Detection of Weak Transient Signals Using a Broadband Subspace Approach

S. Weiss[1], C. Delaosa[1], J. Matthews[2], I.K. Proudler[1], and B.A. Jackson[3]

[1]Dept. of Electronic & Electrical Eng., University of Strathclyde, Glasgow, Scotland
[2]PA Consulting, Global Innovation and Technology Centre Cambridge, Melbourn SG8 6DP, UK
[3]UK Defence Science & Technology Laboratory (Dstl), Porton Down, Salisbury SP4 0JQ, UK

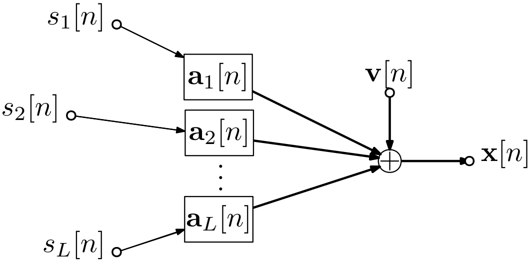
SSPD'21, Edinburgh, 14+15 September 2021

with thanks to: Peter Relph (PA),
Zoe Duffey, Graeme Reid, Emily Goodwin, James Haward (Dstl)

This work was supported by the DASA Invisible Shields Programme, and by the Engineering and Physical Sciences Research Council (EPSRC) Grant number EP/S000631/1 and the MOD University Defence Research Collaboration in Signal Processing.

Problem & Model

- A number of broadband stationary sources $s_\ell[n]$, $\ell = 1, \dots, L$, illuminate an M-element sensor array;
- each transfer path is modelled by a vector of impulse responses $\mathbf{a}_\ell[n] \in \mathbb{C}^M$;
- stationary additive, spatially and temporally uncorrelated noise $\mathbf{v}[n] \in \mathbb{C}^M$;

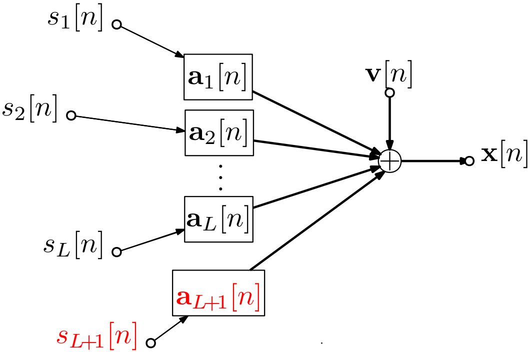


$$\mathbf{x}[n] = \sum_{\ell=1}^{L} \mathbf{a}_\ell[n] * s_\ell[n] + \mathbf{v}[n]$$

Problem & Model

University of Strathclyde Engineering

- A number of broadband stationary sources $s_\ell[n]$, $\ell = 1, \ldots, L$, illuminate an M-element sensor array;
- each transfer path is modelled by a vector of impulse responses $\mathbf{a}_\ell[n] \in \mathbb{C}^M$;
- stationary additive, spatially and temporally uncorrelated noise $\mathbf{v}[n] \in \mathbb{C}^M$;
- a broadband transient signal $s_{L+1}[n]$ enters the scene at some point in time;
- aim: we want to detect the onset of this transient signal, which may be weak in power [12];
- assumption: $M > L$.

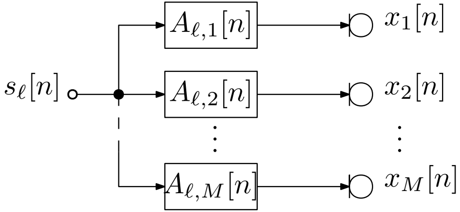


$$\mathbf{x}[n] = \sum_{\ell=1}^{L+1} \mathbf{a}_\ell[n] * s_\ell[n] + \mathbf{v}[n]$$

