Introduction to Spatial Filtering / Array Processing

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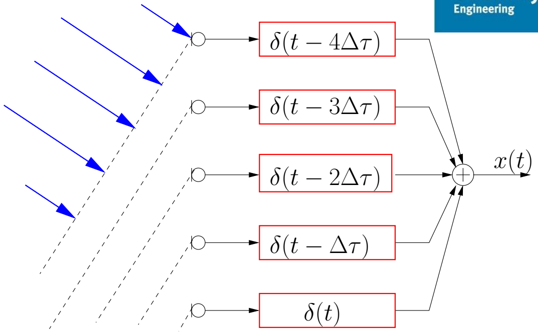
Introduction to Spatial Filtering / Array Processing — Overview

Contents:

1.1 Intuitive Beamforming

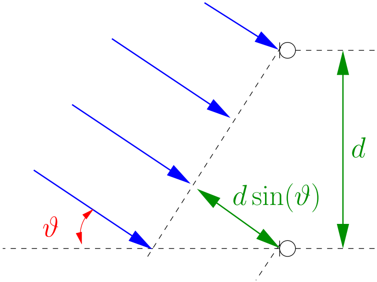
- A farfield wavefront arrives at a linear uniformly spaced sensor array:
- due to the direction of arrival (DOA) and finite propagation speed, the wavefront will arrive at different sensors with a delay $\Delta\tau$;



- with appropriate processing (beamforming), the sensor signals can be aligned to create constructive interference at the output $x(t)$;
- the above is a simple delay-and-sum beamformer [16, 42, 64, 43].

1.2 Spatial Sampling

- For unambiguous spatial sampling, we need to take at least two samples per wavelength of the highest frequency component in the array signals [42];
- analogy from temporal sampling (Nyquist): take at least two samples per period (relating to the highest frequency component);
- Wavelength λ and frequency f are related by the propagation speed c in the medium: $\lambda = \frac{c}{f}$;



- maximum sensor distance

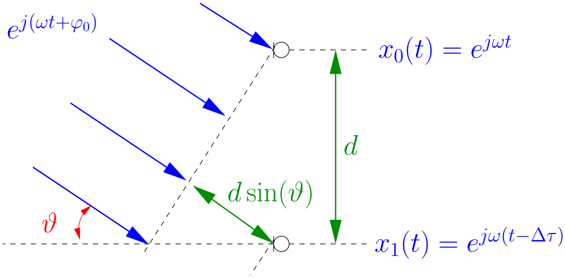
$$d = \frac{\lambda_{\max}}{2} = \frac{c}{2f_{\max}} .$$

- time delay between sensors

$$\Delta\tau = \frac{d\sin(\vartheta)}{c} = \frac{\sin(\vartheta)}{2f_{\max}} .$$

Spatial and Temporal Sampling

- Consider the array signals $x_0(t)$ and $x_1(t)$ due to a source $\mathrm{e}^{\mathrm{j}(\omega t+\varphi_0)}$:



- sampling with $t = nT_\mathrm{s}$ leads to

$$x_0[n] = \mathrm{e}^{\mathrm{j}\omega nT_\mathrm{s}} \qquad \text{and} \qquad x_1[n] = \mathrm{e}^{\mathrm{j}\omega(nT_\mathrm{s}-\Delta\tau)}$$

- with $f_\mathrm{max} = \frac{f_\mathrm{s}}{2} = \frac{1}{2T_\mathrm{s}}$ and normalised angular frequency $\Omega = \omega T_\mathrm{s}$,

$$x_0[n] = \mathrm{e}^{\mathrm{j}\Omega n} \qquad \text{and} \qquad x_1[n] = \mathrm{e}^{\mathrm{j}\Omega n} \cdot \mathrm{e}^{-\mathrm{j}\Omega\sin(\vartheta)} \; .$$

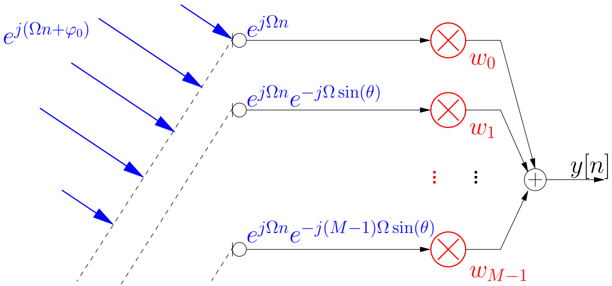
1.3 Steering Vector

- A narrowband source with normalised angular frequency Ω illuminates an M-element linear array of equi-spaced sensors:

$$\mathbf{x}[n] = \begin{bmatrix} x_0[n] \\ x_1[n] \\ \vdots \\ x_{M-1}[n] \end{bmatrix} = \mathrm{e}^{\mathrm{j}\Omega n} \cdot \begin{bmatrix} 1 \\ \mathrm{e}^{-\mathrm{j}\Omega \sin(\vartheta)} \\ \vdots \\ \mathrm{e}^{-\mathrm{j}(M-1)\Omega \sin(\vartheta)} \end{bmatrix} = \mathrm{e}^{\mathrm{j}\Omega n} \cdot \mathbf{s}_{\Omega,\vartheta}$$

- the vector $\mathbf{s}_{\Omega,\vartheta}$ characterises the phase shifts of waveform with frequency Ω and DOA ϑ measured at the array sensors;
- since a narrowband signal $\mathrm{e}^{\mathrm{j}\Omega n}$ only causes phase shifts rather than delays, constructive interference can be accomplished by a set of complex multipliers rather than processors $\delta(t - m\Delta\tau)$, $m = 0, 1, \ldots (M-1)$;
- beamforming problem: how to select the set of complex coefficients?

1.4 Data Independent Beamformer



- Challenge:
  find a set of complex multipliers w_m, $m = 0, 1, \ldots (M-1)$:

- to steer the array characteristic towards this source, the output

$$y[n] = [w_0 \;\; w_1 \;\; \ldots \;\; w_{M-1}]\mathrm{e}^{j\Omega n}\begin{bmatrix} 1 \\ \mathrm{e}^{-\mathrm{j}\Omega\sin(\vartheta)} \\ \vdots \\ \mathrm{e}^{-\mathrm{j}(M-1)\Omega\sin(\vartheta)} \end{bmatrix} = \mathrm{e}^{\mathrm{j}\Omega n}\mathbf{w}^{\mathrm{H}}\mathbf{s}_{\Omega,\vartheta}$$

should satisfy $y[n] = \mathrm{e}^{\mathrm{j}\Omega n}$, leading to $\mathbf{w}^{\mathrm{H}}\mathbf{s}_{\Omega,\vartheta} = 1$.

Coefficient Vector

- For later convenience and compatibility, the Hermitian transpose operator $\{\cdot\}^{\mathrm{H}}$ is used to denote the coefficient vector

$$\mathbf{w}^{\mathrm{H}} = [w_0 \;\; w_1 \;\; \ldots \;\; w_{M-1}] \; ;$$

- as a result, the vector $\mathbf{w}$ hold the complex conjugates of the coefficients,

$$\mathbf{w} = \begin{bmatrix} w_0^* \\ w_1^* \\ \vdots \\ w_{M-1}^* \end{bmatrix} \; ;$$

- to access the actual unconjugated coefficients, the beamforming vector $\mathbf{w}^*$ has to be considered
- note that

$$\mathbf{w}^{\mathrm{H}} \mathbf{s}_{\Omega,\vartheta} = 1 \qquad \longrightarrow \qquad \mathbf{s}^{\mathrm{H}}_{\Omega,\vartheta} \mathbf{w} = 1 \; .$$

Narrowband Beamforming — Single Source

- The expression $\mathbf{s}^{\mathrm{H}}_{\Omega,\vartheta}\mathbf{w} = 1$ forms a system with one equation and M unknowns

$$\boxed{\mathbf{s}^{\mathrm{H}}_{\Omega,\vartheta}} \; \boxed{\mathbf{w}} = \boxed{1}$$

- general solution to an underdetermined system $\mathbf{A}\mathbf{x} = \mathbf{b}$ is the right pseudo-inverse $\mathbf{A}^{\dagger}$ [14],

$$\mathbf{x} = \mathbf{A}^{\dagger}\mathbf{b} = \mathbf{A}^{\mathrm{H}}(\mathbf{A}\mathbf{A}^{\mathrm{H}})^{-1}\mathbf{b} \; ;$$

- here:

$$\mathbf{w} = (\mathbf{s}^{\mathrm{H}}_{\Omega,\vartheta})^{\dagger} \cdot 1 = \mathbf{s}_{\Omega,\vartheta} \cdot (\mathbf{s}^{\mathrm{H}}_{\Omega,\vartheta}\mathbf{s}_{\Omega,\vartheta})^{-1} \cdot 1 = \frac{\mathbf{s}_{\Omega,\vartheta}}{\|\mathbf{s}_{\Omega,\vartheta}\|_2^2} = \frac{1}{M}\mathbf{s}_{\Omega,\vartheta} \; ;$$

- the complex conjugation for $\mathbf{w}^*$ inverts and therefore compensates the phase of the steering vector, which could have been foreseen
- the formulation via the pseudo-inverse will be powerful for more complicated cases.

