Poisson multi-Bernoulli mixture filters for multi-target tracking using sonars

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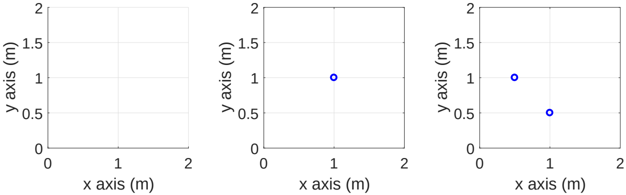
Challenges in Bayesian multi-target tracking using sonars

- In this talk, we will address the following challenges for Gaussian multi-target tracking (MTT) using sonars.
- Bayesian MTT filter:
  - Poisson multi-Bernoulli mixture (PMBM) filter [1, 2].
    - It computes the filtering posterior density.
    - State-of-the-art multiple hypothesis (MHT) tracking algorithm [3].
- Gaussian updates with nonlinear/non-Gaussian measurement models:
  - Suitable modelling of direction-of-arrival measurements [4].
  - Iterated posterior linearisation filter (IPLF) [5, 6].
  - Accurate approximation of normalising constants [7].
- Specific sonar measurement model.

Multiple target Bayesian filtering

- In multiple target tracking, we do not know the number of targets
  - Targets can appear and disappear.
- It is suitable to represent the state as a set $X_k = \left\{x_k^1, ..., x_k^{n_k}\right\}$



- We use random finite sets (RFS) to deal with uncertainty on sets of objects [8].

Multi-target filtering recursion

- X_k evolves with a transition density $f_{k|k-1}(\cdot|X_{k-1})$.
- It is observed at time step k by measurements Z_k with conditional density $f(\cdot|X_k)$.
- All information about X_k is included in the posterior

$$
\begin{aligned}
f_{k|k-1}(X_k) &= \int f_{k|k-1}(X_k|X_{k-1})\, f_{k-1|k-1}(X_{k-1})\,\delta X_{k-1} \\
f_{k|k}(X_k) &= \frac{f(Z_k|X_k)\, f_{k|k-1}(X_k)}{\int f(Z_k|Y)\, f_{k|k-1}(Y)\,\delta Y}
\end{aligned}
$$

Standard point target model (I)

- Standard measurement model
  - For a given multi-target state X_k at time k, each target state $x \in X_k$ is either detected with probability $p^D(x)$ and generates one measurement with density $l(\cdot|x)$, or missed with probability $1 - p^D(x)$.
  - The measurement Z_k is the union of the target-generated measurements and Poisson clutter with intensity $\lambda^C(\cdot)$.

Standard point target model (II)

- Standard dynamic model
  - Given the current multitarget state X_k, each target $x \in X_k$ survives with probability $p^S(x)$ and moves to a new state with a transition density $g(\cdot\,|x)$, or dies with probability $1 - p^S(x)$.
    - For example, $g(\cdot\,|x)$ can model nearly constant velocity model, a random walk or an Ornstein Uhlenbeck process.
  - Target birth model is a Poisson RFS with intensity $\lambda^B(\cdot)$.

Poisson multi-Bernoulli mixture (PMBM) filter

- The posterior and the predicted density are PMBMs [1, 2].

$$f_{k'|k}\left(X_{k'}\right) = \sum_{Y \uplus W = X_{k'}} f^{p}_{k'|k}\left(Y\right) f^{mbm}_{k'|k}\left(W\right)$$

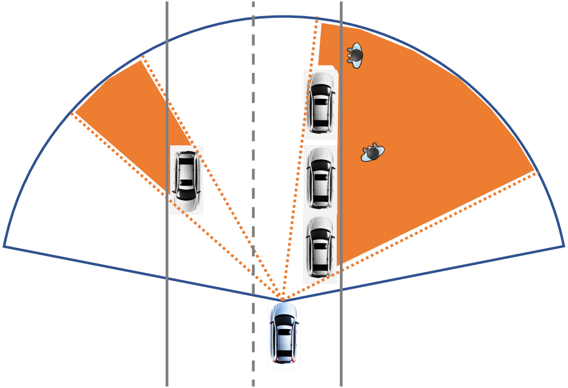
$$f^{p}_{k'|k}\left(X_{k'}\right) = e^{-\int \lambda_{k'|k}(x)dx} \left[\lambda_{k'|k}\left(\cdot\right)\right]^{X_{k'}}$$

$$f^{mbm}_{k'|k}\left(X_{k'}\right) = \sum_{a \in \mathcal{A}_{k'|k}} w^{a}_{k'|k} \underbrace{\sum_{\uplus_{l=1}^{n_{k'|k}} X^{l} = X_{k'}} \prod_{i=1}^{n_{k'|k}} f^{i,a^i}_{k'|k}\left(X^i\right)}_{\text{Multi}-\text{Bernoulli}} .$$

where $k' \in \{k, k+1\}$ and $f^{i,a^i}_{k'|k}\left(\cdot\right)$ is a Bernoulli density (existence $r^{i,a^i}_{k'|k}$ and single target density $p^{i,a^i}_{k'|k}\left(\cdot\right)$).