Model

- Each source, $s_\ell[n]$, contributes to the data vector $\mathbf{x}[n] = [x_1[n], \ \ldots \ , x_M[n]]^{\mathrm{T}}$ via a steering vector $\mathbf{a}_\ell[n] = [A_{\ell,1}[n], \ldots A_{\ell,M}[n]]^{\mathrm{T}}$;
- compact with $\mathbf{A}[n] = [\mathbf{a}_1[n] \ldots \mathbf{a}_L[n]]$ and $\mathbf{s}[n] = [s_1[n], \ldots, s_L[n]]^{\mathrm{T}}$:

$$\mathbf{x}[n] = \mathbf{A}[n] * \mathbf{s}[n] + \mathbf{v}[n] \ ;$$



- space-time covariance: $\mathbf{R}[\tau] = \mathcal{E}\{\mathbf{x}[n]\mathbf{x}^{\mathrm{H}}[n-\tau]\}$:

$$\begin{aligned} \mathbf{R}[\tau] &= \mathbf{A}[\tau] * \mathcal{E}\{\mathbf{s}[n]\mathbf{s}^{\mathrm{H}}[n-\tau]\} * \mathbf{A}^{\mathrm{H}}[-\tau] + \mathcal{E}\{\mathbf{v}[n]\mathbf{v}^{\mathrm{H}}[n-\tau]\} && (1) \\ &= \mathbf{A}[\tau] * \mathbf{\Gamma}[\tau] * \mathbf{A}^{\mathrm{H}}[-\tau] + \sigma_v^2 \mathbf{I}_M \delta[\tau] \ . && (2) \end{aligned}$$

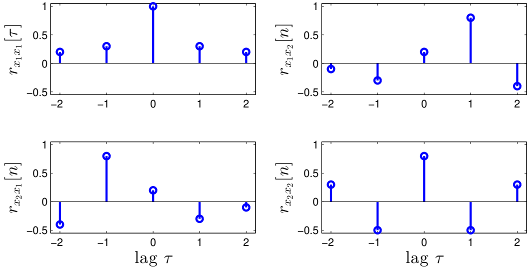
Cross-Spectral Density Matrix

- Transfer function matrix $\boldsymbol{A}(z) = \sum_n \mathbf{A}[n] z^{-n}$ (short $\boldsymbol{A}(z) \bullet\!\!-\!\!\circ\, \mathbf{A}[n]$) is a polynomial in $z \in \mathbb{C}$;
- cross-spectral density $\boldsymbol{R}(z) \bullet\!\!-\!\!\circ\, \mathbf{R}[\tau]$:

$$\boldsymbol{R}(z) = \boldsymbol{A}(z)\boldsymbol{\Gamma}(z)\boldsymbol{A}^{\mathrm{P}}(z) + \sigma_v^2 \mathbf{I}_M \ ;$$

- parahermitian property:

$$\boldsymbol{R}^{\mathrm{P}}(z) = \boldsymbol{R}^{\mathrm{H}}(1/z^*) = \boldsymbol{R}(z) \ ;$$



- when evaluated for a specific normalised angular frequency Ω_0: $\mathbf{R}_0 = \boldsymbol{R}(z)|_{z=\mathrm{e}^{\mathrm{j}\Omega_0}}$;
- $\mathbf{R}_0$ is a constant matrix that describes a *narrowband* problem;
- $\mathbf{R}_0$ is Hermitian $\longrightarrow$ eigenvalue decomposition (EVD) $\mathbf{R}_0 = \mathbf{Q}_0 \boldsymbol{\Lambda}_0 \mathbf{Q}_0^{\mathrm{H}}$.

