Beyond the Ambiguity Function

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UDRC Themed Meeting on Signal Processing for the Electromagnetic Environment, 25 November 2020

Outline

- The Electromagnetic Environment
- The Ambiguity Function
- The Ambiguity Function for Bistatic Radar
- Spectral Containment
- Cognitive Radar
- Conclusions

The RF spectrum

A spectrum allocation plan from 1927.

It was all so simple then …

RADIOTELEGRAPH REGULATIONS, WASHINGTON, 1927. [Art. 5.

Frequencies in Kilocycles per Second (kc/s.)	Approximate Wave-lengths in Metres.	Services.
550–1,300[4]	545–230[4]	Broadcasting.
1,300–1,500	230–200	(*a*) Broadcasting. (*b*) Maritime mobile services, wave of 1,365 kc/s (220 m.) exclusively.
1,500–1,715	200–175	Mobile services.
1,715–2,000	175–150	Mobile services. Fixed services. Amateurs.
2,000–2,250	150–133	Mobile services and fixed services.
2,250–2,750	133–109	Mobile services.
2,750–2,850	109–105	Fixed services.
2,850–3,500	105–85	Mobile services and fixed services.
3,500–4,000	85–75	Mobile services. Fixed services. Amateurs.
4,000–5,500	75–54	Mobile services and fixed services.
5,500–5,700	54–52·7	Mobile services.
5,700–6,000	52·7 –50	Fixed services.
6,000–6,150	50 –48·8	Broadcasting.
6,150–6,675	48·8 –45	Mobile services.
6,675–7,000	45 –42·8	Fixed services.
7,000–7,300	42·8 –41	Amateurs.
7,300–8,200	41 –36·6	Fixed services.
8,200–8,550	36·6 –35·1	Mobile services.
8,550–8,900	35·1 –33·7	Mobile services and fixed services.
8,900–9,500	33·7 –31·6	Fixed services.
9,500–9,600	31·6 –31·2	Broadcasting.
9,600–11,000	31·2 –27·3	Fixed services.
11,000–11,400	27·3 –26·3	Mobile services.
11,400–11,700	26·3 –25·6	Fixed services.
11,700–11,900	25·6 –25·2	Broadcasting.
11,900–12,300	25·2 –24·4	Fixed services.
12,300–12,825	24·4 –23·4	Mobile services.
12,825–13,350	23·4 –22·4	Mobile services and fixed services.
13,350–14,000	22·4 –21·4	Fixed services.
14,000–14,400	21·4 –20·8	Amateurs.
14,400–15,100	20·8 –19·85	Fixed services.
15,100–15,350	19·85–19·55	Broadcasting.
15,350–16,400	19·55–18·3	Fixed services.
16,400–17,100	18·3 –17·5	Mobile services.
17,100–17,750	17·5 –16·9	Mobile services and fixed services.
17,750–17,800	16·9 –16·85	Broadcasting.
17,800–21,450	16·85–14	Fixed services.
21,450–21,550	14 –13·9	Broadcasting.
21,550–22,300	13·9 –13·45	Mobile services.
22,300–23,000	13·45–13·1	Mobile services and fixed services.
23,000–28,000	13·1 –10·7	Not reserved.
28,000–30,000	10·7 –10	Amateurs and experiments.
30,000–56,000	10 –5·35	Not reserved.
56,000–60,000	5·35–5	Amateurs and experiments.
Above 60,000	Below 5	Not reserved.

[4] Mobile services may use the band 550 to 1,300 kc/s (545–230 m.) on condition that such use does not interfere with the services of a country which uses this band exclusively for broadcasting.

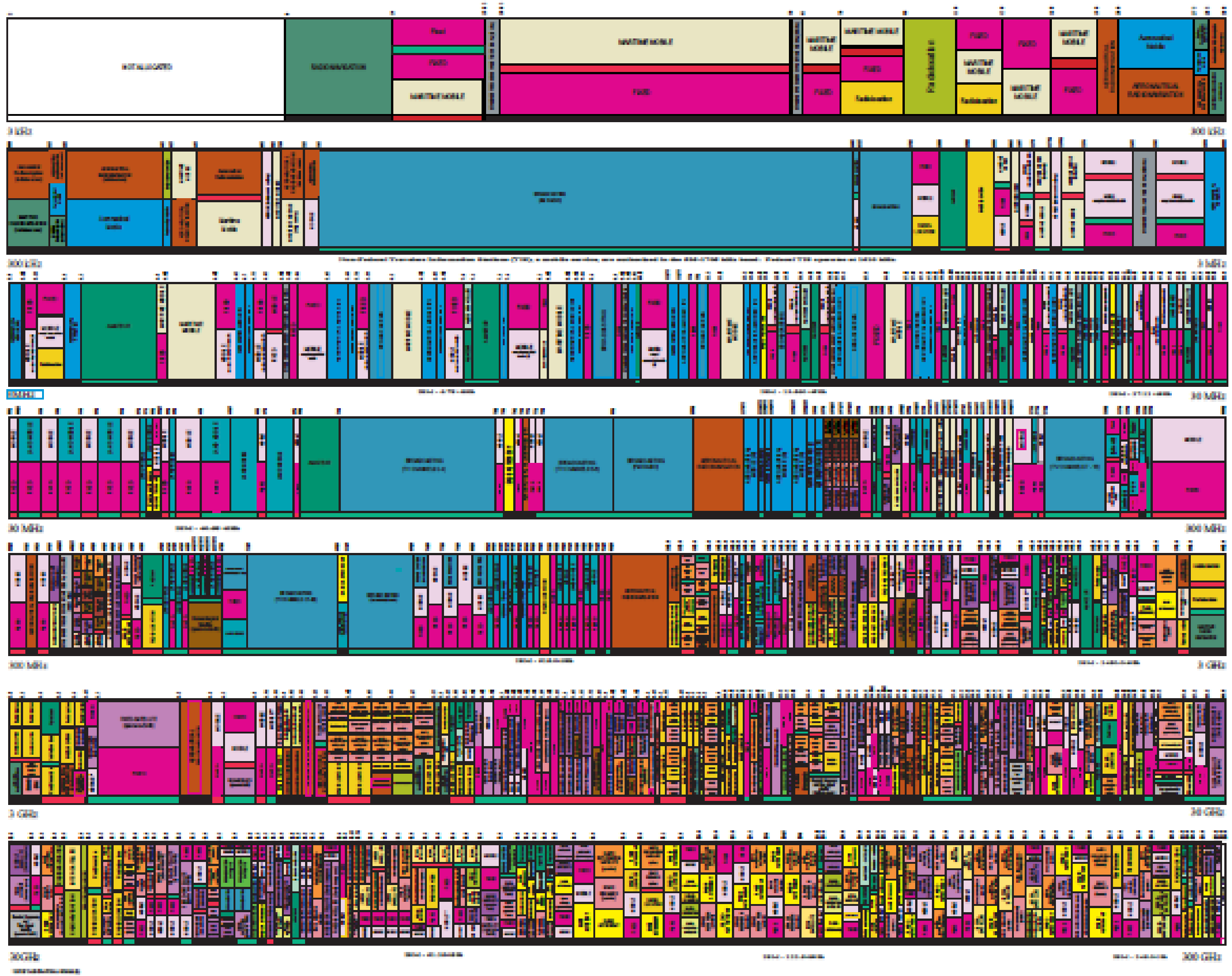
NOTE.—It is recognized that short waves (frequencies from 6,000 to 23,000 kc/s approximately—wave lengths from 50 to 13 m. approximately) are very efficient for long distance communications. It is recommended that as a general rule this band of waves should be reserved for that purpose, in services between fixed points.

41

General Regulations annexed to the International Radiotelegraph Convention (Washington,1927) http://search.itu.int/history/HistoryDigitalCollectionDocLibrary/1.4.48.en.100.pdf#search=frequency%20allocation

