# Poisson point process tracking and intentionality modelling

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# Outline

1. Background: Non-homogeneous Poisson process (NHPP) tracker

2. Scalable association-based NHPP tracker

★ 3. Time-varying rate estimation for both targets and clutter

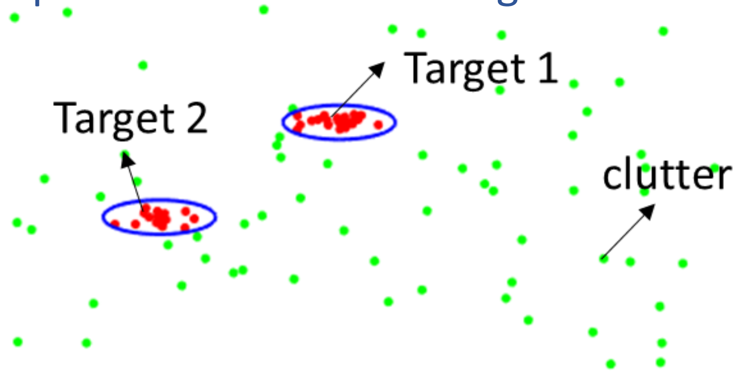
4. Extensions
   - Shape estimation for group/extended targets
   - intentionality modelling in multiple target tracking
   - α-stable Lévy state-space models for highly maneuverable objects
   - Tracking of a time-varying number of objects using reversible jump MCMC

# Non-homogeneous Poisson process (NHPP) tracker

## NHPP measurement model

Intensity $\lambda(Z_n|X_n) = \sum_{i=0}^{K} \lambda_i(Z_n|X_{i,n})$

a superposition of NHPPs of targets and clutter




$X_n = [X_{1,n}, ..., X_{K,n}]^T$ joint state of K objects

$Z_n = [Z_{1,n}, ..., Z_{M,n}]$. M observations

for each object/clutter: $\lambda_i(Z_{j,n}|X_{i,n}) = \Lambda_i p(Z_{j,n}|X_{i,n})$

Poisson mean/rate: $\Lambda_i$

Measurement number $m_{i,n}$ ~ Poisson($\Lambda_i$)

Measurement Locations are i.i.d. samples from $p(Z_{j,n}|X_{i,n}) = \begin{cases} p(Z_{j,n}|X_{i,n}) & i \neq 0; \text{ object} \\ \frac{1}{V} & i = 0; \text{ clutter} \end{cases}$

## NHPP-based multi-target tracker[1]:

Objective: $p(X_n|Z_{0:n})$

Measurement likelihood $p(Z_n|X_n) \propto \prod_{j=1}^{M} \sum_{i=1}^{K} \lambda_i(Z_{j,n}|X_{i,n}) \propto \prod_{j=1}^{M} (\frac{\Lambda_0}{V} + \sum_{i=1}^{K} \Lambda_i p(Z_{j,n}|\hat{X}_{i,n}))$

Inference scheme Sampling methods, e.g., Particle filtering

[1] K. Gilholm, S. Godsill, S. Maskell, and D. Salmond, "Poisson models for extended target and group tracking," in Signal and Data Processing of Small Targets 2005, vol. 5913

# Scalable association-based multi-target NHPP tracker[2]

**Data Association** $\theta_n = [\theta_{1,n}, ..., \theta_{M,n}]$ M observations, K objects

$$\theta_{j,n} = i \begin{cases} i = 0 & \text{clutter} \\ i \in \{1, ..., K\} & \text{measurement j is from target i} \end{cases}$$

**Objective:** $p(X_{n-1:n}, \theta_n | Z_{0:n})$

$$X_n = [X_{1,n}, ..., X_{K,n}]^T.$$
$$Z_n = [Z_{1,n}, ..., Z_{M,n}].$$

**Scalable inference scheme**

Online Gibbs sampling
SMCMC scheme →

independent parallel draws

At iteration $m$:

For every measurement $j = 1, ..., M$,

Sample each association variable $\theta_{j,n}^{(m)}$

For every object $i = 1, ..., K$,

Sample each target state $X_{i,n-1}^{(m)}$ $X_{i,n}^{(m)}$

[2] Q. Li, J. Liang and S. Godsill, "A scalable sampling-based scheme for data association and multi-target tracking under mixed Poisson measurement process", ICASSP 2022, Singapore, submitted

# Scalable association-based multi-target NHPP tracker

## Rao-Blackwellisation scheme

$p(\theta_{0:n}|Z_{0:n})$ Is approximated by samples $\{\theta_{0:n}^{(p)}\}_{p=1}^{N_p}$

$$p(X_n|Z_{0:n}) \approx \frac{1}{N_p}\sum_{p=1}^{N_p} \boxed{p(X_n|\theta_{0:n}^{(p)}, Z_{0:n}),}$$

Kalman update

linear Gaussian

**Standard RB-scheme**

Objective: $p(\theta_{0:n}|Z_{0:n})$

At iteration $m$:

1. Sample $\theta_{j,n}^{(m)}$ from $\boxed{p(\theta_{j,n}|\theta_{-j,n}, \theta_{0:n-1}, Z_{0:n})}$

$j = 1, \ldots, M,$

2. Sample $\theta_{0:n-1}^{(m)}$ from $p(\theta_{0:n-1}|\theta_n, Z_{0:n})$

**RB-scheme with auxiliary variable**

Objective: $p(\theta_{0:n}, X_n|Z_{0:n}) \longrightarrow p(\theta_{0:n}|Z_{0:n})$

At iteration $m$:

1. Parallel sample $\theta_{j,n}^{(m)}$ from $\boxed{p(\theta_{j,n}|Z_n, X_n^{(m-1)})}$

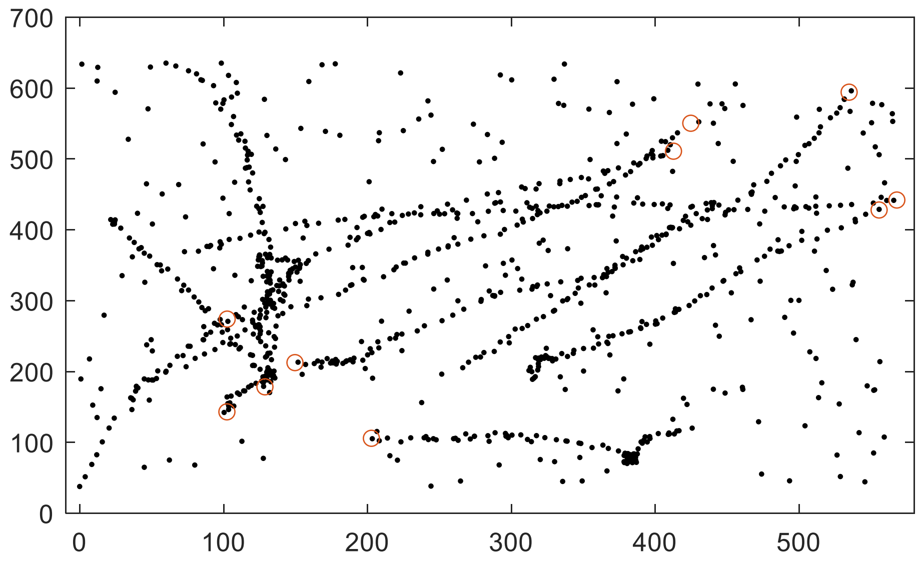
$j = 1, \ldots, M,$

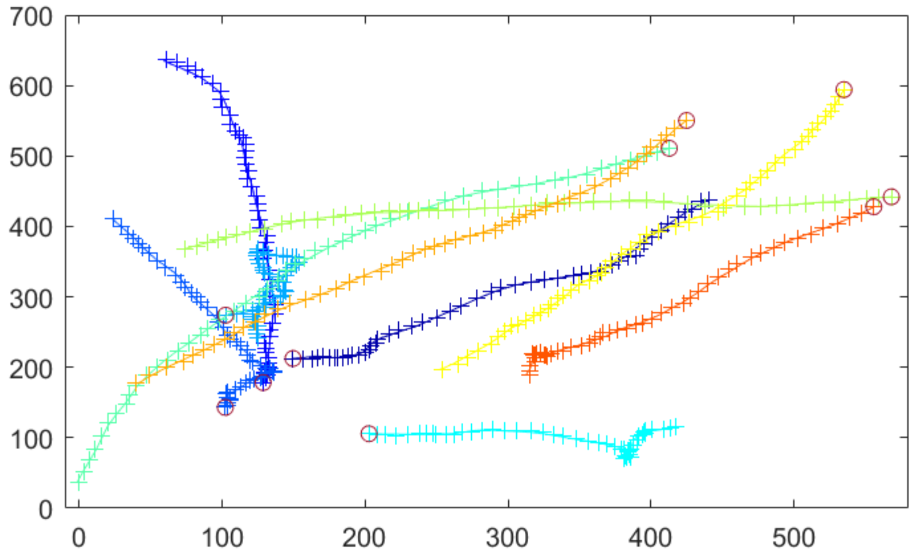
2. Sample $\theta_{0:n-1}^{(m)}$ from $p(\theta_{0:n-1}|\theta_n, X_n, Z_{0:n})$