Narrowband EVD and Subspace Decomposition

- We assume an ordered EVD $\mathbf{R}_0 = \mathbf{Q}_0 \mathbf{\Lambda}_0 \mathbf{Q}_0^{\mathrm{H}}$, where $\mathbf{\Lambda}_0 = \mathrm{diag}\{\lambda_1, \ldots, \lambda_M\}$ with $\lambda_\ell \geq \lambda_{\ell+1}$, $\ell = 1, \ldots, (M-1)$;
- partitioning enables a subspace decomposition:

$$\mathbf{R}_0 = \left[\begin{array}{c:c} \mathbf{Q}_{\mathrm{s}} & \mathbf{Q}_{\mathrm{n}} \end{array}\right] \left[\begin{array}{c:c} \mathbf{\Lambda}_s + \sigma_v^2 \mathbf{I}_L & \\ \hdashline & \sigma_v^2 \mathbf{I}_{M-L} \end{array}\right] \left[\begin{array}{c} \mathbf{Q}_{\mathrm{s}}^{\mathrm{H}} \\ \hdashline \mathbf{Q}_{\mathrm{n}}^{\mathrm{H}} \end{array}\right]$$

- source enumeration: eigenvalues above noise floor = number of uncorrelated sources;
- $\mathbf{y}[n] = \mathbf{Q}_{\mathrm{n}}^{\mathrm{H}} \mathbf{x}[n] \in \mathbb{C}^{M-L}$ only contains noise;
- power in $\mathbf{y}[n]$: $\mathcal{E}\{\|\mathbf{y}[n]\|_2^2\} = (M-L)\sigma_v^2$ because of orthonormality of $\mathbf{Q}$.

Broadband EVD

- Space-time covariance $\mathbf{R}[\tau]$ or equivalently the CSD matrix $\boldsymbol{R}(z)$ are only diagonalised by the EVD for a specific value τ or z;
- for an analytic $\boldsymbol{R}(z)$ that is not derived from multiplexed data, there exists a parahermitian matrix EVD [14, 13]

$$\boldsymbol{R}(z) = \boldsymbol{Q}(z)\boldsymbol{\Lambda}(z)\boldsymbol{Q}^{\mathrm{P}}(z) \; ; \tag{3}$$

- $\boldsymbol{\Lambda}(z)$ is diagonal, parahermitian, analytic, and unique;
- eigenvectors in $\boldsymbol{Q}(z)$ are paraunitary, analytic, and unique up to an arbitrary allpass function;
- paraunitarity $\boldsymbol{Q}(z)\boldsymbol{Q}^{\mathrm{P}}(z) = \boldsymbol{Q}^{\mathrm{P}}(z)\boldsymbol{Q}(z) = \mathbf{I}$ implies losslessness;
- a number of algorithms can approximate (3) [8, 9, 10, 17, 15, 16].

Broadband Subspace Decomposition

University of Strathclyde Engineering

- The parahermitian matrix EVD $\boldsymbol{R}(z) = \boldsymbol{Q}(z)\boldsymbol{\Lambda}(z)\boldsymbol{Q}^{\mathrm{P}}(z)$ enables a broadband subspace decomposition:

$$\boldsymbol{R}(z) = \left[\begin{array}{c:c} \boldsymbol{Q}_{\mathrm{s}}(z) & \boldsymbol{Q}_{\mathrm{n}}(z) \end{array}\right] \left[\begin{array}{c:c} \begin{array}{c}\boldsymbol{\Lambda}_s(z) \\ +\sigma_v^2 \mathbf{I}_L\end{array} & \\ \hdashline & \sigma_v^2 \mathbf{I}_{M-L} \end{array}\right] \left[\begin{array}{c} \boldsymbol{Q}_{\mathrm{s}}^{\mathrm{P}}(z) \\ \hdashline \boldsymbol{Q}_{\mathrm{n}}^{\mathrm{P}}(z) \end{array}\right]$$

- $\mathbf{Q}[n] \circ\!\!-\!\!\bullet\, \boldsymbol{Q}(z)$ describes a lossless filter bank;
- data vector component in the noise-only subspace: $\mathbf{y}[n] = \mathbf{Q}_{\mathrm{n}}^{\mathrm{H}}[-n] * \mathbf{x}[n]$;
- again, it can be shown that ideally $\mathcal{E}\{\|\mathbf{y}[n]\|_2^2\} = (M-L)\sigma_v^2$.

