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Modelling bi-static uncertainties in sequential Monte Carlo with the GLMB model

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Project BLUE

Motivation

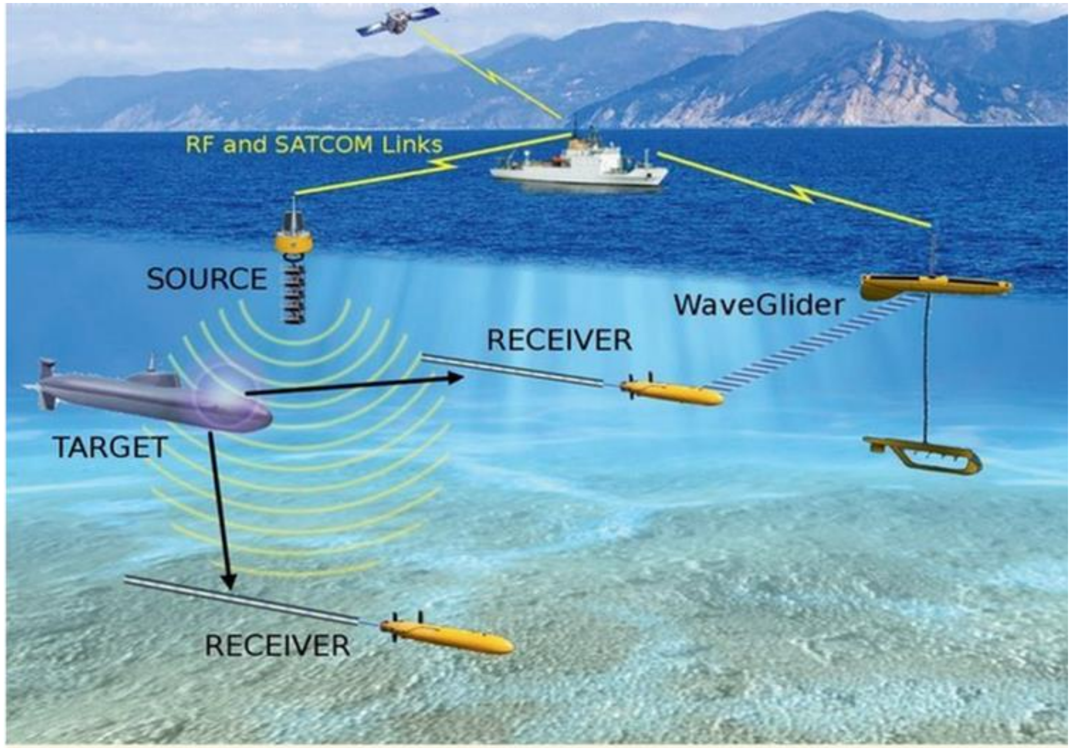

Figure. CMRE Multi-static sonar network. (see e.g., De Magistris et .al "Selective Information Transmission using Convolutional Neural Networks for Cooperative

Underwater Surveillance," Fusion'20).

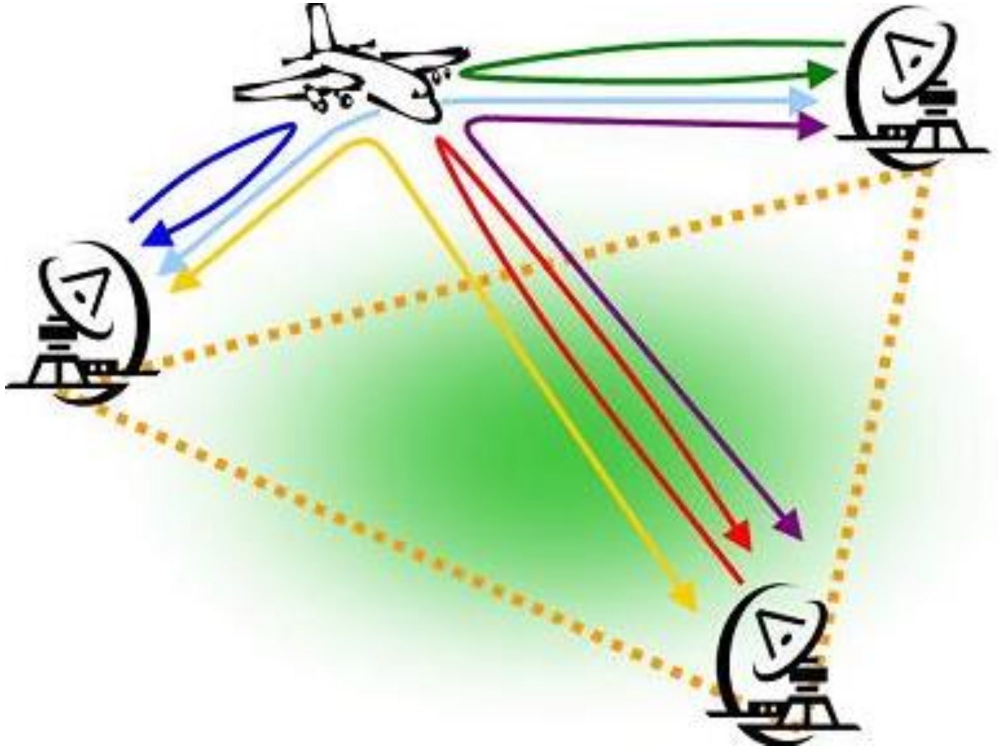

Figure. Example multi-static radar configuration(https://en.wikipedia.org/wiki/Multistatic_radar).