3. Parallel sample $X_{i,n}^{(m)}$ from $p(X_n|\theta_{0:n-1}, \theta_n, Z_{0:n})$

$i = 1, \ldots, K$

# Simulation results




Measurements of 10 targets over 50 time steps

Target rate: 1
Clutter rate: 5

\+ estimated trajectories;
— ground truth

100 iterations

# Time-varying rate estimation for both targets and clutter[2]

**Assumption** Poisson rates are unknown and/or time-varying

**Measurement process** Poisson mixture process

Measurement number $m_{i,n}$ ~ Poisson mixture distribution

$p(m_{i,n}|\Lambda_{i,n})$ ~ Pois$(\Lambda_{i,n})$ Random variable

$$p(m_{i,n}) = \int_0^\infty p(m_{i,n}|\Lambda_{i,n}) \boxed{g(\Lambda_{i,n})} d\Lambda_{i,n}.$$ Prior

**Poisson rate Prior** Generalized inverse Gaussian (GIG) family

$$\mathcal{GIG}(\Lambda; a, b, p) = \frac{(a/b)^{p/2}}{2\mathcal{K}_p(\sqrt{ab})}\Lambda^{p-1}\exp\left(-\frac{a}{2}\Lambda - \frac{b}{2\Lambda}\right)$$

Special cases of the GIG:

| | |
|---|---|
| Gamma | $\mathcal{GIG}(\Lambda; \sqrt{2a}, 0, p).$ |
| inverse Gaussian | $\mathcal{GIG}(\Lambda; a, b, -\frac{1}{2})$ |
| Inverse Gamma | $\mathcal{GIG}(\Lambda; 0, \sqrt{2a}, -p)$ |

$g(\Lambda_{i,n})$
- Time-independent GIG prior: $p(\Lambda_n|\Lambda_{0:n-1}) = p(\Lambda_n),$
- GIG Markov chain prior: $p(\Lambda_n|\Lambda_{0:n-1}) = p(\Lambda_n|\Lambda_{n-1}),$

[2] Q. Li, J. Liang and S. Godsill, "A scalable sampling-based scheme for data association and multi-target tracking under mixed Poisson measurement process", ICASSP 2022, Singapore, submitted

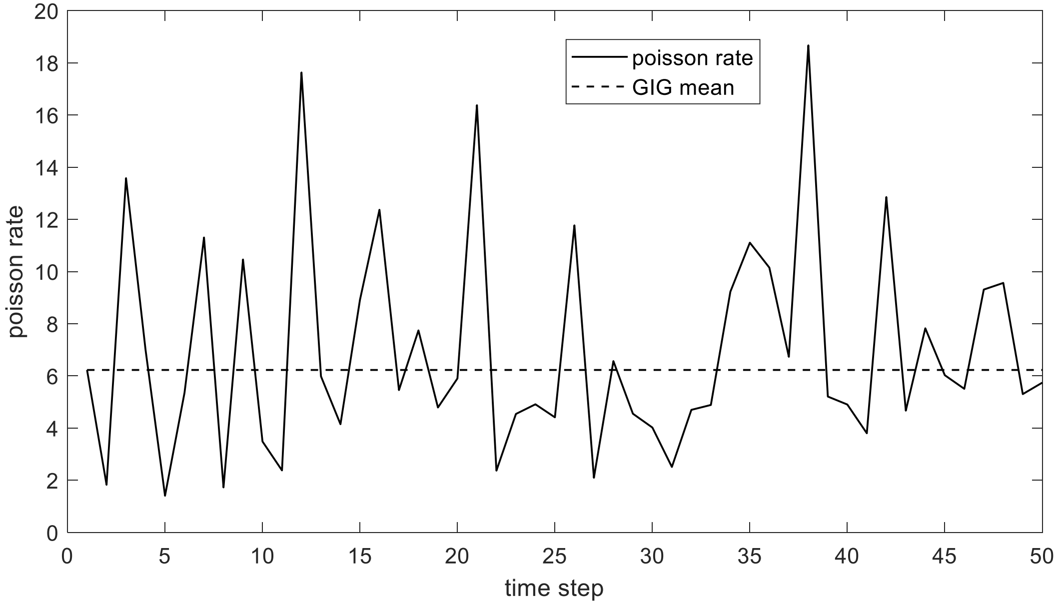
# Time-varying rate estimation for both targets and clutter

## Poisson rate with a time independent GIG prior

**Objective:** $p(\theta_n, \Lambda_n, X_{n-1:n}|Z_{0:n}).$

**Poisson rate GIG prior:** $p(\Lambda_n|\Lambda_{0:n-1}) = p(\Lambda_n),$

$$\Lambda_{i,n} \sim \mathcal{GIG}(\Lambda_{i,n}; a_i, b_i, p_i)$$




**Scalable inference scheme**

Sequential MCMC with Gibbs sampling steps

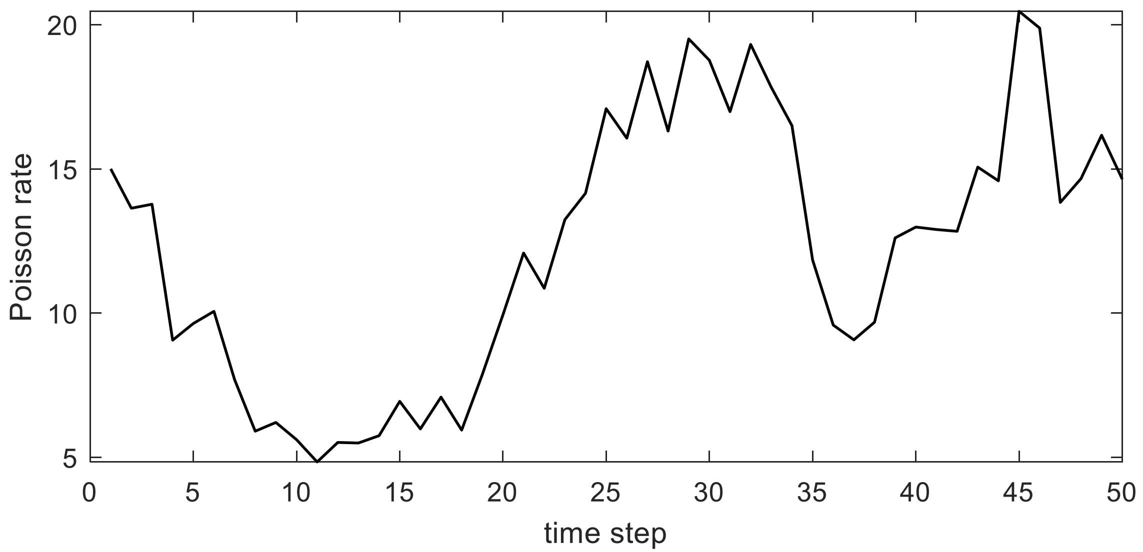
## Poisson rate with a GIG Markov chain prior