'Syndrome' Idea

- We estimate $\boldsymbol{R}(z) \bullet\!\!-\!\!\circ \mathbf{R}[\tau]$ over a window of data, with $L < M$ stationary sources present;
- compute parahermitian matrix EVD, perform source enumeration, and determine the eigenvectors spanning the noise-only subspace, $\boldsymbol{Q}_{\mathrm{n}}(z)$;
- if an additional source $s_{L+1}[n]$ enters the scene, it will likely protrude into the noise-only subspace;
- we therefore monitor the syndrome vector

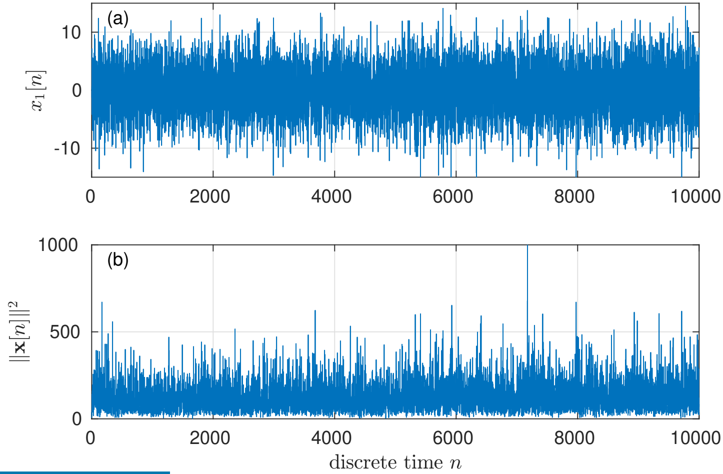
$$\mathbf{y}[n] = \mathbf{Q}_{\mathrm{n}}^{\mathrm{H}}[-n] * \mathbf{x}[n] \tag{4}$$

for a change in power, or for any structured / correlated components.

$$\mathbf{x}[n] \;\circ\!\xrightarrow[M]{\;/\;}\; \boxed{\mathbf{Q}_{\mathrm{n}}^{\mathrm{H}}[-n]} \;\xrightarrow[M-L]{\;/\;}\!\circ\; \mathbf{y}[n]$$

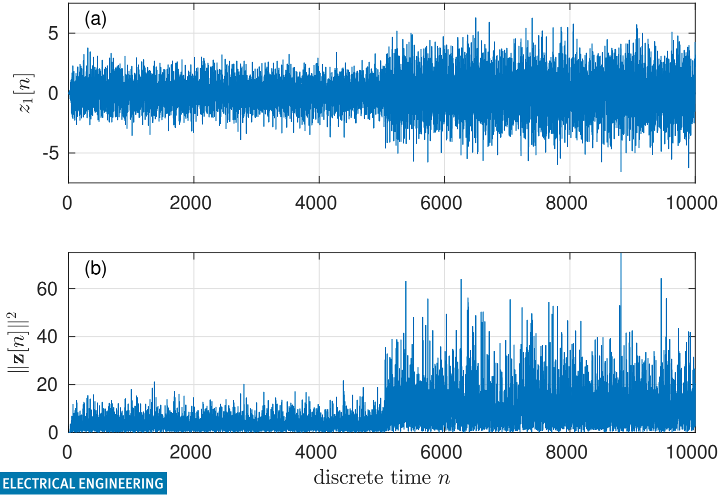
Intuitive Example I

- $M = 6$ sensors, $L = 3$ stationary sources; weak transient source at $n = 5000$;
- monitoring a sensor output $x_1[n]$:



Intuitive Example II

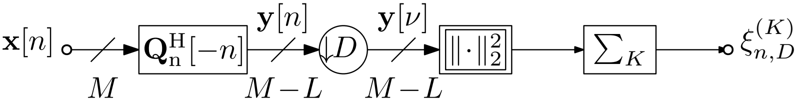
- $M = 6$ sensors, $L = 3$ stationary sources; weak transient source at $n = 5000$;
- monitoring a syndrome element $y_1[n]$:



Proposed Approach

University of Strathclyde Engineering

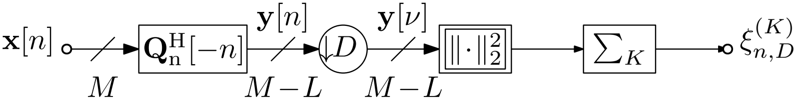
- We use the statistics and evaluated parahermitian matrix EVD of a previous time window, and utilise the broadband noise-only subspace spanned by the columns of $\boldsymbol{Q}_{\mathrm{n}}(z)$;
- being analytic, $\boldsymbol{Q}_{\mathrm{n}}(z)$ can typically be approximated well by low-order polyomials, and is relatively inexpensive to implement;



- because of the processing, elements of the syndrome vector $\mathbf{y}[n]$ are spatially and temporally correlated;
- decimation by D can break temporal correlation and further reduces complexity;
- we can average over consecutive syndrome vectors to increase discrimination;
- $\xi_{n,D}^{(K)}$ is generalised χ^2 distributed if temporal correlation is suppressed [11, 2].

Decimated Processor

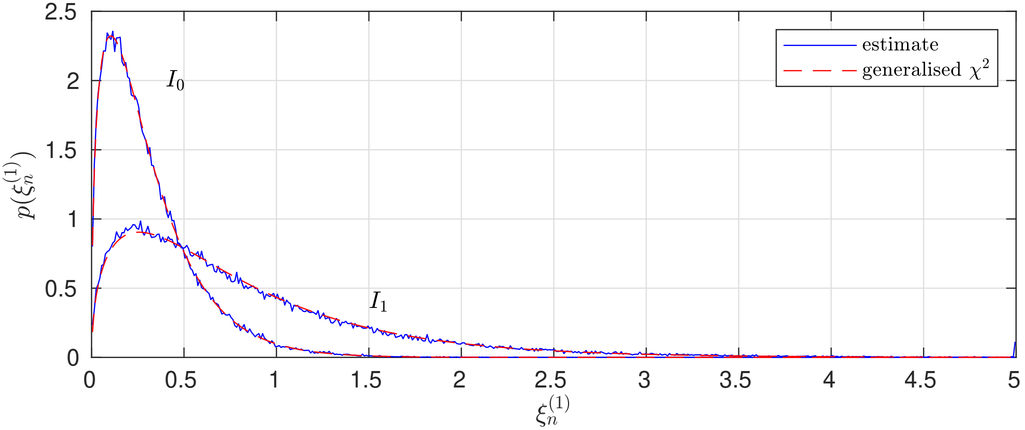
- The proposed subspace projection is followed by a decimation by D:



- cost advantage: a polyphase implementation integrates the decimation with the processor, reducing operations by a factor of D;
- temporal decorrelation: if the temporal correlation does not exceed D lags, the decimation will temporally decorrelate susequent snapshots of the syndrome vector $\mathbf{y}[\nu]$.