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Contents

- **Bi-static detections and modelling bistatic measurement uncertainties**
- **Multi-object tracking using the proposed likelihood model**
- **Example**
- **Conclusion**

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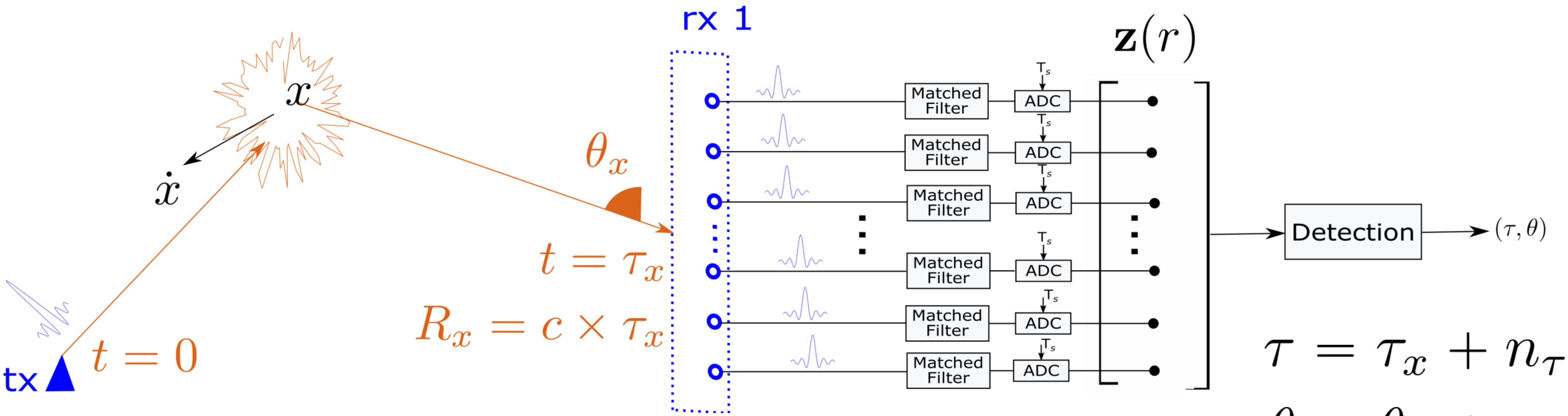
Modelling bi-static measurement uncertainties (1/5)



$$\tau = \tau_x + n_\tau$$

$$\theta = \theta_x + n_\theta$$

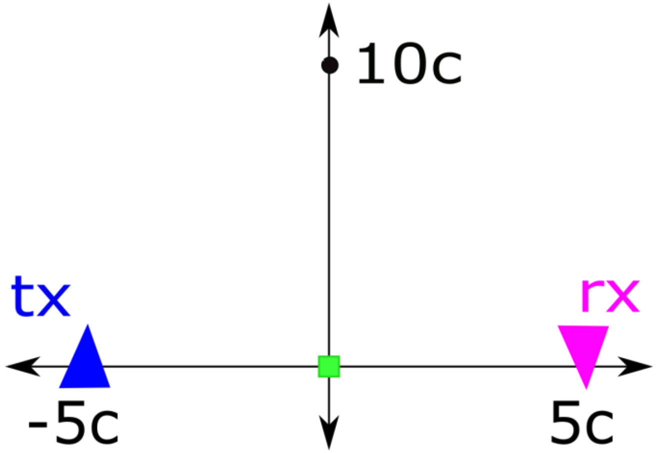
Figure. Bi-static detections and time-of-flight (ToF) and angle-of-arrival measurements.

Modelling assumptions:

1) Estimation errors in measuring time-of-flight τ and angle-of-arrival θ are zero mean Gaussian distributed.
2) False detections are uniformly distributed over $[\tau_{min}, \tau_{max}] \times [0, \pi]$ with a Poisson rate λ.

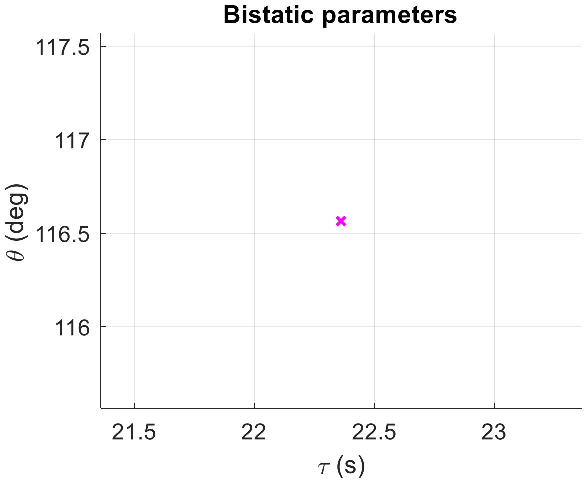
Modelling bi-static measurement uncertainties(2/5)

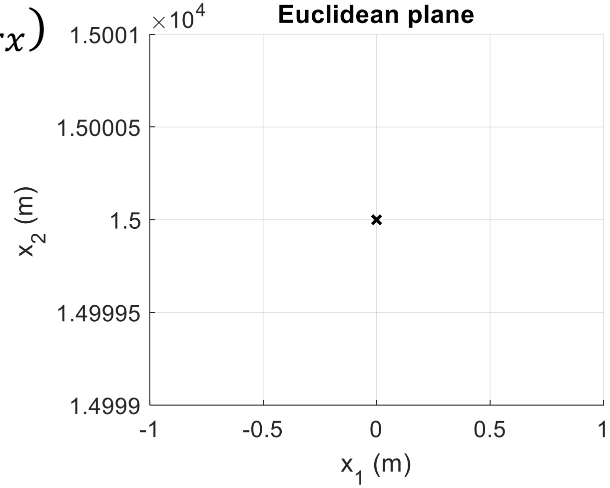
Figure. Bi-static measurements of a reflector at (0,10c) where c is the propagation speed.