**Objective:** $p(\theta_n, \Lambda_{n-1,n}, X_{n-1:n}|Z_{0:n})$

**Poisson rate GIG prior:** $p(\Lambda_n|\Lambda_{0:n-1}) = p(\Lambda_n|\Lambda_{n-1}),$

mean of $p(\Lambda_{i,n}|\Lambda_{i,n-1}) = \Lambda_{i,n-1}$

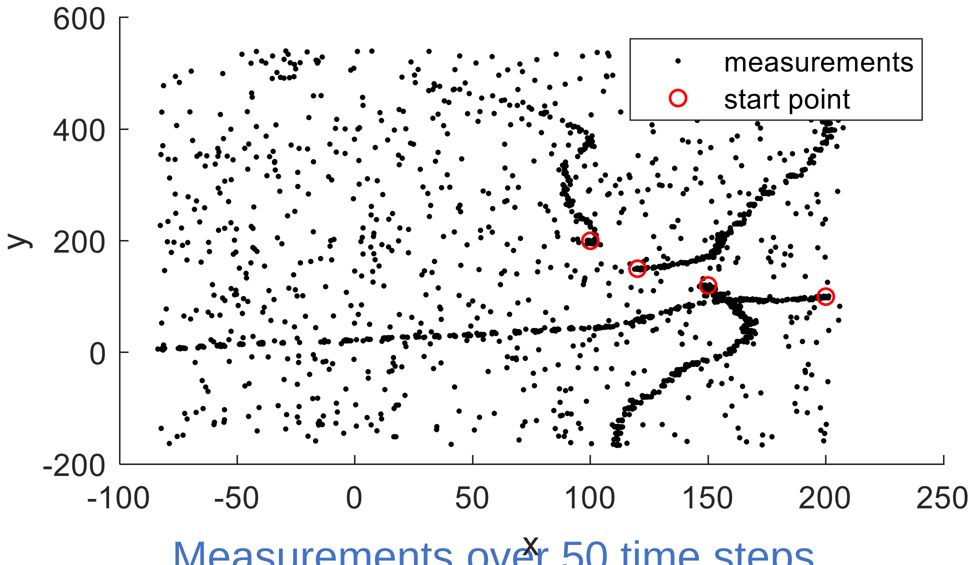
$$\Lambda_{i,n}|\Lambda_{i,n-1} \sim \mathcal{GIG}\left(\Lambda_{i,n}; \frac{r_c r_B}{\Lambda_{i,n-1}}, \frac{r_c \Lambda_{i,n-1}}{r_B}, p_i\right)$$



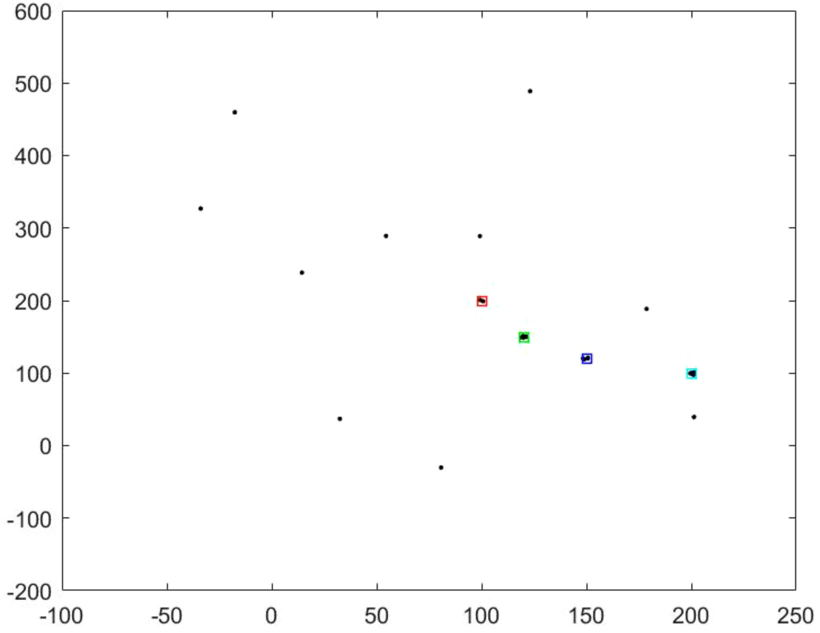

**Scalable inference scheme**

Sequential MCMC with Gibbs sampling steps

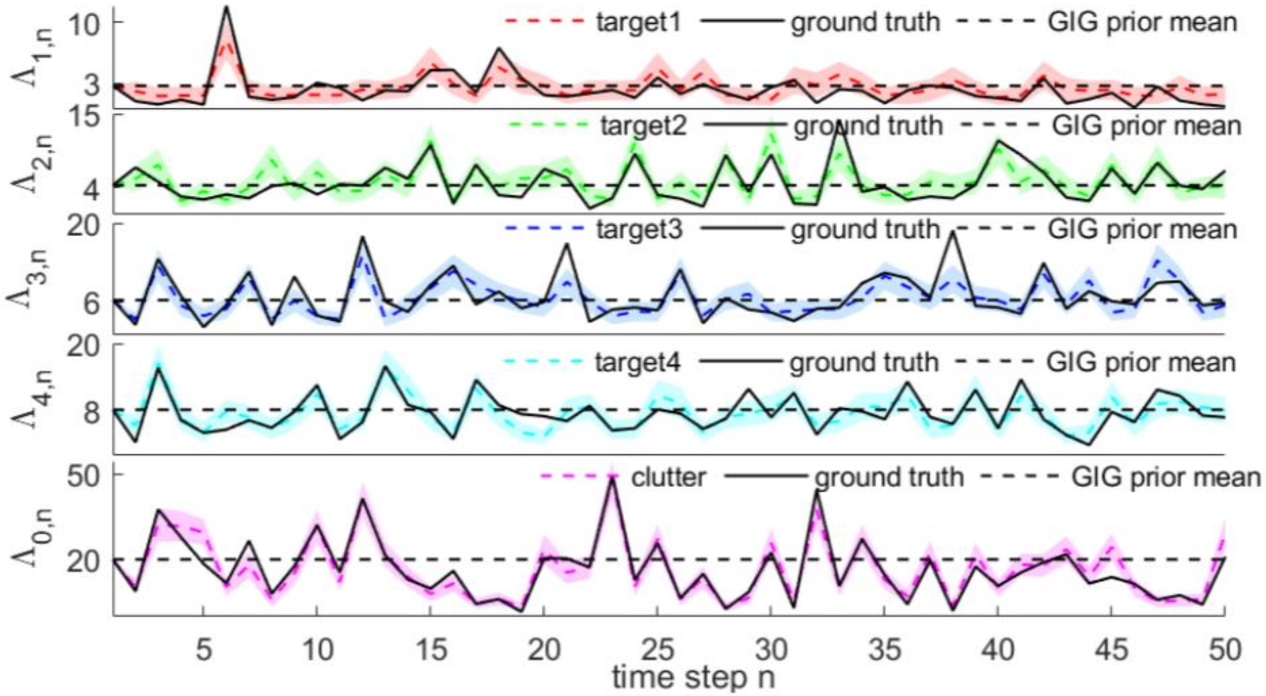
# Simulation - time independent GIG prior