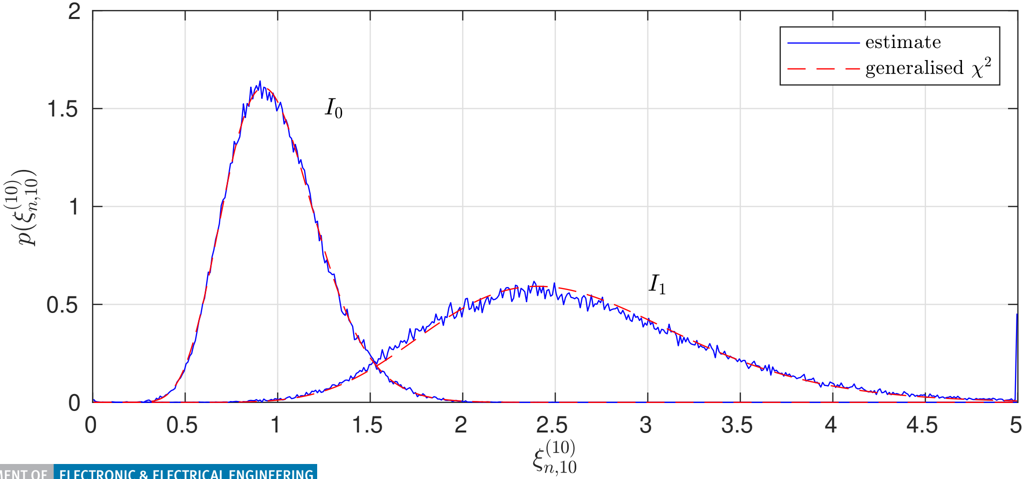
Results I — Statistics

- $M = 6$ sensors, $L = 2$ stationary sources, transfer functions determined by radio propagation model for dense urban environment (polynomial order $\approx$ 40);
- statistics of output for I_0: no transient versus I_1: transient present; $K = 1$;



Results I — Statistics

- $M = 6$ sensors, $L = 2$ stationary sources, transfer functions determined by radio propagation model for dense urban environment (polynomial order ≈ 40);
- statistics of output for I_0: no transient versus I_1: transient present; $K = 10$;



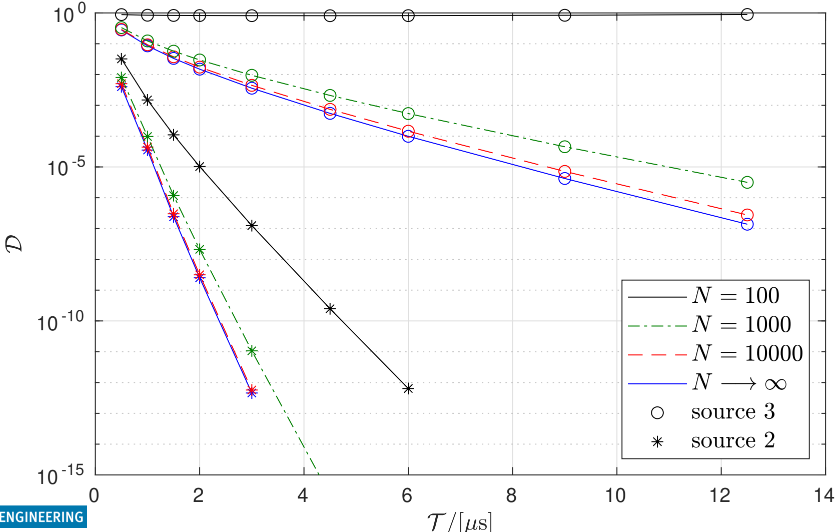
Results II — Sources and Propagation Environment

- Power of contributions for realistic channel scenario:

signal	power
source 1	0.0000 dB
source 2	-4.3494 dB
source 3	-13.2865 dB
noise	-16.2865 dB

Results III — Discrimination vs Decision Time

- Averaging increasingly separates the distributions for I_0 and I_1 — measured as discrimination $\mathcal{D}$: derived from the ROC [6];
- averaging also increases the time to compute $\xi_{n,D}^{(T)} \longrightarrow$ decision time $\mathcal{T}$ (for a 20MHz channel);
- N here is the window over which the space-time covariance is estimated [3, 4, 5]



Summary

- We have proposed a broadband subspace approach to detect the presence of weak transient signals;
- this is based on second order statistics of sensor array data — the space-time covariance matrix — and a polynomial matrix EVD;
- this covariance matrix and its decomposition can be computed off-line; for low-cost implementations, see e.g. [1, 7]
- a subspace decomposition for the noise-only subspace determines a syndrome vector;
- in the absence of a transient signal, this syndrome only contains noise;
- a transient signal is likely to protrude into the noise-only subspace, and a change in energy can be detected even if the signal is weak;
- discrimination can be traded off against decision time;
- further work: (i) impact of time-varying channels, and (ii) forensic investigation of the transient source once detected.

References I

[1] F. K. Coutts, I. K. Proudler, and S. Weiss.
Efficient implementation of iterative polynomial matrix evd algorithms exploiting structural redundancy and parallelisation.
IEEE Transactions on Circuits and Systems I: Regular Papers, 66(12):4753–4766, Dec. 2019.

