An Approximate Likelihood Ratio Detector for QTMS Radar and Noise Radar

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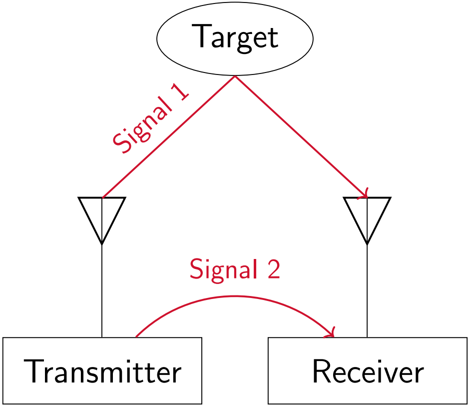
Introductory remarks

- Radar stands for **ra**dio **d**etection **a**nd **r**anging
- Modern radars support **many more functions** than just detection and ranging, but these are still basic capabilities of radars
  - Other functions include target tracking, clutter suppression, synthetic aperture radar, and many more
- In this presentation, we focus on the **detection** aspect

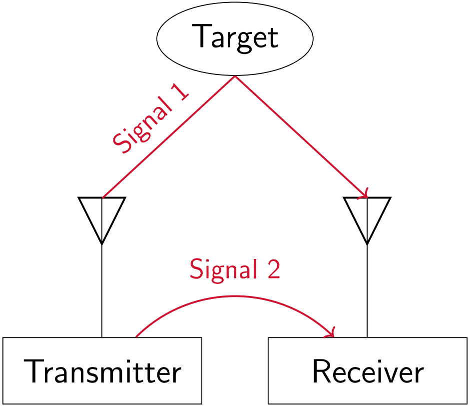
(Generalized) matched filtering



We consider radars that work as follows:

1. Produce two microwave signals.
2. Transmit one. Keep the other.
3. Receive and measure a signal.
4. Use the received and retained signals to decide whether there is a target.

(Generalized) matched filtering



- This is essentially **matched filtering**
- However, it is **more general** than the strict definition of the matched filter, $h[n] = x^*[-n]$
  - Reference signal need not be exactly the same as the transmit signal
  - Filter need not be linear

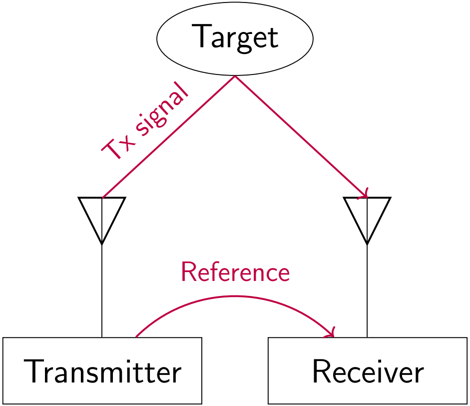
The radar detection problem

- As far as detection is concerned, radars are machines for **hypothesis testing**
- We wish to distinguish between:

$$H_0 : \text{Target absent}$$
$$H_1 : \text{Target present}$$

- We need a **test statistic**—that is, a **detector function**—to distinguish between H_0 and H_1
- Choice of detector function depends on the exact nature of the radar signals

Noise radar



My research focuses on **noise radars** which work like this:

1. Produce a microwave **noise** signal.
2. Transmit the signal, retaining a copy as a reference.
3. Receive a signal from free space.
4. Correlate the received signal with the reference.

Mathematical background

- RF signals are described by a pair of real-valued time series: **in-phase voltages** $I(t)$ and **quadrature voltages** $Q(t)$
- For a noise radar, we consider two signals (four time series):
  - received signal: $I_1(t)$ and $Q_1(t)$
  - reference signal: $I_2(t)$ and $Q_2(t)$
  - Note: no need to consider the *transmitted* signal
- For noise radar, each signal is a **random process**
- We assume that all the signals are Gaussian white noise (so we can drop the time variable t)

The noise radar covariance matrix

$$\begin{bmatrix} \sigma_1^2 & 0 & \rho\sigma_1\sigma_2 \cos\phi & \rho\sigma_1\sigma_2 \sin\phi \\ 0 & \sigma_1^2 & -\rho\sigma_1\sigma_2 \sin\phi & \rho\sigma_1\sigma_2 \cos\phi \\ \rho\sigma_1\sigma_2 \cos\phi & -\rho\sigma_1\sigma_2 \sin\phi & \sigma_2^2 & 0 \\ \rho\sigma_1\sigma_2 \sin\phi & \rho\sigma_1\sigma_2 \cos\phi & 0 & \sigma_2^2 \end{bmatrix}$$

- It can be shown that I_1, Q_1, I_2, Q_2 are characterized by the above covariance matrix
- σ_1^2, σ_2^2 are signal powers for the received and reference signals; ϕ is the phase between them