Measurements over 50 time steps



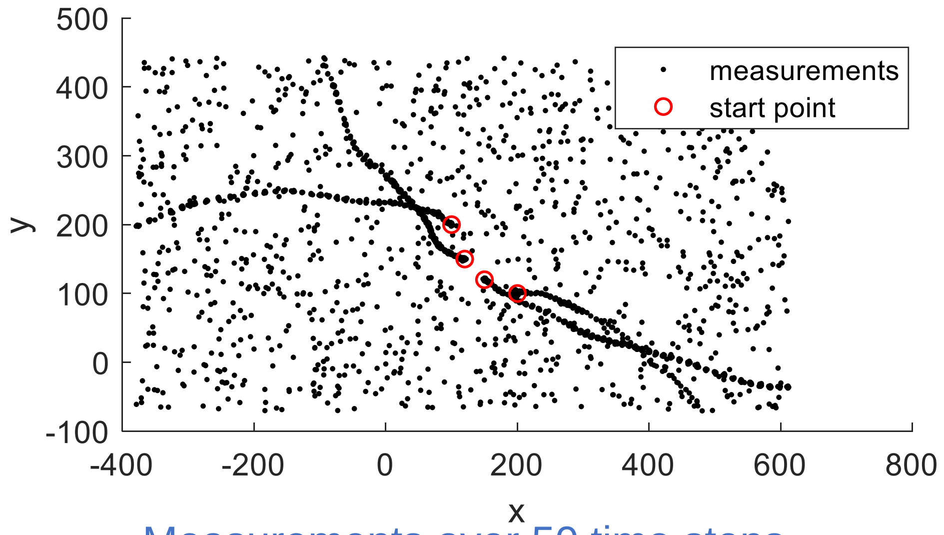

the estimated track of four objects



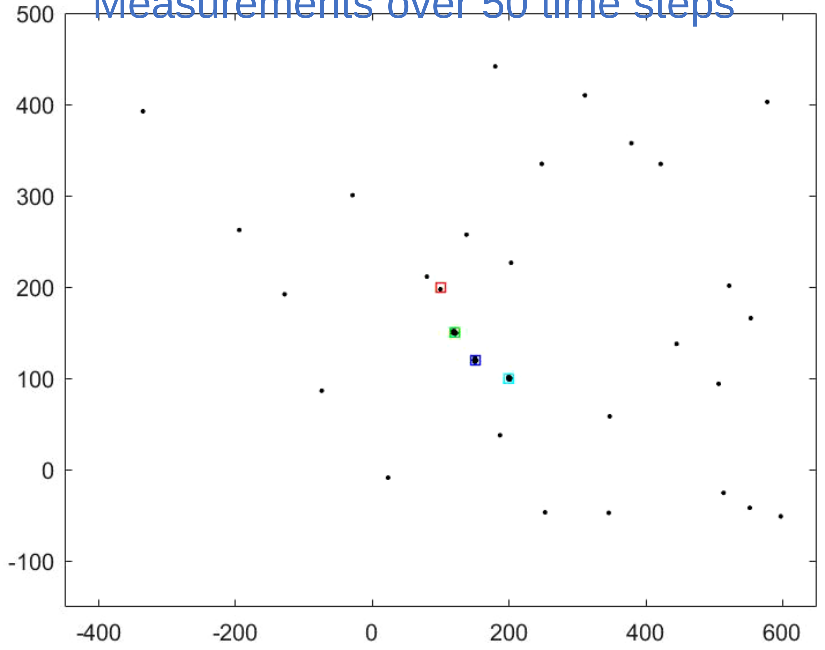

Poisson rate of four objects and clutter over time

The shaded areas represent estimated Poisson rates $\pm 1\sigma$

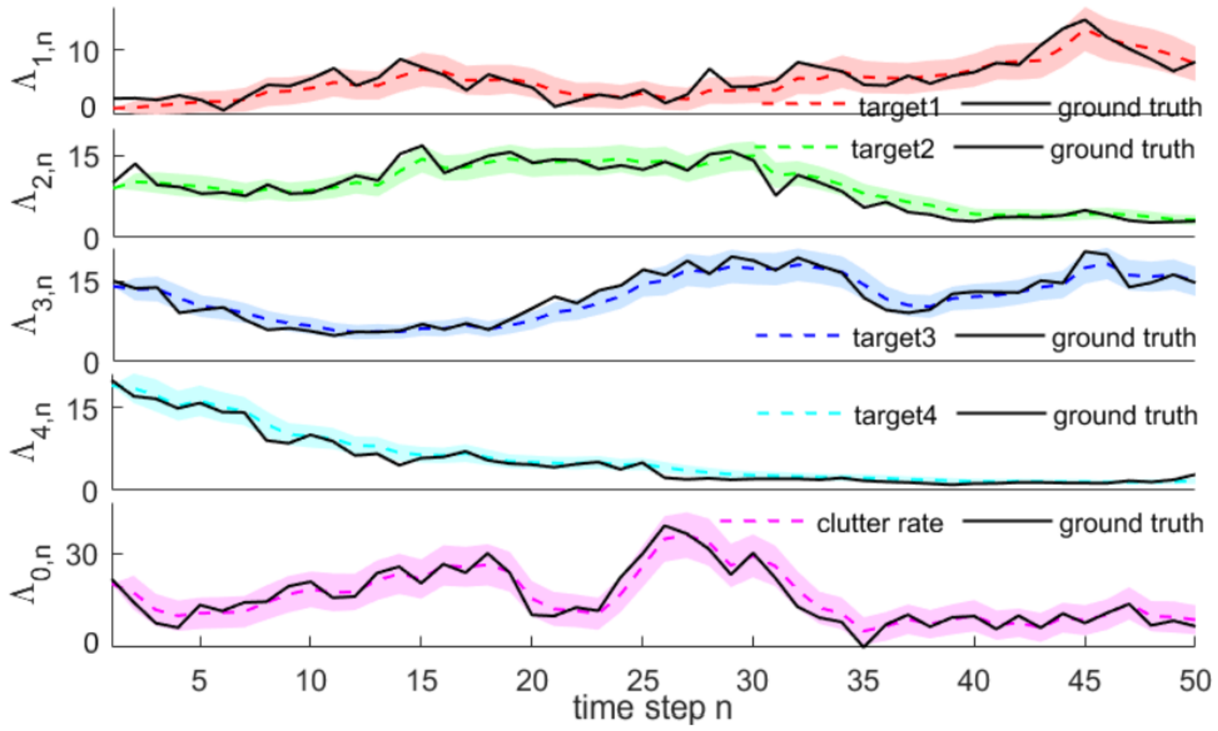
# Simulation - GIG Markov chain prior




Measurements over 50 time steps



the estimated track of four objects




Poisson rate of four objects and clutter over time

the shaded areas represent estimated Poisson rates ±1σ

# Connecting NHPP and probabilistic multi-hypothesis tracker (PMHT)

## Standard PMHT

$X_n = [X_{1,n}, ..., X_{K,n}]^T$. joint state of K objects

$Z_n = [Z_{1,n}, ..., Z_{M,n}]$. M observations

$\theta_n = [\theta_{1,n}, ..., \theta_{M,n}]$. $\theta_{j,n} = i \quad i \in \{0, 1, ..., K\}$. Association

$\Pi_n = [\pi_{0,n}, ..., \pi_{K,n}]$ detection probabilities

Assumptions: the detection probabilities are unknown constants

Measurement likelihood $p(Z_n|X_n) \propto \prod_{j=1}^{M}(\frac{\pi_0}{V} + \sum_{i=1}^{K} \pi_i p(Z_{j,n}|\hat{X}_{i,n}))$

Relationship to NHPP-based multi-target tracker

Conditional on measurement number $\pi_{i,n} = \frac{\Lambda_{i,n}}{\sum_i^K \boxed{\Lambda_{i,n}}}$.

Poisson rate

probabilistic multi-hypothesis sampler (PMHS)

Sequential MCMC $p(\theta_n, \Pi_n, X_n|Z_{0:n})$

# Other Extensions

- Shape estimation for group/extended targets
- Intentionality modelling in group target tracking
- α-stable Lévy State-Space Models for highly maneuverable objects
- Tracking of a time-varying number of objects using reversible jump MCMC

# Shape estimation for group/extended targets

Assumptions: ellipsoidal shape

target model: Gaussian distribution target model

$$p(Z_n|x_n) = \mathcal{N}(Z_n; Hx_n, \boxed{\Sigma})$$

target extent

Objective: $p(\theta_n, X_{n-1:n}, \Lambda_{n-1:n}, \Sigma_{n-1:n}|Z_{0:n})$.