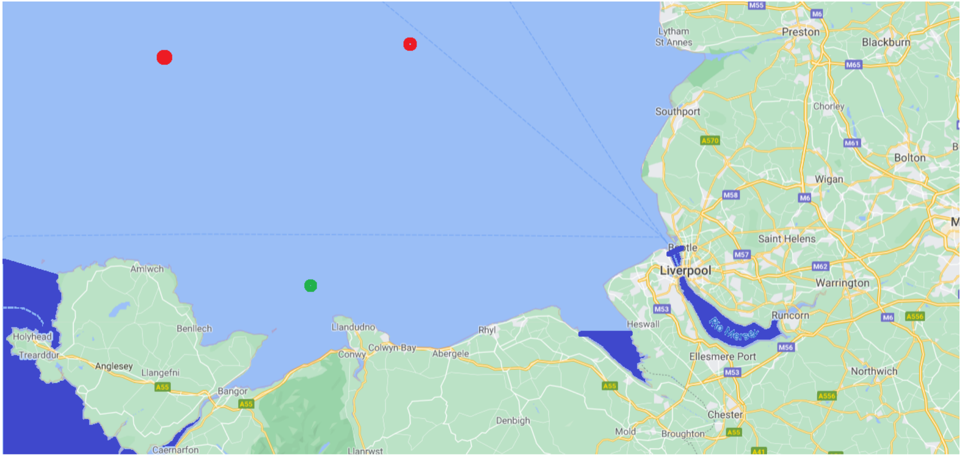
- Union of an independent Poisson RFS $f^{p}_{k'|k}\left(\cdot\right)$ and a multi-Bernoulli mixture RFS $f^{mbm}_{k'|k}\left(\cdot\right)$.
  - Poisson RFS includes information on existing targets that have never been detected.
    - Useful information in autonomous vehicles, sonar, and search-and-track sensor management [9].
  - Multi-Bernoulli mixture (MBM) includes information on existing targets that have been detected at some point.

Possibility of undetected objects in occluded areas



This figure was obtained from the YouTube/edX course: L. Svensson, K. Granström: “Multiple Object Tracking”, https://www.youtube.com/channel/UCa2-fpj6AV8T6JK1uTRuFpw

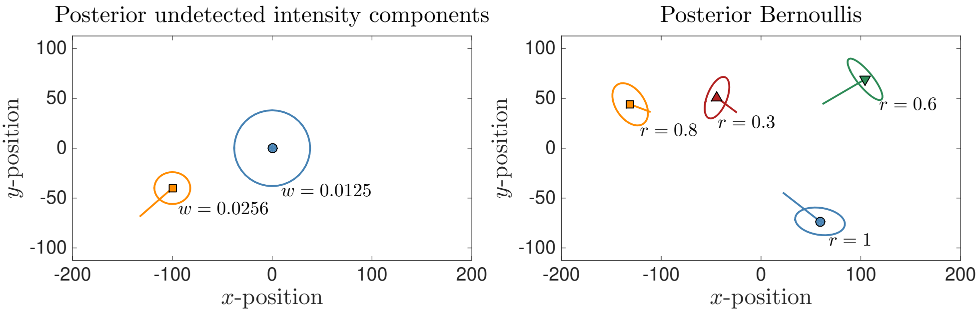
Possibility of undetected objects in occluded areas (sonar)



Interpretation MBM

- Each measurement at each time step gives rise to a new potentially detected target (a Bernoulli RFS)
    - A new measurement can be the first detection of a target,
    - It can also correspond to another previously detected target or clutter, in which case there is no new target.
- For each Bernoulli component, there are single target association history hypotheses (local hypotheses), which are data-to-data.
- A global hypothesis is represented by one of the multi-Bernoulli RFSs in the mixture.

