Decoupling and Parallelisation for Broadband Multichannel Problems

Stephan Weiss

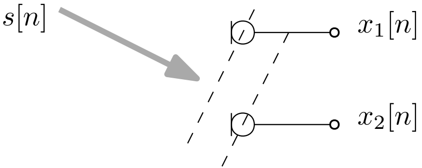
Dept. of Electronic & Electrical Eng., University of Strathclyde, Glasgow, Scotland
stephan.weiss@strath.ac.uk

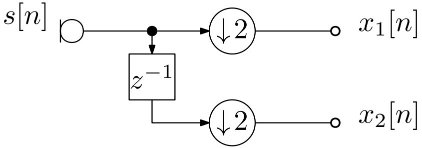
UDRC Themed Meeting on Algorithm Implementations and Low Swap Challenges,
November 30. 2022

This work was supported by the Engineering and Physical Sciences Research Council (EPSRC) Grant number EP/S000631/1 and the MOD University Defence Research Collaboration in Signal Processing.

Background: Broadband Multichannel Signals

- Imagine data acquired by an array of M sensors;
- for spatially propagating signals, directional information is embedded in the relative delay;

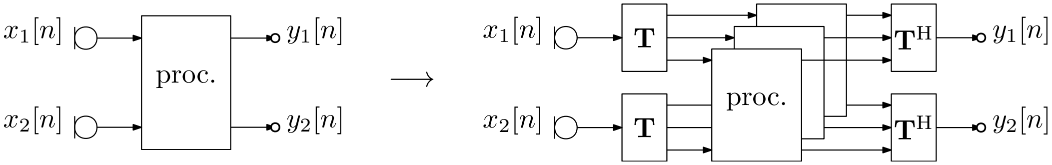




- we typically need filters, not just complex gain factors, to process the signals $x_m[n]$, $m = 1, \ldots, M$;
- multichannel signals may also arise from single channel data through demultiplexing — example: high speed analogue-to-digital converters.

Task of Parallelisation

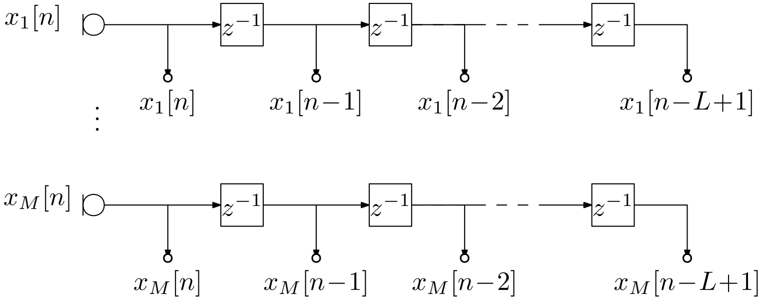
- Typically we want to perform some processing on the data: angle of arrival estimation, beamforming, signal separation, enhancement, etc.;



- the aim is to transform the data to enable the execution of parallel processing tasks;
- parallisation can lead to computational efficiency, and also potentially a reduction of the challenges.

Broadband Processing

- To resolve broadband signals, a tap delay line of length L is applied to each sensor output:



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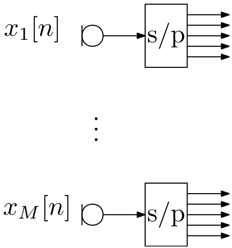
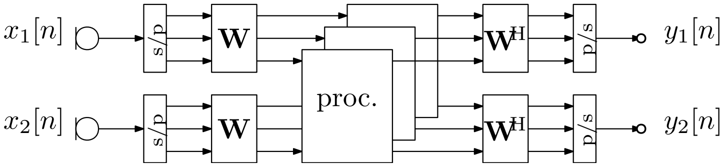


Discrete Fourier Transform-Based Processing

- A popular approach is to perform L-point DFT matrices $\mathbf{W}$, and then perform independent processing in individual frequency bins;
- We apply an L-point DFT matrix $\mathbf{W}$ to the tap delay lines:

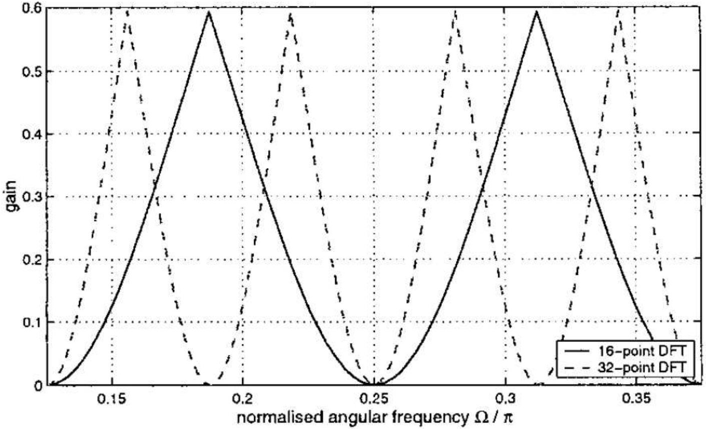


- bins are treated independently; processing typically will be narrowband;
- optimal in terms of computational complexity;
- spectral coherence is neglected: suboptimal in terms of performance; reconstruction can suffer.