Narrowband Beamformer Example

- Source parameters: $\Omega = \frac{\pi}{2}$ and $\vartheta = 30^\circ$; array parameter: $M = 5$;
- steering vector (with $\Omega \sin(\vartheta) = \frac{1}{4}\pi$):

$$\mathbf{s}^{\mathrm{T}}_{\Omega,\vartheta} = [1 \;\; \mathrm{e}^{-\mathrm{j}\frac{1}{4}\pi} \quad \ldots \quad \mathrm{e}^{-\mathrm{j}\frac{4}{4}\pi}]$$

- coefficient vector is given by $\mathbf{w} = (\mathbf{s}^{\mathrm{H}}_{\Omega,\vartheta})^{\dagger}$;
- numerical solution in Matlab;

```
Omega=1/4; theta = pi/6; M=5;
s = exp(-sqrt(-1)*Omega*sin(theta)*(0:(M-1)'));
w = pinv(s');
```

- `angle([s conj(w)])/pi` yields:

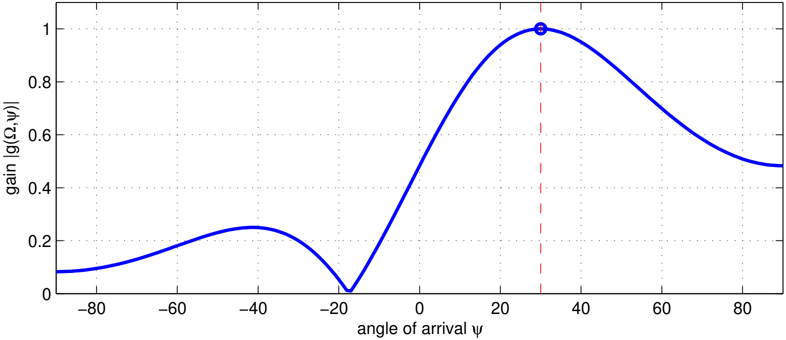
```
-0.00000   0.00000
-0.25000   0.25000
-0.50000   0.50000
-0.75000   0.75000
-1.00000   1.00000
```

Beam Pattern I

- The beamformer has a unit gain towards a source with frequency Ω and DoA θ; what is its gain response towards other angles of arrival?
- the beam pattern measures the response of a beamformer by sweeping the angle ψ of a source with frequency Ω

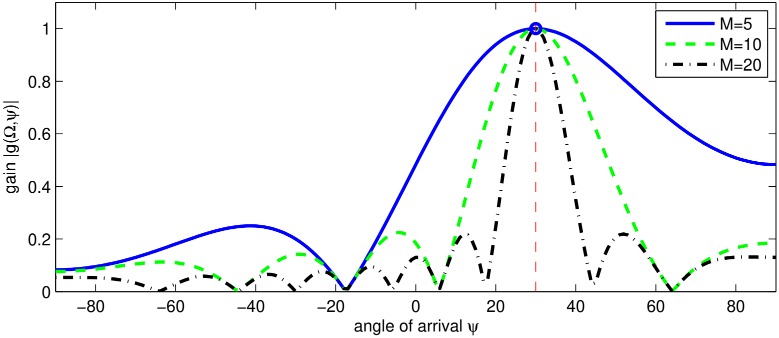
$$g(\Omega, \psi) = \mathbf{w}^{\mathrm{H}} \mathbf{s}_{\Omega,\psi}$$

- beam pattern for the previous example:



Beam Pattern II

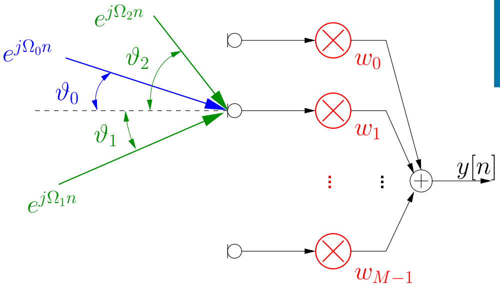
- Beam patterns for $\Omega = \frac{\pi}{2}$ and $\vartheta = 30^\circ$ with variable M:



- increasing the sensor number M narrows the main beam, and increases the number of spatial zeros;
- analogous characteristic in the time domain: increased temporal support leads to higher frequency resolution.

Interference

- Many scenarios contain a source of interest and a number of interferers:
  signal of interest:
  $\{\Omega_0, \vartheta_0\}$
  two interferers:
  $\{\Omega_1, \vartheta_1\}$, $\{\Omega_2, \vartheta_2\}$



- we would like to control the beampattern to place spatial nulls in the directions of the interfering sources;
- Problem formulation and solution :

$$\begin{bmatrix} \mathbf{s}^{\mathrm{H}}_{\Omega_0,\vartheta_0} \\ \mathbf{s}^{\mathrm{H}}_{\Omega_1,\vartheta_1} \\ \mathbf{s}^{\mathrm{H}}_{\Omega_2,\vartheta_2} \end{bmatrix} \mathbf{w} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \longrightarrow \qquad \mathbf{w} = \begin{bmatrix} \mathbf{s}^{\mathrm{H}}_{\Omega_0,\vartheta_0} \\ \mathbf{s}^{\mathrm{H}}_{\Omega_1,\vartheta_1} \\ \mathbf{s}^{\mathrm{H}}_{\Omega_2,\vartheta_2} \end{bmatrix}^{\dagger} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} .$$

Narrowband BF Example — Multiple Sources

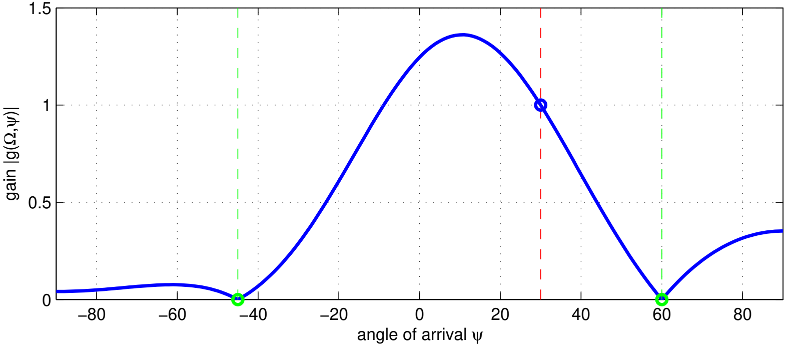
- The signal of interest illuminates an $M = 5$ element array at a frequency $\Omega_0 = \frac{\pi}{2}$ with a DoA $\vartheta_0 = 30^\circ$
- two interferers at $\Omega_1 = \Omega_2 = \Omega_0$ are present with DoA $\vartheta_1 = -45^\circ$ and $\vartheta_2 = 60^\circ$
- results via right pseudo-inverse of steering vectors

$\angle \mathbf{s}_{\Omega_0,\vartheta_0}$	$\angle \mathbf{s}_{\Omega_1,\vartheta_1}$	$\angle \mathbf{s}_{\Omega_2,\vartheta_2}$	$\angle \mathbf{w}^*$	$\lvert\mathbf{w}\rvert$
0.00	0.00	0.00	-42.81	0.3172
45.00	63.64	-77.94	-105.01	0.3004
90.00	127.28	-155.89	-90.00	0.2343
135.00	-169.08	126.17	-74.99	0.3004
180.00	-105.44	48.23	-137.19	0.3172

- the angle of $\mathbf{w}$ is no longer intuitive; also note that the coefficients in $\mathbf{w}$ no longer have the same modulus
- amongst a manifold of solutions, the right pseudo-inverse provides the minimum norm solution.

Multiple Source Example — Beampattern

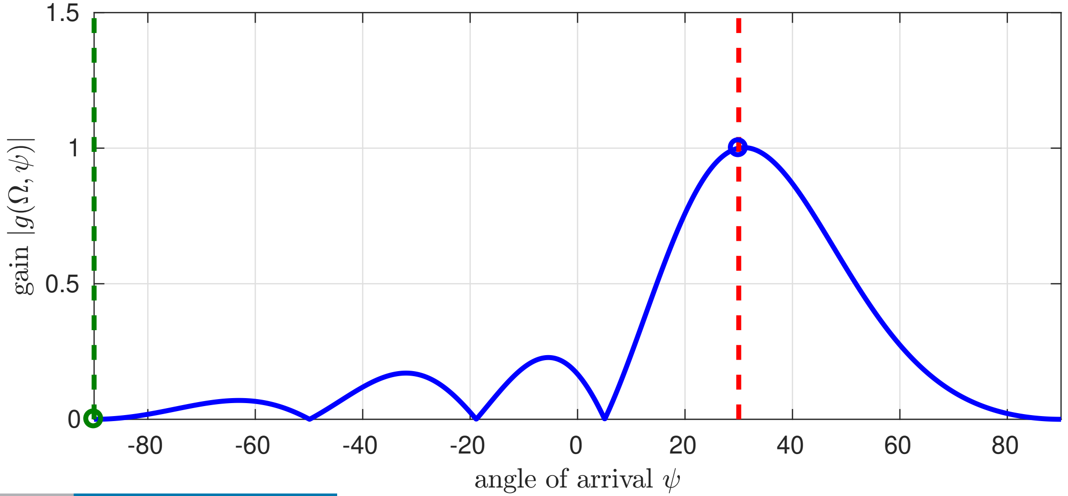
- Beam pattern one source of interest and two interferers:



- the pseudo-inverse is the minimum norm solution, keeping the general gain response as low as possible;
- the minimum norm property protects against spatially white noise.