Example of posterior visualisation

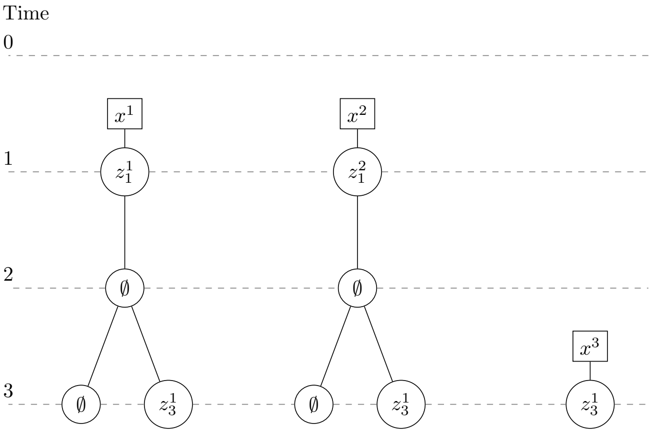


Figures obtained from https://www.youtube.com/channel/UCa2-fpj6AV8T6JK1uTRuFpw

Illustrative example

- Three time steps, measurement sets:
  - $Z_1 = \{z_1^1, z_1^2\}$.
  - $Z_2 = \emptyset$.
  - $Z_3 = \{z_3^1\}$.
- At time $k = 0$, the prior is a Poisson RFS.

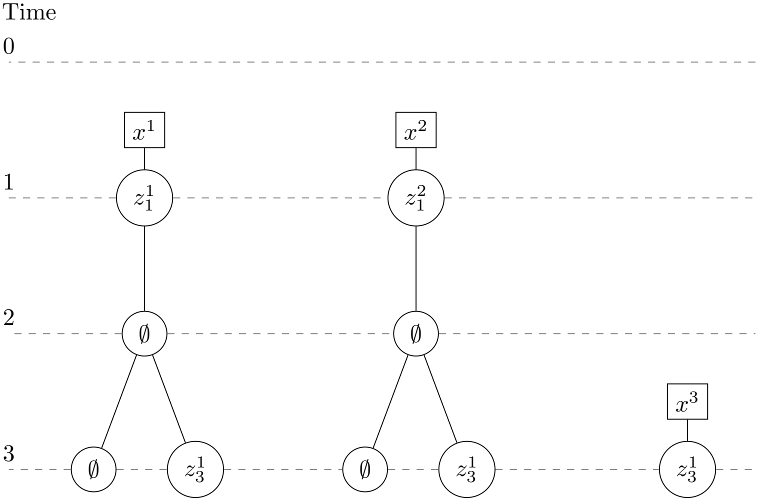
Illustrative example: local hypotheses



For each leaf node, we have $r_{k'|k}^{i,a^i}$ and $p_{k'|k}^{i,a^i}(\cdot)$

PMBM global hypotheses

Total number of MBs (global hypotheses): 3



Look-up table:

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

For simplicity: local hypotheses indexed from left to right.

Each Bernoulli is not represented in each MB (global hypothesis)

PMBM update

- There are four steps
  - Update the PPP.
  - Update single target hypotheses.
    - Misdetection.
    - Update with each received measurement.
  - Create new Bernoulli components.
  - Construct new global hypotheses.

PMBM single-target update (I)

- In a Gaussian implementation, each Bernoulli single-target density is of the form

$$p_{k'|k}^{i,a^i}(x) = \mathcal{N}\left(x; \overline{x}_{k'|k}^{i,a^i}, P_{k'|k}^{i,a^i}\right),$$

- The single target update

$$q_{k'|k}^{i,a^i}(x) = \frac{l(z|x)\,p^D(x)\,p_{k'|k}^{i,a^i}(x)}{l(z)}$$

where the normalising constant is

$$l(z) = \int l(z|x)\,p^D(x)\,p_{k'|k}^{i,a^i}(x)\,dx$$

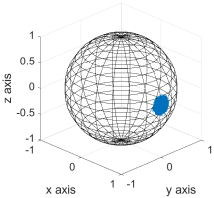
- In sonar measurement modelling, $l(z|x)$ may be non-linear/non-Gaussian.