[2] A. Das and W. S. Geisler.
A method to integrate and classify normal distributions.
arXiv e-prints, page arXiv:2012.14331, Dec. 2020.

[3] C. Delaosa, F. K. Coutts, J. Pestana, and S. Weiss.
Impact of space-time covariance estimation errors on a parahermitian matrix EVD.
In *10th IEEE Workshop on Sensor Array and Multichannel Signal Processing*, pages 1–5, July 2018.

[4] C. Delaosa, J. Pestana, N. J. Goddard, S. Somasundaram, and S. Weiss.
Sample space-time covariance matrix estimation.
In *ICASSP 2019 - 2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pages 8033–8037, May 2019.

[5] C. Delaosa, J. Pestana, N. J. Goddard, S. D. Somasundaram, and S. Weiss.
Support estimation of a sample space-time covariance matrix.
In *Sensor Signal Processing for Defence*, Brighton, UK, March 2019.

[6] J. A. Hanley and B. J. McNeil.
The Meaning and Use of the Area under a Receiver Operating Characteristic (ROC) Curve.
Radiology, 143:26–36, 1982.

[7] F. A. Khattak, S. Weiss, and I. K. Proudler.
Fast givens rotation approach to second order sequential best rotation algorithms.
In *International Conference in Sensor Signal Processing for Defence*, pages 40–44, Edinburgh, Scotland, September 2021.

[8] J. G. McWhirter, P. D. Baxter, T. Cooper, S. Redif, and J. Foster.
An EVD Algorithm for Para-Hermitian Polynomial Matrices.
IEEE Transactions on Signal Processing, 55(5):2158–2169, May 2007.

References II

[9] S. Redif, J. McWhirter, and S. Weiss.
Design of FIR paraunitary filter banks for subband coding using a polynomial eigenvalue decomposition.
IEEE Transactions on Signal Processing, 59(11):5253–5264, November 2011.

[10] S. Redif, S. Weiss, and J. McWhirter.
Sequential matrix diagonalization algorithms for polynomial EVD of parahermitian matrices.
IEEE Transactions on Signal Processing, 63(1):81–89, January 2015.

[11] H. Solomon and M. A. Stephens.
Distribution of a sum of weighted chi-square variables.
Journal of the American Statistical Association, 72(360):881–885, 1977.

[12] S. Weiss, C. Delaosa, J. Matthews, I. Proudler, and B. Jackson.
Detection of weak transient signals using a broadband subspace approach.
In *International Conference on Sensor Signal Processing for Defence*, pages 65–69, Edinburgh, Scotland, Sept. 2021.

[13] S. Weiss, J. Pestana, I. Proudler, and F. Coutts.
Corrections to "on the existence and uniqueness of the eigenvalue decomposition of a parahermitian matrix".
IEEE Transactions on Signal Processing, 66(23):6325–6327, Dec 2018.

[14] S. Weiss, J. Pestana, and I. K. Proudler.
On the existence and uniqueness of the eigenvalue decomposition of a parahermitian matrix.
IEEE Transactions on Signal Processing, 66(10):2659–2672, May 2018.

[15] S. Weiss, I. Proudler, F. Coutts, and J. Deeks.
Extraction of analytic eigenvectors from a parahermitian matrix.
In *International Conference on Sensor Signal Processing or Defence*, E, 2020.

References III

[16] S. Weiss, I. K. Proudler, and F. K. Coutts.
Eigenvalue decomposition of a parahermitian matrix: Extraction of analytic eigenvalues.
IEEE Transactions on Signal Processing, 69:722–737, 2021.

[17] S. Weiss, I. K. Proudler, F. K. Coutts, and J. Pestana.
Iterative approximation of analytic eigenvalues of a parahermitian matrix EVD.
In *IEEE International Conference on Acoustics, Speech and Signal Processing*, Brighton, UK, May 2019.