UNITED STATES FREQUENCY ALLOCATIONS

THE RADIO SPECTRUM




THE RADIO SPECTRUM
RADIO SERVICES COLOR LEGEND
AERONAUTICAL MOBILE
INTER-SATELLITE
RADIO ASTRONOMY
AERONAUTICAL MOBILE SATELLITE
LAND MOBILE
RADIODETERMINATION SATELLITE
AERONAUTICAL RADIONAVIGATION
LAND MOBILE SATELLITE
RADIOLOCATION
AMATEUR
MARITIME MOBILE
RADIOLOCATION SATELLITE
AMATEUR SATELLITE
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RADIONAVIGATION
BROADCASTING
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METEOROLOGICAL AIDS
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EARTH EXPLORATION SATELLITE
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SPACE RESEARCH
FIXED
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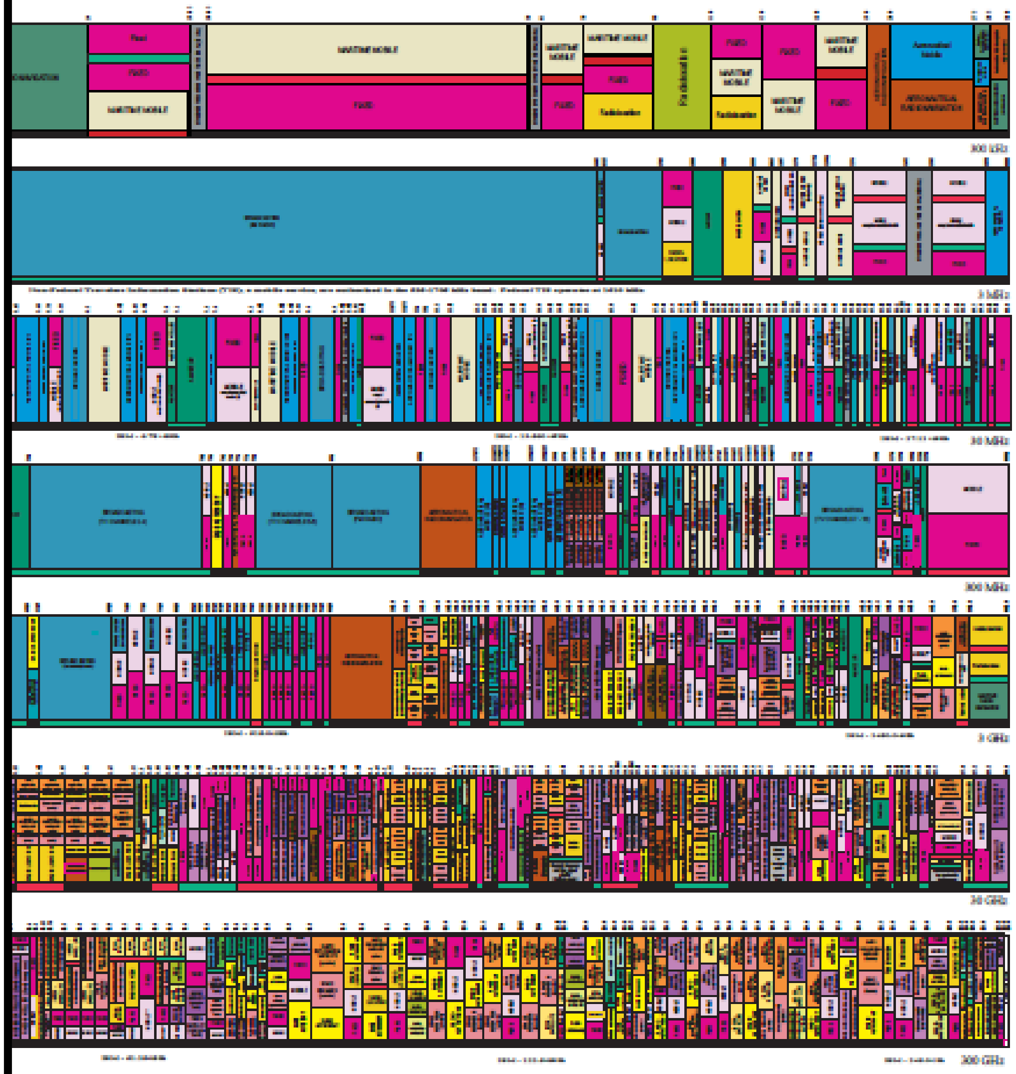
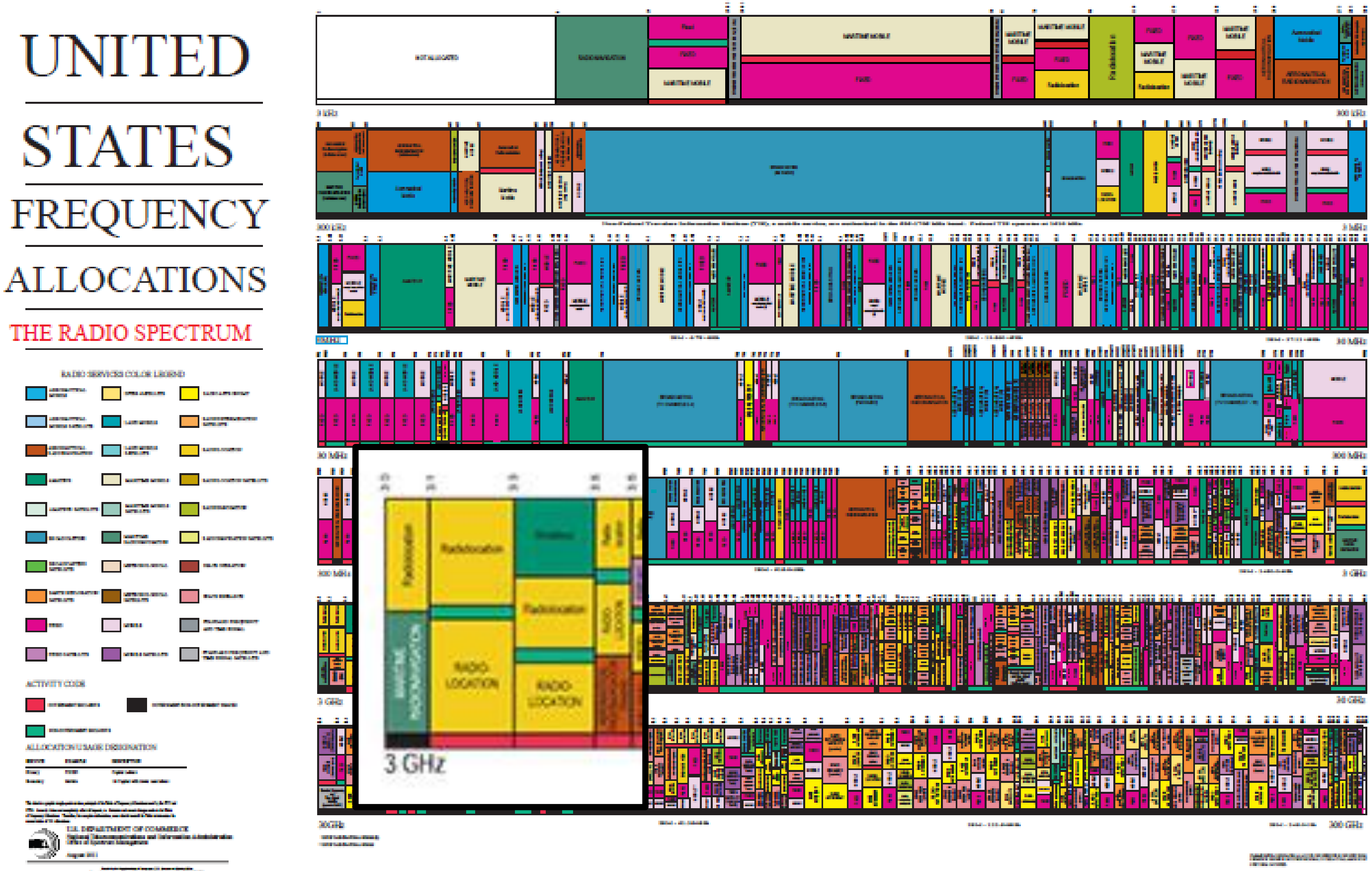
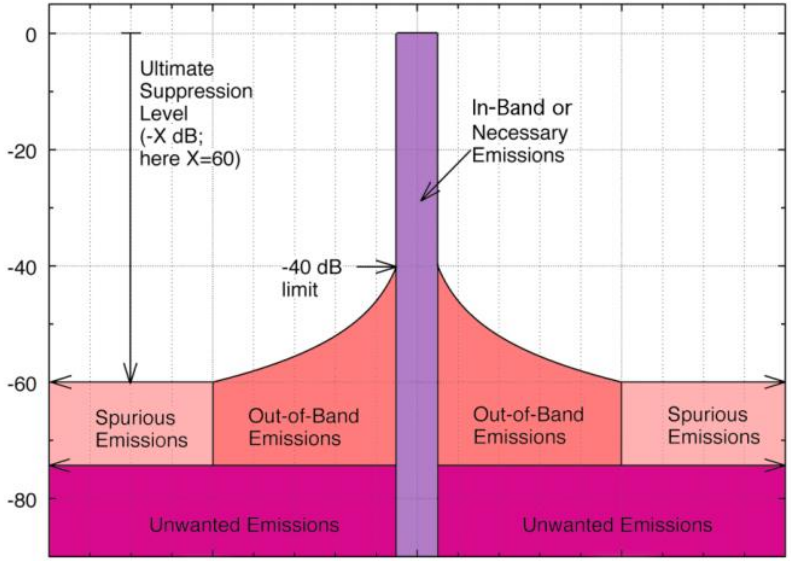

300 kHz
3 MHz
30 MHz
300 MHz
3 GHz
30 GHz
300 GHz


UNITED
STATES
FREQUENCY
ALLOCATIONS
THE RADIO SPECTRUM
Radiolocation
RADIO LOCATION
3 GHz

Radar Waveforms

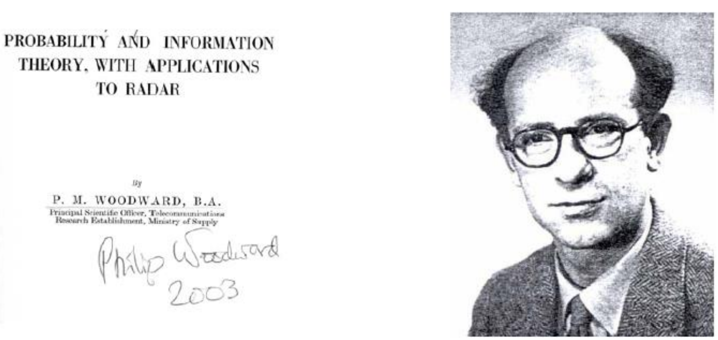
- Conventionally, pulsed or chirped waveforms
- Ambiguity function
- **Modern digital signal processing now allows us to generate precise, wide bandwidth waveforms of arbitrary form, and to vary them on a pulse-by-pulse basis in response to a dynamically-changing target scene**
- Ultra-low range sidelobes
- Spectrally-clean waveforms