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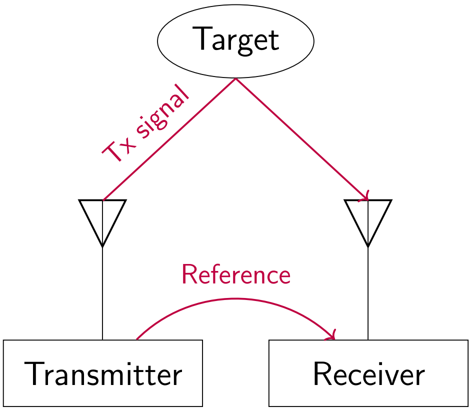
- It can be shown that I_1, Q_1, I_2, Q_2 are characterized by the above covariance matrix
- σ_1^2, σ_2^2 are signal powers for the received and reference signals; ϕ is the phase between them
- ρ characterizes the **correlation** between the two signals

The importance of ρ

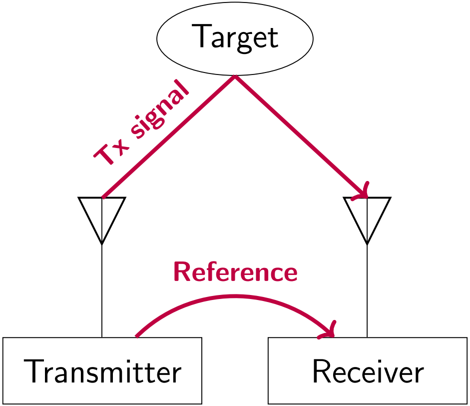


- ρ tells us whether there is any **correlation** between the received and reference signals
- $\rho > 0$ at receiver $\implies$ target **present**
- $\rho = 0$ at receiver $\implies$ target **absent**
- Target detection problem = hypothesis testing on ρ

Why introduce ρ?

- ρ has previously been little-studied in the context of radars
- Not as familiar as SNR
- One advantage: the target detection problem is easily formulated as a hypothesis test in terms of ρ
- ρ takes into account **imperfections in the reference signal**, which is ignored when we deal only with SNR
  - We will return to this point when we discuss *quantum two-mode squeezing radar*

Hypothesis testing on ρ



- Recall: ρ tells us the **correlation** between the received and reference signals
- In terms of ρ, the target detection problem requires us to distinguish between these hypotheses:

$$H_0 : \rho = 0 \quad \text{Target absent}$$
$$H_1 : \rho > 0 \quad \text{Target present}$$

A note about choosing hypotheses

$$\begin{aligned} H_0 &: \rho = 0 \quad \text{Target absent} \\ H_1 &: \rho > 0 \quad \text{Target present} \end{aligned}$$

- Note that H_1 is a **composite** hypothesis
- We **do not know** what ρ would be if a target is present
  - ρ is a function of range, radar cross section, background noise, ...
- We are **not** testing between $\rho = 0$ and $\rho = \kappa$ for some known value of κ
- The Neyman-Pearson lemma does not apply here

Detector functions

- There are many detector functions which could be used for this target detection problem

- A detector function is used for radar detection as follows:

  1. Set a **threshold**
  2. Calculate the detector function using the given I_1, Q_1, I_2, and Q_2 samples
  3. **Declare a detection** if the value exceeds the threshold

- One natural detector function: **the (generalized) likelihood ratio**

Nuisance parameters

$$\begin{bmatrix} \sigma_1^2 & 0 & \rho\sigma_1\sigma_2\cos\phi & \rho\sigma_1\sigma_2\sin\phi \\ 0 & \sigma_1^2 & -\rho\sigma_1\sigma_2\sin\phi & \rho\sigma_1\sigma_2\cos\phi \\ \rho\sigma_1\sigma_2\cos\phi & -\rho\sigma_1\sigma_2\sin\phi & \sigma_2^2 & 0 \\ \rho\sigma_1\sigma_2\sin\phi & \rho\sigma_1\sigma_2\cos\phi & 0 & \sigma_2^2 \end{bmatrix}$$

- Unfortunately, the covariance matrix has three nuisance parameters: σ_1, σ_2, and ϕ
- In order to simplify our problem, we assume $\sigma_1 = 1$, $\sigma_2 = 1$, and $\phi = 0$
- Future work: estimate these parameters or deal with them in a more principled manner

The (simplified) noise radar target detection problem

Given N samples of the radar signals I_1, Q_1, I_2, and Q_2 such that

$$\begin{bmatrix} I_1 \\ Q_1 \\ I_2 \\ Q_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & \rho & 0 \\ 0 & 1 & 0 & -\rho \\ \rho & 0 & 1 & 0 \\ 0 & -\rho & 0 & 1 \end{bmatrix} \right),$$

decide which of the following is true:

$$\begin{aligned} H_0 &: \rho = 0 \quad \text{Target absent} \\ H_1 &: \rho > 0 \quad \text{Target present} \end{aligned}$$

