} - \mathbf{Q}_{\mathbf{n}_{n'}}(\mathbf{Q}_{\tilde{\mathbf{y}}_{n'}} + \mathbf{Q}_{\mathbf{n}_{n'}})^{-1} \textit{ for } \mathbf{Q}_{\mathbf{x}} = \mathrm{E}\left[\mathbf{x}\mathbf{x}^{\mathrm{H}}\right]$$

Reconstruction Error:

$$\|\mathbf{X} - \hat{\mathbf{X}}\|_{\mathrm{F}} \leq \|g\|\|(\mathbf{I} - \mathbf{H})\,\mathrm{vec}(h(\mathbf{X}))\|_{\mathrm{F}} + \|g\|\|\mathbf{H}\,\mathrm{vec}(\mathbf{N})\|_{\mathrm{F}}$$

which can be compared with

$$\|\mathbf{X} - \hat{\mathbf{X}}\|_{\mathrm{F}} \leq \|g\|\|\mathbf{N}\|_{\mathrm{F}}$$

RFI-aware Sparse Image Formation

RFI-aware Sparse Image Formation

$$\hat{\mathbf{X}} = \underset{\mathbf{X}}{\text{minimise}} \ \|\mathbf{X}\|_1$$
$$\text{subject to } \|\mathbf{Y} - h(\mathbf{X})\|_{\mathbf{Q_N}^{-1}} \leq \epsilon,$$

$$\text{where, } \|\mathbf{A}\|_{\mathbf{Q}} = \text{vec}(\mathbf{A})^{\text{H}}\mathbf{Q}\,\text{vec}(\mathbf{A})$$

Motivation Theory: Compressive Sensing

Modified Restricted Isometry Property:

$$C(1-\delta_{2S})\|\mathbf{X}_{2S}\|_{\text{F}}^2 \leq \|h(\mathbf{X}_{2S})\|_{\mathbf{Q_N}^{-1}}^2 \leq C(1+\delta_{2S})\|\mathbf{X}_{2S}\|_{\text{F}}^2$$

Reconstruction Error Bound:

$$\|\mathbf{X} - \hat{\mathbf{X}}\|_{\text{F}} \leq C_{1,S}\|\mathbf{N}\|_{\mathbf{Q}_\mathbf{N}^{-1}} + C_{2,S}S^{-1/2}\|\mathbf{X} - \mathbf{X}_S\|_1,$$

Outline

Numerical Experiments

VHF/UHF SAR simulation Parameters

parameter	value
carrier frequency (ω_0)	$2\pi \times 308$ Mrad/s
altitude	7000 m
number of targets	20
chirp bandwidth ($2\alpha T$)	$2\pi \times 324$ Mrad/s
stand-off distance	7000 m
number of interferes	80
IF bandwidth	$2\pi \times 60$ Mrad/s
aperture length	7000 m
signal to noise ratio (SNR)	60 dB
scene radius (L)	75 m
number of aperture samples	300
signal to interference ratio (SIR)	-30 dB
transmit notch centre frequencies	175, 330, 389, 416 and 448 $\times 2\pi$ Mrad/s
transmit notch bandwidths	15, 7, 13, 20 and 10 $\times 2\pi$ Mrad/s

Numerical Experiments

RFI-aware Sparse Image Formation Implementation

Estimate Noise Covariance:

Estimate $\mathbf{Q_N}$ using ten "dead-time" measurements.

Assume elements of **N** *are independent.*

Unconstrained Optimisation:

$$\hat{\mathbf{X}} = \underset{\mathbf{X}}{\text{minimise}} \ \|\mathbf{X}\|_1 + \lambda(\|\mathbf{Y} - \text{diag}\,(\mathbf{w})h(\mathbf{X})\|_{\mathbf{Q_N}^{-1}} - \epsilon)$$

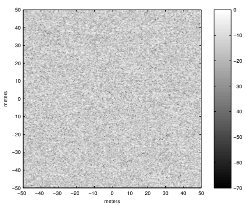
Approximately solved using thirty iterations of a fast iterative shrinkage thresholding algorithm.

Project onto Domain of $h(\cdot)$:

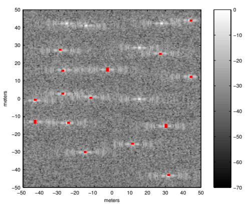
$$\hat{\mathbf{X}} \leftarrow g(h(\hat{\mathbf{X}}))$$

Numerical Experiments

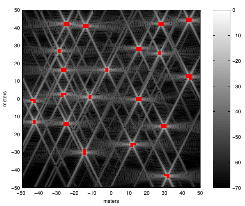
CFAR Results



Filtered back-projection



Wiener filtered followed by filtered back-projection



RFI-aware sparse image formation

Conclusions and Future Work

Conclusions

- Theoretically near optimal performance for ideally sparse SAR images.
- Improved performance for simulated UWB SAR.

Future Work

- Balance trade off between image quality and computational complexity (parallel processing?).
- Propose a modified scheme for systems with interference that is not stationary.
- Investigate algorithm on real UWB SAR data.

Thank you for your attention!

Questions?