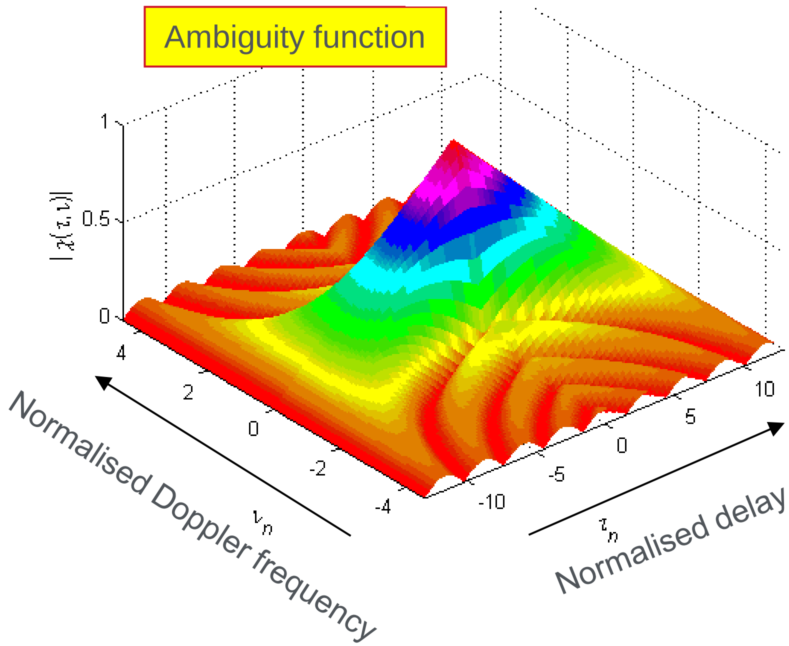


The Ambiguity Function

$$|\chi(\tau, f)|^2 = \left|\int u(t)u^*(t+\tau)\exp(j2\pi ft)\,dt\right|^2$$

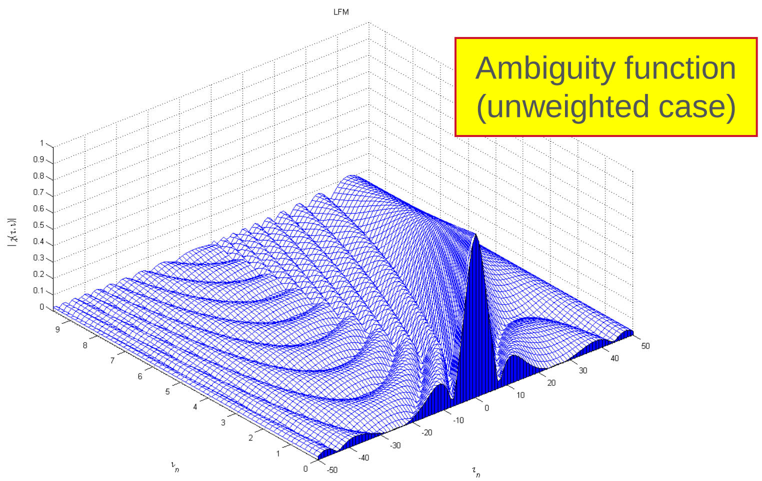
P.M.Woodward:
Probability and information theory, with applications to radar (1953).

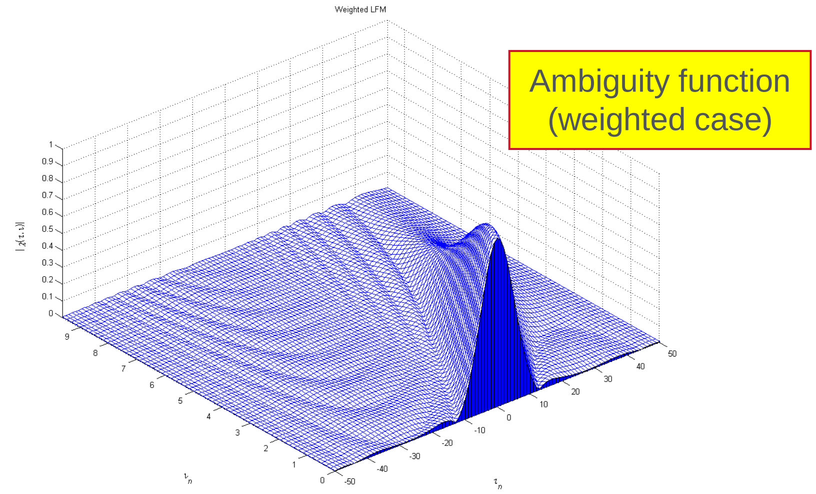


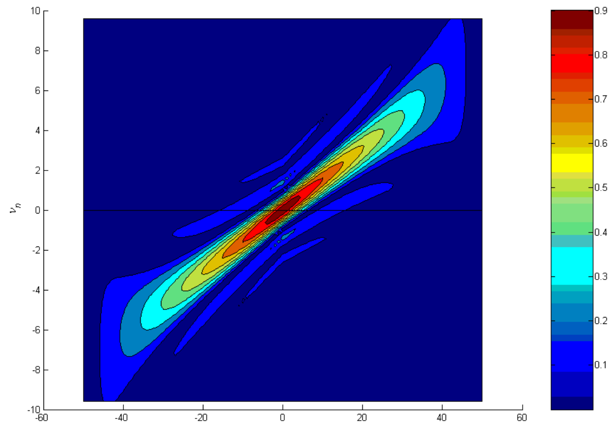


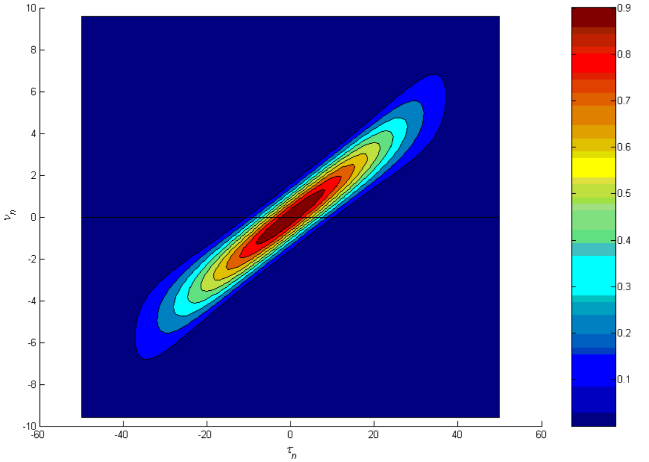
LFM and Weighted LFM Pulses

Low sidelobes can be obtained through a suitable Chebishev weighting in the time domain.



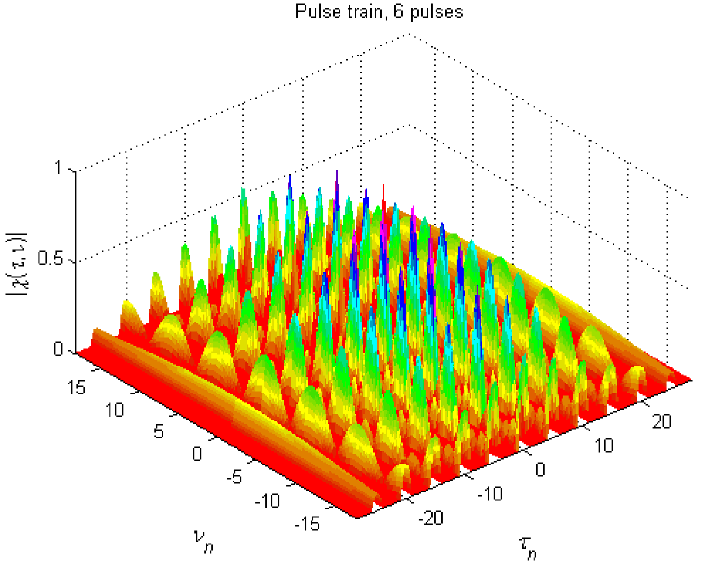




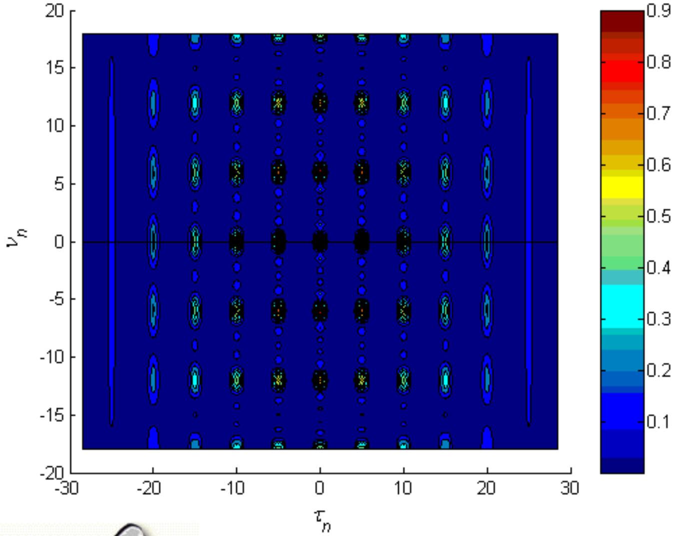


Ambiguity function contours

Ambiguity Function: regular pulse train

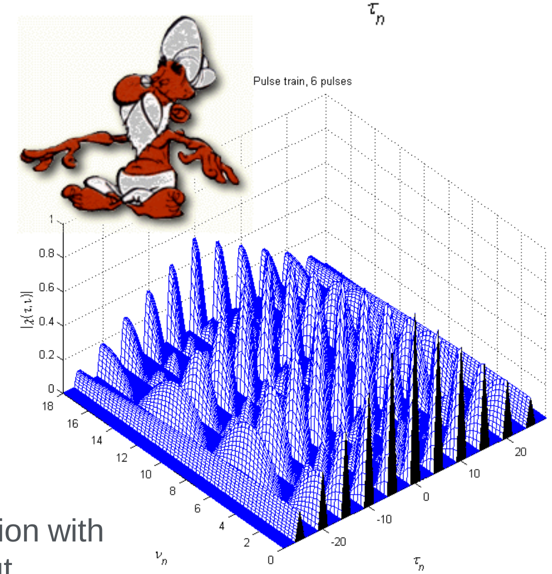


Ambiguity function



BED OF NAILS

It achieves good resolution in both delay and Doppler but there are both range and Doppler ambiguities.

