Recent advances in multi-object estimation

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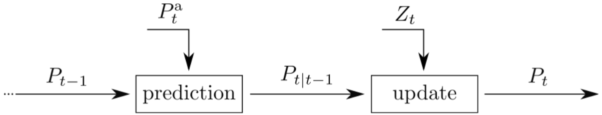
UDRC Summer School - June 2016 @ Edinburgh University, UK

1. Multi-object filtering framework: basics
2. Traditional solutions: strengths and weaknesses
   - The track-based approach
   - The RFS-based approach
3. Stochastic populations for multi-object filtering
   - Multi-object estimation framework
   - Bayesian filtering
   - The DISP filter
   - Information gain for sensor management

Bayesian filtering

Principle



- P_t: *propagated* "information" on objects of interest or *targets*
- P_t^{a}: *a priori* "information" on appearing targets
- Z_t: observations produced by the sensor system at time t

Sensor system for target tracking

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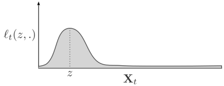
Stochastic description

Likelihood $\ell_t(z, x)$: how likely is obs. z to come from a target with state x?



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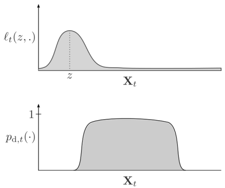

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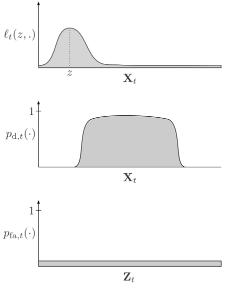
Probability of false alarm $p_{\mathrm{fa},t}(z)$: how likely is the sensor to produce a false alarm with state z?



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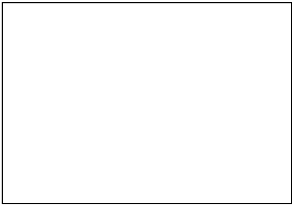

$\mathbf{Z}_t$

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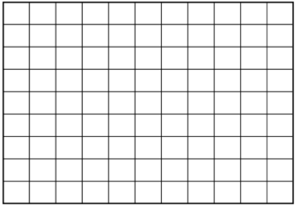

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- *Discrete* observation space $\mathbf{Z}_t$

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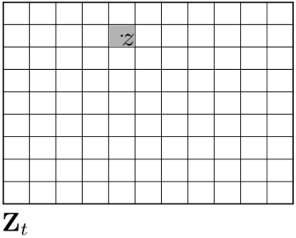

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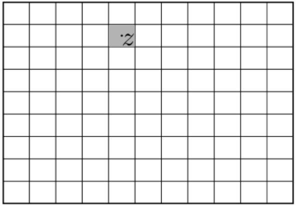

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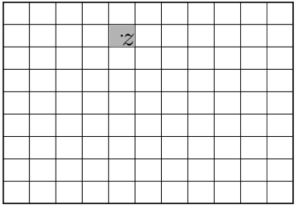

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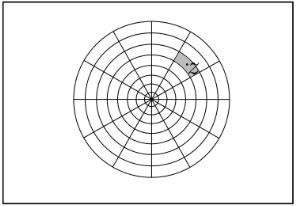

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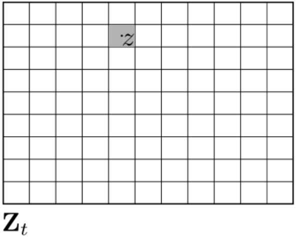

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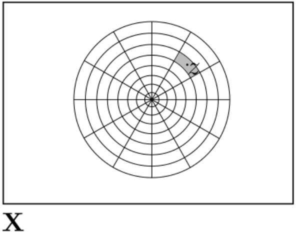
- $\mathbf{Z}_t$ projected onto $\mathbf{X}$ shapes the sensor field of view (FoV)
- Outside of the sensor FoV, $p_{\mathrm{d},t}$ is always zero (i.e. no target detection)



Multi-object filtering: common assumptions

Common assumptions (time t)

1. Targets behave independently
2. Observations are produced independently
3. At most one observation per target (if none, target is *miss-detected*)
4. At most one target per observation (if none, obs. is a *false alarm*)

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The assumptions above...

- 1. ... simplify the estimation problem (notably the data association)
- 2. ... will be used in the context of this presentation
- 3. ... are *not* necessary in the general multi-object estimation framework

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General principle

"A potential target = one track."

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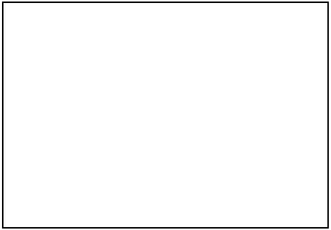

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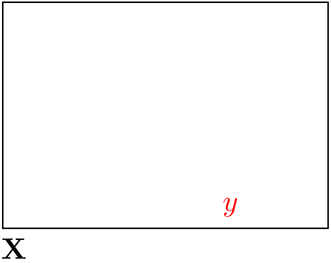

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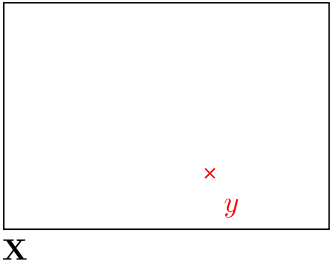

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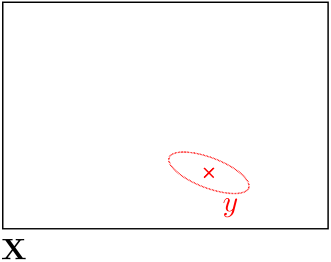

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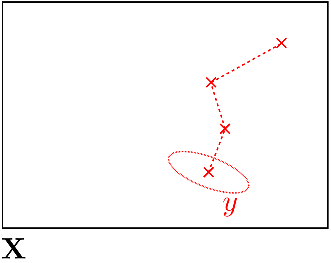

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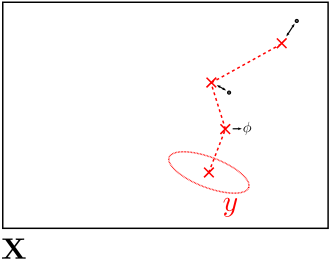
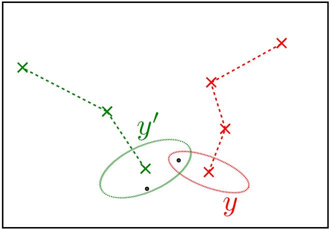


A track y is...