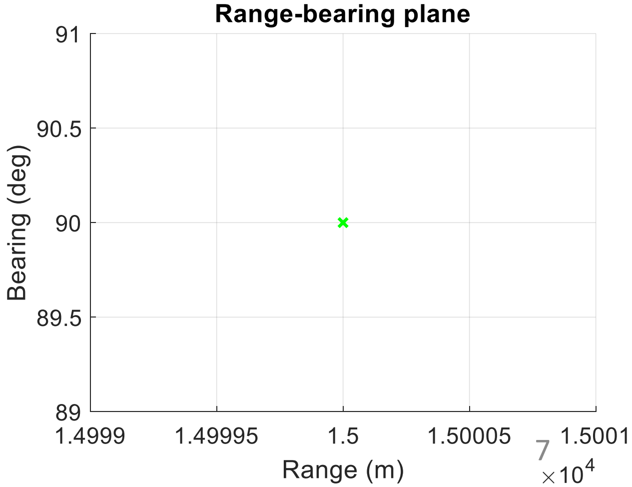


$$(\tau, \theta) \xrightarrow{B(\tau, \theta, x_{tx}, x_{rx})} (x_1, x_2) \longrightarrow (\tilde{R}, \tilde{\theta})$$

$$(x_1, x_2) \xrightarrow{B^{-1}(x_1, x_2, x_{tx}, x_{rx})} (\tau, \theta) \qquad (\tilde{R}, \tilde{\theta}) \longrightarrow (x_1, x_2)$$

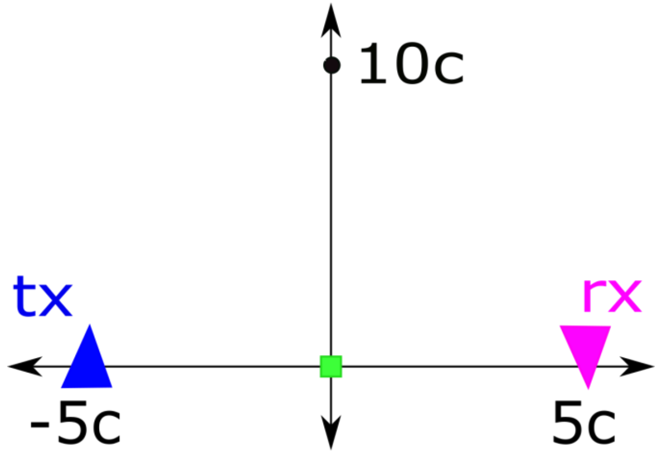




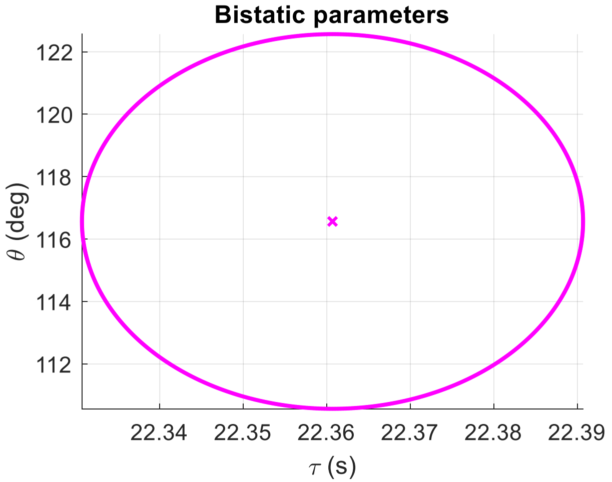


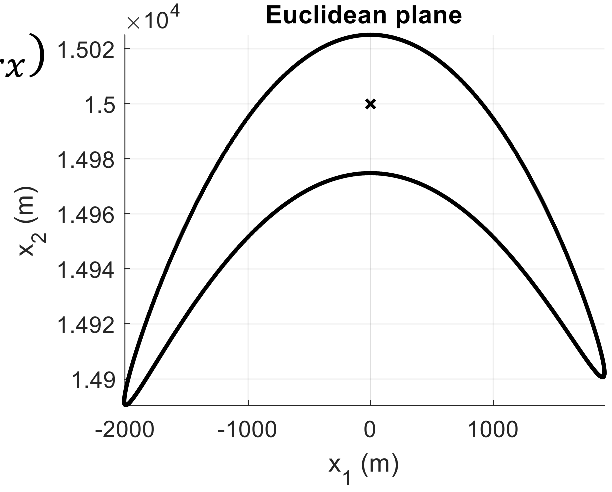
Modelling bi-static measurement uncertainties (3/5)

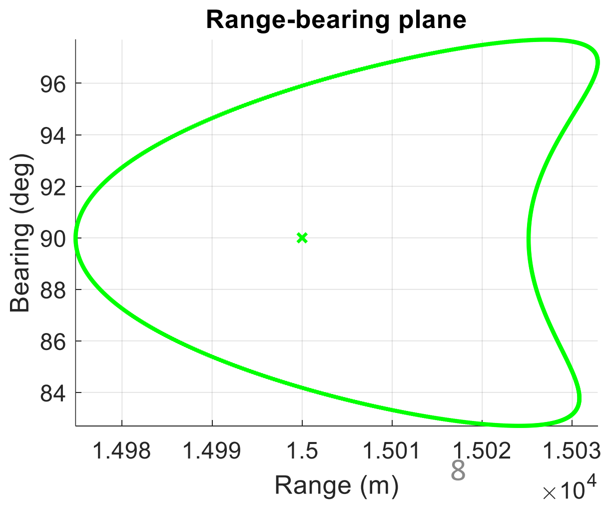
Figure. Bi-static measurements of a reflector at (0,10c) where c is the propagation speed.