Beamforming Example — Variable Interferer I

- $M = 5$ sensors, source of interest towards $\theta_0 = 30°$, interferer variable:



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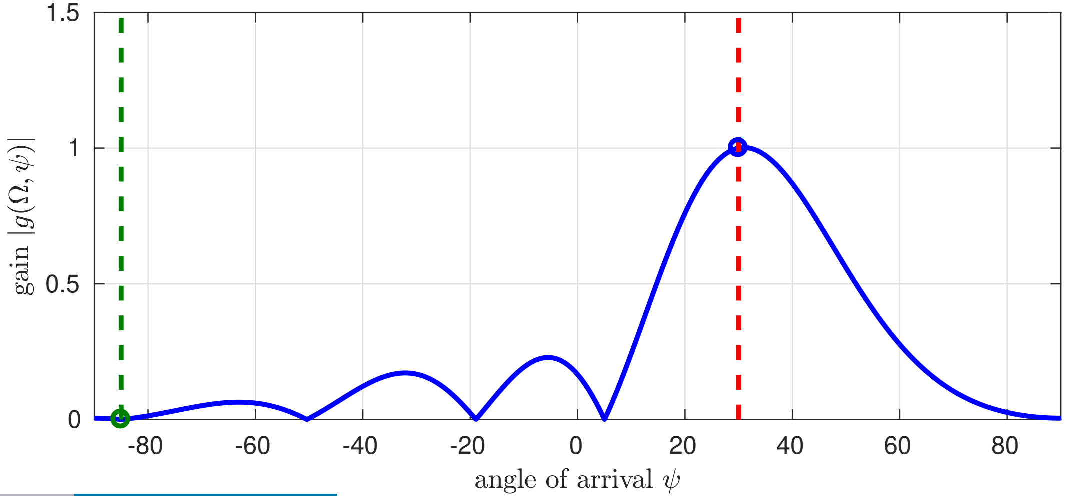

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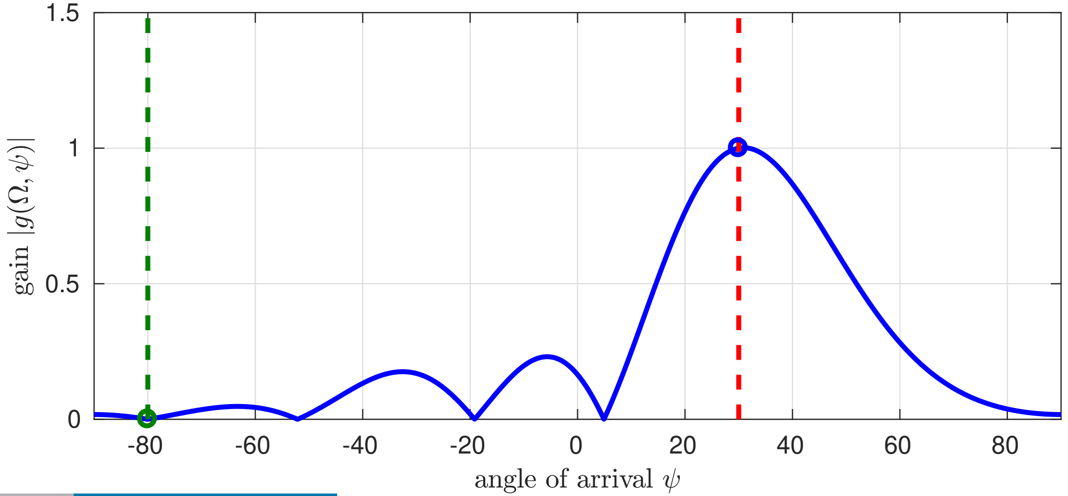

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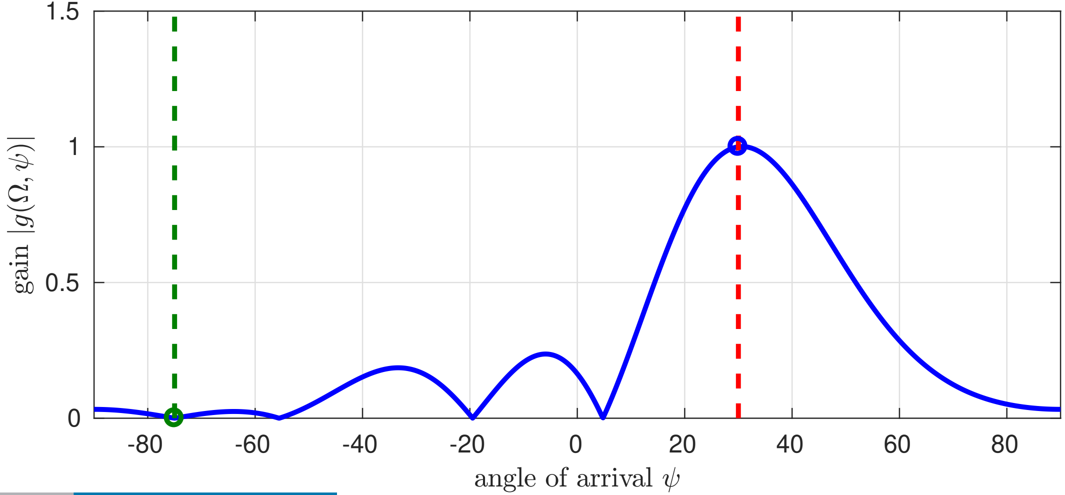

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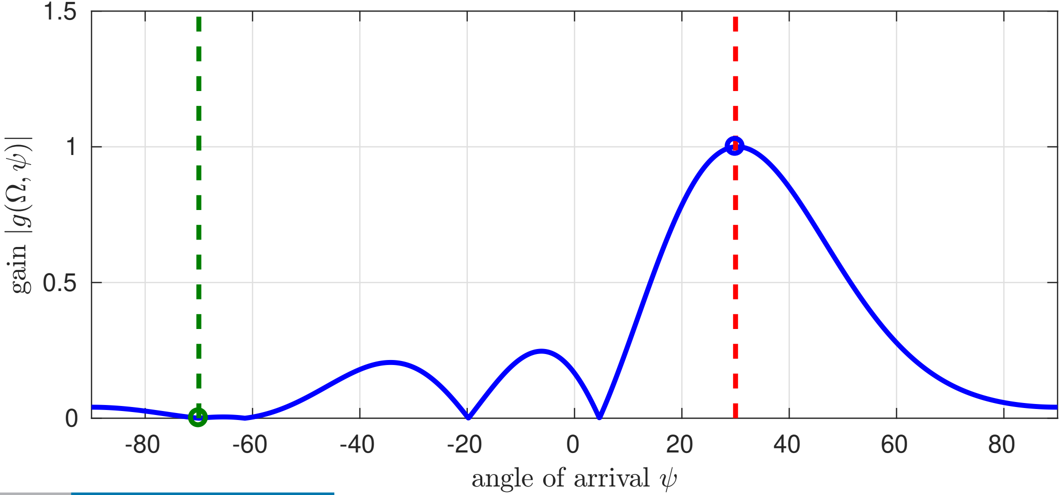

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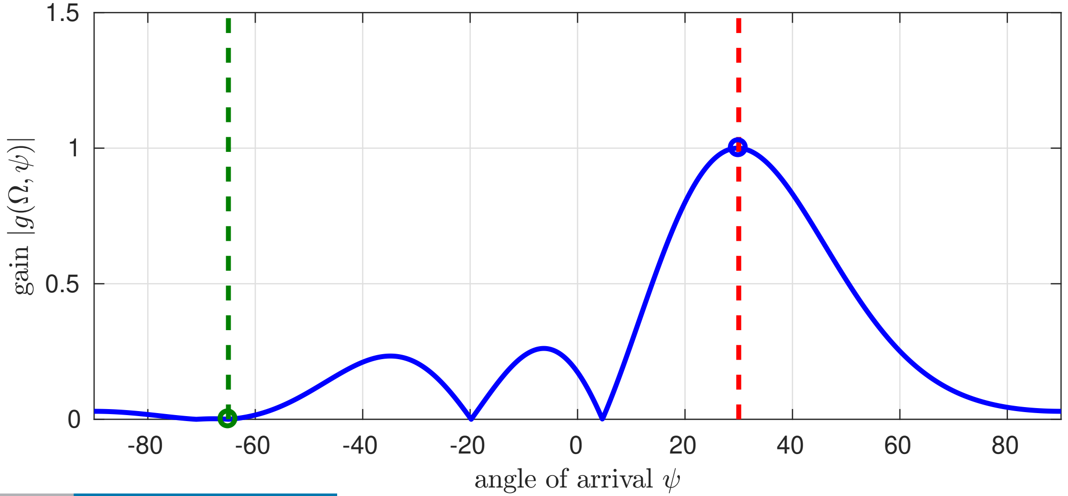

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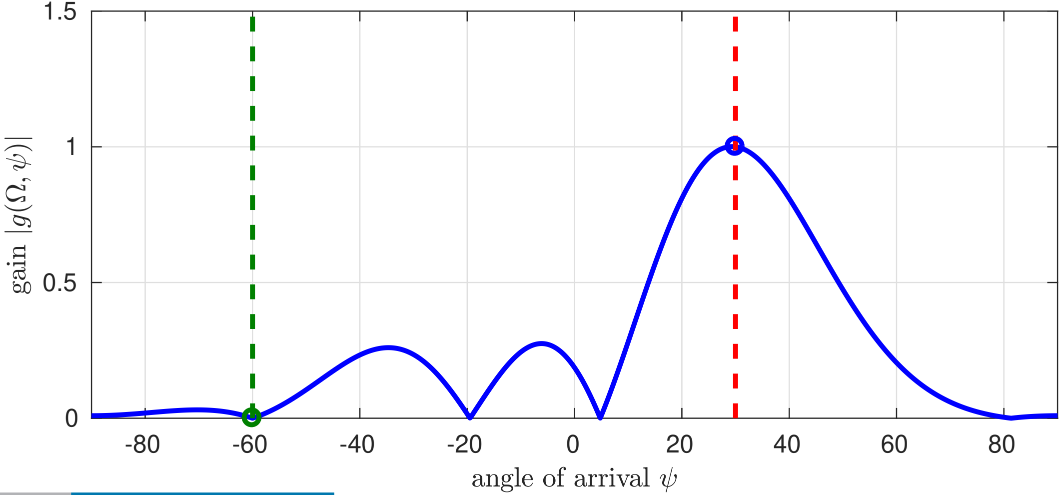