$X_n = [X_{1,n}, ..., X_{K,n}]^T$. joint state of K objects

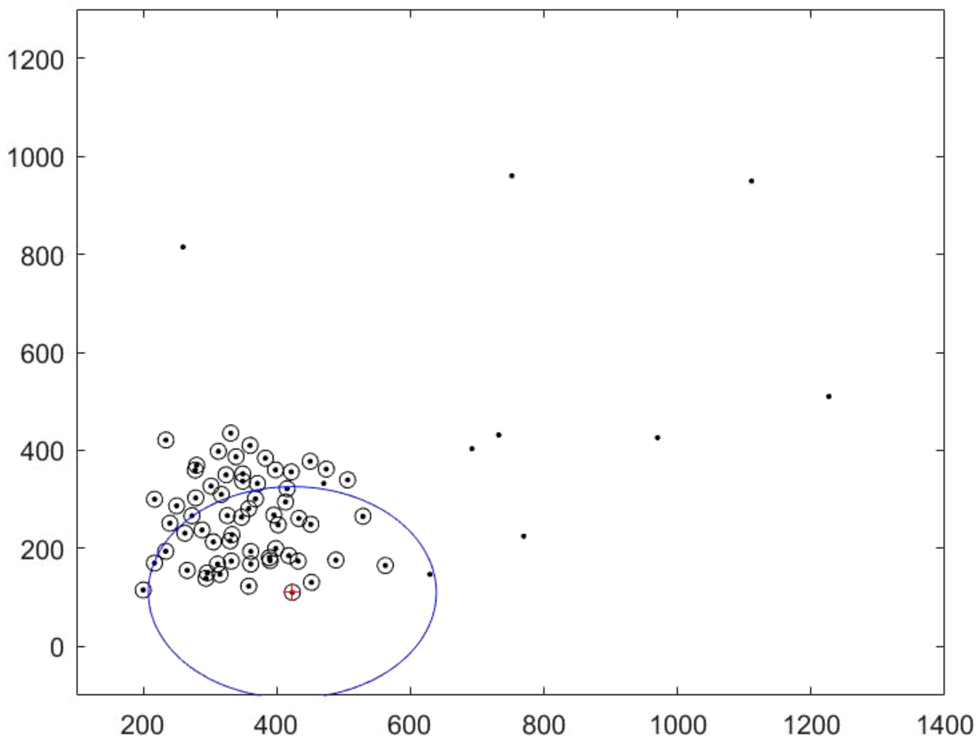
$Z_n = [Z_{1,n}, ..., Z_{M,n}]$. M observations

$\theta_n = [\theta_{1,n}, ..., \theta_{M,n}]$ $\theta_{j,n} = i$ $i \in \{0, 1, ..., K\}$. Association

$\Sigma_{n-1:n}$ target extent

$\Lambda_{n-1:n}$ Poisson rate

Scalable inference scheme → Sequential MCMC with Gibbs sampling steps




a group of 61 fishes; clutter rate: 10
Dots are measurements; circles are fishes;
plus is estimated group centre;
ellipse is estimated group extent

# Intentionality modelling in group target tracking (in clutter)

Assumptions: The objects move driven by the common intent/destination

Objective: $p(X_n, \theta_n | Z_{0:n})$

$$\theta_n = [\theta_{1,n}, \ldots, \theta_{M,n}]$$ Association

$$X_n = [\underbrace{X_{1,n}, \ldots X_{K,n}}_{\text{Group object states}}, \underbrace{r_{X,n}}_{\text{intent}}]^T$$

Dynamical model: For each objects in a group

SDE: $d\dot{X}_i(t) = \eta(r_X(t) - X_i(t))dt - \rho\dot{X}_i(t)dt + dB_i(t)$

For group intent/destination

$dr_X(t) = dB_r(t)$ Brownion motion

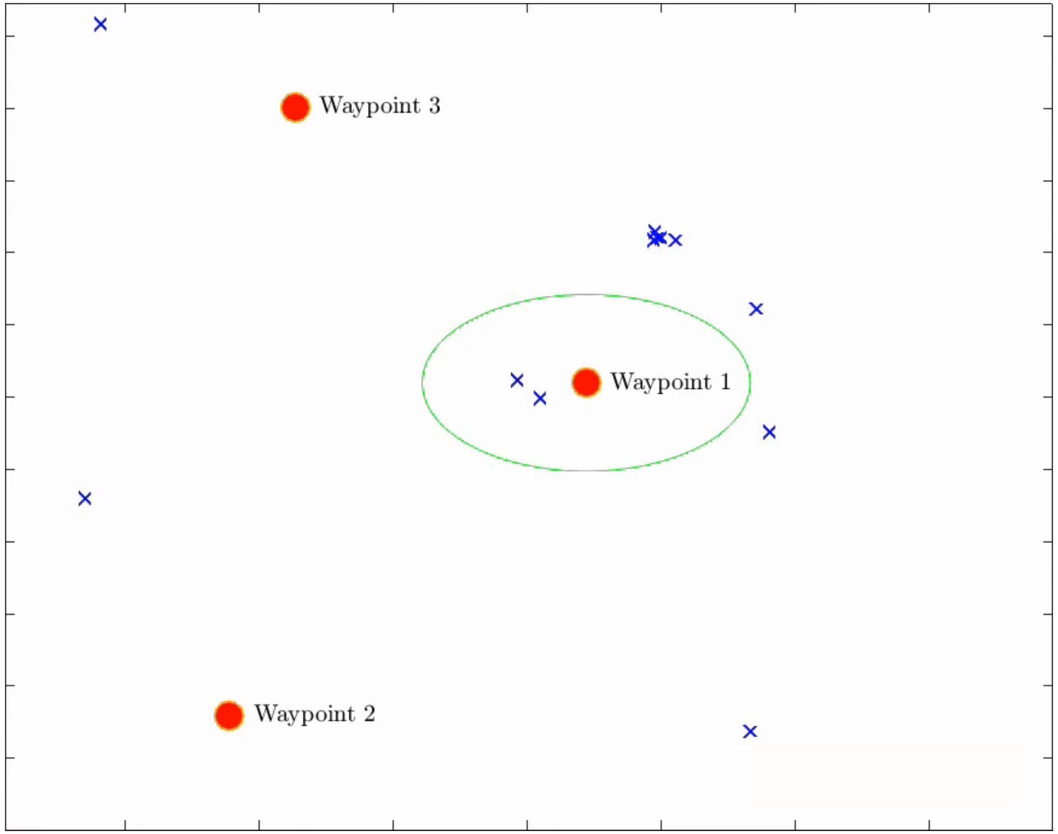
Scalable inference scheme → Sequential MCMC with Gibbs sampling steps →

At iteration $m$:

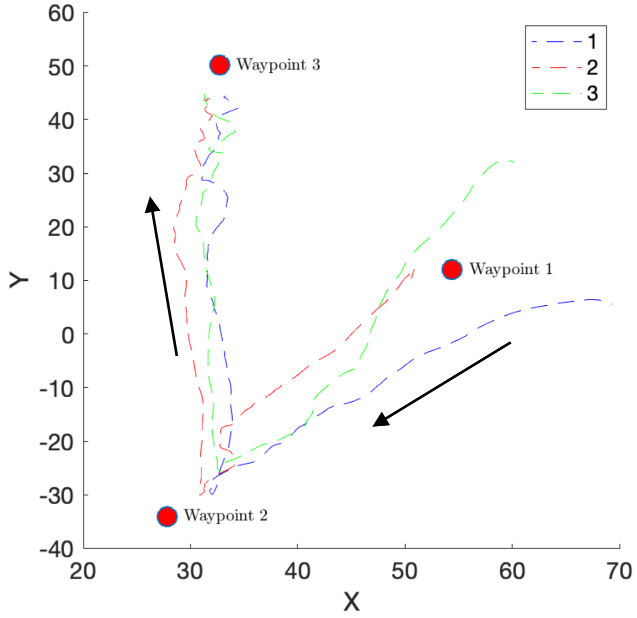
1. Parallel sample each association variable $\theta_{j,n}^{(m)}$
2. Sample past sequence $\theta_{0:n-1}^{(m)}$
3. Sample target state $X_n^{(m)}$

