Range Doppler SAR Processing Using the Fractional Fourier Transform

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***Abstract*—The Fractional Fourier transform (FrFT), which is a generalized form of the well-known Fourier transform, has opened up the possibility of a new range of potentially promising and useful applications including radar involving the use and detection of chirp signals, pattern recognition and Synthetic Aperture Radar (SAR) image processing. In this paper the Fractional Fourier transform is applied to the Range Doppler Algorithm in order to obtain a better result in terms of resolution. The proposed technique takes advantage from the property of the FrFT to resolve with high precision chirp signals. Preliminary results are encouraging and confirms that the FrFT can be useful to perform high resolution SAR processing.**

I. Introduction

Synthetic Aperture Radar (SAR) is an imaging radar for earth observation from satellite and airborne manned/unmanned platforms; it is currently operational in recently launched polar-orbiting platforms such as TerraSAR-X, RadarSAT-2 and Cosmo SkyMed as well as in previous missions. Applicatons are tailored to disaster observation and management, mapping of renewable resources, geological mapping, snow/ice mapping and strategic surveillance of military sites. Moreover, the scientific community is more and more oriented to a wide range of applications where the first step is the focalization of SAR data [1]. The use of new signal processing techniques is a good way to achieve better resolution especially using high resolution sensors where the feature of the smaller scatterer increases its importance. In [2], [3] the Fractional Fourier Transform has been applied to the Chirp Scaling Algorithm [4] to obtain good results in terms of resolution. The remainder of this paper is organized as follows. Section 2 provides some background on the SAR acquisition geometry , in section 3 an overview of the Range Doppler Algorithm in given. Section 4 introduces the Fractional Fourier Transform and in section 5 the Fractional RDA is introcuced. Sections 5 and 6 include some preliminary results and conclusions.

II. SAR geometry

The basic geometry of a SAR system is shown in Figure 1. The antenna system, looking across the flight direction, transmits a short chirped waveform, with a pulse repetition time 1/PRF much longer than the waveform duration; the echoes reflected by the earth surface are received through the antenna pattern and digitized line by line at each platform position as a two-dimensional array of samples. The Azimuth direction refers to the along-track direction, i.e. parallel to the flight direction

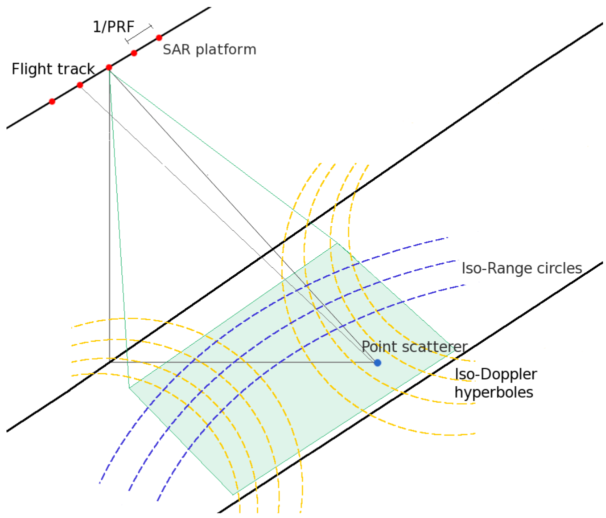

Figure 1. Syntetic Aperture Radar system geometry

while Range refers to the across-track direction. The equirange surfaces are concentric spheres whose intersection with the flat terrain generates concentric circles. Surfaces with identical Doppler shift are coaxial cones with the flight line as the axis and the radar platform as the apex. The intersection of these cones with the flat terrain generates hyperbolae. Objects lying along the same hyperbola will provide equi-Doppler returns. Since the received signal can be viewed as a superposition of returns from elementary scatters, the processing problem comprises a deconvolution of the received signal that is spread out in both range and azimuth directions.