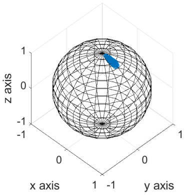
PMBM single-target update (II)

- Standard non-linear Kalman filters linearise measurement function w.r.t. the prior.
  - Analytical linearisation: Extended Kalman filter [10].
  - Statistical linear regression: unscented Kalman filter, cubature Kalman filter [10, 11, 12]
- These approaches have difficulties with sufficiently high non-linearities or high prior uncertainty [13].
- We seek an optimal linearisation using iterated statistical linear regressions w.r.t. the current posterior approximation:
  - Iterated posterior linearisation filter [5, 6].
  - We need $\mathrm{E}\left[z|x\right]$ and $\mathrm{C}\left[z|x\right]$.
  - We can deal with direction-of-arrival data from first principles, e.g., via von-Mises Fisher (VMF) distributions [4, 14].

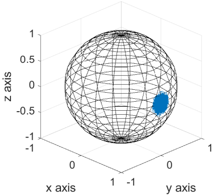
Gaussian versus VMF direction-of-arrival measurements

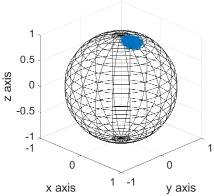
- With additive Gaussian noise





- With VMF angular noise

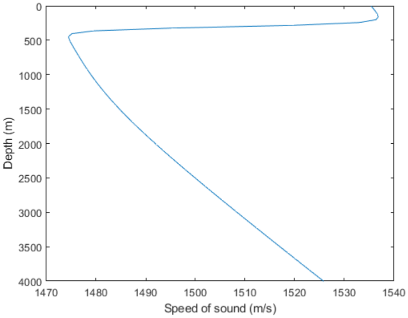


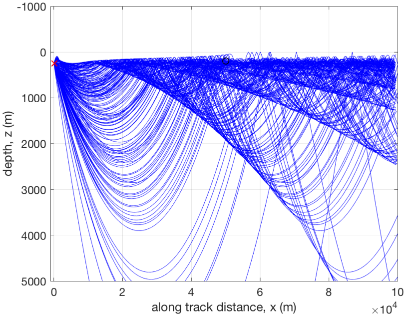


Sonar scenario

- Two own-ships near the surface of the water, one of which transmits an active sonar pulse
- Pulse bounces off the targets and is received by both receivers after a time delay.
  - Round trip distance.
  - Varying speed of sound in the water.
  - Multi-sensor data fusion.
- Returns give information on the direction of arrival (subject to refraction) and time delay.

- Speed of sound depending on depth according to the Leroy-Robinson-Goldsmith equation
  - Path of the sound wave is bent according to Snell's law





Sonar scenario (III)

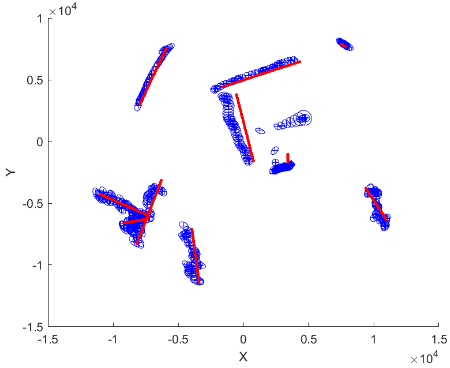
- In our model, we suppose that the received time delay is the delay we would expect for the round trip distance with a nominal constant sound speed, plus a bias for each target and measurement noise.
- Target state with bias

$$x_t = \left(p_{x,t}, \dot{p}_{x,t}, p_{y,t}, \dot{p}_{y,t}, p_{z,t}, \dot{p}_{z,t}, \{\phi_t^{\mathrm{bias},i}, \theta_t^{\mathrm{bias},i}, \Delta t_t^{\mathrm{bias},i}\}_{i=1}^{N_R} \right)^T$$

- Direction-of-arrival measurements are von-Mises Fisher distributed.
- We are simulating the bias from a model that involves physics, but using a tracker that considers it to be a statistical bias.
- Using the bias model explicitly would be possible, but would require knowledge of the environment.

Sonar scenario (IV)

- Results with 10 targets in x y plane.
- 10 clutter detections per scan.



Conclusions

- The Poisson multi-Bernoulli mixture filter is a fully Bayesian MHT algorithm.
  - It computes the filtering posterior density.
  - Information on undetected targets.
  - Probabilistic target existence in each hypothesis.
  - Single and multi-frame assignment implementations are possible [15].
- A Gaussian implementation with highly nonlinear/non-Gaussian measurements can be achieved via the iterated posterior linearisation filter.
- Particle filter implementations are also possible [16, 17].
  - Efficient algorithms for out-of-sequence data [18].
- We have provided some results with a sonar model.

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