- ... *identified* by its state distribution (e.g. mean + covariance)
- ... *described* by its history of past estimates
- ... *characterised* by its observation path

Track update

Data association



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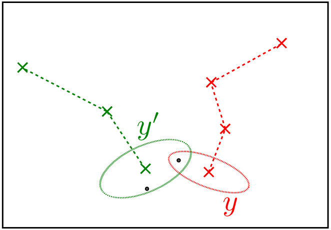

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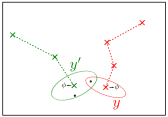

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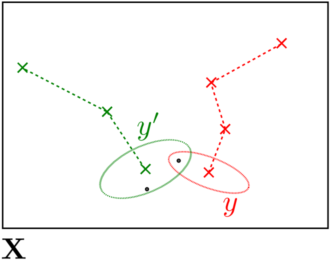

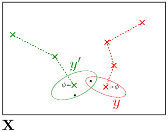

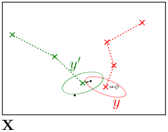

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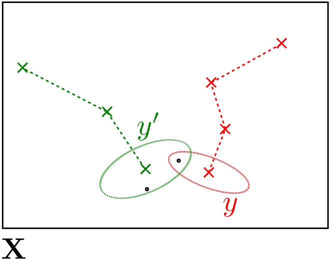

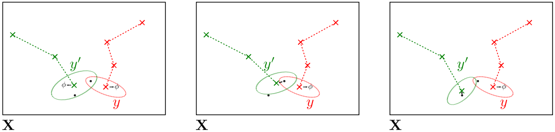

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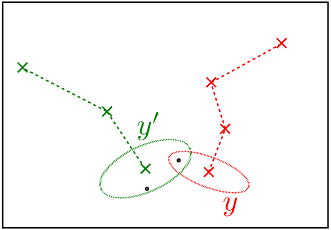

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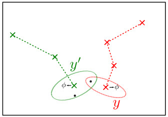

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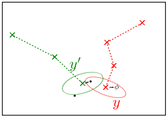

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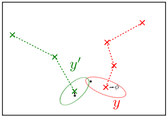

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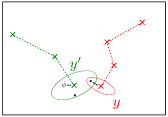

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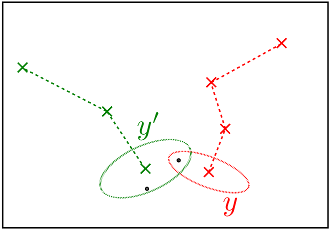

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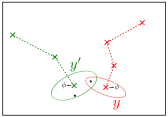

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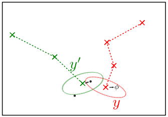

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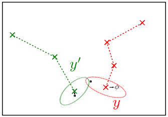

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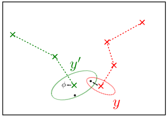

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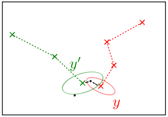

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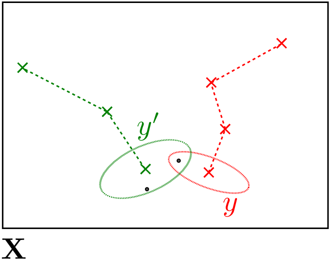

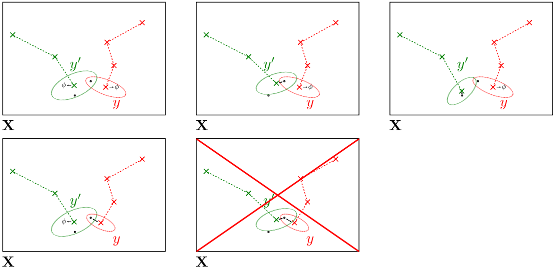

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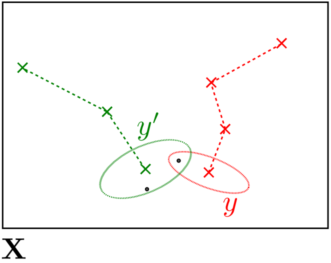

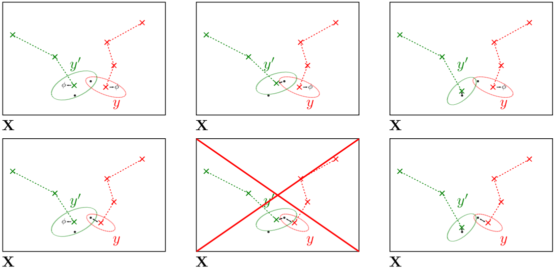

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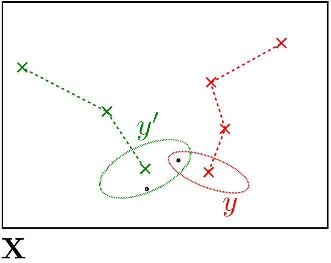

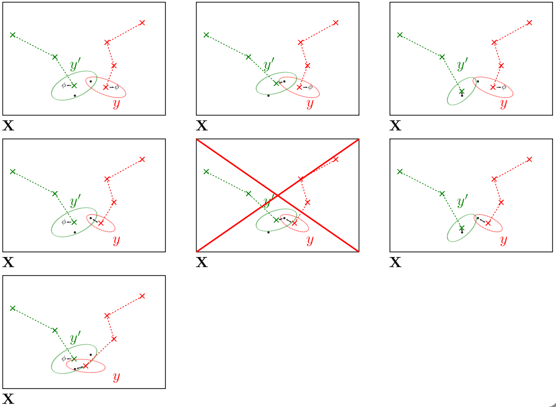

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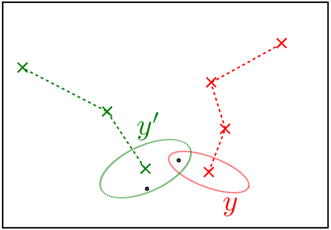

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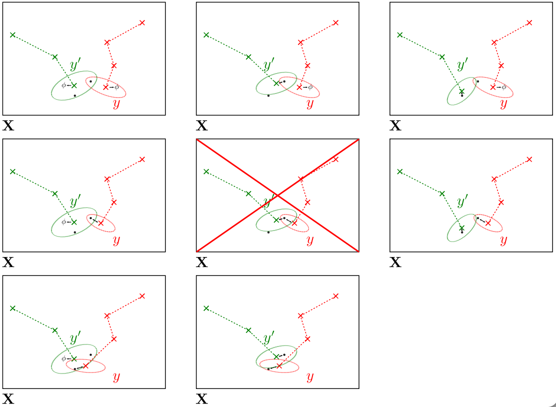

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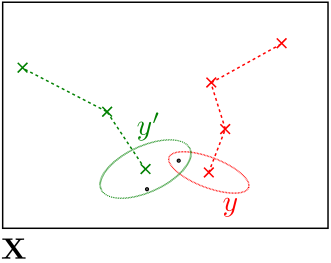

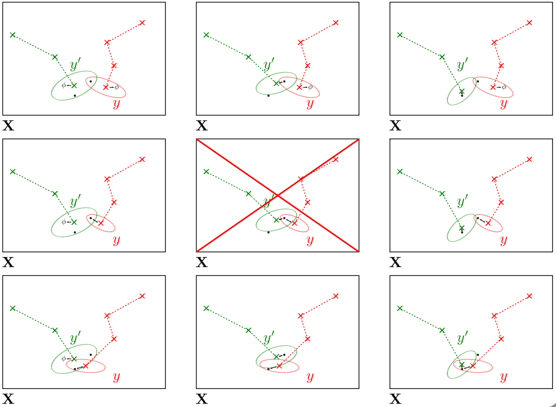

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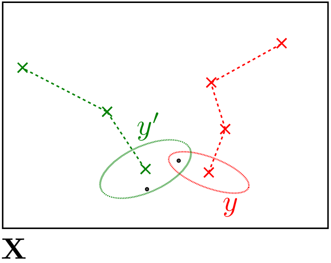