III. Range Doppler Algorithm Overview

SAR data processing consists of a set of procedures for obtaining the final spatial and radiometric resolutions from the instrument. It should satisfy requirements of accuracy, computational complexity and technical feasibility. In the relatively small set of available techiques for SAR data processing (also referred to as SAR focusing), the range-Doppler algorithm and its variants is one of the most widely used. It was first developed by MacDonald Dettwiler and Associates (MDA) and the Jet Propulsion Lab (JPL) in 1979 for the processing of SEASAT data [5], [6]. The algorithm is designed to achieve block processing efficiency, using frequency domain operations in both range and azimuth, while maintaining the simplicity of the one-dimensional operations. Block processing efficiency is also achieved for the Range Cell Migration Correction (RCMC), operation because it is performed in the so called *range–Doppler plane*, as explained later in this section. The sequence of core steps of the algorithm are shown in Figure 2 where it is worth noting that the deconvolution is split in the range and azimuth directions. This is possible because the range between the radar and a given point target is assumed fixed for a given pulse (start-stop approximation). The range focusing is a filtering stage carried out in three

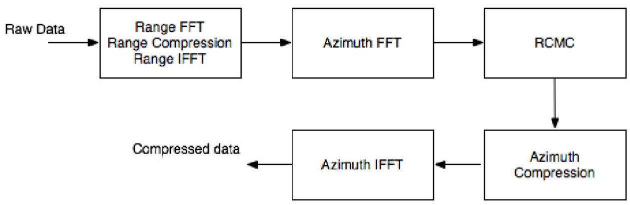

Figure 2. Range-Doppler SAR processing algorithm

steps:

1. the **Range FFT**, that is a set of one–dimensional Fast Fourier Transforms in the range direction, one for each acquisition of the sensor at the time n/PRF.
2. the **Range Compression**, namely a matched filtering obtained by multiplying the range transformed data by the range reference function in the frequency domain.
3. **Range IFFT**, the one-dimensional set of inverse FFTs to re–obtain data in the time domain.

The azimuth focusing is a range dependent matched filtering stage that includes RCMC. It includes:

1. the **Azimuth FFT** that is a set of one–dimensional Fast Fourier Transforms in the azimuth direction; the resulting data lie in the range–Doppler plane.
2. **Range Cell Migration Correction** In the frequency domain, variations of the point–to–radar distance induce a range-dependent Doppler frequency shift in the received signal.This stage includes range shifts and interpolation operations to track the exact azimuth trajectories of the received echoes.
3. **Azimuth Compression** namely a matched filtering obtained by multiplying the azimuth transformed data by the range–dependent azimuth reference function in the frequency domain.
4. **Azimuth IFFT** the inverse one–dimenional set of FFT to obtain the final image.

IV. The Fractional Fourier Transform

A Fourier transformation (FT) maps a one-dimensional time signal $x(t)$ into a one-dimensional frequency function $X(f)$, the signal spectrum. The Fourier transform operator can be visualized as a change in representation of the signal corresponding to a counter clockwise rotation of the axis by an angle $\pi/2$. Although the Fourier transform provides the spectral content of the signal, it fails to indicate the time location of the spectral components, which is of great importance when we consider non-stationary or time-variant signals. In order to describe and analyze such signals, time-frequency representations (TFRs) are used. A TFR maps a onedimensional time signal into a two-dimensional function of time and frequency. The fractional Fourier transform (FrFT) which belongs to the class of linear TFRs, introduced by Namias in 1980 [7], then rediscovered in optics [8], [9], [10], [11] and introduced to the signal processing community by Almeida in 1994 [12]. The fractional Fourier transform, which is a generalization of the ordinary Fourier transform, can be considered as a rotation by an angle (not multiple of $\pi/2$) in the time-frequency plane or a decomposition of the signal in terms of chirps. It also serves as an orthonormal signal representation for chirp signals. The fractional Fourier transform is also called rotational Fourier transform or angular Fourier transform [13]. The fractional Fourier transform is computed using the angle of rotation in the time-frequency plane as the fractional power of the ordinary Fourier transform. Letting $x(u)$ be an arbitrary signal, its ath-order FrFT is defined as [10]:

$$\mathbf{X}_a(u) = \int K_a(u,u')x(u')du' \tag{1}$$

where a is the fractional transformation order (corresponding to a rotation angle $\alpha = a\frac{\pi}{2}$ with $a \in \mathbf{R}$) and $K_a(u,u')$ is the FrFT kernel and is defined as:

$$K_a(u,u') = \begin{cases} A_0 \exp[j\pi(u^2+u'^2)\cot\alpha - 2uu'\csc\alpha] \\ \text{ if } \alpha \text{ is not a multiple of } \pi \\ \delta(u-u') \text{ if } \alpha \text{ is a multiple of } 2\pi \\ \delta(u+u') \text{ if } \alpha+\pi \text{ is a multiple of } 2\pi \end{cases} \tag{2}$$