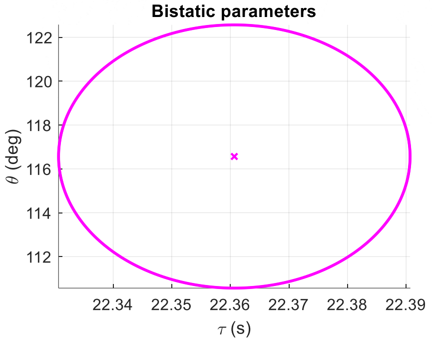
$$(\tau,\theta) \underset{B^{-1}(x_1,x_2,x_{tx},x_{rx})}{\overset{B(\tau,\theta,x_{tx},x_{rx})}{\rightleftarrows}} (x_1,x_2) \rightleftarrows (\tilde{R},\tilde{\theta})$$

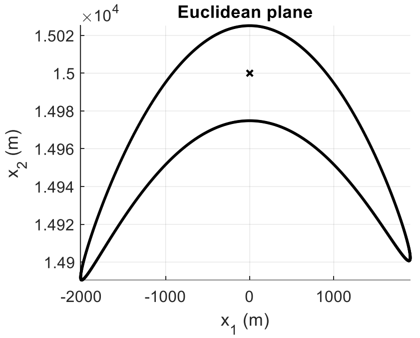


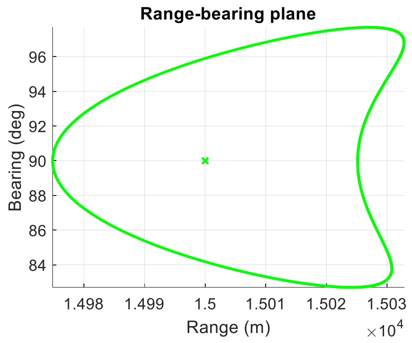




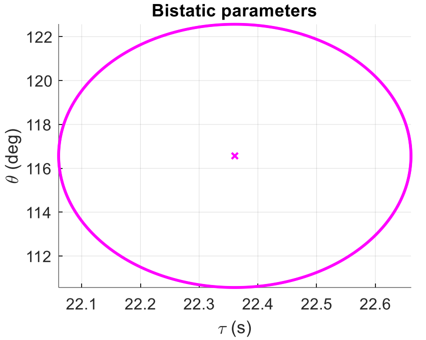
(τ, θ) $\rightleftarrows$ (x_1, x_2) $\rightleftarrows$ $(\tilde{R}, \tilde{\theta})$

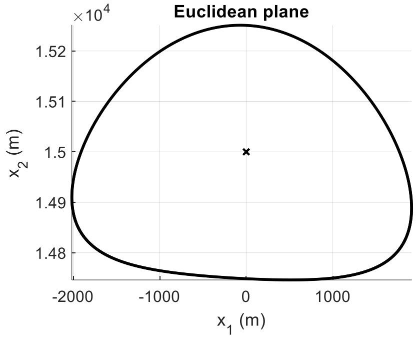


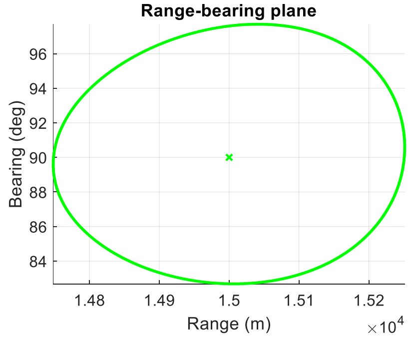




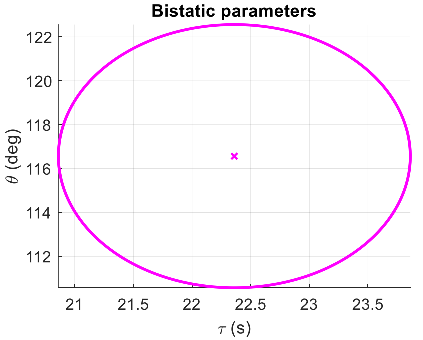
The top figures are for $\sigma_\tau = 0.01s, \sigma_\theta = 2^\circ$ and c $= 1500\ m/s$.

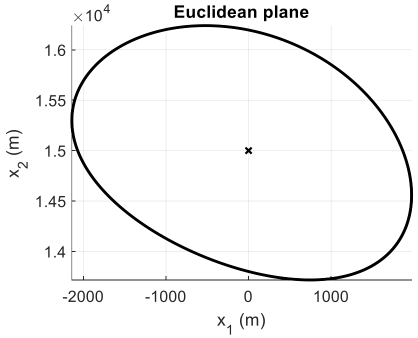


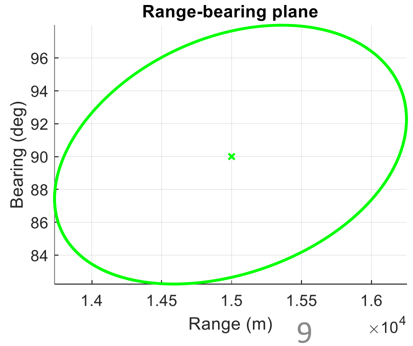




The middle figures are for $\sigma_\tau = 0.1s, \sigma_\theta = 2^\circ$ and c $= 1500\ m/s$.







The bottom figures are for $\sigma_\tau = 0.5s, \sigma_\theta = 2^\circ$ andc $= 1500\ m/s$.