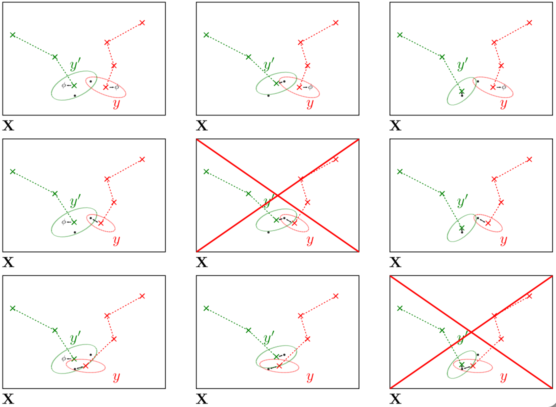

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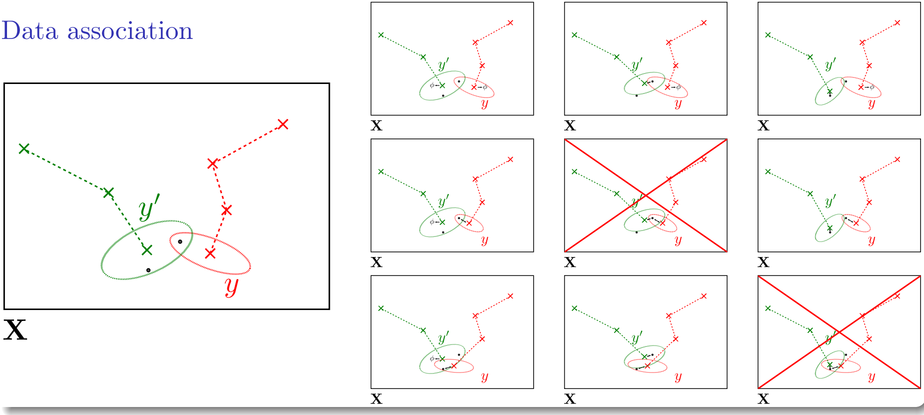

What is propagated to the next step?

Track update

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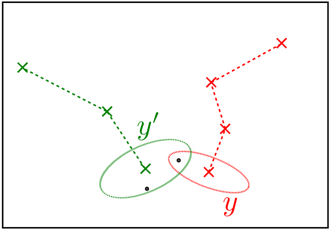

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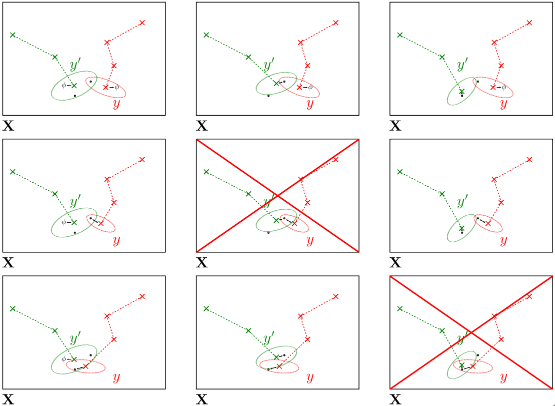

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All configurations, with associated probabilities → MHT filter

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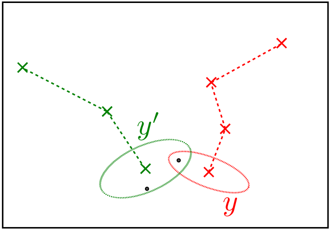

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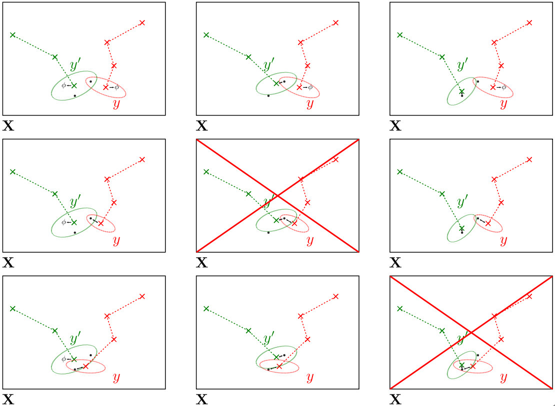

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A weighted combination of all configurations → JPDA filter

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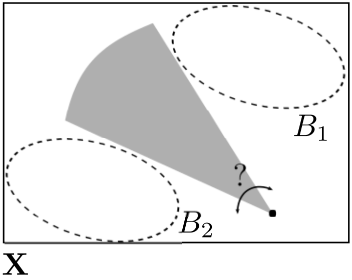

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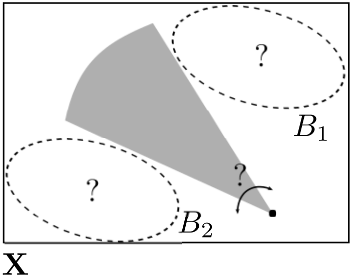

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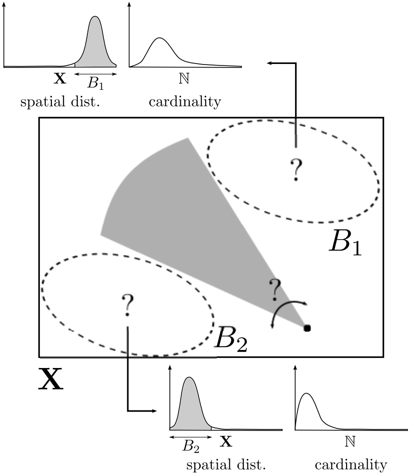

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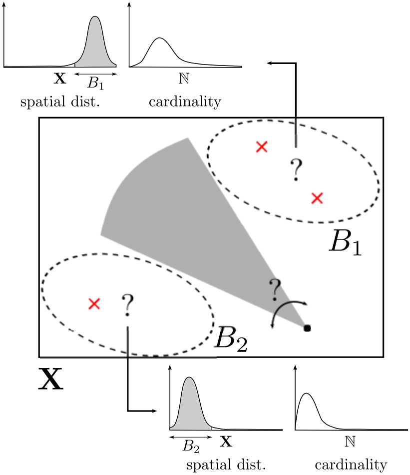
Suppose prior information on target *population* in B_1 and B_2 is available

Challenges

Since tracks are created upon detections, how to model *yet-to-be-detected* targets?



Sensor management problem: explore B_1 or B_2?

Suppose prior information on target *population* in B_1 and B_2 is available

How many tracks to create? Where?

Challenges (cont.)

Track creation/deletion

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Track creation/deletion

- When to create tracks?

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Track creation/deletion

- When to create tracks? For every new observation?

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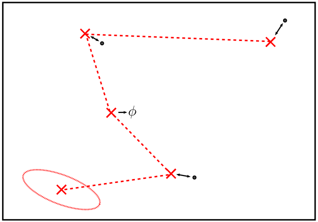

$\mathbf{X}$

Because it is "unlikely" to represent a target?

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Track creation/deletion

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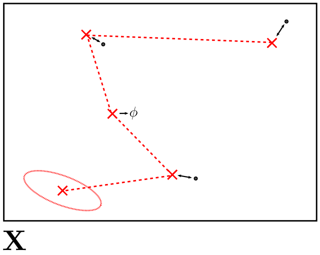

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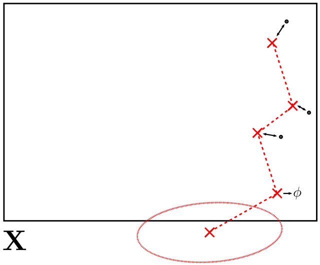

Because the target it represents has "probably" left the scene?