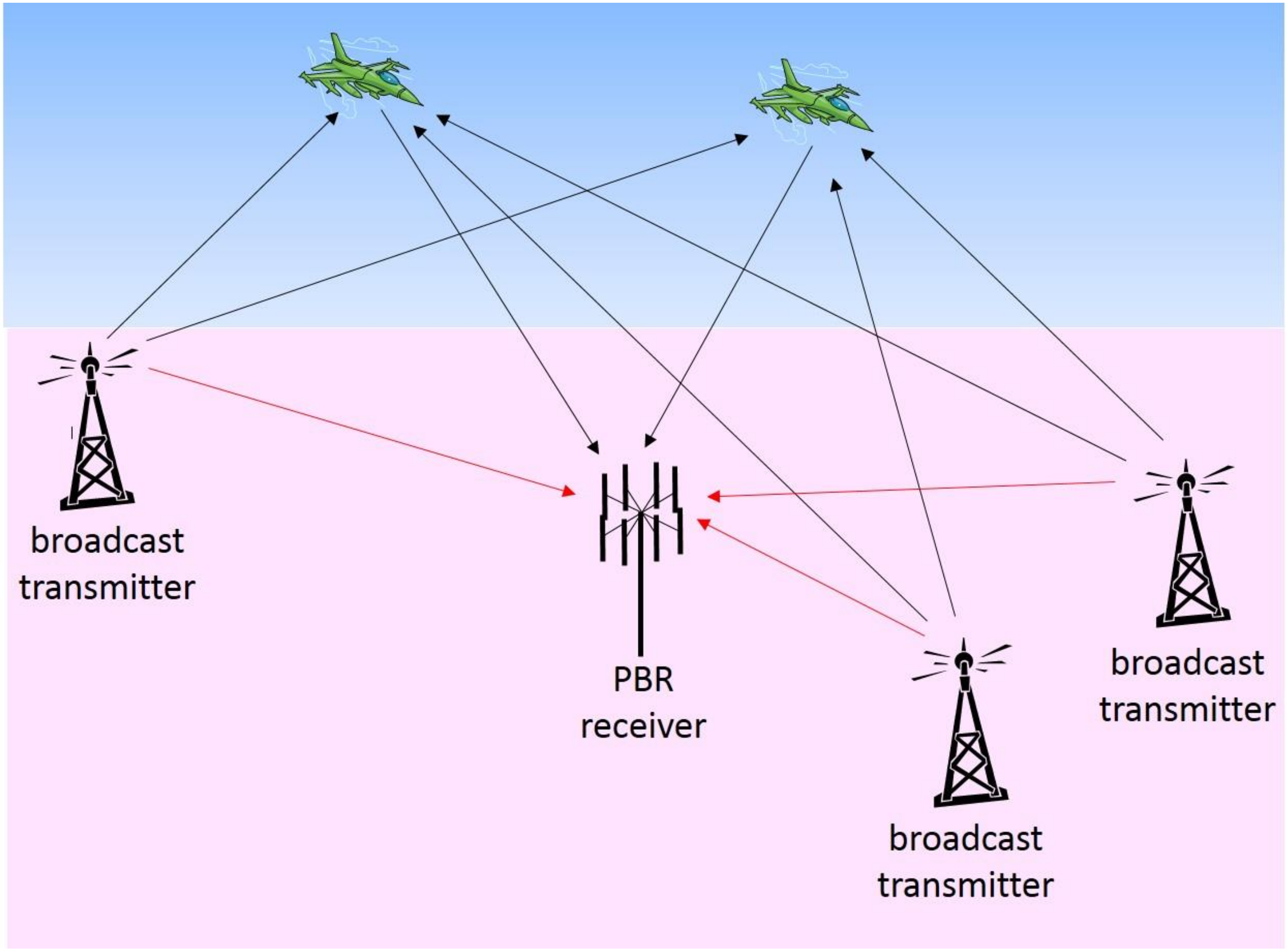


Partial Ambiguity function with Zero-Doppler cut

But …

- The ambiguity function does not tell the whole story
- It says nothing about spectral content
- In the case of a bistatic radar, it depends not only on the waveform but also on the bistatic geometry
- In the case of a Passive Bistatic Radar it depends both on the instantaneous modulation of the waveform and on the bistatic geometry
- But because these dependences are deterministic it should be possible to select the optimum transmissions to use to track a given target on a dynamic basis

Passive Bistatic Radar



Terrestrial analogue TV

measured
digital) TV

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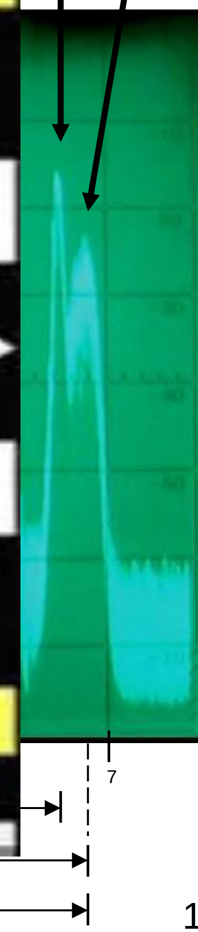
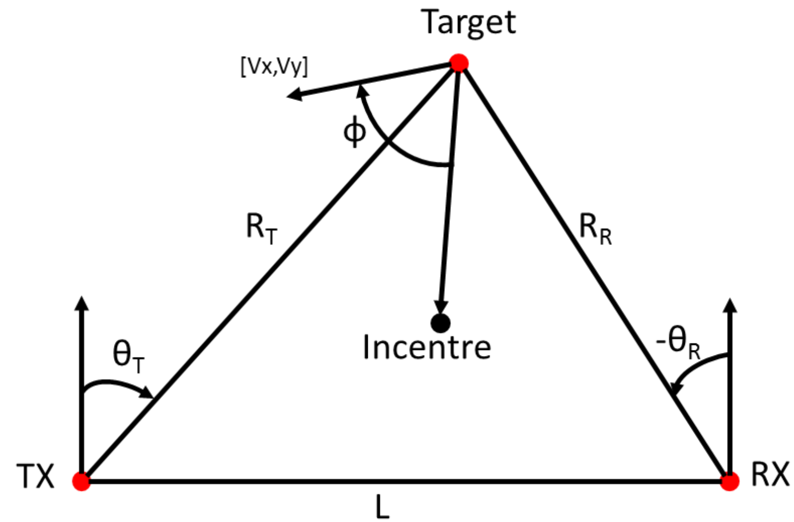


Bistatic Ambiguity Function

- Depends on the geometry of the TX-RX pair relative to the target
- The time-delay and Doppler shift of a signal measured by a receiver will also depend on the bistatic geometry
- Simple rule of thumb:

  <u>The closer the target is to the baseline the more ambiguous the range-Doppler estimate</u>



$$|\chi(\tau_h, \tau_a, vh, va)|^2 = \left| \int_{-\infty}^{\infty} s(t-\tau_a)s^*(t-\tau_h)e^{j2\pi(va-vh)t}dt \right|^2$$

$$\tau_h = \frac{R_R = \sqrt{(R_R)^2 + L^2 + 2RRLsin\theta_R}}{c}$$

$$v_h = \frac{2fc}{c}Vcos\phi\sqrt{\frac{1}{2} + \frac{R_R + Lsin\theta_R}{2\sqrt{(R_R)^2 + L^2 + 2R_RLsin\theta_R}}}$$

Previous Research

- Previous research at the University of Pisa* considered the following to construct the bistatic range-Doppler AF
  - Pulsed LFM parameters: PRF and BW
  - Relative locations of each transmitter, receiver and target
- CRLB of the bistatic AF used to estimate range-Doppler performance of each bistatic configuration
- They aimed to find the optimal bistatic configuration within a multistatic network for range-Doppler estimation of a target

*M.S. Greco, P. Stinco, F. Gini and A. Farina, 'Cramer-Rao Bounds and selection of bistatic channels for multistatic radar systems', *IEEE Trans. AES*, Vol.47, No.4, pp2934 - 2948, 2011.

The inverse problem

- The Fisher information matrix can be calculated from the measurement likelihood function

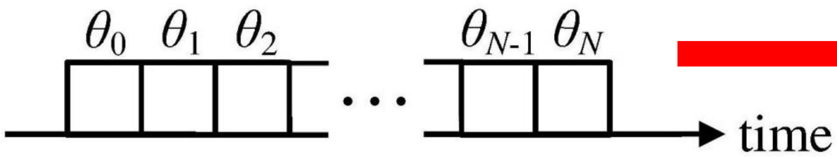
$$\boldsymbol{J} = -E\left[\left[\nabla_X \Lambda(\tau,\omega)\right]\left[\nabla_X \Lambda(\tau,\omega)\right]'\right] = \begin{bmatrix} \frac{\partial^2 \left|\ln\Lambda(\tau,\omega)\right|}{\partial\tau^2} & \frac{\partial^2 \left|\ln\Lambda(\tau,\omega)\right|}{\partial\tau\partial\omega} \\ \frac{\partial^2 \left|\ln\Lambda(\tau,\omega)\right|}{\partial\tau\partial\omega} & \frac{\partial^2 \left|\ln\Lambda(\tau,\omega)\right|}{\partial\omega^2} \end{bmatrix}$$

- The ambiguity function is the log likelihood function excluding the effect of signal attenuation and noise. The inverse of the Fisher information is the Cramér-Rao Lower Bound which bounds the error variance of the estimates produced from the radar measurements.

$$\boldsymbol{C} = E\left[\left[x_k - \hat{x}_{k|k}\right]\left[x_k - \hat{x}_{k|k}\right]' \middle| Z^k\right] \geq \boldsymbol{J}^{-1}$$