Modelling bi-static measurement uncertainties (5/5)

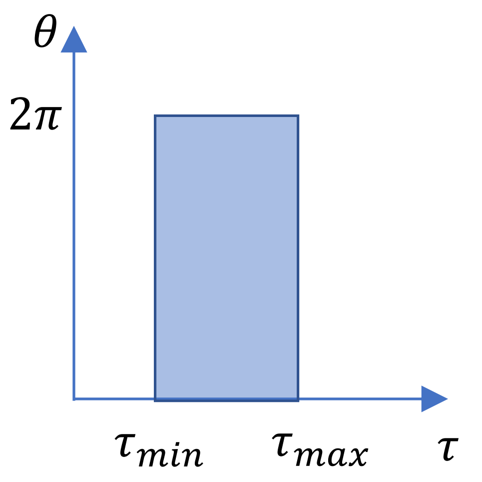
- Conversion of bi-static parameters to range-bearing surrogates poses additional modelling issues in specifying
  - the likelihood of Doppler measurements (when available),
  - the false alarm (clutter) distribution,
  - the signal-to-noise ratio and the probability of detection.

- We propose to use
  - the endogenous bi-static likelihood,

    $$l(z=(\tau,\theta)|x,x_{rx},x_{tx}) = \mathcal{N}(z; B(x;x_{tx},x_{rx}),\Sigma),$$

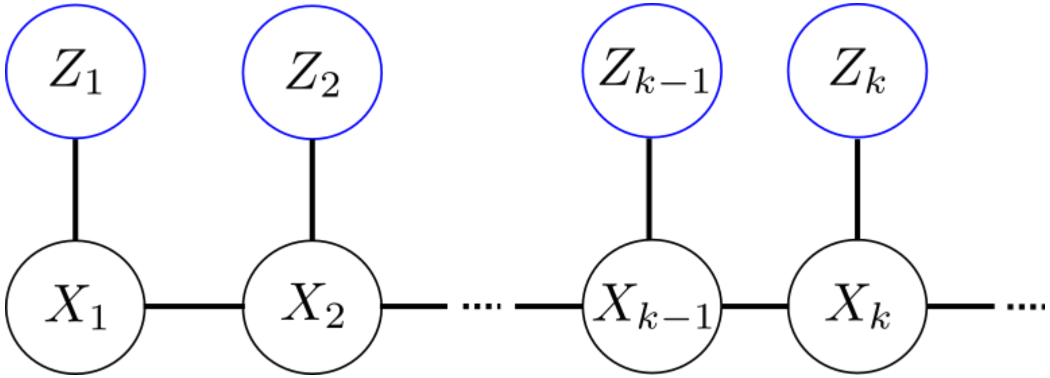
  - The clutter density; $\kappa(z) = Pois(\lambda, \mathcal{U}_{[\tau_{min},\tau_{max}]\times[0,2\pi)})$,
  - Probability of detection;

$$P_D(x) = \begin{cases} 0, & \tau(x;x_{tx},x_{rx}) \leqslant \tau_{min}, \\ pd(x;x_{tx},x_{rx}), & \tau_{min} < \tau(x;x_{tx},x_{rx}) \leqslant \tau_{max}. \end{cases}$$



Multi-object tracking with the proposed likelihood

- Tracking aims to estimate the number of objects and their trajectories based on sensor measurements
- Often solved as Bayesian filtering over a state space model



$$p(X_1, X_2, \ldots, X_k, Z_1, Z_2, \ldots, Z_k) = p(X_1) \prod_{l=2}^{k} p(X_l|X_{l-1}) \prod_{l=1}^{k} g(Z_l|X_l),$$

$$g(Z_l = z_{1:N}|X_l = x_{1:M}) = \sum_{\pi \in \Pi_{N,M}} \underline{\kappa(z_{(i,\varnothing)\in\pi})} \prod_{(i,j)\in\pi} \underline{P_D(x_j) l(z_i = (\tau_i, \theta_i)|x_j, x_{rx}, x_{tx})} \prod_{(\varnothing,j)\in\pi} (1 - \underline{P_D(x_j)}).$$

Multi-object tracking with the proposed likelihood: The GLMB state space model

- In the (generalised) labelled multi-Bernoulli model, $X_k = \{(x_1, l_1), \ldots, (x_{M_k}, l_{M_k})\}$ with $\{l_1, \ldots, l_{M_k}\} \sim p(L)$, $\quad C \sim p\left(C \middle| \{l_1, \ldots, l_{M_k}\}\right)$, $x_i \sim p(x \mid l_i, C)$ for $i = 1, \ldots, M_k$:

$$p(X_k|Z_1, Z_2, \ldots, Z_k) = \frac{g(Z_k|X_k)}{p(Z_k|Z_{1:k-1})} \int p(X_k|X_{k-1}) p(X_{k-1}|Z_1, Z_2, \ldots, Z_{k-1}) \mu(\mathrm{d}X_{k-1}).$$

- Detailed formulae for the Bayesian filtering update above for the GLMB model.*
- The growing number of Bernoulli components are capped using Murty's algorithm.**
- The proposed multi-object likelihood $g(Z|X)$ is easily used with sequential Monte Carlo.

* B-N Vo, B-T Vo, and D Phung, "Labeled random finite sets and the Bayes multi-target tracking filter," IEEE TSP, 2014.
**K. G. Murty, "An algorithm for ranking all the assignments in order of increasing cost," Oper. Res., vol. 16, no. 3, pp. 682–687, 1968.