The track-based approach: pros and cons

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Can handle spatial information on tracks "optimally"

- Data association allows optimal single-observation/single-track update (e.g. Kalman filter)
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Describes targets on the *individual* level, not on the *collective* level

- When to create a track, especially in high clutter environment?
- When and why to delete a track (i.e. what is an "unlikely" track?)
- No representation of populations of "non-separable" targets (e.g., *yet-to-be-detected* ones)

The RFS-based approach

General principle

“The population of potential targets = one random finite set (RFS).”

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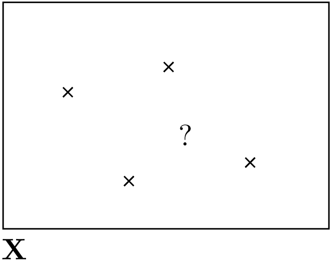
RFS representation



The target RFS Ξ...

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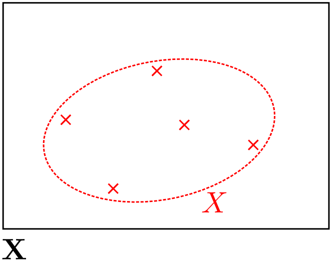

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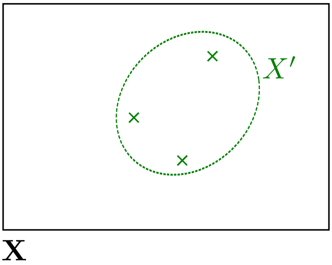

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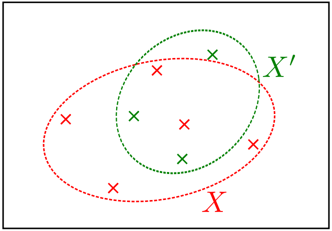

$\mathbf{X}$

$$P_\Xi(X) = 0.1$$
$$P_\Xi(X') = 0.03$$
$$\dots$$

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- ... whose realizations $X = \{x_1, \dots, x_n\}$ describe potential multi-target configurations
- ... is described by *probability distribution* P_Ξ

First-moment density for usual RFS-based filters

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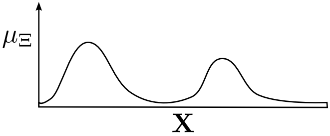
First-moment density μ_Ξ



Approximate description of RFS Ξ

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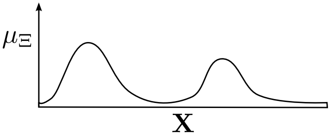

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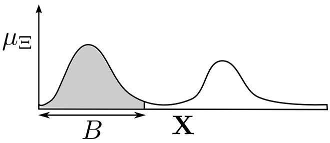

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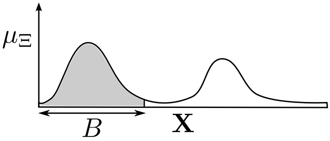

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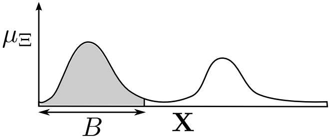

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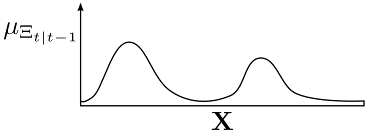

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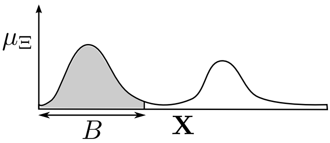

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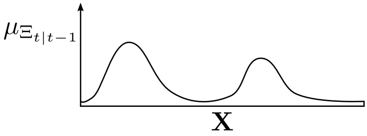

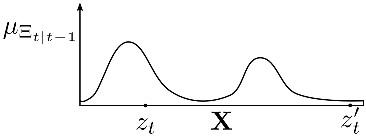

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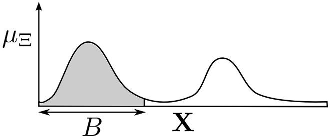

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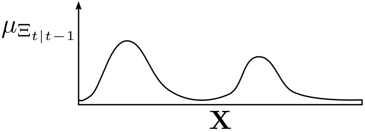

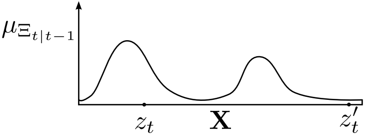

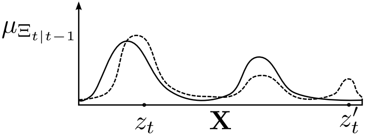

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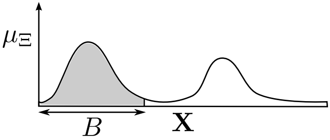

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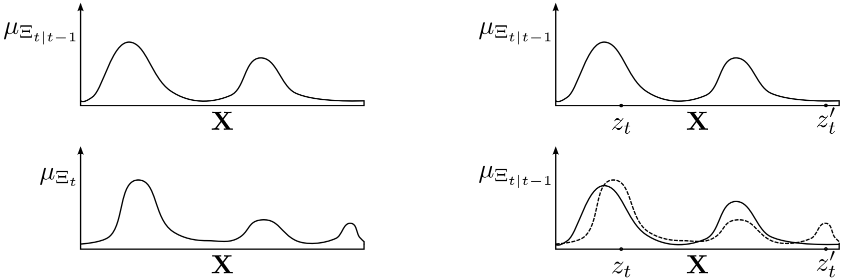

Challenges

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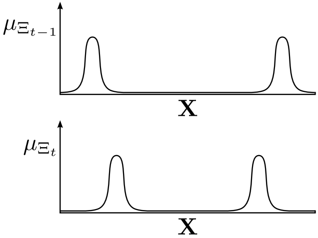
No equivalent of track history: what are the consequences?



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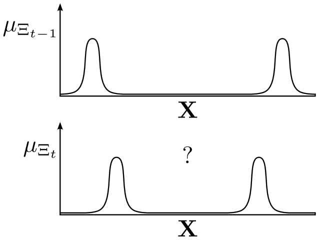

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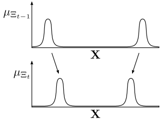

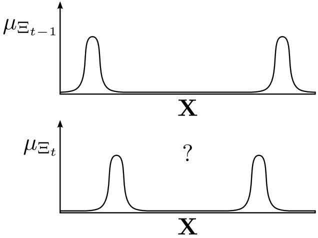

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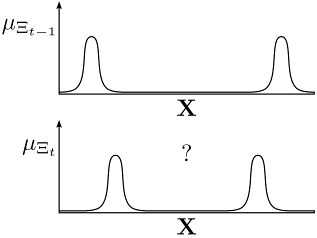

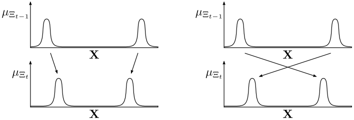

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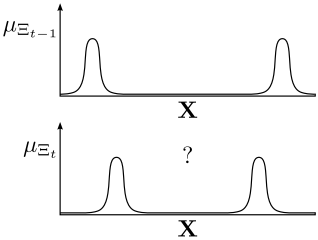