Spectral Containment

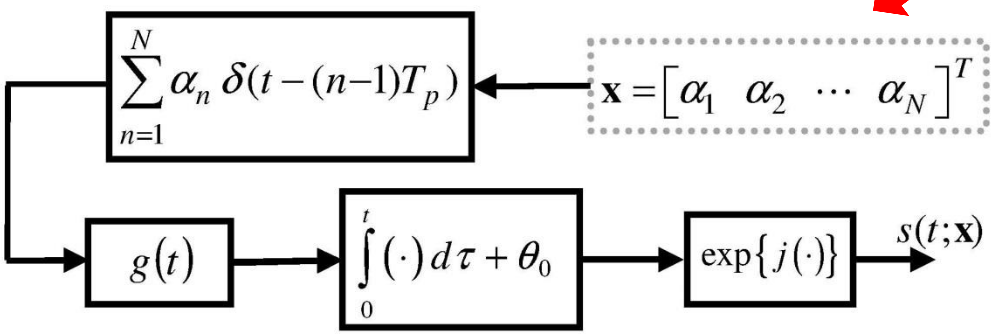
polyphase code => NLFM

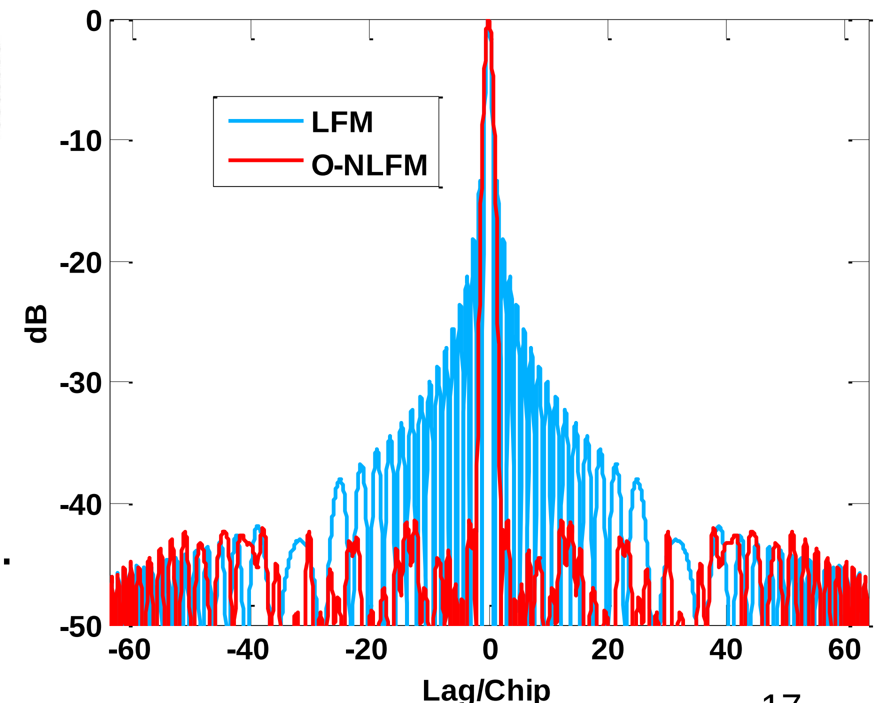


$$-\pi \le \theta_n \le \pi$$

$$\tilde{\alpha}_n = \theta_n - \theta_{n-1} \quad \text{for } n = 1, \cdots, N$$

$$\alpha_n = \begin{cases} \tilde{\alpha}_n & \text{if } |\tilde{\alpha}_n| \le \pi \\ \tilde{\alpha}_n - 2\pi \,\mathrm{sgn}(\tilde{\alpha}_n) & \text{if } |\tilde{\alpha}_n| > \pi \end{cases}$$



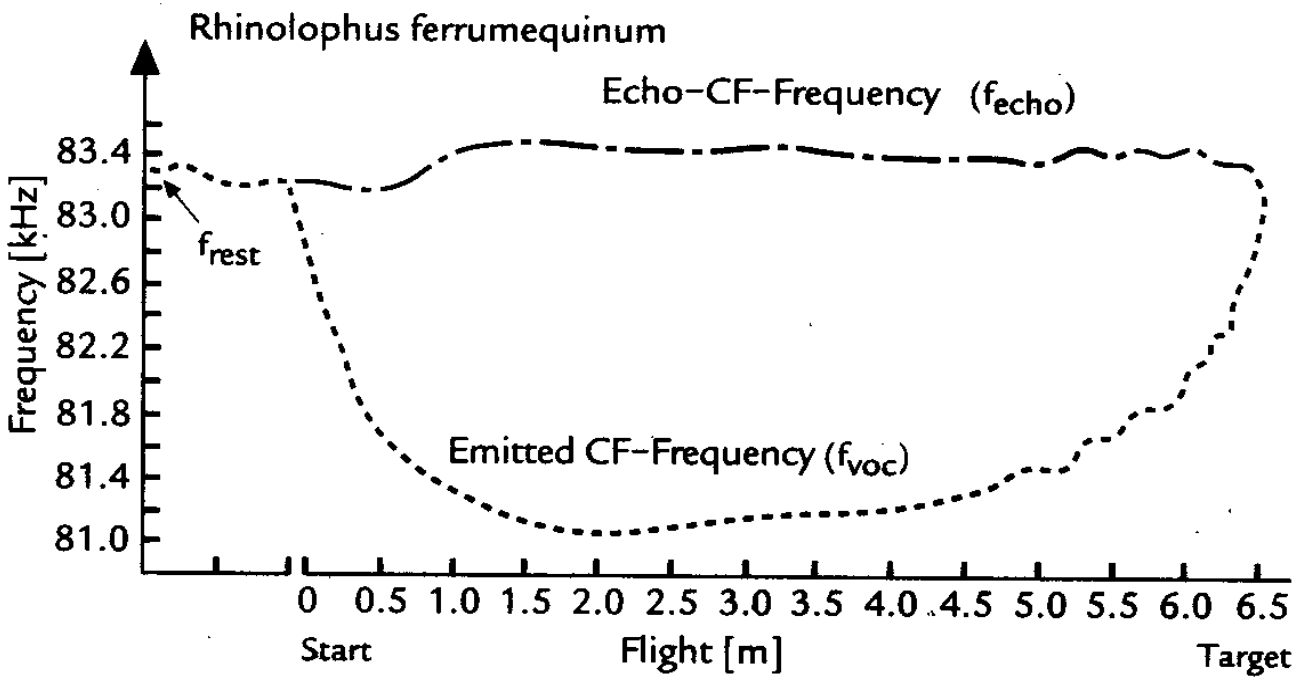


CPM implementation converts discrete polyphase code into a continuous NLFM waveform with good spectral containment.