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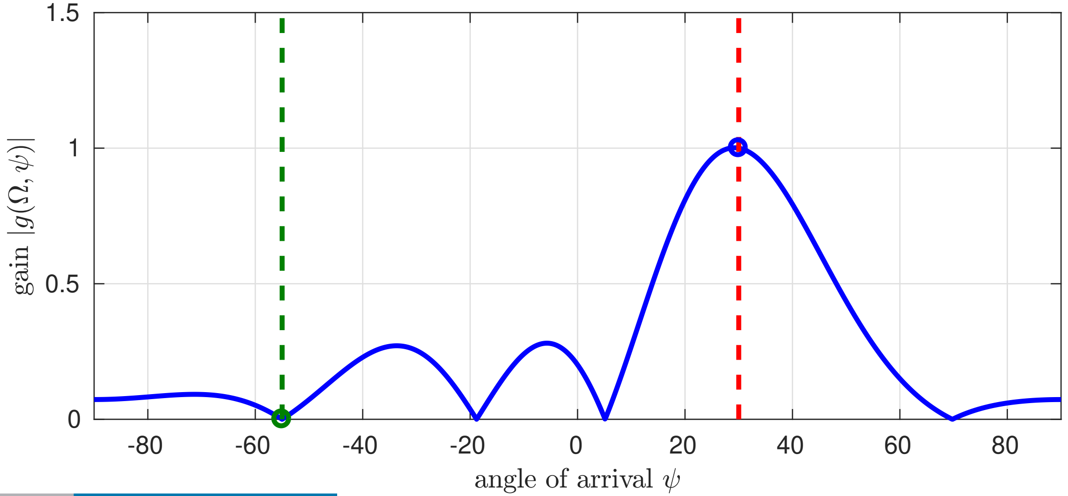

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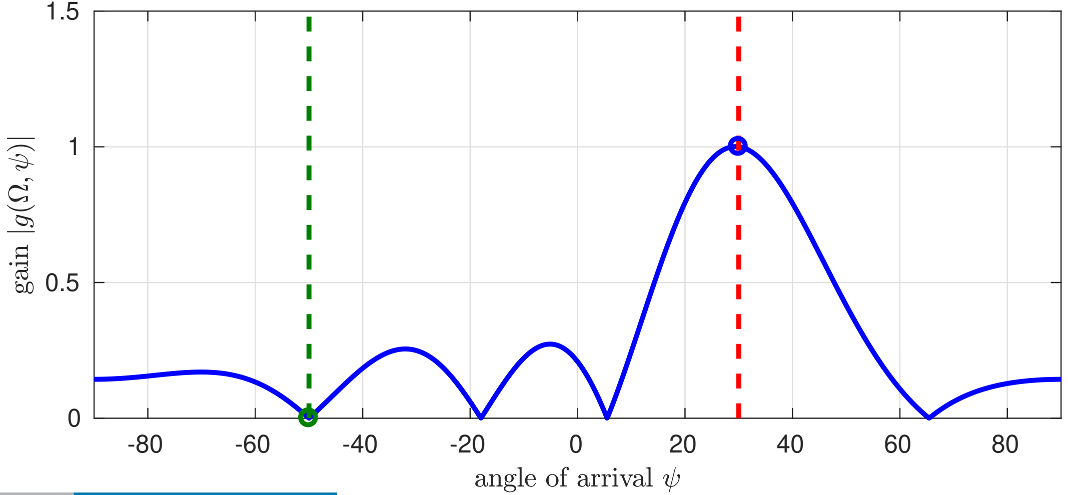

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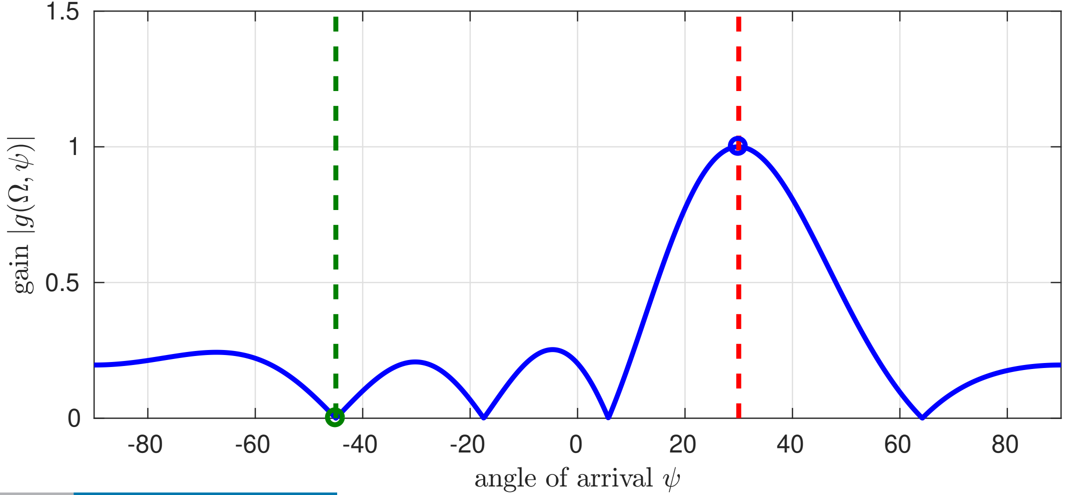

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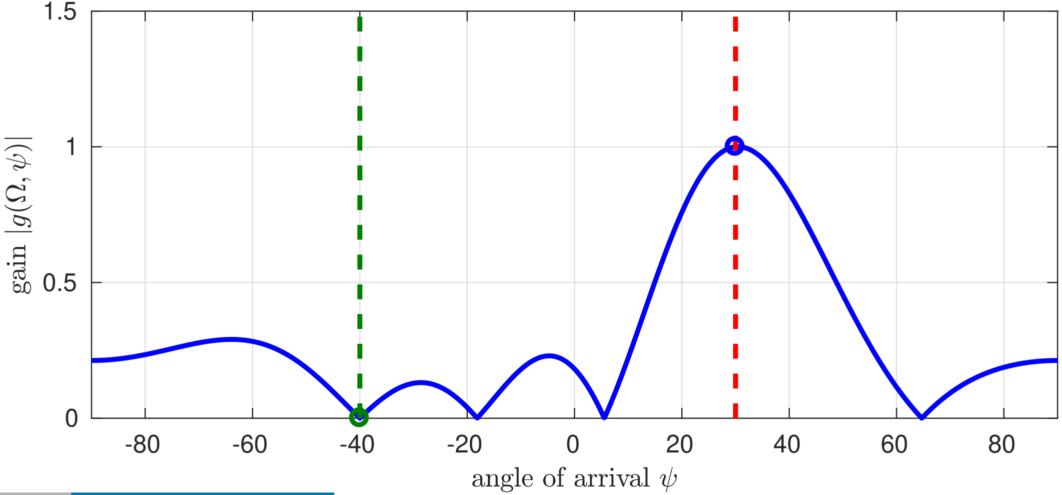

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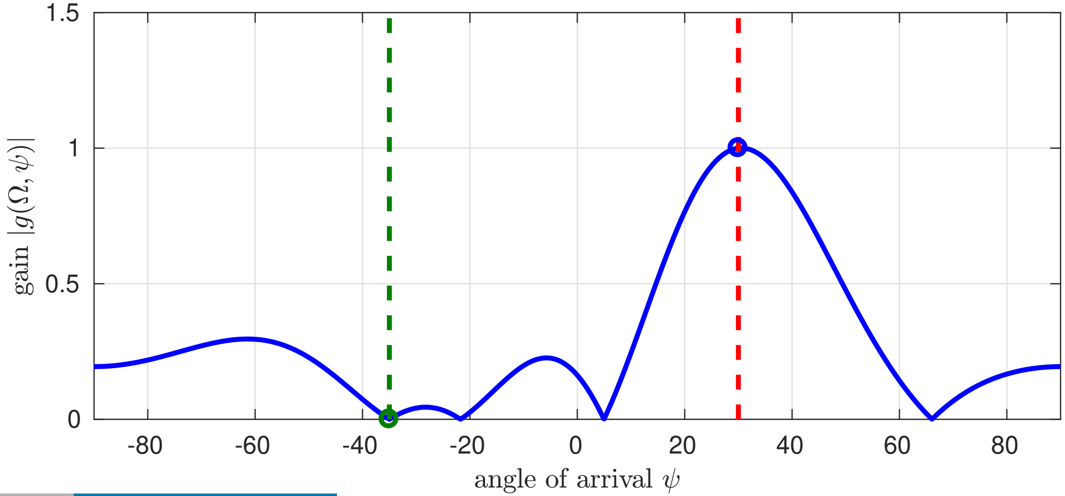

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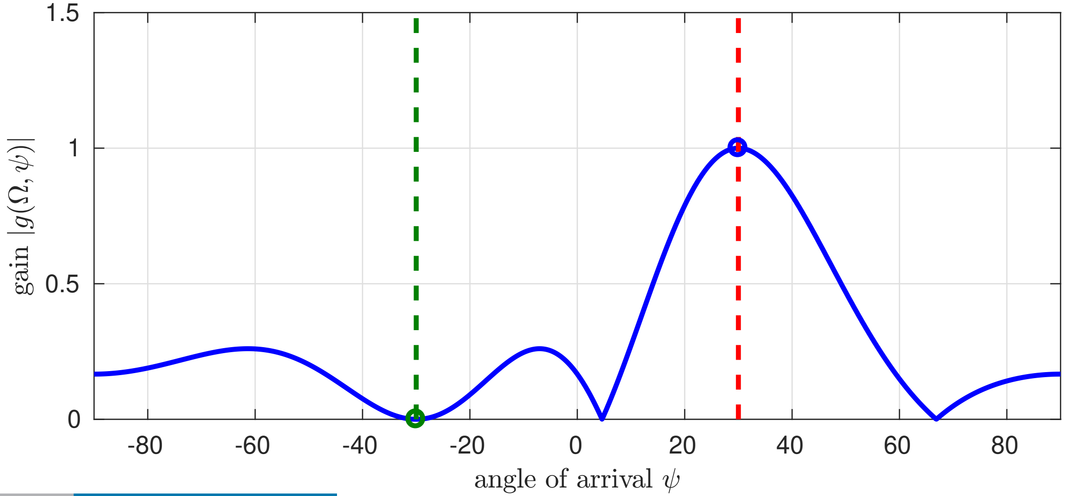