The GLR detector

$$D_{\mathrm{GLR}} = N\left[\frac{2\bar{D}_1\hat{\rho} - \bar{P}_{\mathrm{tot}}\hat{\rho}^2}{1-\hat{\rho}^2} - 2\ln(1-\hat{\rho}^2)\right]$$

- $P_{\mathrm{tot}} \equiv I_1^2 + Q_1^2 + I_2^2 + Q_2^2$
- $D_1 \equiv I_1 I_2 - Q_1 Q_2$
- Line over expression = sample mean over N samples
- $\hat{\rho}$ = maximum likelihood estimate of ρ (complicated)

Approximate GLR detector

$$D_{\mathrm{GLR}} = N\left[\frac{2\bar{D}_1\hat{\rho} - \bar{P}_{\mathrm{tot}}\hat{\rho}^2}{1-\hat{\rho}^2} - 2\ln(1-\hat{\rho}^2)\right]$$

- This is very complicated
- But to second order in ρ, this reduces to

$$D_{\mathrm{GLR}} \approx \frac{N\bar{D}_1^2}{\bar{P}_{\mathrm{tot}} - 2}$$

- Much simpler to calculate (e.g. on a digital signal processor)
- Can obtain closed-form formula for the ROC curve

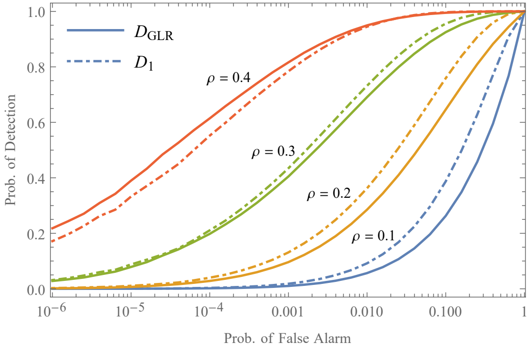
Connection to a previous detector

- $D_1 = I_1 I_2 - Q_1 Q_2$ has itself been used as a detector function

- It was called "Detector 1" in the world's first journal paper on experimental microwave quantum radar
  - D. Luong, C. W. S. Chang, A. M. Vadiraj, A. Damini, C. M. Wilson and B. Balaji, "Receiver Operating Characteristics for a Prototype Quantum Two-Mode Squeezing Radar," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 56, no. 3, pp. 2041-2060, June 2020 (accepted Sept. 2019)

- It is equivalent to the "digital receiver" in the world's second journal paper on experimental microwave quantum radar
  - S. Barzanjeh, S. Pirandola, D. Vitali, and J. M. Fink, "Microwave quantum illumination using a digital receiver," *Science Advances*, vol. 6, no. 19, p. eabb0451, May 2020 (accepted Feb. 2020)

ROC curve comparison



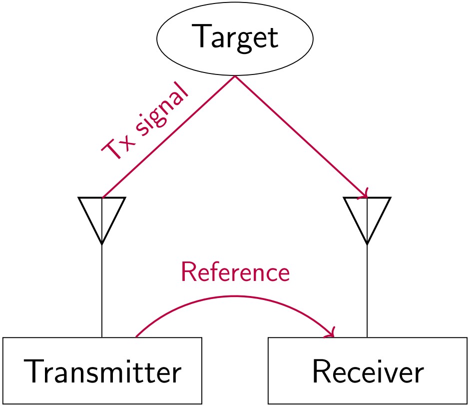
- $\rho = 0.1, 0.2, 0.3, 0.4$
- $N = 50$

- Performance of D_{GLR} is similar to D_1
- D_1 slightly better when ρ is small; D_{GLR} better when ρ is slightly larger
- Neither one is "optimal"
- As expected, overall radar performance improves as ρ increases

What happened to **quantum** radar?

- So far, I have spoken only about **noise radar**
- What happened to *quantum two-mode squeezing* (QTMS) radar?

Noise radar recap



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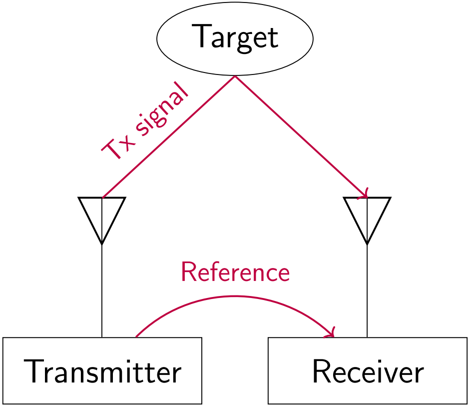
QTMS radar



Quantum two-mode squeezing radars work (roughly) like this:

1. Produce a microwave noise signal.
2. Transmit the signal, retaining a **better** copy as a reference.
3. Receive a signal from free space.
4. Correlate the received signal with the reference.