Biologically-inspired cognitive sensing

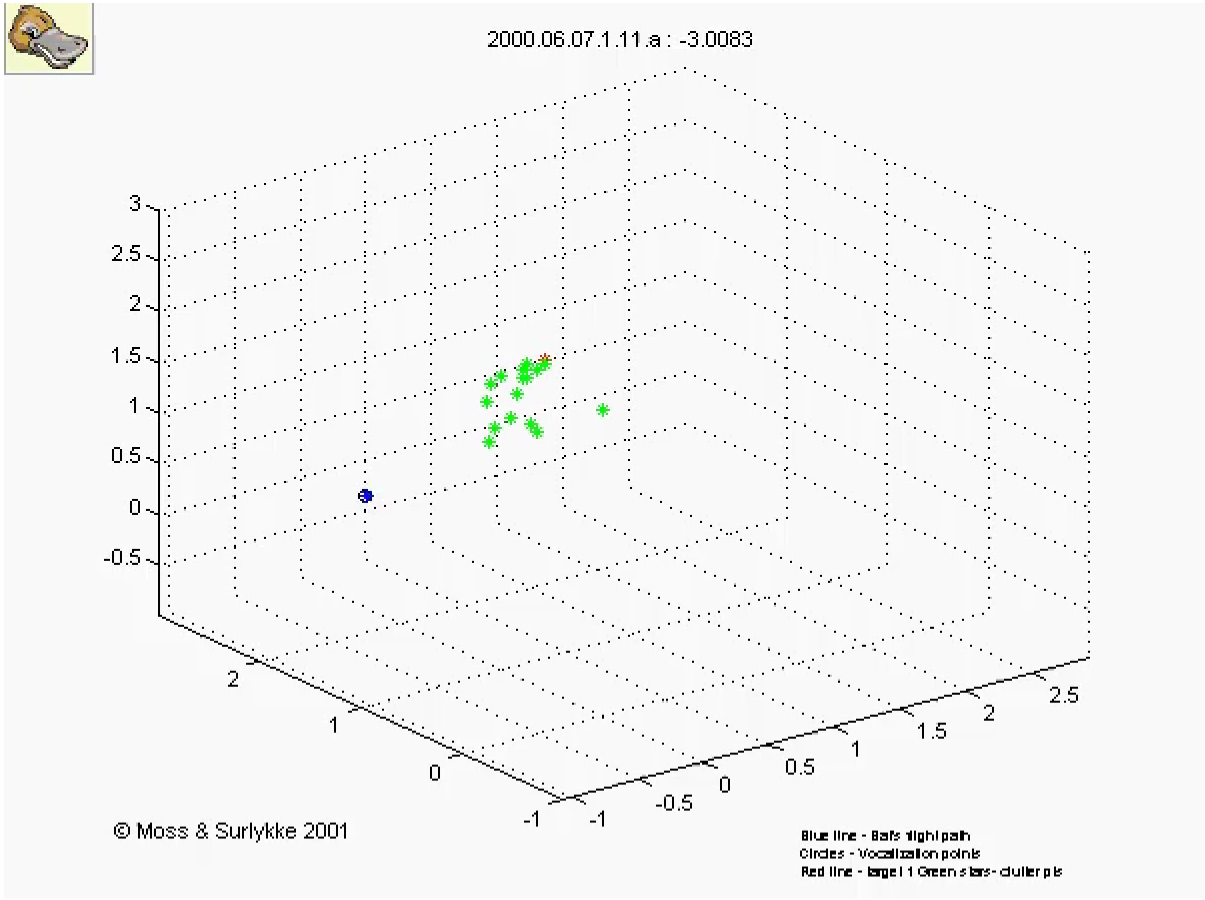
progdata.umflint.edu

Doppler



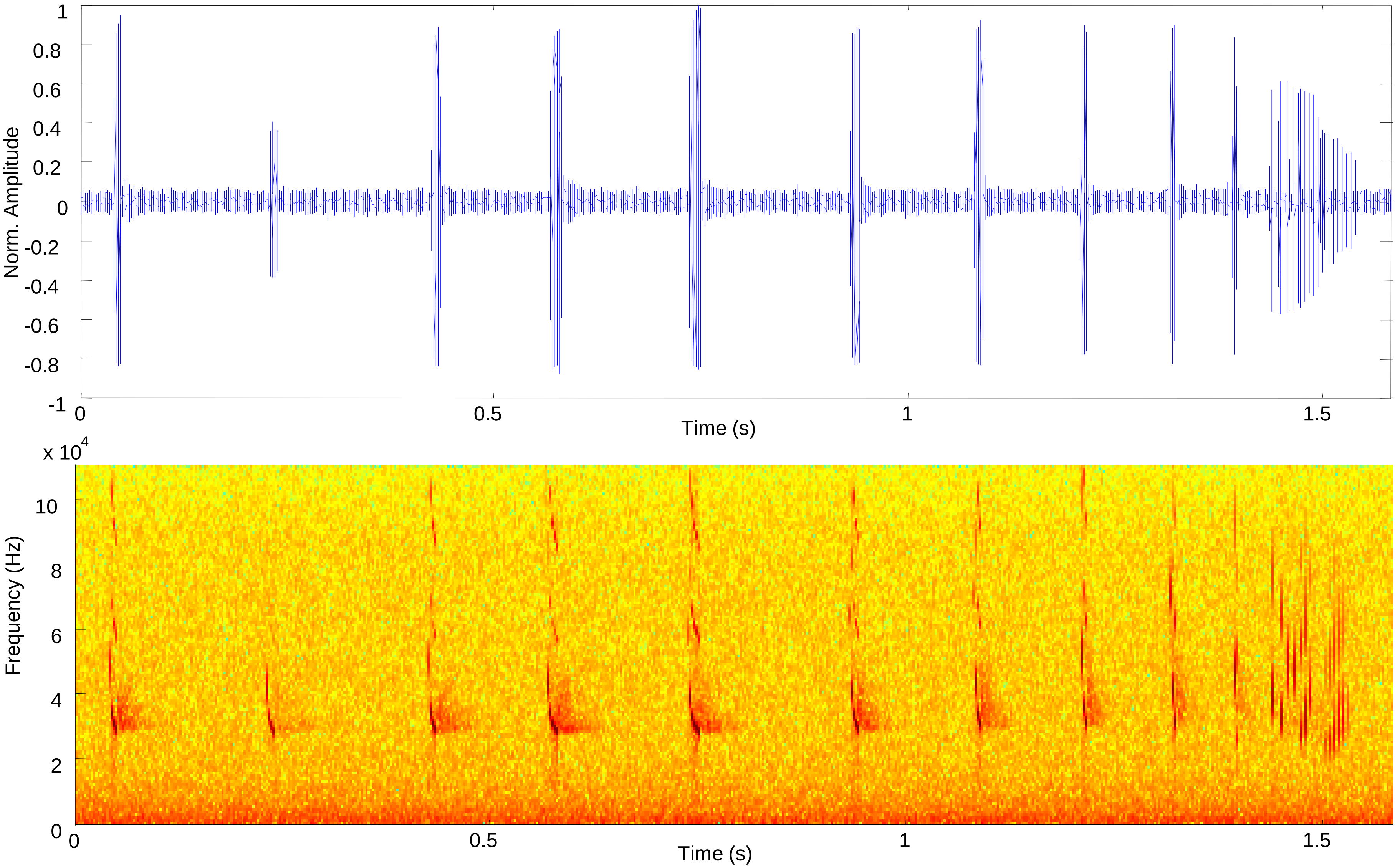
During flight, the emitted frequency is lowered as the bat flies faster so that the bat compensates for Doppler shifts induced by its flight speed, and hence the frequency returning to the bat is relatively constant at the frequency of the acoustic fovea

Attack of prey

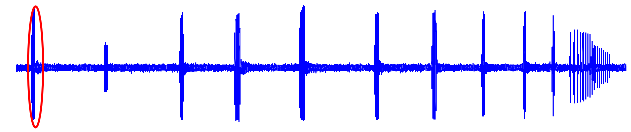


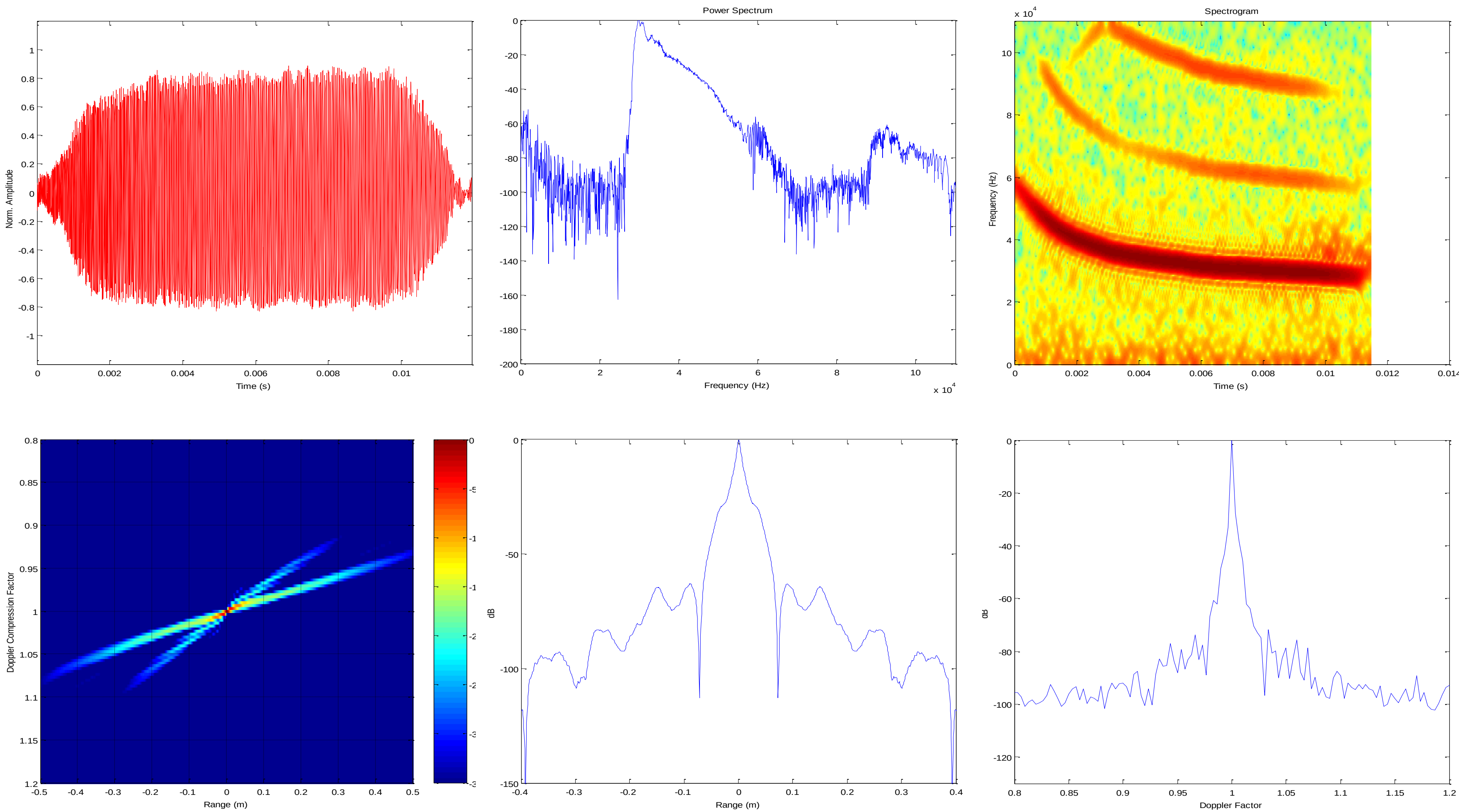
http://www.bsos.umd.edu/psyc/batlab

Eptesicus Nilsonii



Eptesicus Nilsonii





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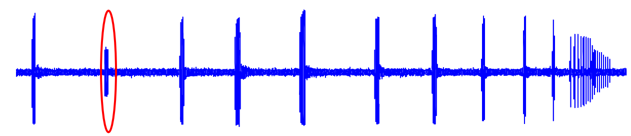