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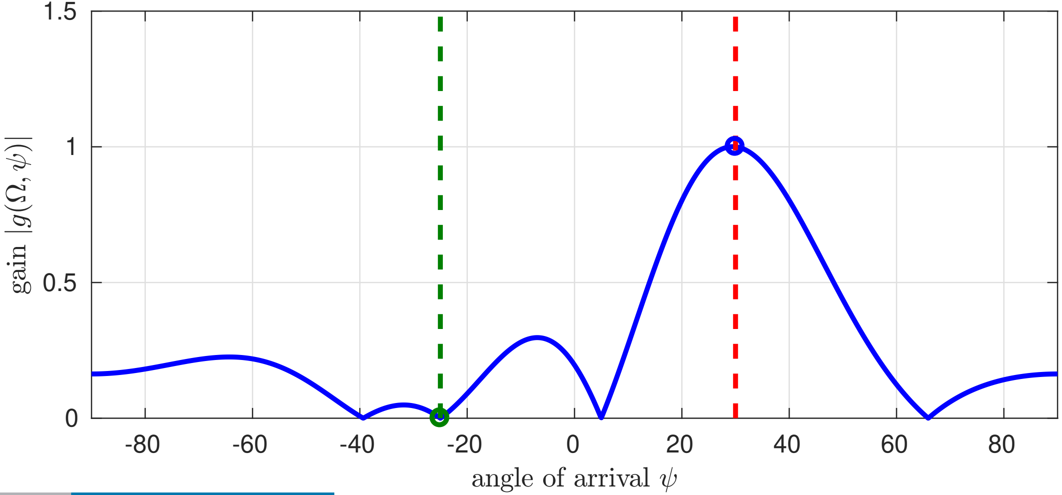

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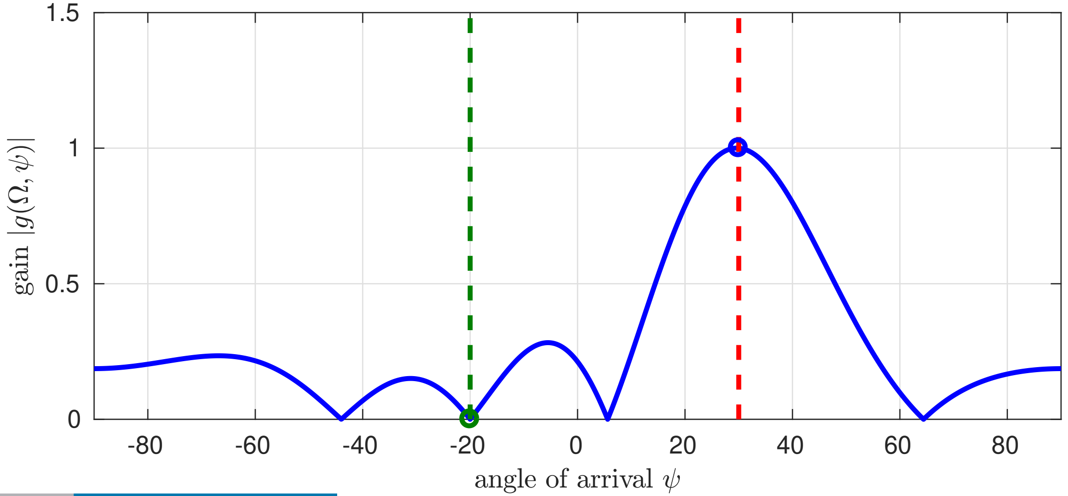

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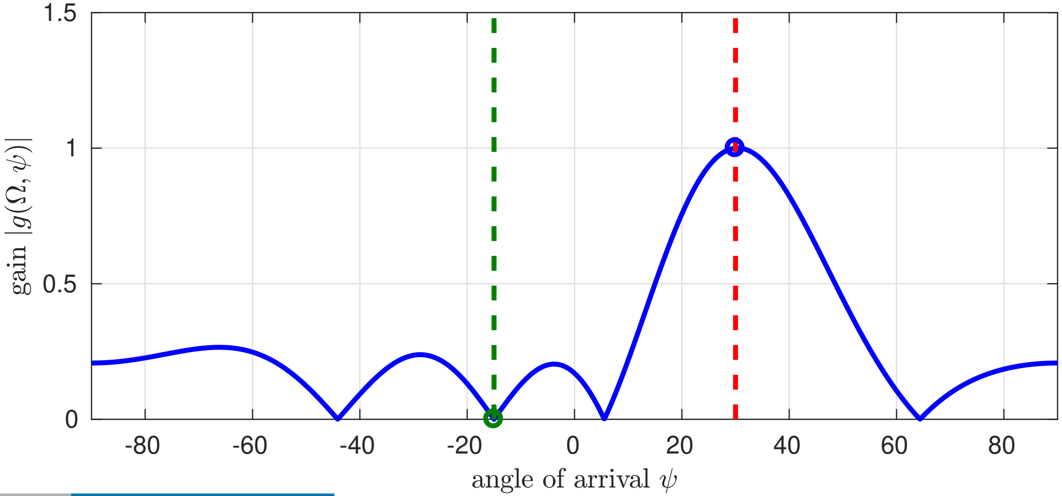

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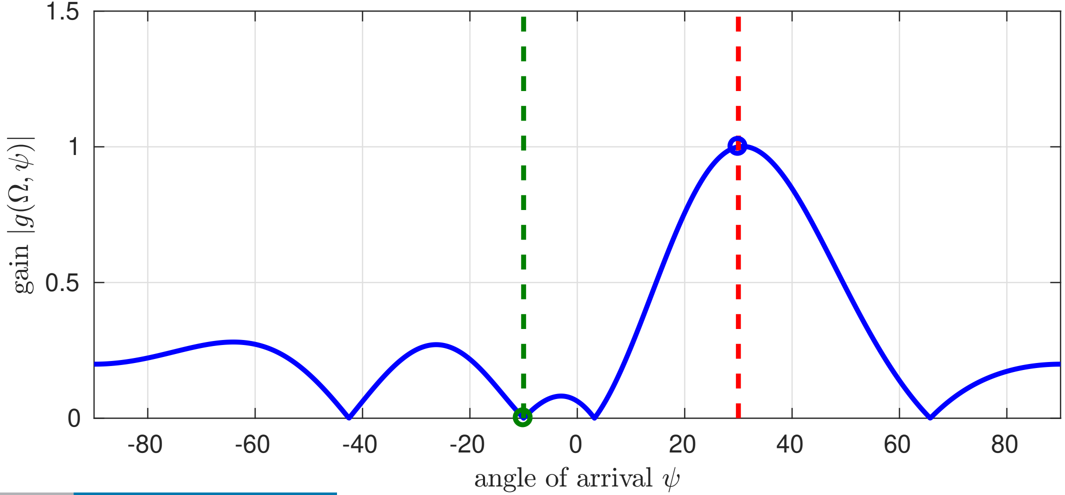

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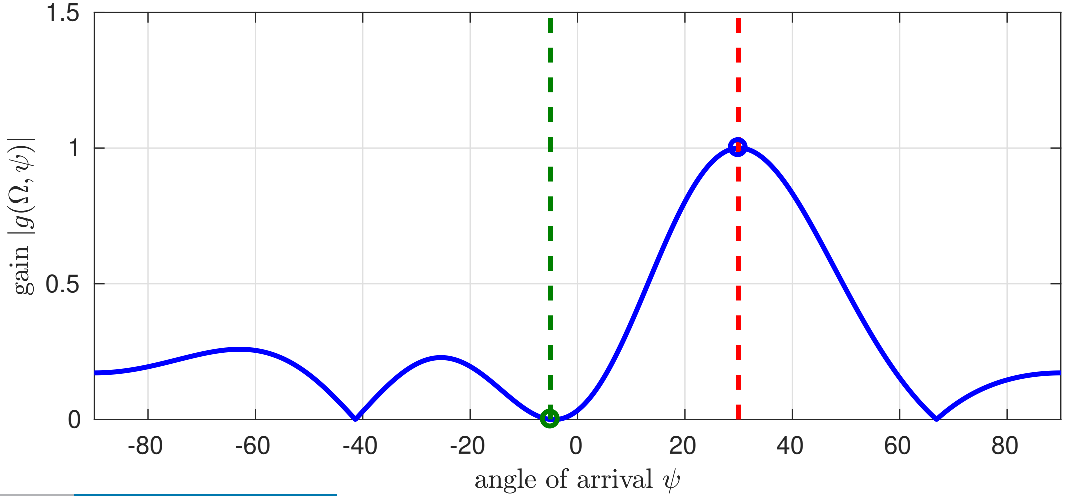