Example

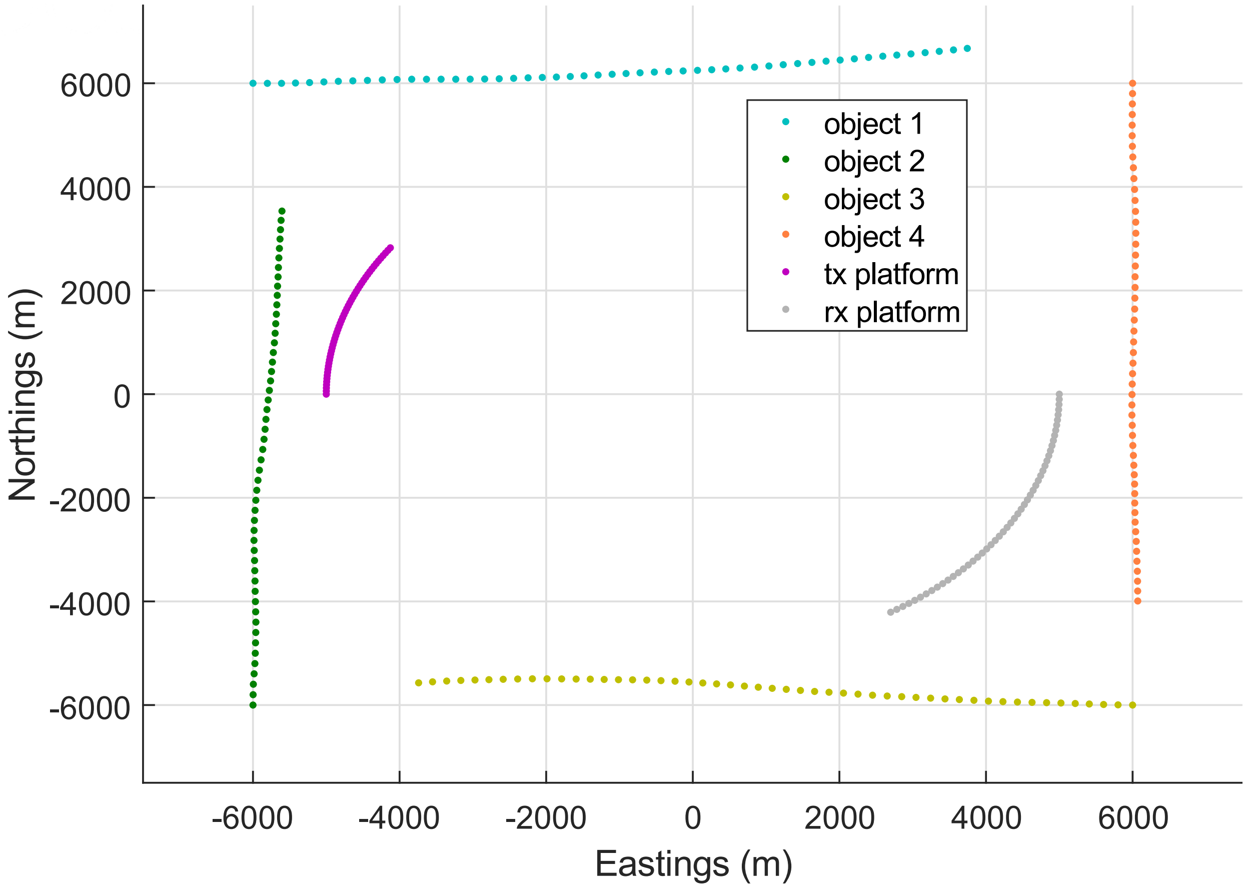


Figure. Example bi-static sensing scenario.

- Mobile tx and tx plaforms, underwater acoustic wave propagation (c=1490m/s), scan period 20s, 50 scans.