where $A_0 = \frac{e^{j\frac{\alpha}{2}}}{\sqrt{j\sin\alpha}}$

The fractional Fourier transform is able to process chirp signals better than the ordinary Fourier transform. This is because a chirp signal forms a line in the time-frequency plane, and therefore, there exists an order of transformation in wich such signals are compact. Chirp signals are not compact in the time or spatial domain. Thus we can extract the signal easily in an appropiate (or we can say optimum) fractional Fourier domain. When it is not possible to separate the signal and noise in the spatial or frequency domain [14].

The extra degree of freedom introduced by the choice of the fractional order of transformation (angle of rotation) gives the FrFT a potential improvement in any application where the ordinary FT is used.

V. Fractional RDA

The basic idea of the proposed technique lies in the fact that the optimum Fractional Fourier transform for a chirp signal allows to obtain a good resolution in the time-frequency domain. In example two different chirp signals, that overlap in time and frequency can be distinguished using the FrFT.

The two dimensional chirp behaviour [1] of the received SAR signal allows to develop a simple and intuitive algorithm to demonstrate the capabilities of the Fractional Fourier Transform in the SAR processing.
The compression is performed throught the use of the opitmum transformation angles for both the range and azimuth direction. The optimum transformation angle is defined as [15]:

$$a_{opt} = -\frac{2}{\pi} \tan^{-1} \left(\frac{F_s^2/N}{2k} \right) \tag{3}$$

Where F_s is the sampling frequency, in the range direction is the receiver sampling frequency while in the azimuth direction is the PRF. The number of samples is represented by N that in the azimuth direction is the number of acquired echoes while in range is the number of samples acquired in the gate. k is the chirp rate that is different for the azimuth and for the range direction.
The proposed algorithm is illustrated in Figure 3:

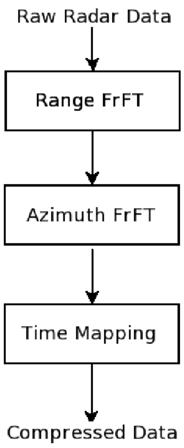

Figure 3. Fractional Range-Doppler SAR processing algorithm

1. The first step in the algorithm is the use of the Fractional Fourier Transform along the azimuth gates to perform the range compression step.
2. After this step the same approach is used to focus the data along the range gates obtaining the azimuth compression.
3. A time mapping operation is needed due to the use of the FrFT to correctly locate the data in the image plane.

In the proposed version of the algorithm the RCMC step is neglected, its future development will increase the algorithm performances.

VI. Results

Some preliminary results have been obtained using simulated data, the working SAR configuration to generate the raw SAR data is reported in Table I: The results obtained with the RDA and the Fractional RDA for two close point scatterers in the scene are presented in Figure 4: For the simulation good focusing is obtained using the FrRDA. The position of the detected scatterers is in accordance with the position of the simulated data and has is a good match to the position of the focused scatterers obtained with the RDA. Moreover a lower side lobe level is obtained both for the range and azimuth directions as illustrated in Figure 5. From the simulation results the FrRDA offers improved resolution of the two different scatterers because of the narrower shape of the peak.

PRF	600 Hz
Carrier Frequency	4.5 GHz
Pulse Duration	2.5 μs
Flight Duration	3 s
Pulse Bandwidth	400 MHz
Platform Velocity	200 m/s