A better reference signal

$$\begin{bmatrix} \sigma_1^2 & 0 & \rho\sigma_1\sigma_2 \cos\phi & \rho\sigma_1\sigma_2 \sin\phi \\ 0 & \sigma_1^2 & -\rho\sigma_1\sigma_2 \sin\phi & \rho\sigma_1\sigma_2 \cos\phi \\ \rho\sigma_1\sigma_2 \cos\phi & -\rho\sigma_1\sigma_2 \sin\phi & \sigma_2^2 & 0 \\ \rho\sigma_1\sigma_2 \sin\phi & \rho\sigma_1\sigma_2 \cos\phi & 0 & \sigma_2^2 \end{bmatrix}$$

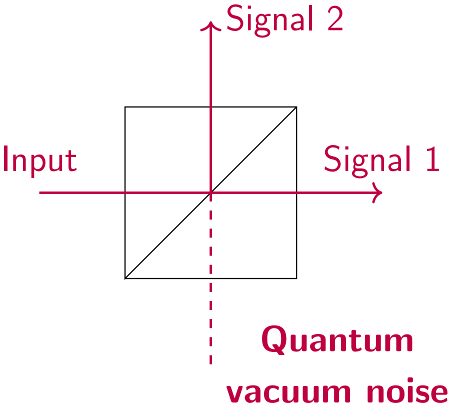
- Mathematically (but not experimentally!), QTMS radars are exactly the same as noise radars except that they achieve **higher values of** ρ
- The reference signal is a **higher-fidelity copy** of the transmit signal

Noise in the reference signal

- Conventional matched filtering assumes **a perfect copy of the Tx signal is available**
- Quantum mechanics says a perfect copy is **impossible**
- There will **always be noise** in I and Q voltage measurements, even in an **theoretically ideal** system

If you think you can achieve 100% correlation,
you've forgotten about quantum mechanics!

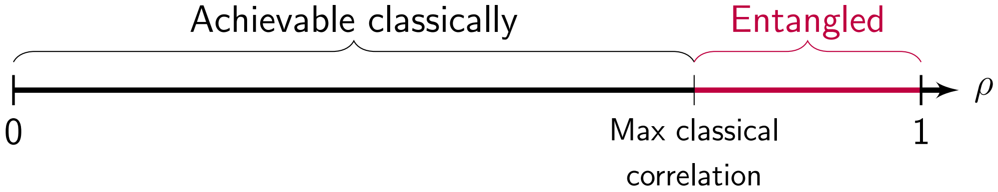
Can't you just split the signal?



- **Vacuum noise** will creep into the beamsplitter, even at absolute zero and in a perfect vacuum

Quantum noise and entanglement

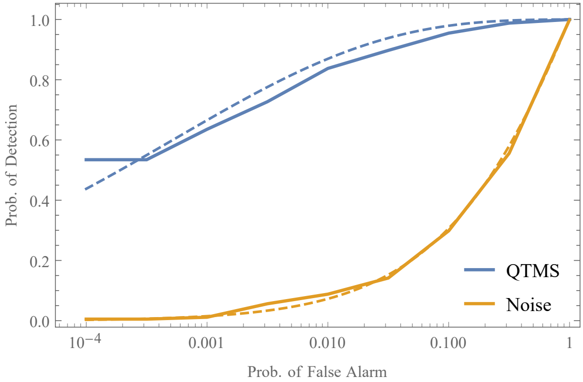
- 100% correlation is **impossible** according to quantum mechanics
- With conventional methods of preparing a reference signal, $\rho <$ classical limit
- Using **entanglement**, can achieve classical limit $< \rho < 1$



Radars are not made out of paper

- No amount of theory can replace an experiment
- If a radar cannot improve ρ for the same signal power, **no point pursuing it**
- A QTMS radar experiment was performed by Wilson et al. at the Institute for Quantum Computing (University of Waterloo)
  - D. Luong, C. W. S. Chang, A. M. Vadiraj, A. Damini, C. M. Wilson and B. Balaji, "Receiver Operating Characteristics for a Prototype Quantum Two-Mode Squeezing Radar," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 56, no. 3, pp. 2041-2060, June 2020

Experimental ROC curves



- Experimental ROC curves fit our theoretically derived ROC curves very well
- Improvement in QTMS radar over standard noise radar corresponds to **increasing** ρ **by a factor of 3**

Conclusion

- Noise radar performance is characterized by the correlation coefficient ρ between the transmit and reference signals
- Can use the **generalized likelihood ratio** to distinguish between $\rho = 0$ and $\rho > 0$
- An approximate GLR works well in most cases
- QTMS radars improve performance by increasing ρ
- This improvement has been **experimentally** demonstrated