# Drone surveillance scenario

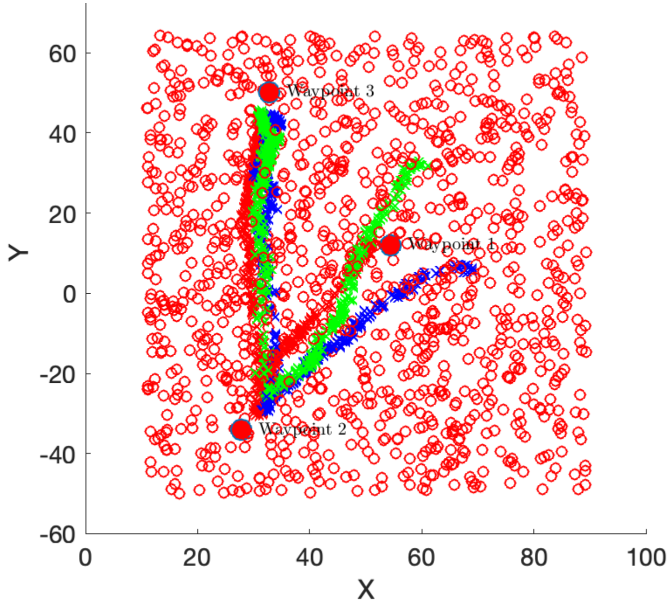
Task: joint estimation of intent(waypoints) and trajectories of 3 drones over 80 time steps




Measurements (blue crosses) and posterior mean filtering estimates (solid lines) with uncertainty ellipses;
Black ellipses are intent estimation uncertainty.




Movement direction

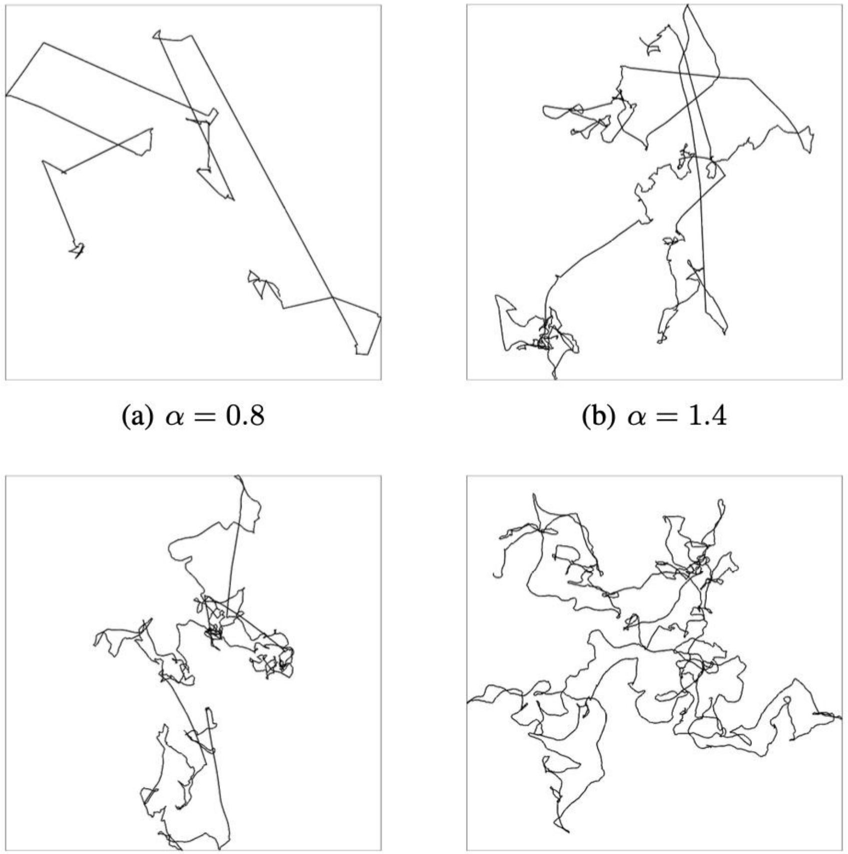



Measurements

Clutter rate: 5
Drones rate: 2

# α-stable Lévy State-Space Models for highly maneuverable objects

Scenario: irregular movement (e.g., sharp turns, target occasionally moves fast to other spatial regions)



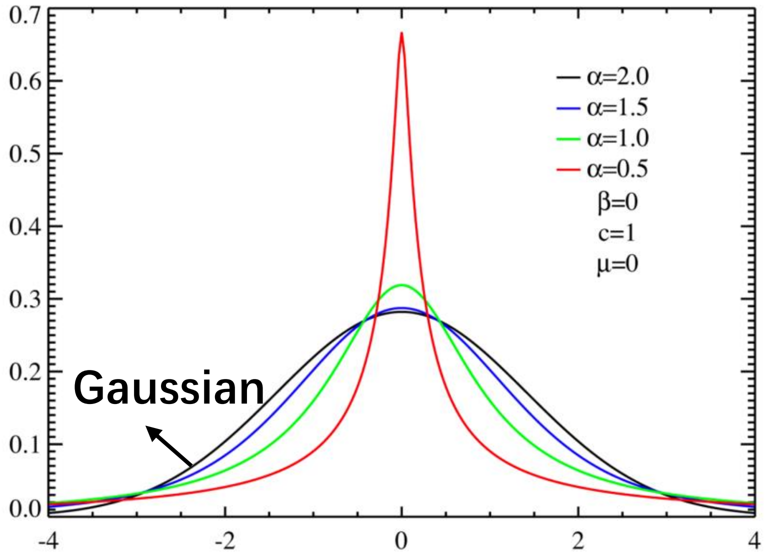
(a) $\alpha = 0.8$ (b) $\alpha = 1.4$

(c) $\alpha = 1.7$ (d) $\alpha = 2$

α-stable Lévy State-Space Models[3] :

Model driven by non-Gaussian Lévy noise

α-stable Lévy *noise*




$\alpha = 2$ corresponds to Gaussian white noise and Brownian motion.

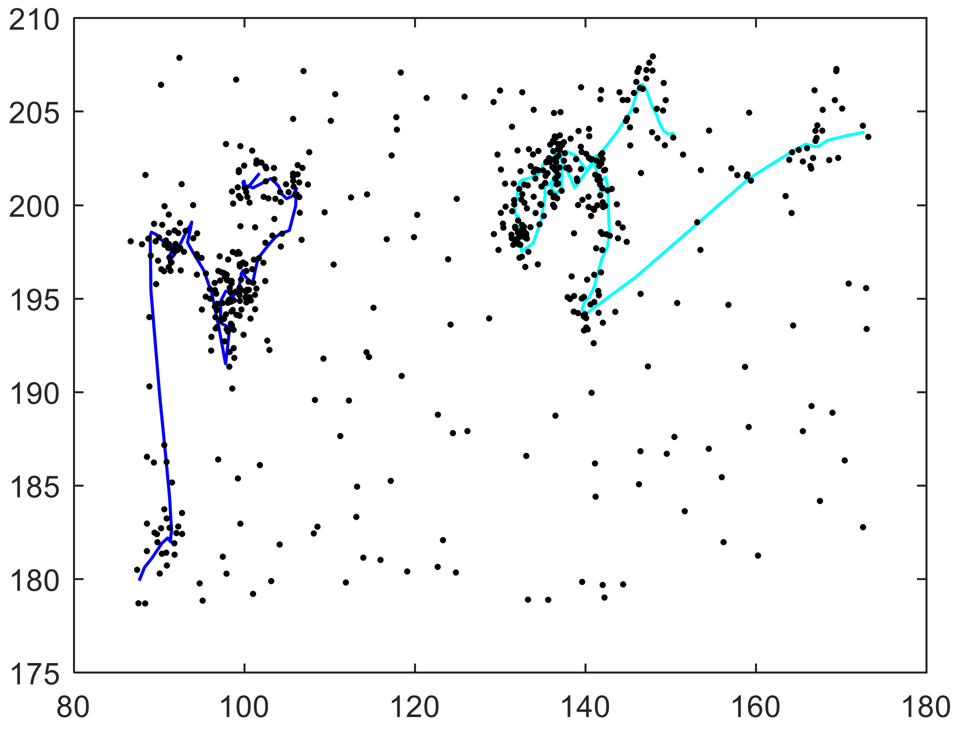
[3]Gan, R., Ahmad, B.I. and Godsill, S.J., 2021. Lévy State-Space Models for Tracking and Intent Prediction of Highly Maneuverable Objects. IEEE Transactions on Aerospace and Electronic Systems, 57(4).