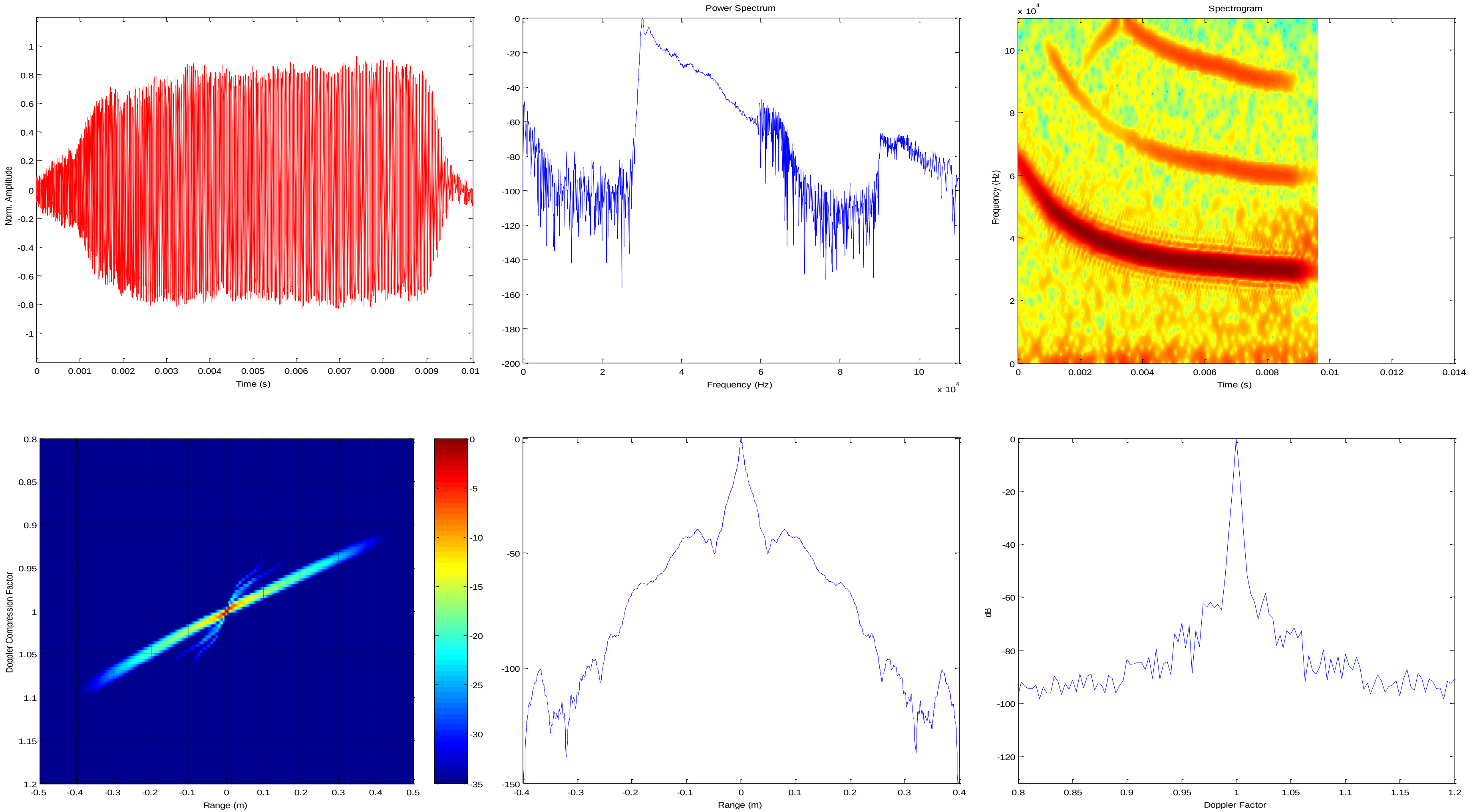

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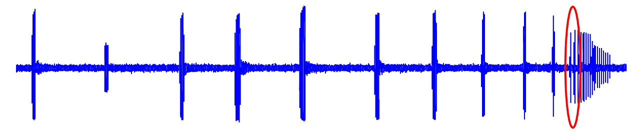