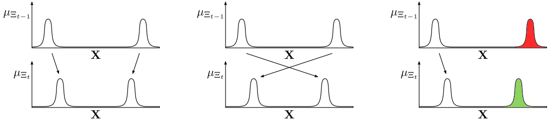

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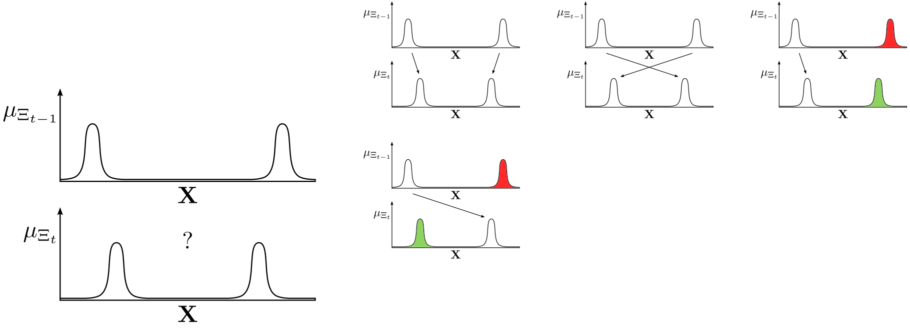

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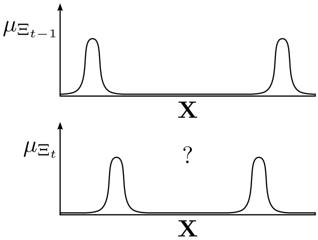

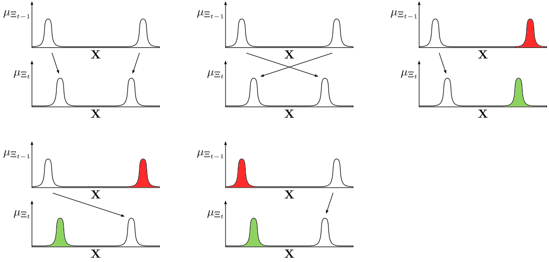

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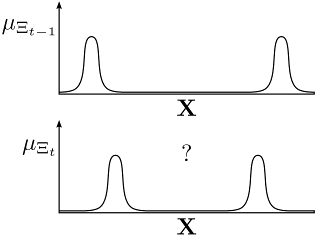

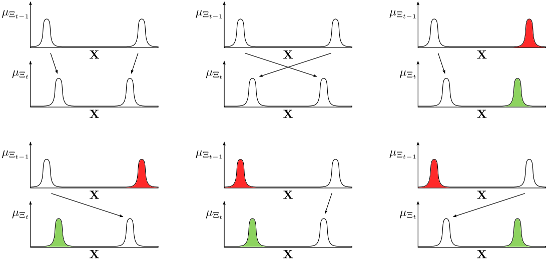

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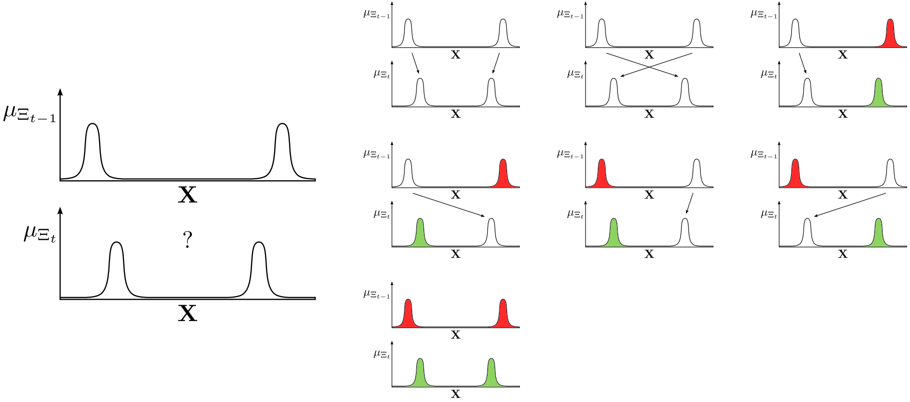

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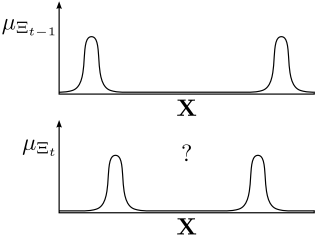

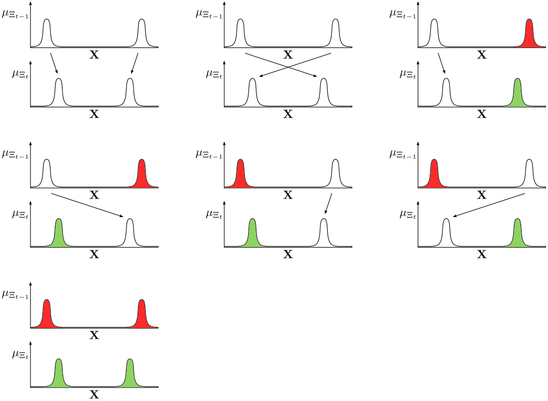

No inherent solution to link individuals from successive populations

Introducing labelling on top of RFS framework recently explored (Labeled Multi-Bernoulli filter, Vo et al.)

The RFS-based approach: pros and cons

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Provides probability framework incorporating all system uncertainties

- Appearing targets, yet-to-be-detected targets, false alarms, etc. described by RFSs
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- Each observation influences the spatial distribution of the whole population (i.e. no “optimal” single-observation/single-target update)
- Track histories or observation paths unavailable (unless through heuristics)

Estimation framework for stochastic populations

General principle

"A potential target is represented by a specific amount of information: not too little, but *not too much either*."

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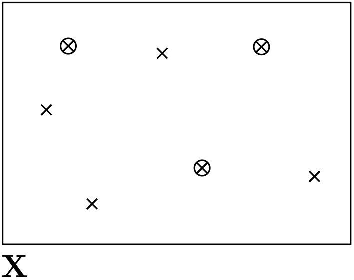
Outline

- Fully probabilistic framework, developed by J. Houssineau (supervisor: D. Clark)
- Level of description depends whether individual is *distinguishable* or *indistinguishable*
- Ongoing developments beyond tracking (e.g. sensor management, sensor calibration, performance assessment, ...)

Target distinguishability



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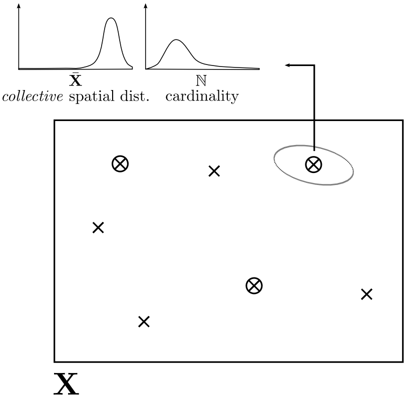

Indistinguishable target $i \in \mathbb{I}^{\circ}$:

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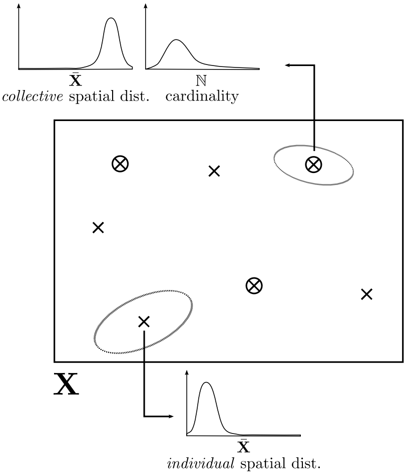

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Distinguishable targets $i \in \mathbb{I}^{\bullet}$:

- individual *characterised* by specific information
- e.g., "*the* potential individual that entered 10 time steps ago and produced observations z^3_{21}, z^1_{25}"

Individual management: where are the targets?