# α-stable Lévy State-Space Models for highly maneuverable objects

## α-stable Lévy Langevin dynamics:

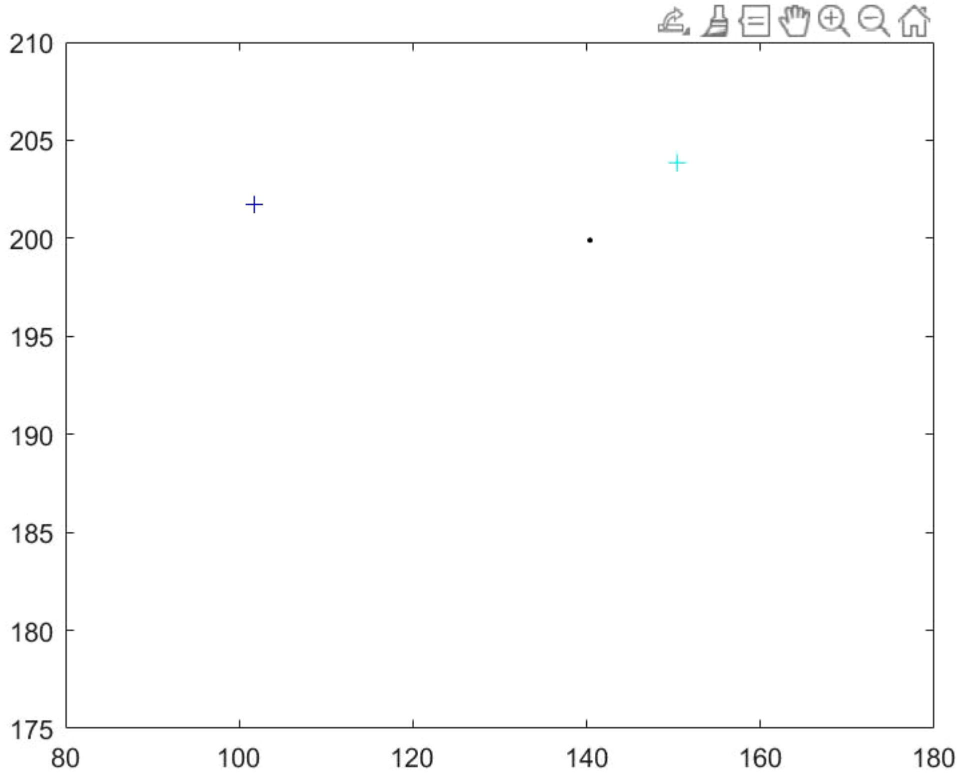
$$d\dot{x}_j(t) = -\lambda \dot{x}_j(t)dt + dW_j(t)$$

Targets cluster in a small region and occasionally moves fast to other spatial regions




\+ estimated trajectories;

— ground truth



# Tracking of a varying number of objects using reversible jump MCMC

Objective: $p(\theta_n, X_{0:n}, \boxed{K_{0:n}} | Z_{0:n})$ the number of objects

Transition density for target number:

$$p(K_n|K_{n-1}) = \begin{cases} P_b & K_n = K_{n-1} + 1; \quad \text{target birth} \\ 1 - P_b - P_d & K_n = K_{n-1}; \quad \text{unchanged} \\ P_d & K_n = K_{n-1} - 1; \quad \text{target death} \end{cases}$$

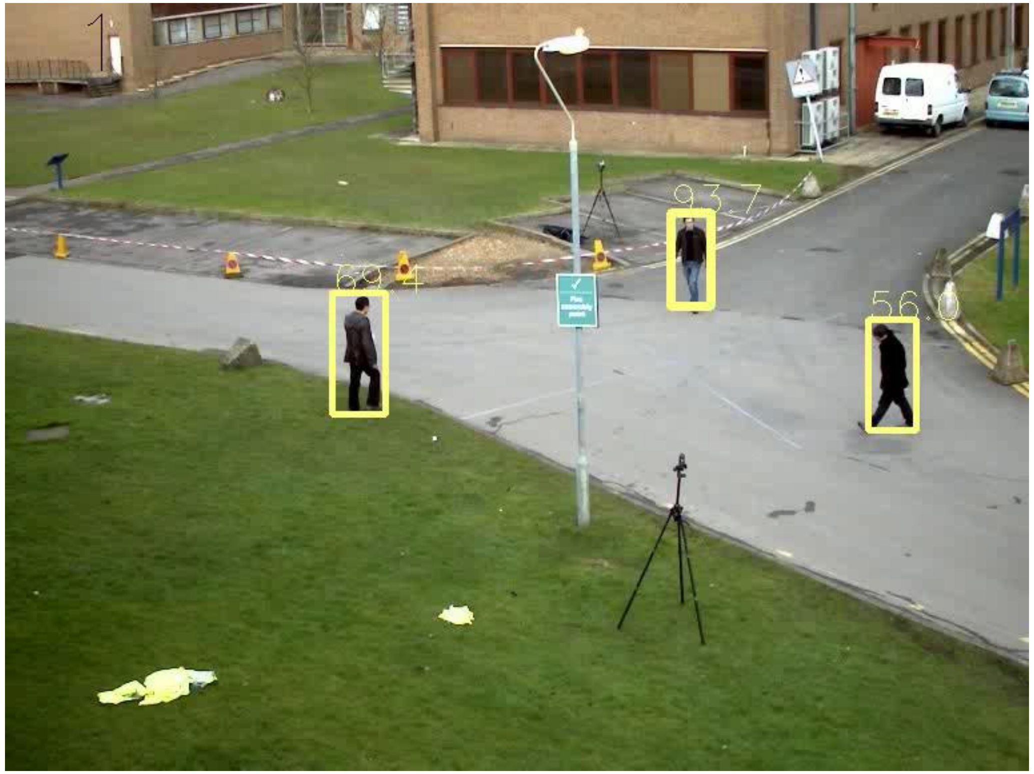
Transition density for target state:

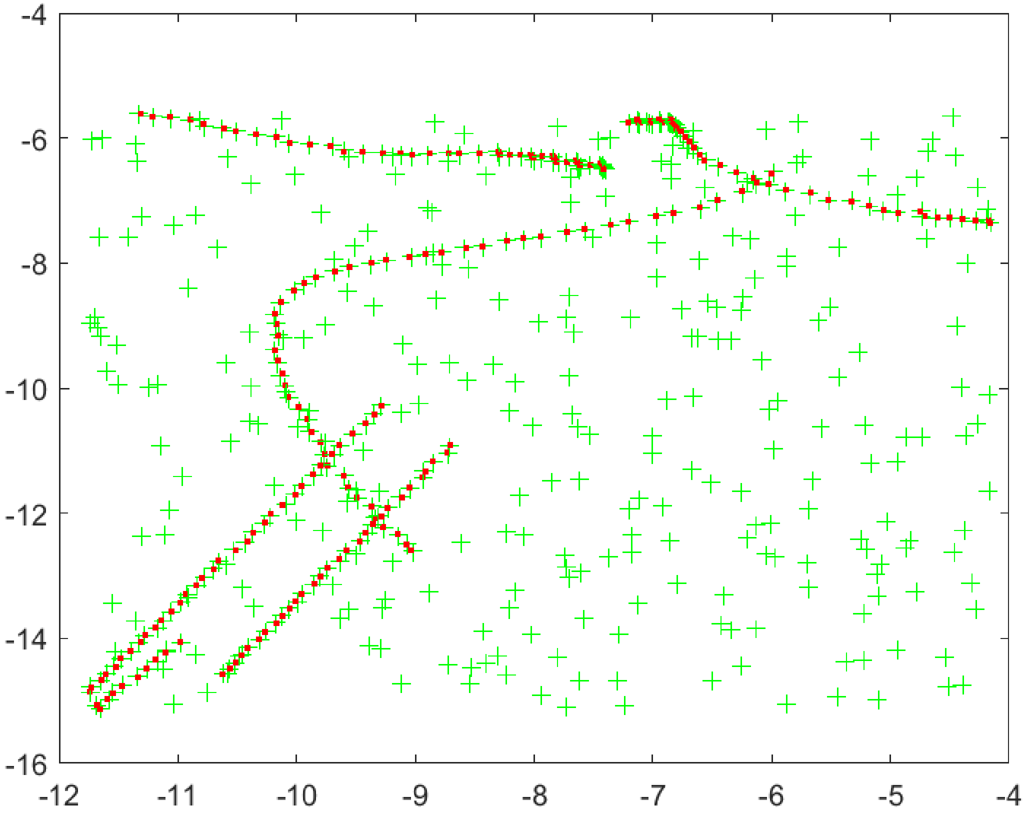
$$p(X_n|X_{n-1}, K_n, K_{n-1}) = \begin{cases} \boxed{p_0(X_{K_n,n})} \prod_{i=1}^{K_{n-1}} p(X_n|X_{n-1}) & K_n = K_{n-1} + 1; \quad \text{target birth} \\ \prod_{i=1}^{K_n} p(X_n|X_{n-1}) & K_n = K_{n-1}; \quad \text{unchanged} \\ \boxed{p_0(i_D)} \prod_{i=1, i \neq i_D}^{K_{n-1}} p(X_n|X_{n-1}) & K_n = K_{n-1} - 1; \quad \text{target death} \end{cases}$$