Are DFT Bins Independent?

- No — broadband signals generally are spectrally coherent;

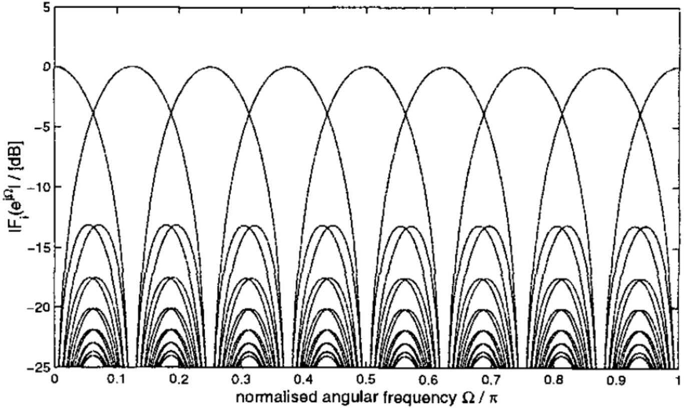
- increasing the length L of the DFT divides the problem into a larger number of smaller problems;
- we have a finer spectral resolution;
- but it does not reduce the worst-case error.



Weiss, Proudler: "Comparing efficient broadband beamforming architectures and their performance trade-offs", DSP, 2002.

Filter Banks

- DFT downsides: (i) processing bins in isolation; (ii) performing narrowband processing despite insufficient frequency selectivity:



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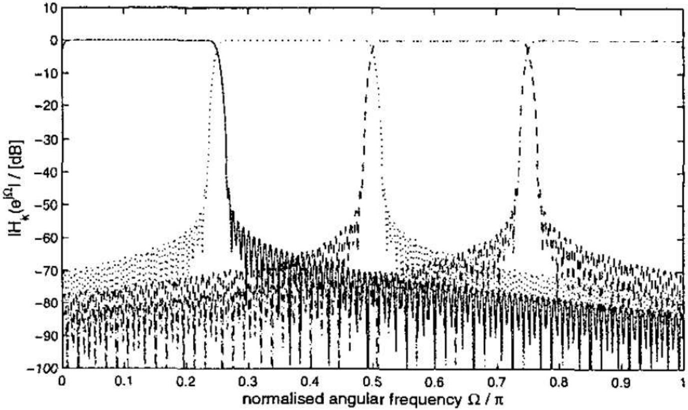


- (ii) can be enhanced by using more selective filter banks, but the computational complexity increases.

Data-Dependent Filter Banks and Space-Time Covariance

- DFT and filter banks are data independent transforms;
- in the narrowband case, the Karhunen-Loeve transform (KLT) is optimum to decouple data;
- space-time covariance matrix including explicit lags $\tau \in \mathbb{Z}$:

$$\mathbf{R}[\tau] = \mathcal{E}\{\mathbf{x}[n]\mathbf{x}^{\mathrm{H}}[n-\tau]\} = \mathbf{R}^{\mathrm{H}}[-\tau] \quad \rightarrow \quad \boldsymbol{R}(z) = \sum_{\tau} \mathbf{R}[\tau] z^{-\tau} = \boldsymbol{R}^{\mathrm{P}}(z) = \boldsymbol{R}^{\mathrm{H}}(1/z^{*})$$

- $\boldsymbol{R}(z)$ is a matrix of functions, and parahermitian — and extension of being symmetric / Hermitian;
- the eigenvalue decomposition of $\mathbf{R}[0]$ yields the KLT — instantaeous decorrelation of data.

Parahermitian Matrix EVD

- Eigenvalue decomposition of a parahermitian matrix $\boldsymbol{R}(z)$ that is analytic in $z \in \mathbb{C}$ [7, 8]:

$$\boldsymbol{R}(z) = \mathbf{Q}(z)\boldsymbol{\Lambda}(z)\boldsymbol{Q}^{\mathrm{P}}(z) \; ; \tag{1}$$

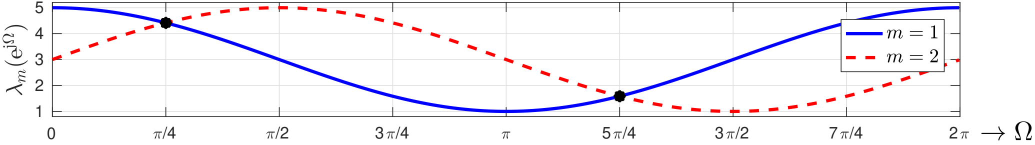
- the diagonal, parahermitian matrix $\boldsymbol{\Lambda}(z) = \mathrm{diag}\{\lambda_1(z), \ldots, \lambda_m(z)\}$ contains the unique analytic eigenvalues;
- the matrix of analytic eigenvectors,

$$\boldsymbol{Q}(z) = [\boldsymbol{q}_1(z), \; \ldots, \; \boldsymbol{q}_M(z)] \; , \tag{2}$$