Example

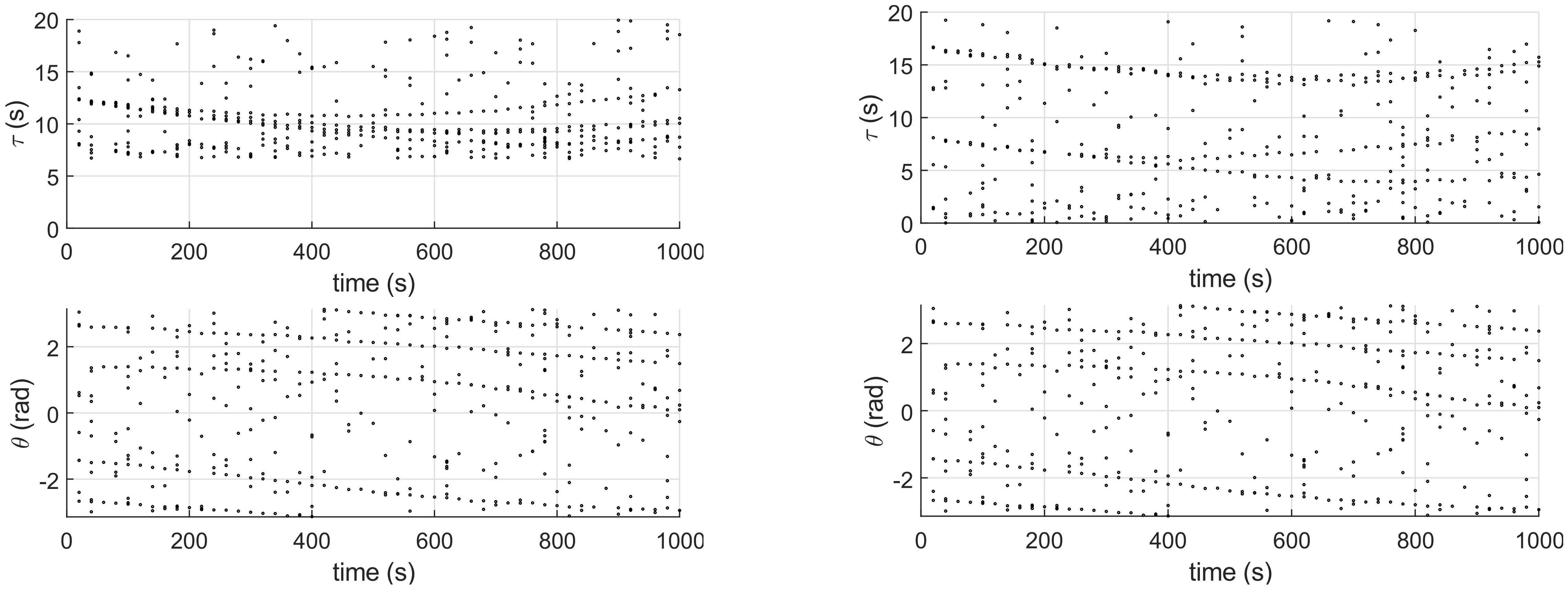
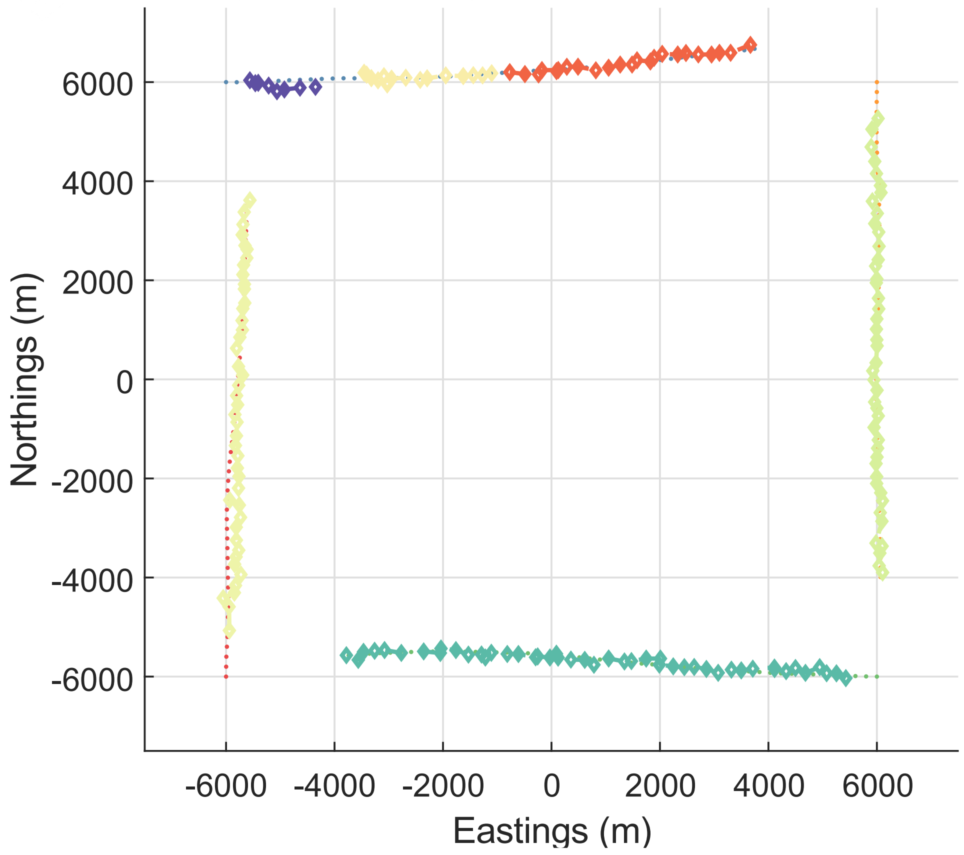


Figure. (left) Bi-static measurements. (right) Monostatic surrogate measurements.

- Bi-static likelihood parameters are $\sigma_\tau = 0.1s, \sigma_\theta = 1^\circ$, false alarm (clutter) rate $\lambda = 5$, pd(x) = 0.95 when x is not on the baseline and inside the field-of-view.