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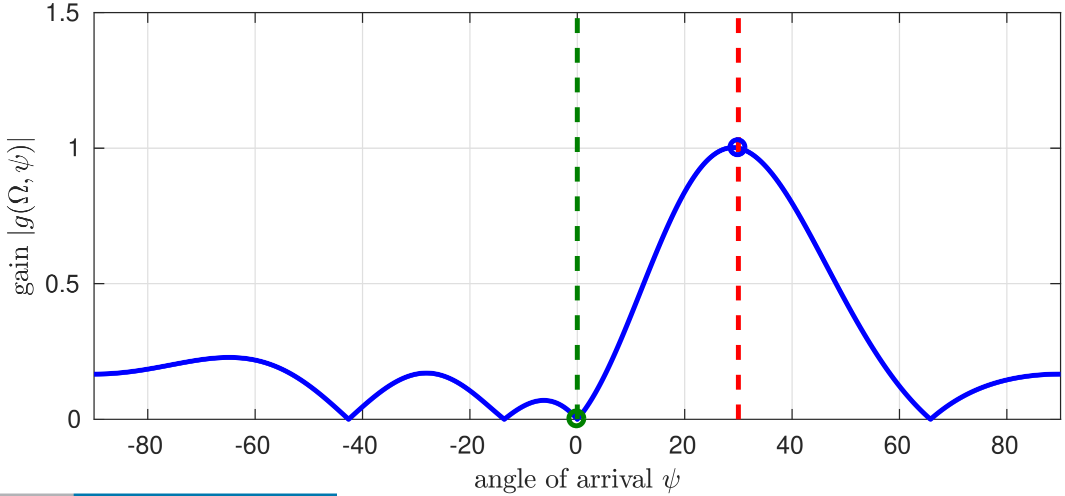

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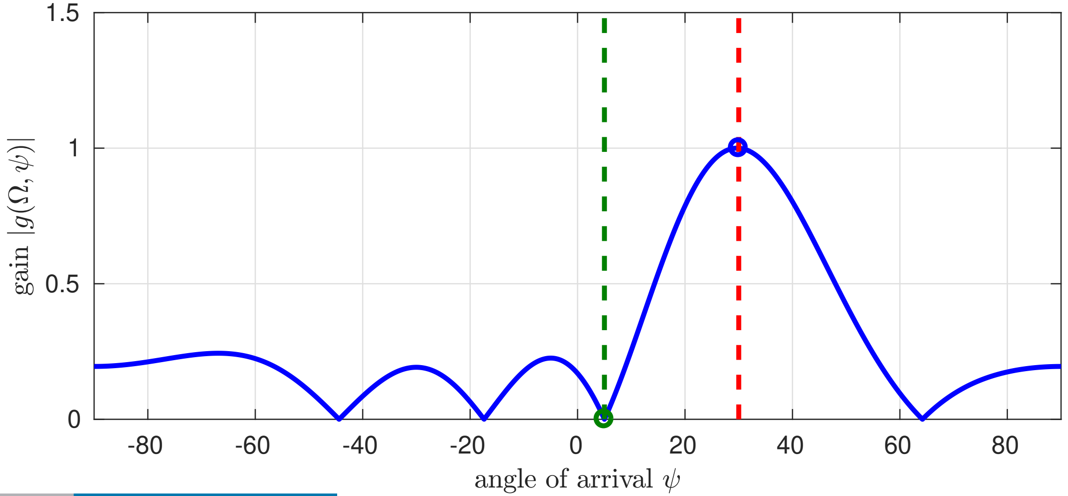

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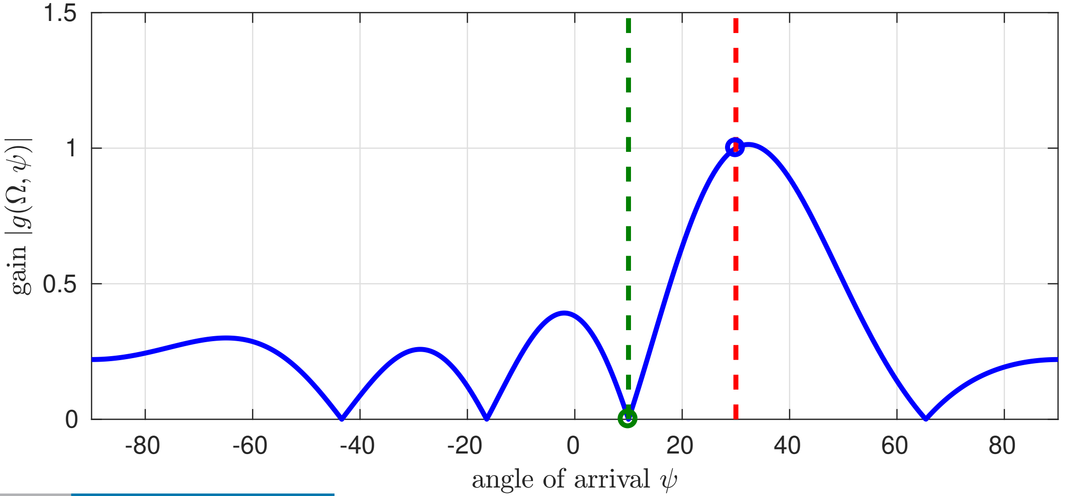

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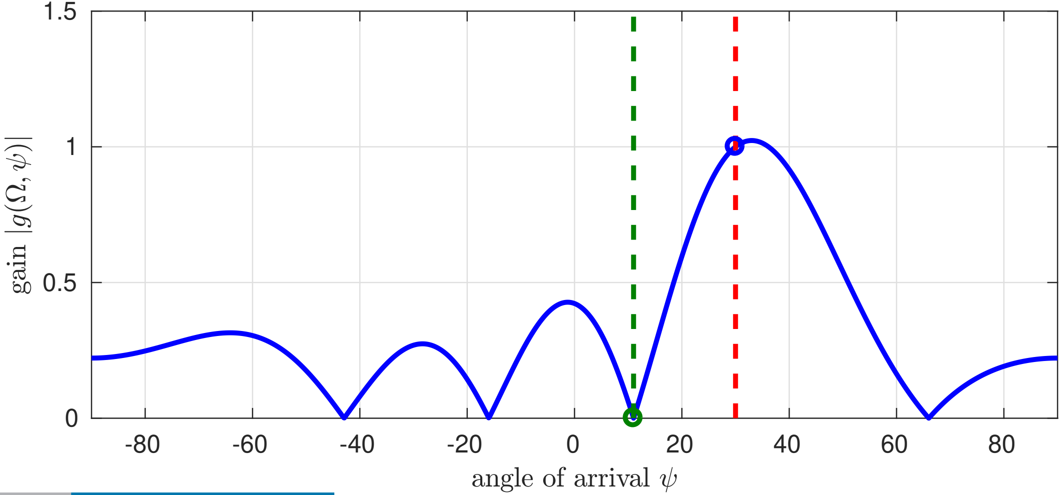

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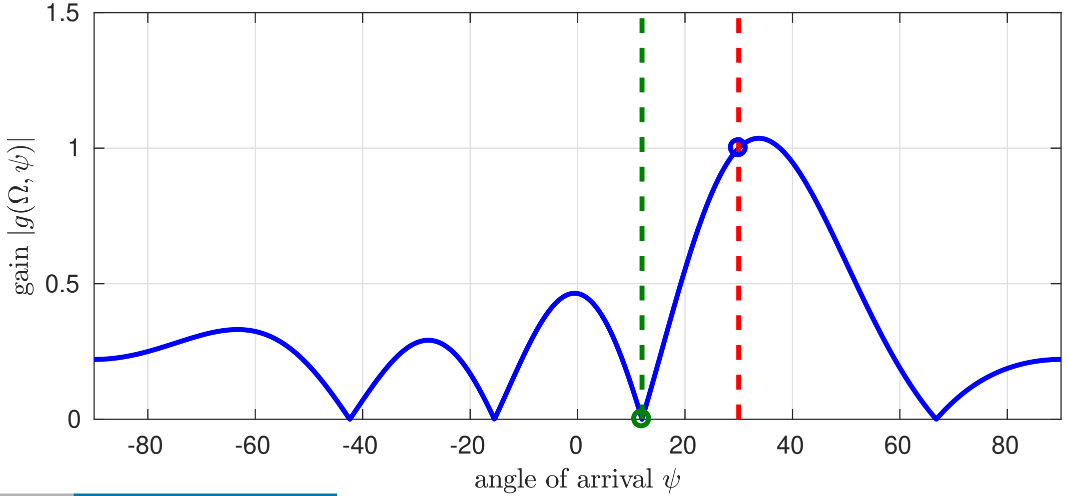

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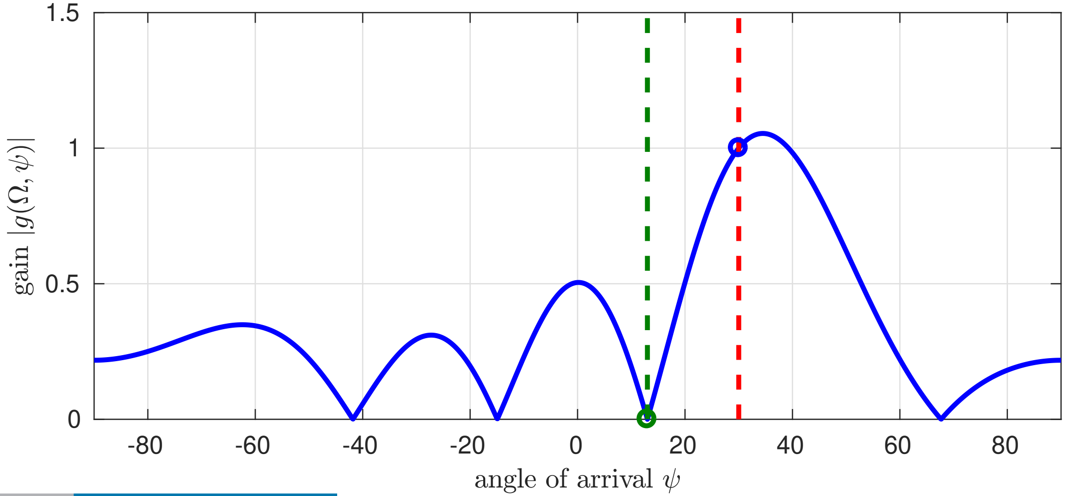