choose one target to delete

Inference: Sequential reversible jump MCMC with Gibbs refinement steps

# Results





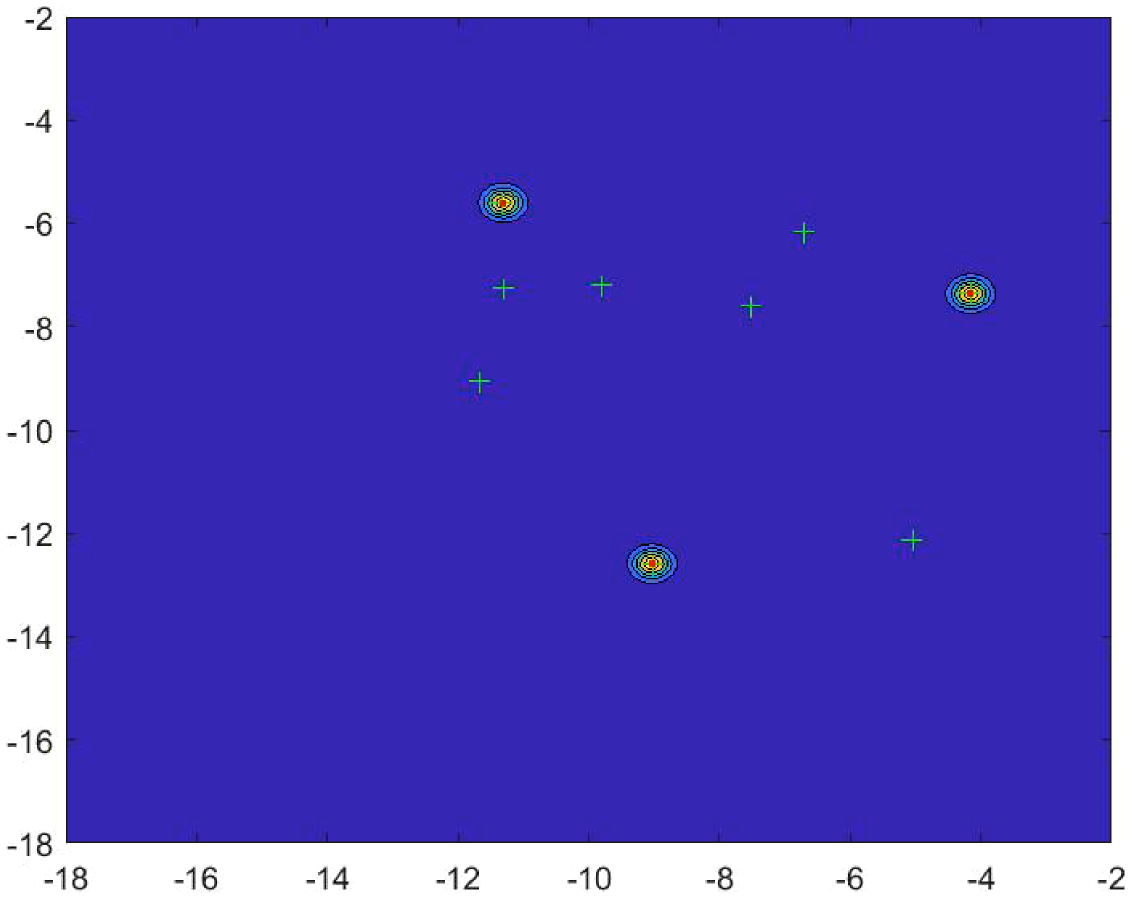

MOT15 benchmark data sets for multiple people tracking; Detection are provided on the MOT Challenge website [4]

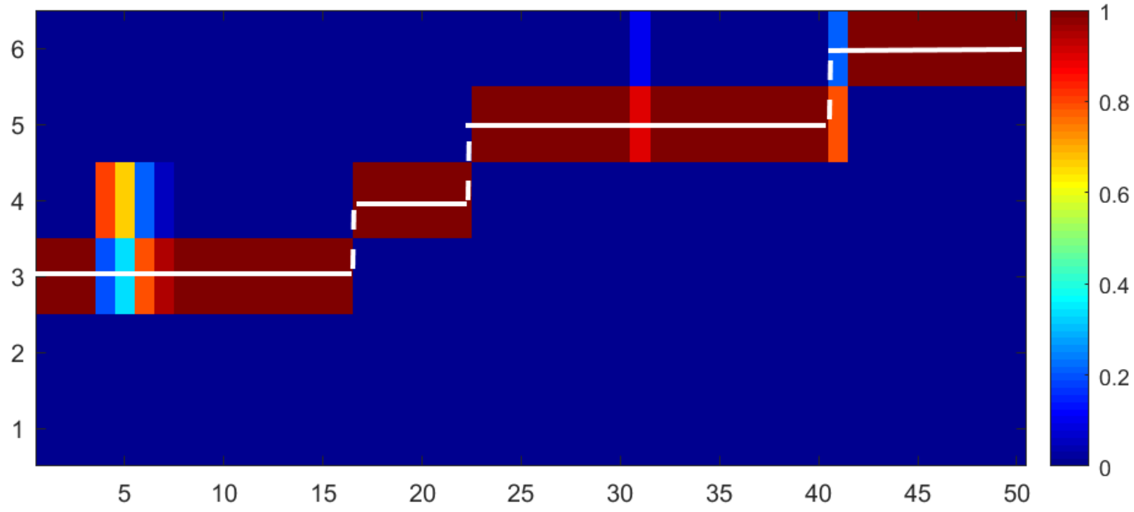
● ground truth + measurements

Clutter rate: 5

[4] https://motchallenge.net/data/MOT17/

# Results




The estimated cardinality(number of target) over time

Colormap shows the probability of cardinality
White lines are ground truth

The estimated track over 50 time step;
contours are level plot of the intensity

+ measurements
● ground truth

# Summary

1. Background: Non-homogeneous Poisson process (NHPP) tracker

2. Scalable association-based NHPP tracker
   - Rao-Blackwellisation scheme

3. Time-varying rate estimation for both targets and clutter
   - Generalized inverse Gaussian (GIG) priors for Poisson rates
   - Connecting NHPP and probabilistic multi-hypothesis tracker (PMHT)

4. Extensions
   - Shape estimation for group/extended targets
   - intentionality modelling in multiple target tracking
   - Lévy state-space models for highly maneuverable objects
   - Tracking of a time-varying number of objects using reversible jump MCMC

Thank you!