Spatial distribution and "empty state" ψ

Individual management: where are the targets?

Spatial distribution and “empty state” ψ

p_t^i

What is the target state?

Individual management: where are the targets?

Spatial distribution and "empty state" ψ

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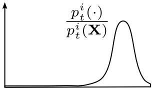

What is the target state?

If the target is the scene,
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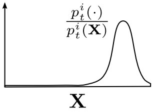
p_t^i	$=$		$+$	$p_t^i(\{\psi\})$
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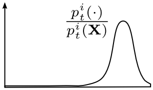
p_t^i = 

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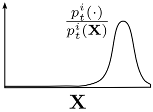
p_t^i = 

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  $\rightarrow$ *first detection* triggers distinguishability

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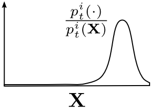
p_t^i = 

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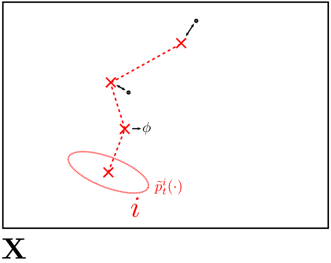
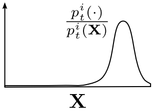
- Associated track i describes observation path and current info on target state



Individual management: where are the targets?

Spatial distribution and "empty state" ψ

p_t^i = 

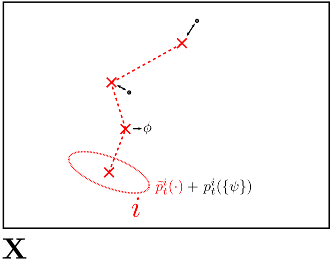
+ $p_t^i(\{\psi\})$

What is the target state? | If the target is the scene, what is its state? | Is the target in the scene?

Distinguishable target and associated track

- "One-target-per-measurement"rule + "One-measurement-per-target" rule

  → *first detection* triggers distinguishability
- Associated track i describes observation path and current info on target state
- Usual notion of track, except probability of absence $p_t^i(\{\psi\})$



Population management: which are the true targets?

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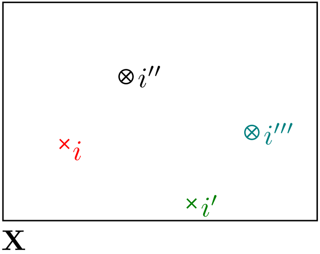

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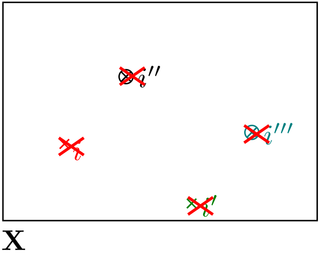

Example:

- $H = \emptyset, \boldsymbol{n}_{i''} = 0, \boldsymbol{n}_{i'''} = 0$

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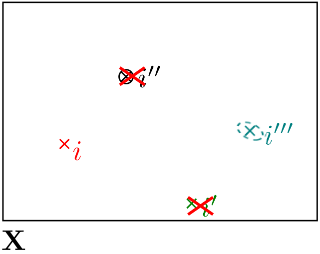

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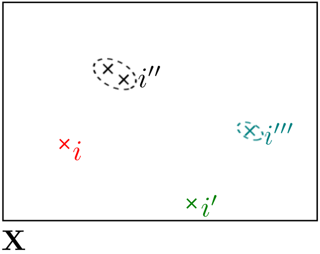

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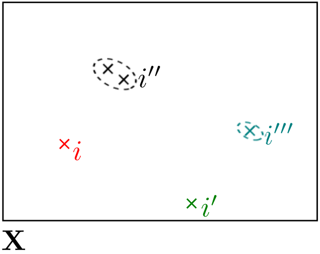

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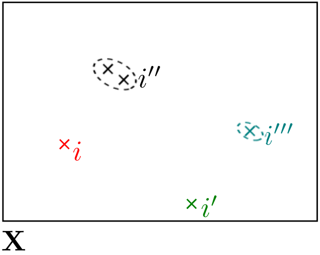

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- ...

Probabilistic representation of population composition:

- P.m.f. $\mathbf{w}_t(H, \boldsymbol{n})$: assesses joint existence of targets in configuration $(H, \boldsymbol{n})$
- $\sum_{H \subseteq I^{\bullet}} \sum_{\boldsymbol{n} \in \mathbb{N}^{I^{\circ}}} \mathbf{w}(H, \boldsymbol{n}) = 1$

Stochastic populations: what can we get out of it?

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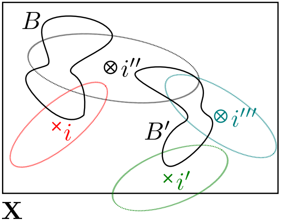
Elementary events



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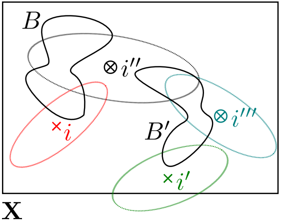

Probability that:

Stochastic populations: what can we get out of it?

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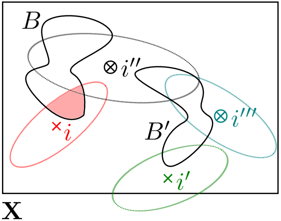

Probability that:

- i exists, and in B, *and*

Stochastic populations: what can we get out of it?

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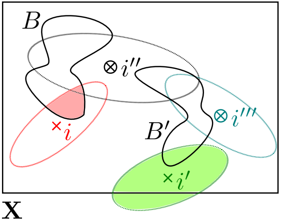

Probability that:

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- i' exists, *and*

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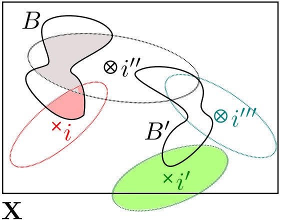

Probability that:

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- i' exists, *and*
- two targets i'', both in B, *and*

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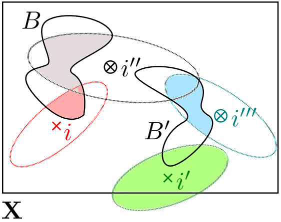

Probability that:

- i exists, and in B, *and*
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- one target i''', and in B'

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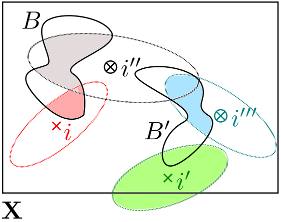

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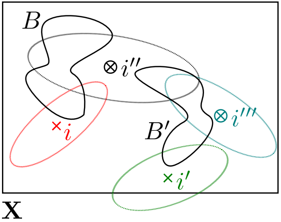

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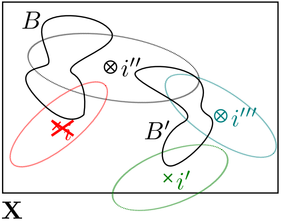

Probability that:

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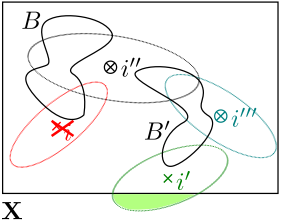

Probability that:

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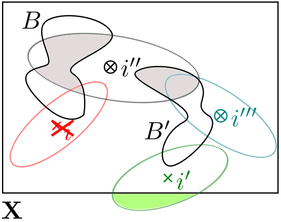

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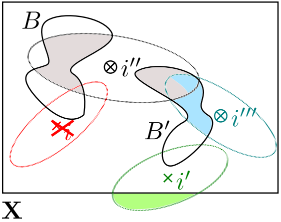

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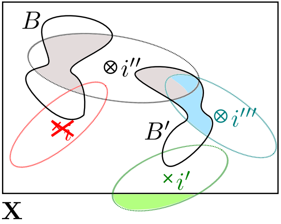

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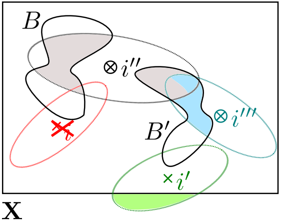

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Statistical moments

Stochastic populations: what can we get out of it?