Example



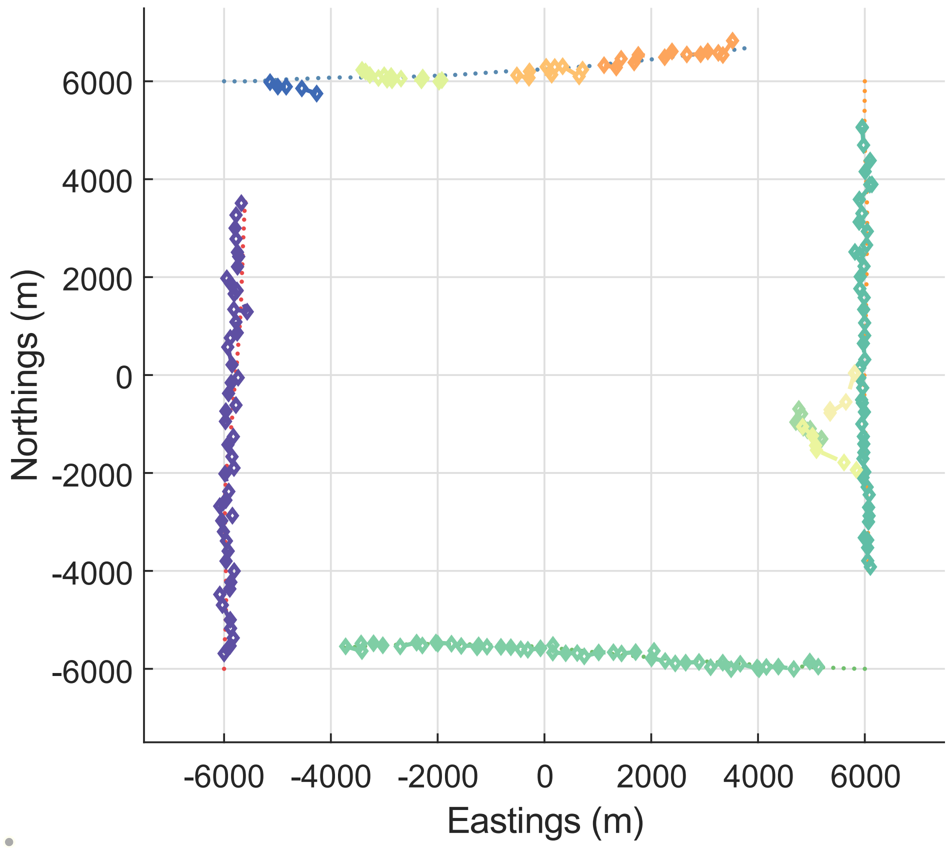


Figure. (left) Track estimates from the proposed MO likelihood model in sequential MC GLMB track filtering. (right) Track estimates with the surrogate likelihood model in the same filter.

Example

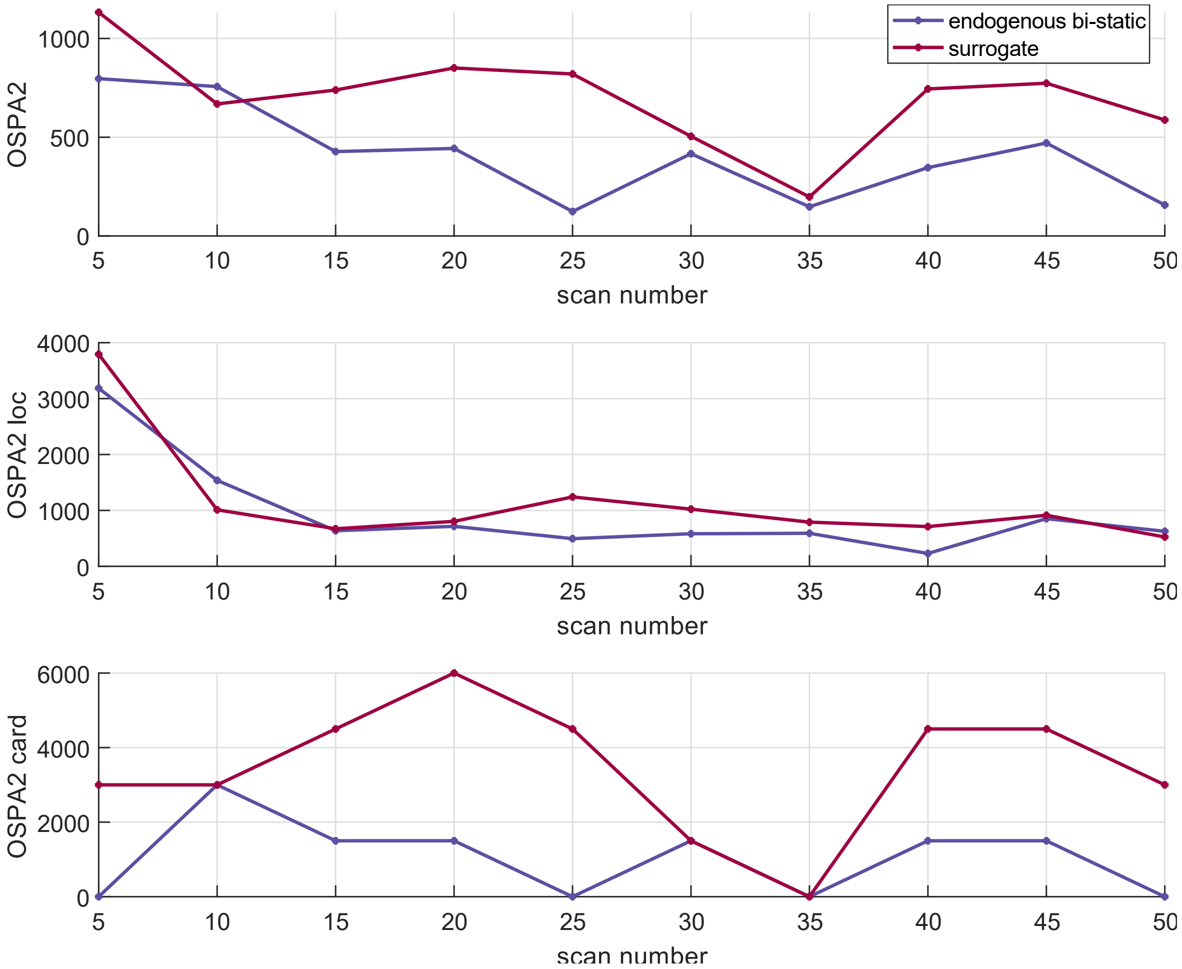


Figure. Track estimation errors quantified using OSPA-squared* of order p=1 and with cut-off c=1500m.

* M. Beard, B. T. Vo, and B. Vo, “A solution for large-scale multi-object tracking,” IEEE TSP, 2020.

Conclusions

- **Introduced a multi-object likelihood model for bi-static measurements.**
- **Demonstrated improved accuracy over widely used surrogates.**
- **Future work includes incorporation of Doppler measurements and probability of detection using Cassini ovals.**

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Thank you for your attention,
Please feel free to ask questions!