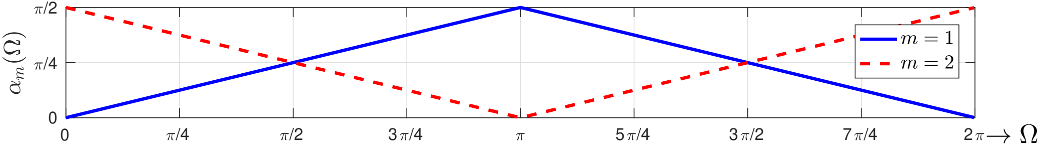
  is paraunitary: $\boldsymbol{Q}(z)\boldsymbol{Q}^{\mathrm{P}}(z) = \boldsymbol{Q}^{\mathrm{P}}(z)\boldsymbol{Q}(z) = \mathbf{I}$;
- eigenvectors unique up to arbitrary allpass functions $\psi_m(z)$: $\psi_m(z)\boldsymbol{q}_m(z)$ is also a valid eigenvector corresponding to $\lambda_m(z)$;
- different from the KLT, this decomposition diagonalised $\boldsymbol{R}(z)$ for every z.

Example for a Parahermitian Matrix EVD

- $\boldsymbol{R}(z)$ with PhEVD factors $\mathbf{q}_{1,2}(z) = [1, \pm z^{-1}]^{\mathrm{T}}/\sqrt{2}$, $\lambda_1(z) = z + 3 + z^{-1}$ and $\lambda_2(z) = \mathrm{j}z + 3 - \mathrm{j}z^{-1}$;
- eigenvalues on the unit circle:



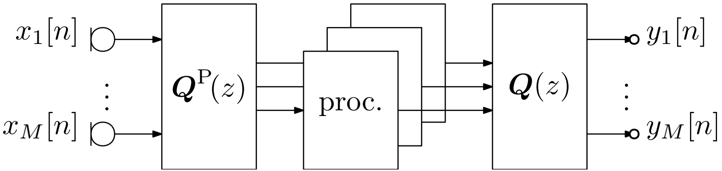
- Hermitian angle of eigenvectors $\cos\alpha_{1,2}(\Omega) = |\mathbf{q}_1^{\mathrm{H}}(\mathrm{e}^{\mathrm{j}0})\mathbf{q}_{1,2}(\mathrm{e}^{\mathrm{j}\Omega})|$:



- note: the eigenvalues have an algebraic multiplicity of two for $\Omega = \pi/4$ and $\Omega = 5\pi/4$.

Processing Based on the Paraunitary Matrix

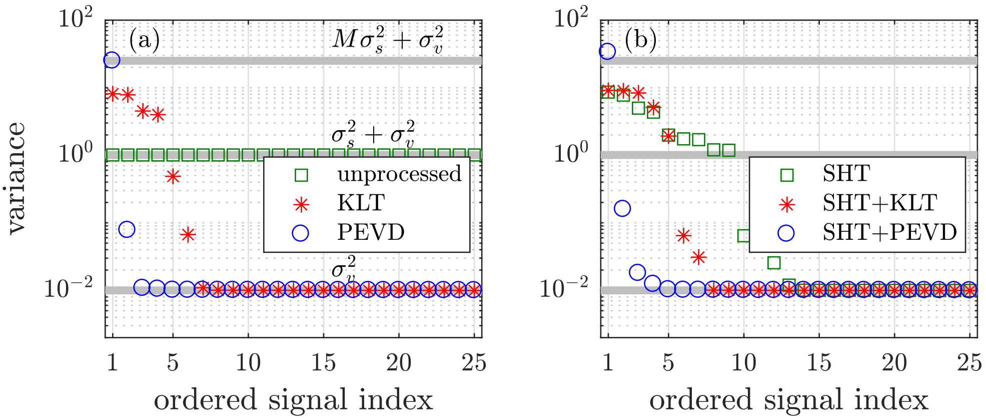
- A number of algorithms exist to compute or approximate the PhEVD [3, 5, 1, 6, 9];
- these can be efficiently implemented [2];
- once computed, the paraunitary matrix can used to spatially decouple the data:



- the output of the processor block $\boldsymbol{Q}^{\mathrm{P}}(z)$ is strongly decorrelated;
- the processing can be performed on decorrelated signal components;
- spectral coherence is preserved — this can provide much enhanced perceptual quality [4].

PhEVD-Based Decoupling — Example

- Spherical microphone array data with $M = 26$ recording a single speaker in omnidirectional noise;
- we compare a spherical harmonic transform (SHT) to KLT and PhEVD:



Summary

- We have considered preprocessing in order to decouple broadband multichannel signal processing tasks;
- processing in independent frequency bins is computationally optimal but suboptimal in terms of performance — the worst-case error is typically does not dependent on the DFT length;
- a parahermitian matrix eigenvalue decomposition can decouple the data; for the instantaneous/narrowband case, this is equivalent to a Karhunen-Loeve transform;
- for a spherical microphone array and a speaker, apart from the decoupling, weak (approximately noise-only) bands have been isolated;
- hence processing tasks can be parallelised, or even be suppressed if the decorrelated subchannels are sufficiently weak in power.

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