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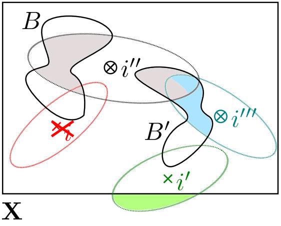

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Statistical moments

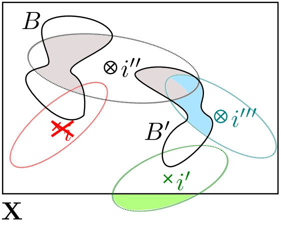
1. Full moments
   - Evaluate targets' *laws*
   - E.g. estimate number of targets whose laws are close to some p

$$\mathrm{M}(F) = \sum_{i \in \mathbb{I}} \mathbf{m}(i) F(p^i),$$

$$\mathrm{Var}(F) = \sum_{i,j \in \mathbb{I}} \mathbf{cov}(i,j) F(p^i) F(p^j).$$

Stochastic populations: what can we get out of it?

Elementary events



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Statistical moments

2. Collapsed moments

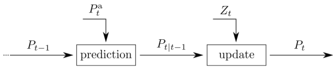
- Evaluate targets' *states*
- E.g. estimate number of targets whose state is in some region B

$$\mathrm{m}(f) = \sum_{i \in \mathbb{I}} \mathbf{m}(i) p^i(f),$$

$$\mathrm{var}(f) = \sum_{i,j \in \mathbb{I}} \mathbf{cov}(i,j) p^i(f) p^j(f).$$

Bayesian filtering with stochastic populations (1/2)

Principle



Maintain and update a dynamical stochastic population reflecting the true multi-target configuration

Bayesian filtering with stochastic populations (1/2)

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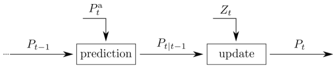

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Filtering design: a few modelling choices

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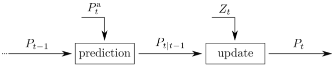

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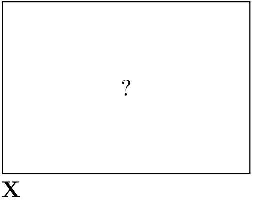

- How to represent incoming targets at time t?

Bayesian filtering with stochastic populations (1/2)

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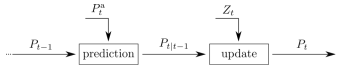

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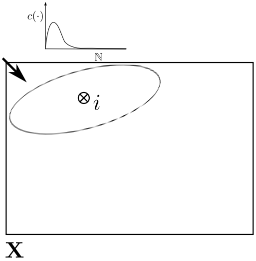

- How to represent incoming targets at time t? One ind. population?

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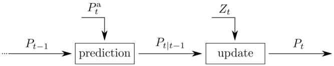

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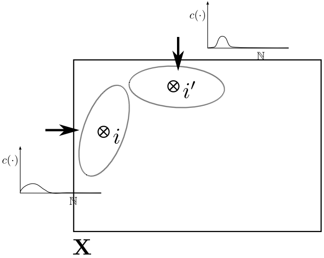

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Bayesian filtering with stochastic populations (1/2)

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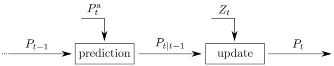

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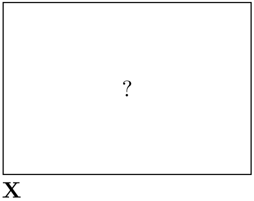
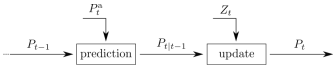


- How to represent incoming targets at time t? One ind. population? Two?
- How to mitigate information loss?

Bayesian filtering with stochastic populations (1/2)

Principle



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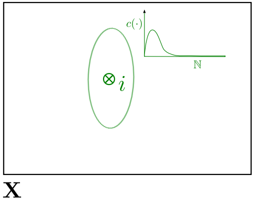
Filtering design: a few modelling choices



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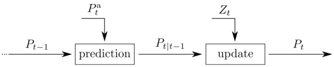
  Merge "close" ind. populations?

Bayesian filtering with stochastic populations (1/2)

Principle



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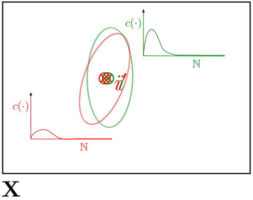
Filtering design: a few modelling choices



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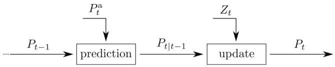

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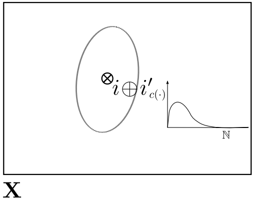

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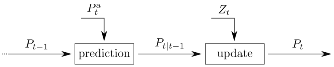

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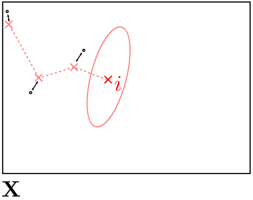

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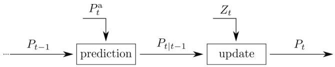
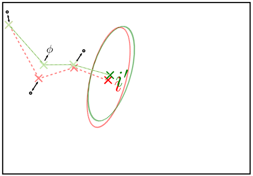
Bayesian filtering with stochastic populations (1/2)

Principle



Maintain and update a dynamical stochastic population reflecting the true multi-target configuration

Filtering design: a few modelling choices



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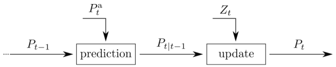

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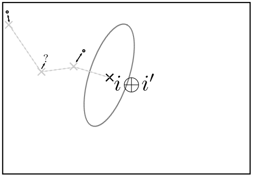

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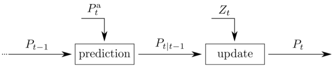

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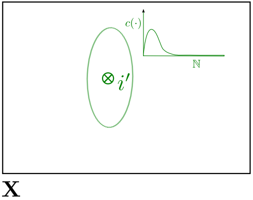

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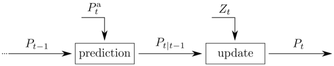

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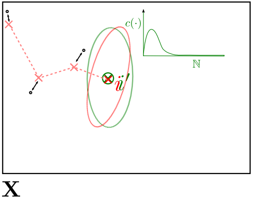

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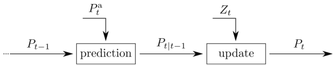

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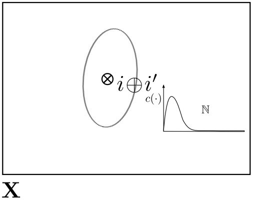