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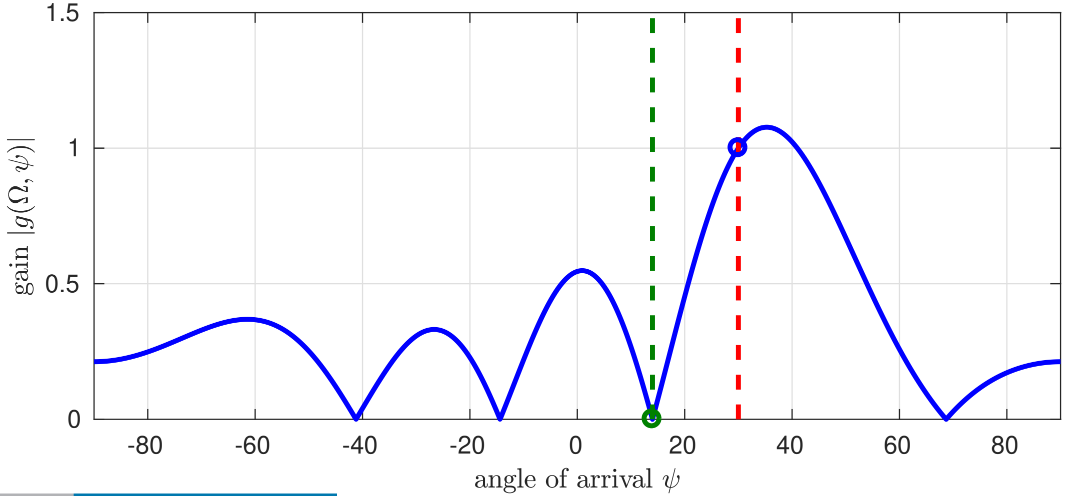

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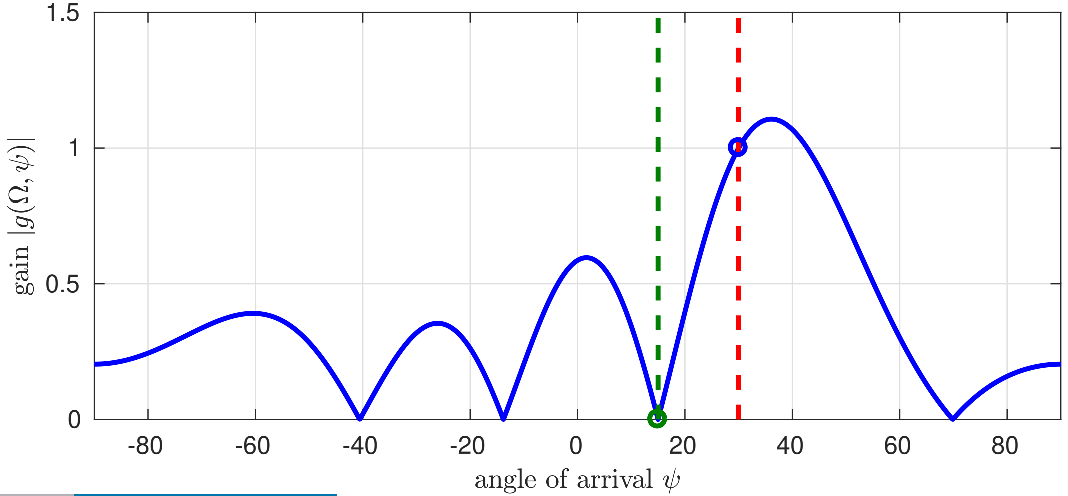

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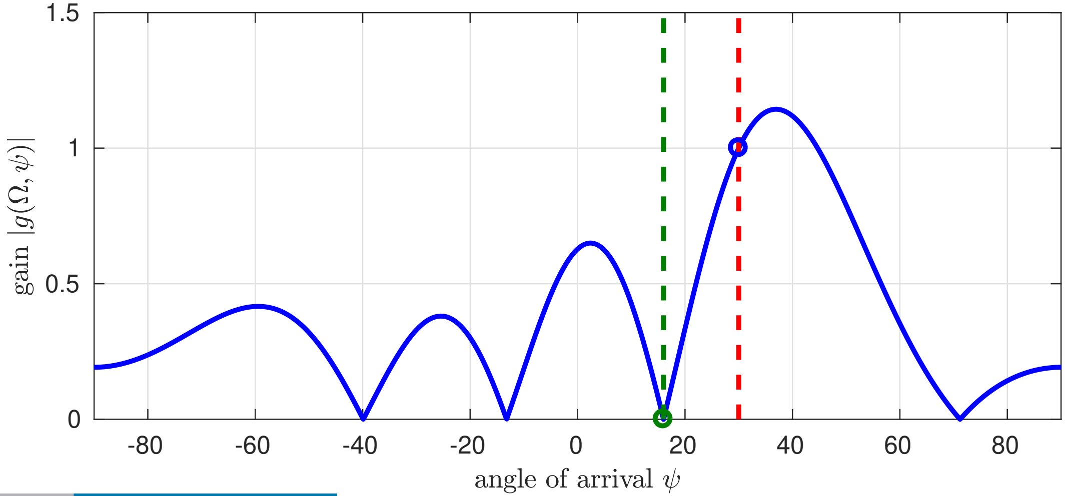

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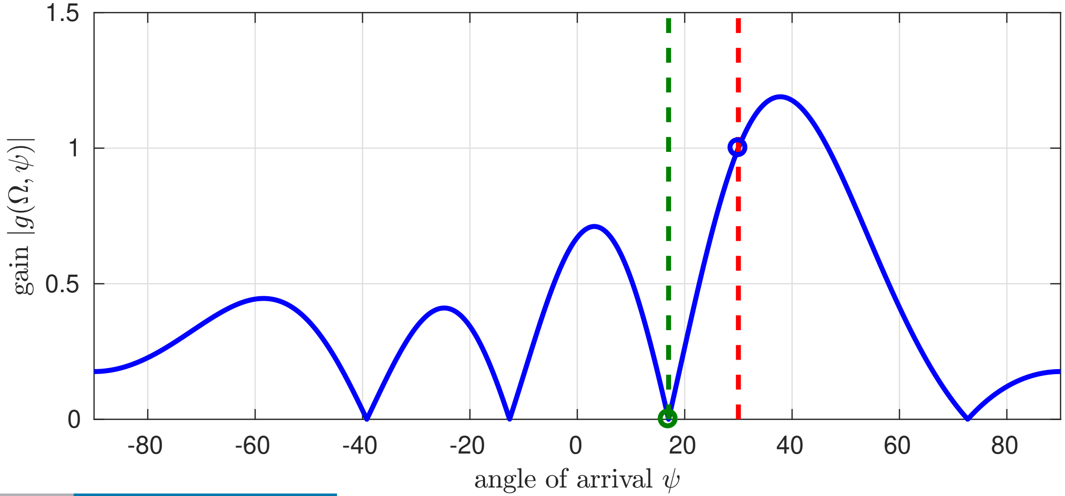

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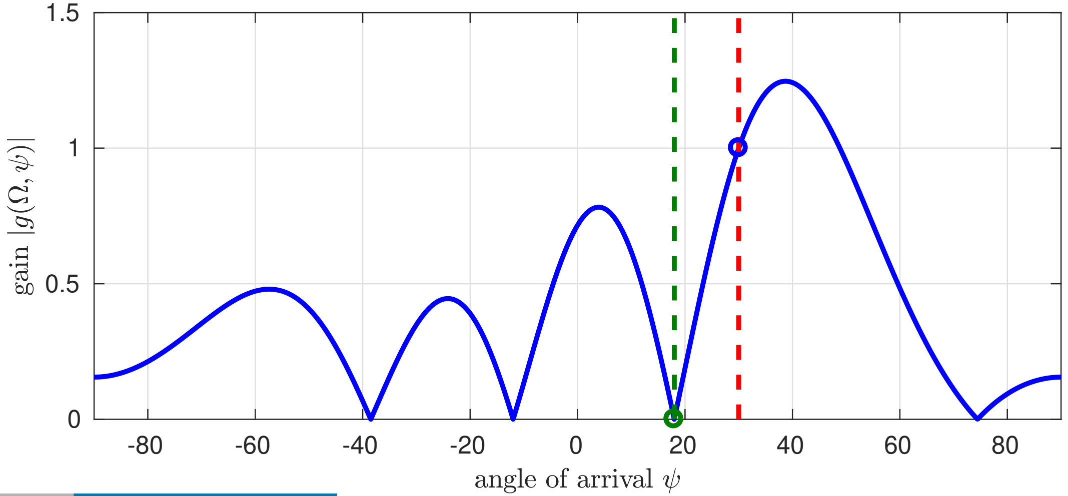

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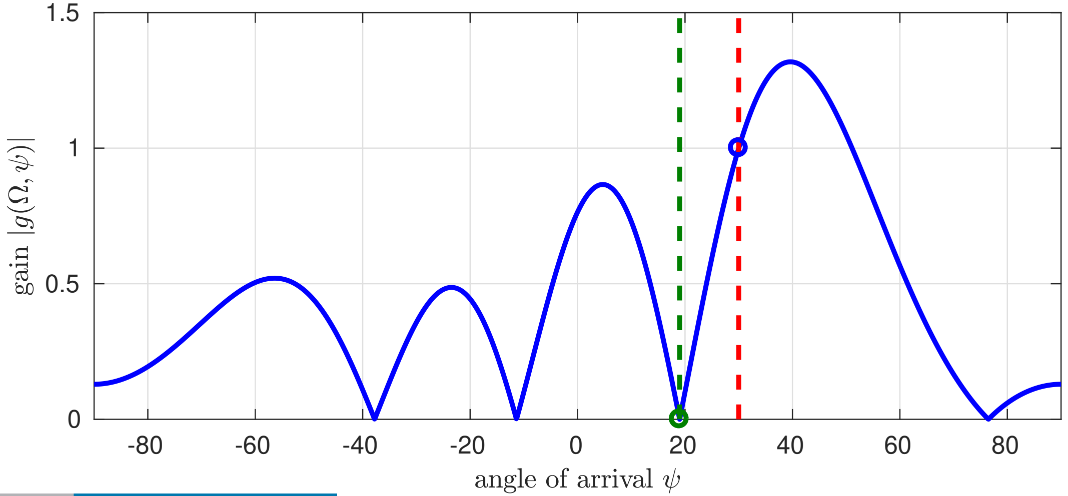