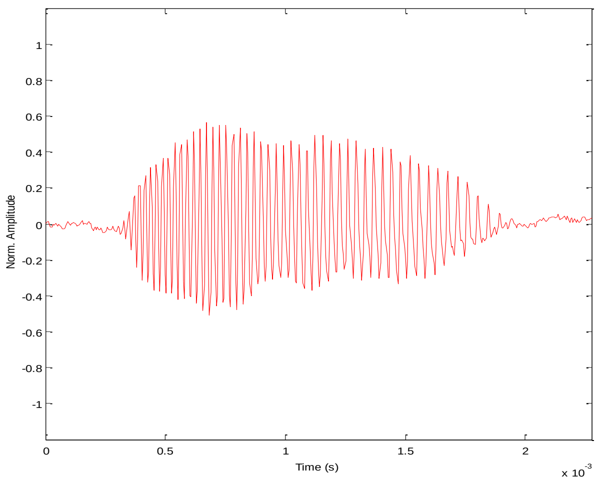

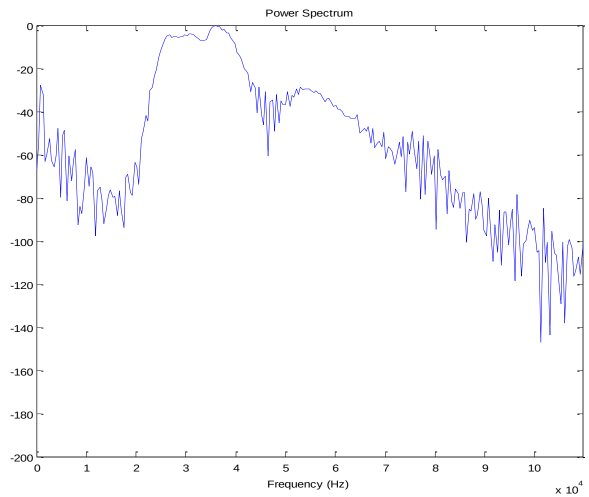

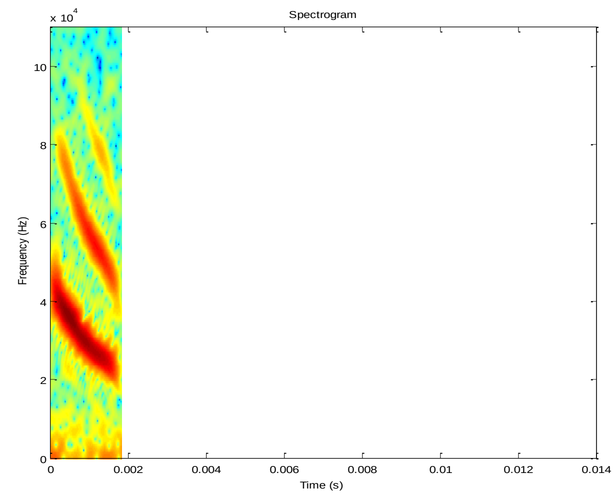

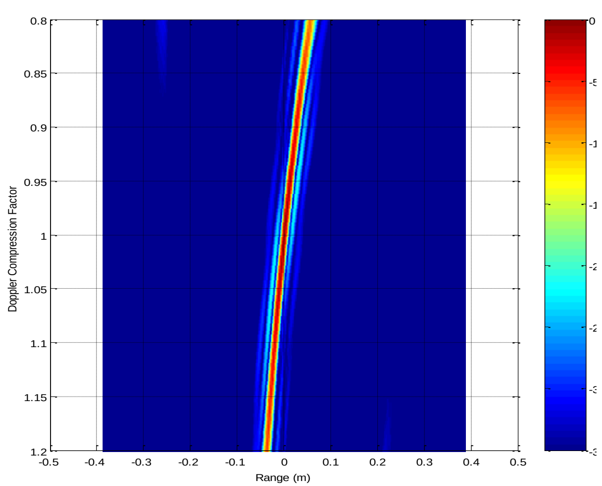

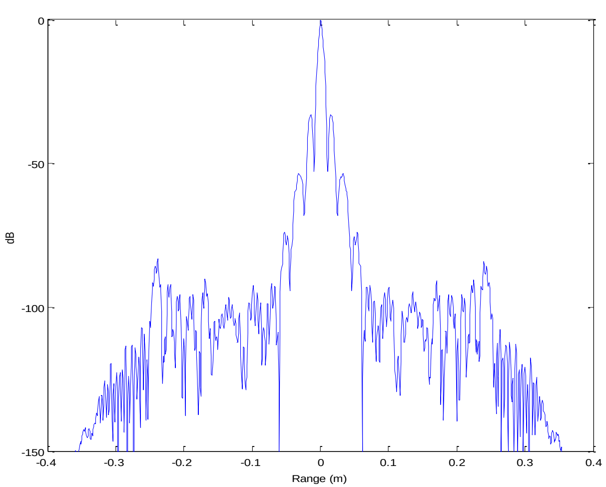

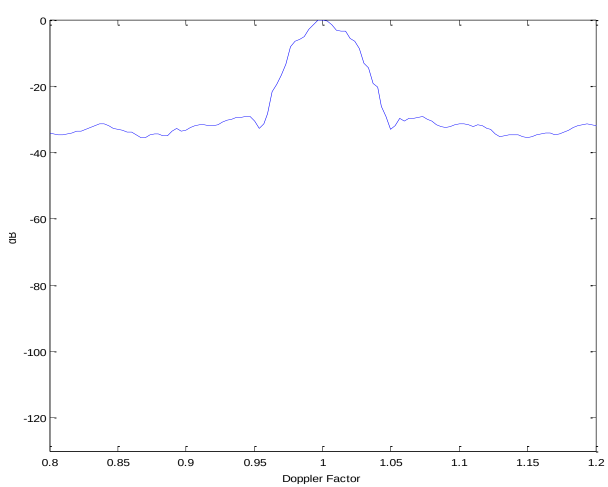

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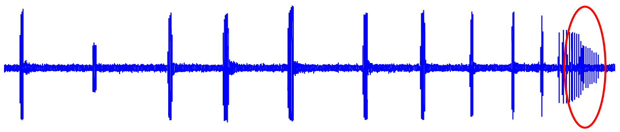
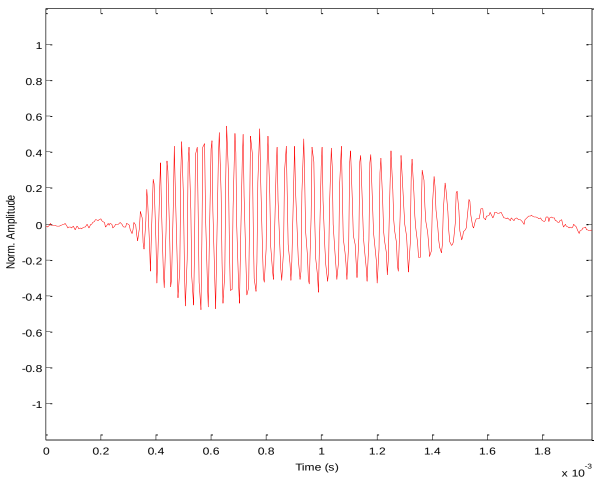
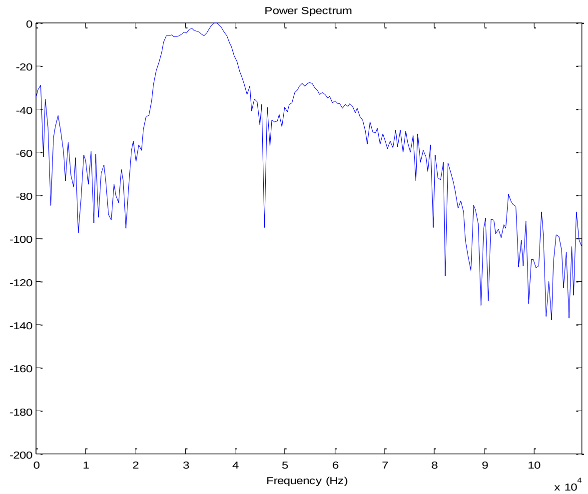
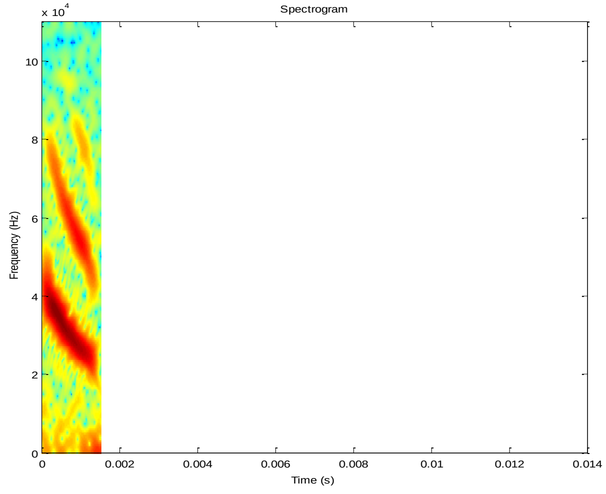
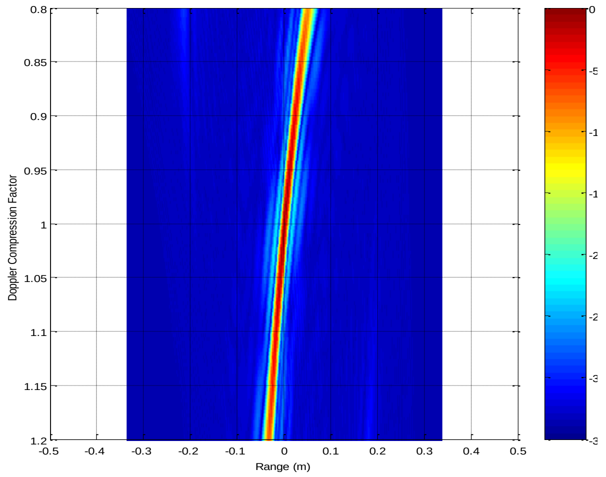
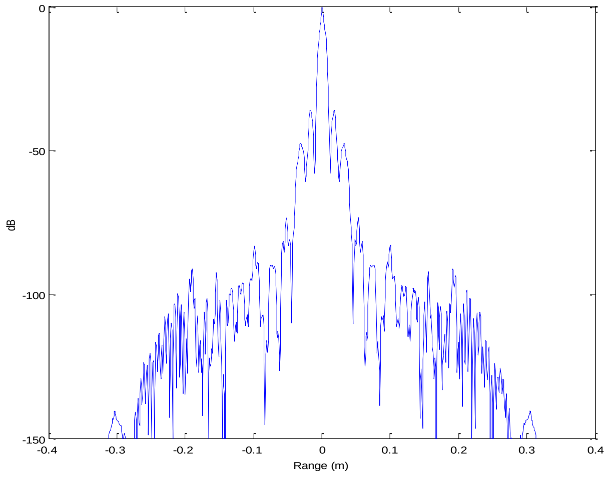


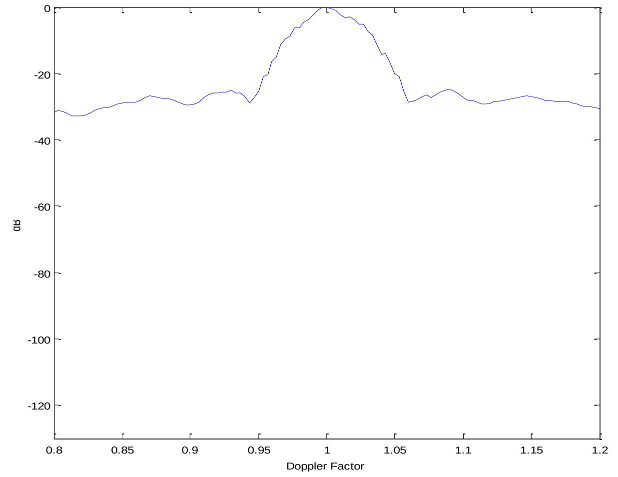




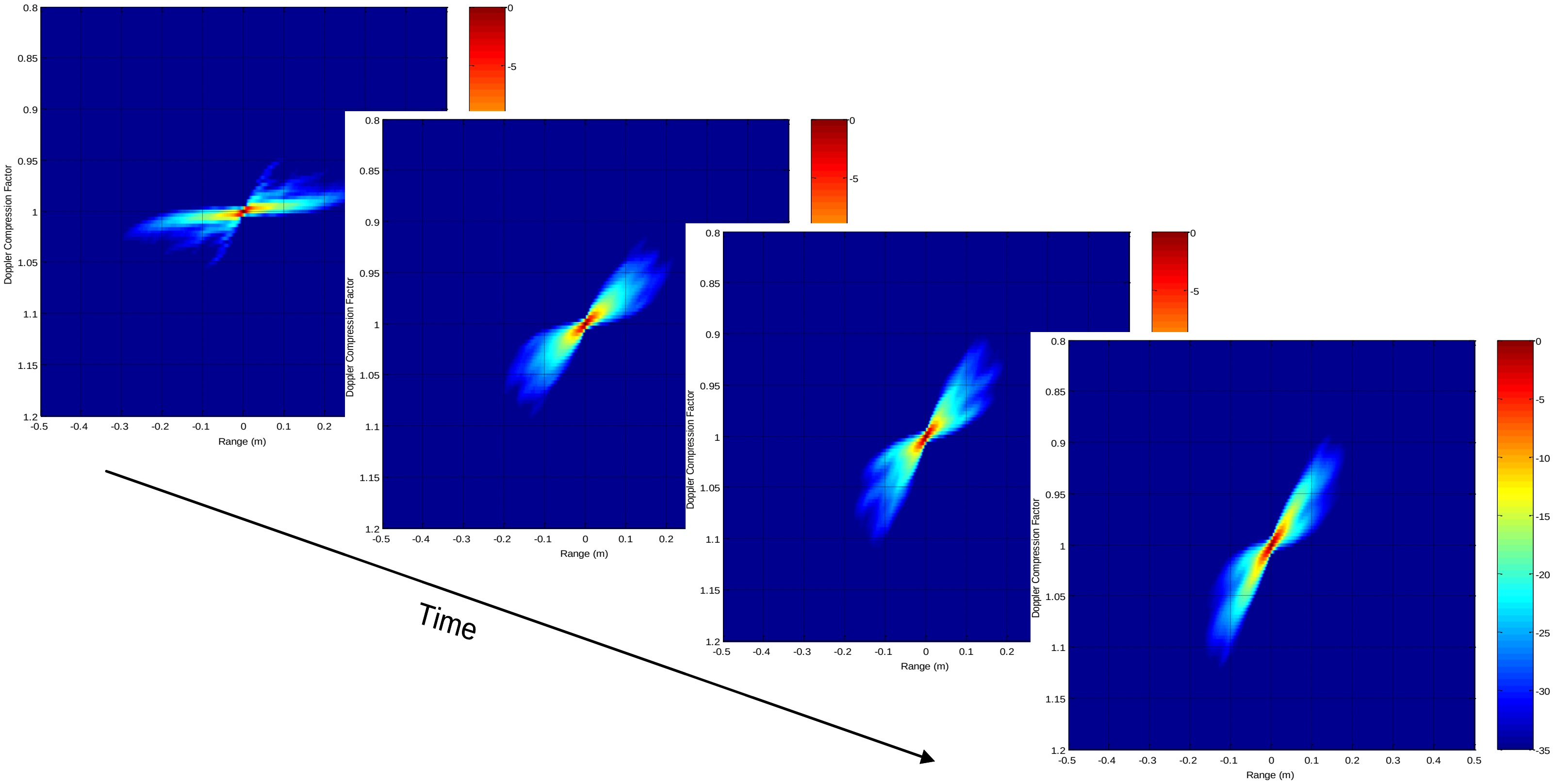








Pipistrellus pygmaeus



The Doppler resolution is progressively deteriorated, suggesting that the prey is tracked by calculating a time sequence of consecutive positions, and exploiting the *a priori* knowledge given by previous measurements.

So the radars of the future will be

- Multifunction
- Distributed
- Spectrally-efficient
- Multistatic
- Intelligent

Our work:

- Passive radar: adaptive selection of transmissions and receivers
- Commensal radar
- Low-sidelobe waveforms with high spectral containment (Shannon Blunt)

Acknowledgements

We express our thanks to those we have worked with and learned from, and who have provided material used in this presentation

Chris Baker
Shannon Blunt
John Chapin
Larry Cohen
Alfonso Farina
Francesco Fioranelli
Eric Mokole
Daniel O'Hagan
Riccardo Palamà
Andy Stove
Craig Tong
Simon Watts
Mike Wicks
Yu Chen