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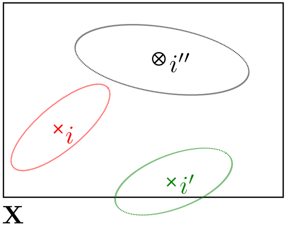

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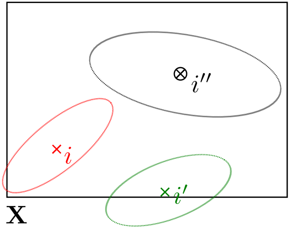

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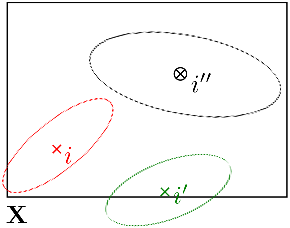

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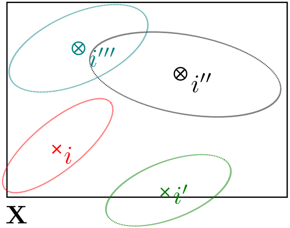

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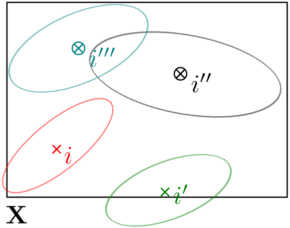

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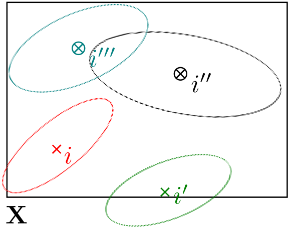

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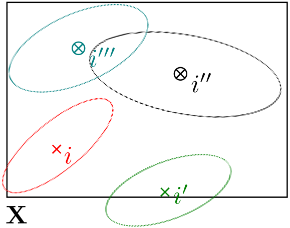

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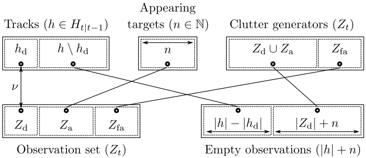

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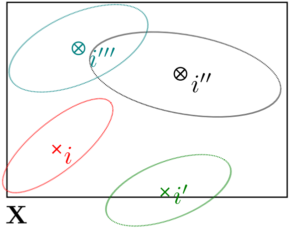

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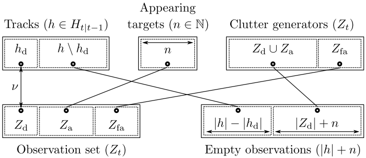

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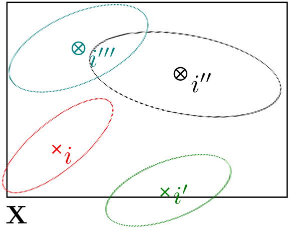

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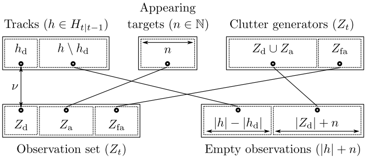

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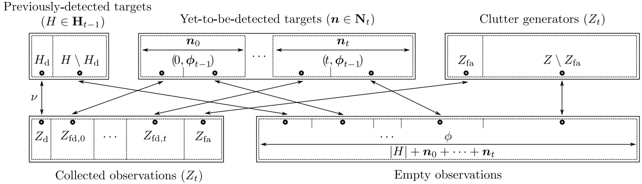

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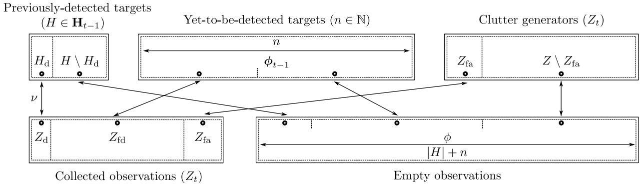

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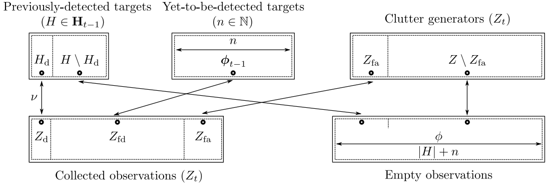

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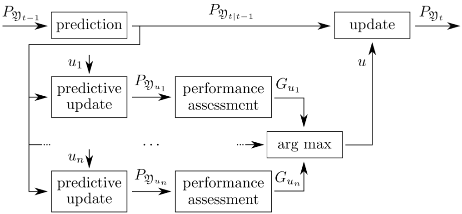

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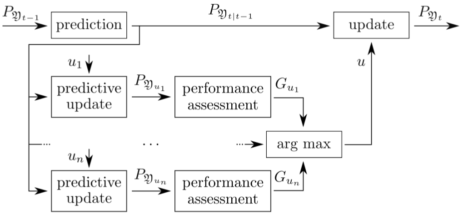

Information gain

Given the predicted information $P_{t|t-1}$, and a set of observations Z_t, can we quantify the information gain from $P_{t|t-1}$ to the updated information P_t?

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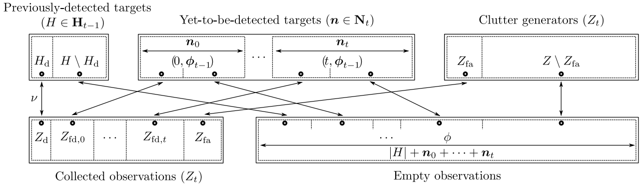

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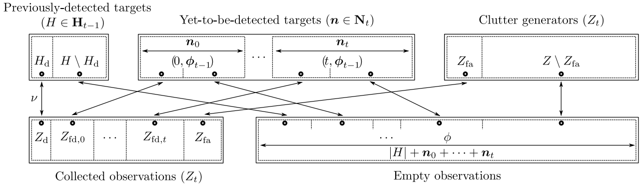

Each association $\mathbf{a} = (H, \boldsymbol{n}, \mathbf{h} \in \mathrm{Adm}_{Z_t}(H, \boldsymbol{n}))$ leads to a unique conf. $(\hat{H}, \hat{\boldsymbol{n}})$:

- Assessed by prob. $v_t^{\mathbf{a}}$ (i.e. how likely is the association producing $(\hat{H}, \hat{\boldsymbol{n}})$?)
- Composed of distinguishable targets
  $\hat{H} = \bigcup_{i \in h_\mathrm{d}} \{i : \nu(i)\} \cup \bigcup_{i \in h \setminus h_\mathrm{d}} \{i : \phi\} \cup \bigcup_{0 \le t' \le t} \bigcup_{z \in Z_{\mathrm{fd},t'}} \{(t', \boldsymbol{\phi}_{t-1} : z)\}$

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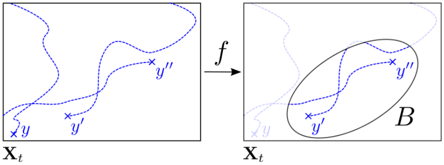

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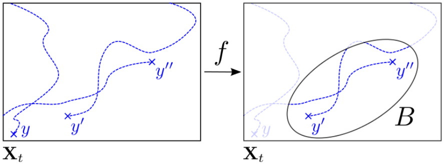

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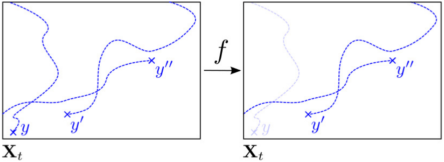
Exclusion of *targets* from information gain



Thank you for your attention!