Table I
SAR SIMULATION PARAMETERS

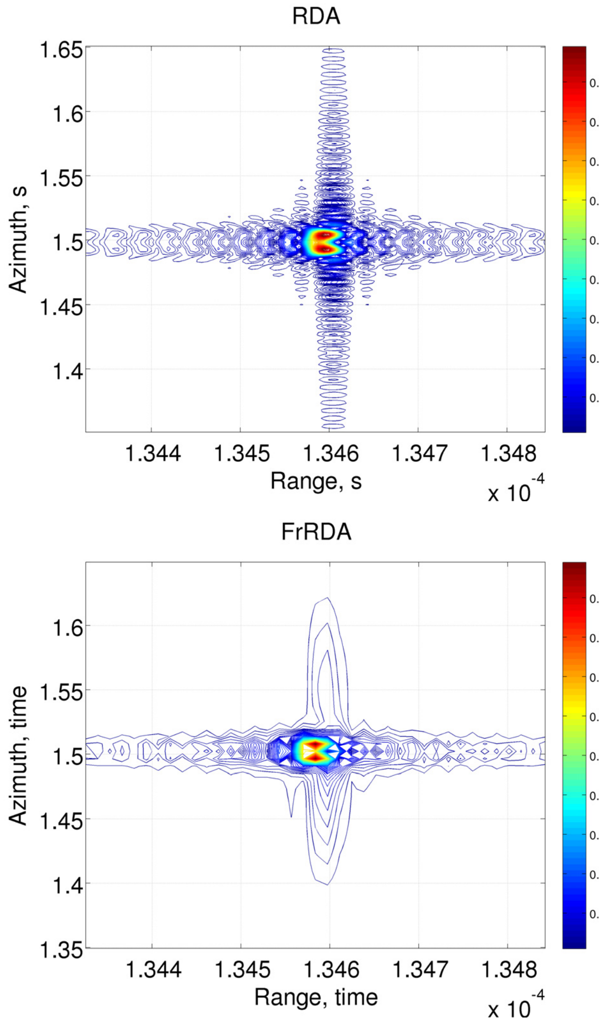

Figure 4. RDA and FrRDA Responses for two point scatterers

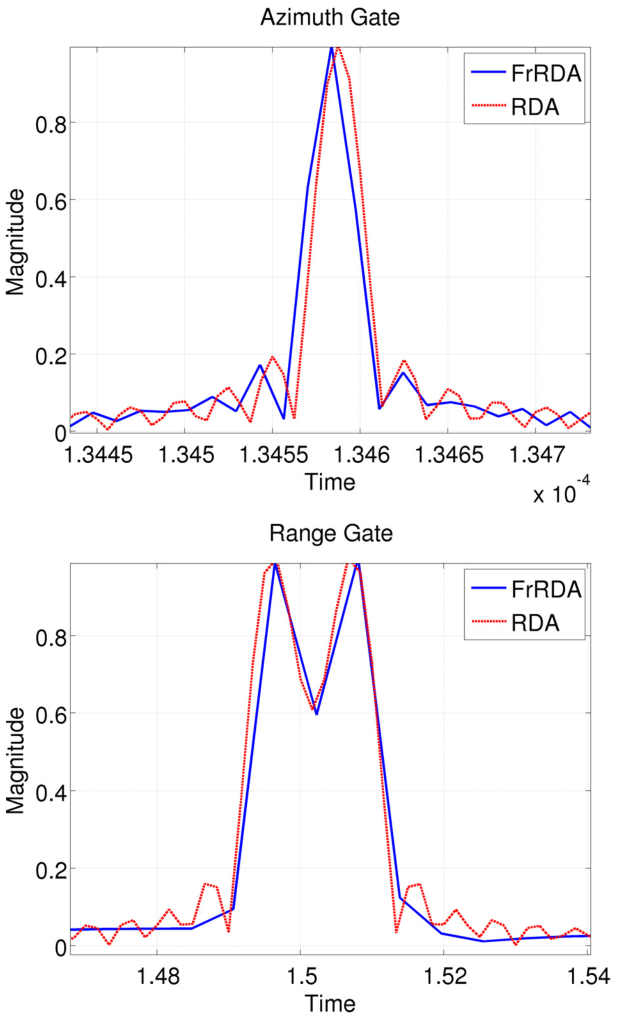

Figure 5. Absolute value of the focused signal for a range gate and an azimuth gate.

VII. Conclusions

In this work a new approach to perform SAR focusing has been proposed using the Fractional Fourier Transform applied to the Range Doppler Algorithm. The resulting FrRDA algorithm uses the property of the Fractional Fourier Transform to resolve chirp signals to increase the resolution of the point scatterers in the scene.

The presented algorithm allows us to focus SAR data correctly and to increase the resolution of the point scatterers. The resulting side lobe level results to be lower than those obtained using the conventional RDA. Further investigations will be conducted to correct the range rell migration effect. Analysis relative to the imaging of extended objects will be also carried out.

Acknowledgements

This work was supported by the Engineering and Physical Research Council (grant N° EP/H012877/1) and the MOD University Defence Research Centre in Signal Processing.

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