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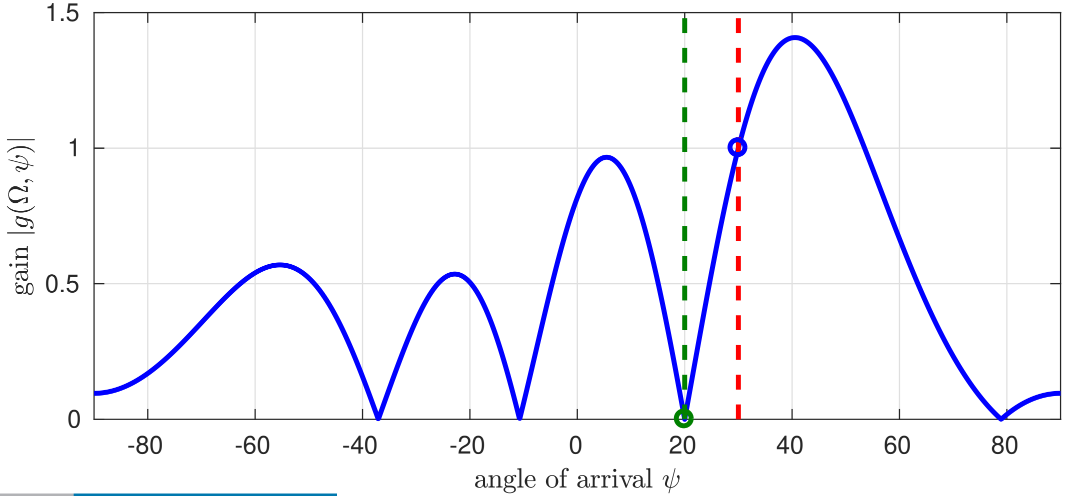
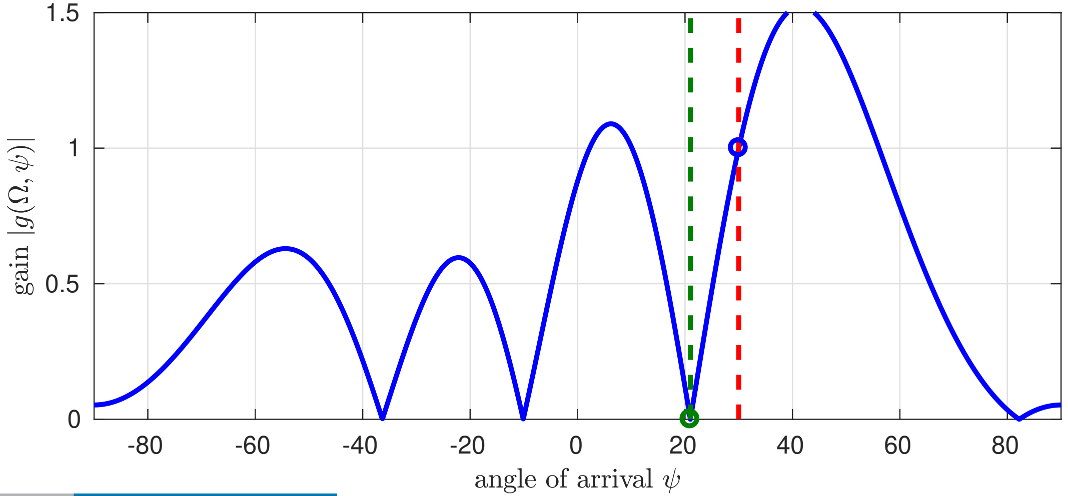


Beamforming Example — Variable Interferer I

- $M = 5$ sensors, source of interest towards $\theta_0 = 30°$, interferer variable:



Beamforming Example — Variable Interferer I

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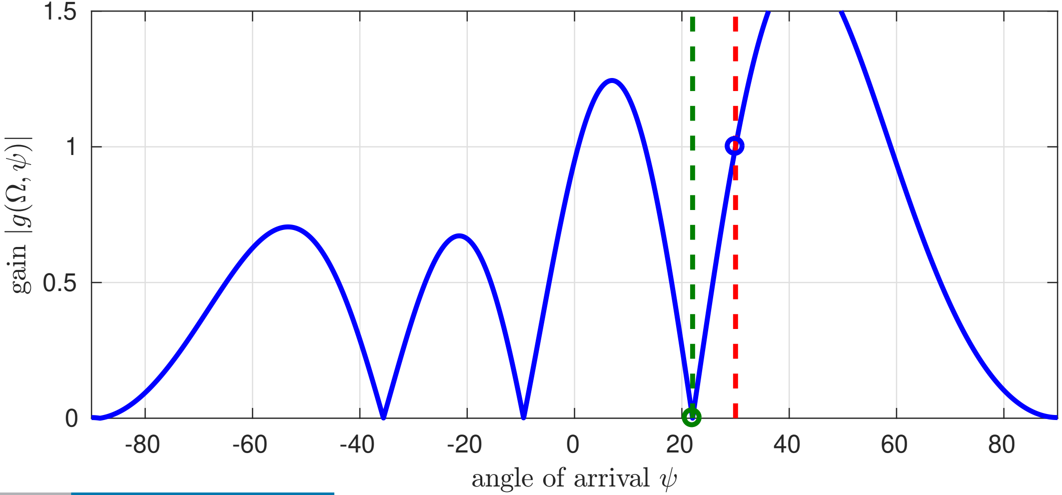

Beamforming Example — Variable Interferer I

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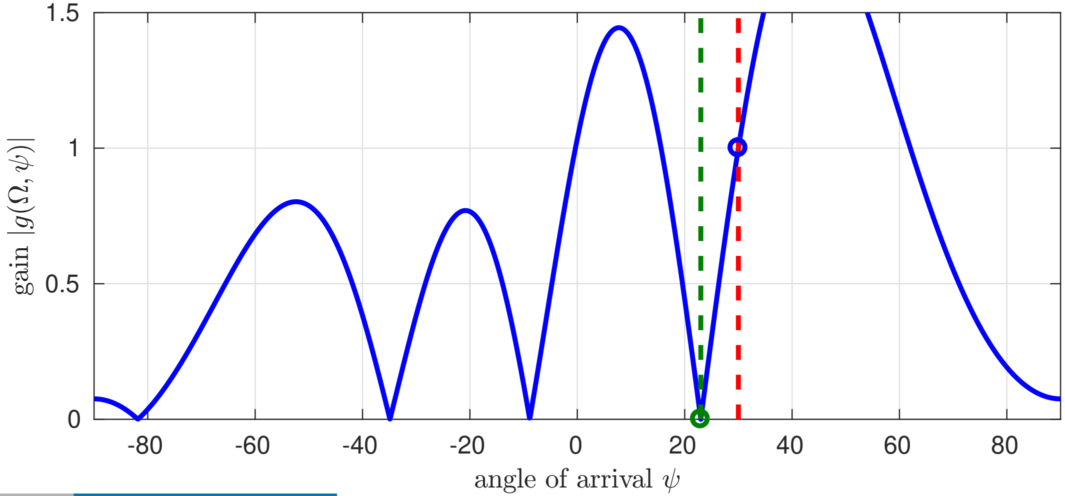

Beamforming Example — Variable Interferer I

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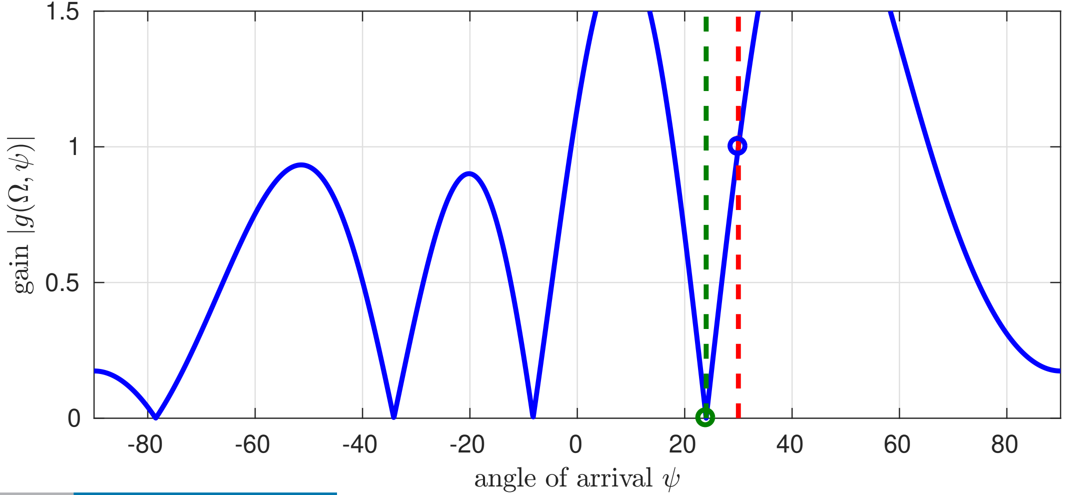

Beamforming Example — Variable Interferer I

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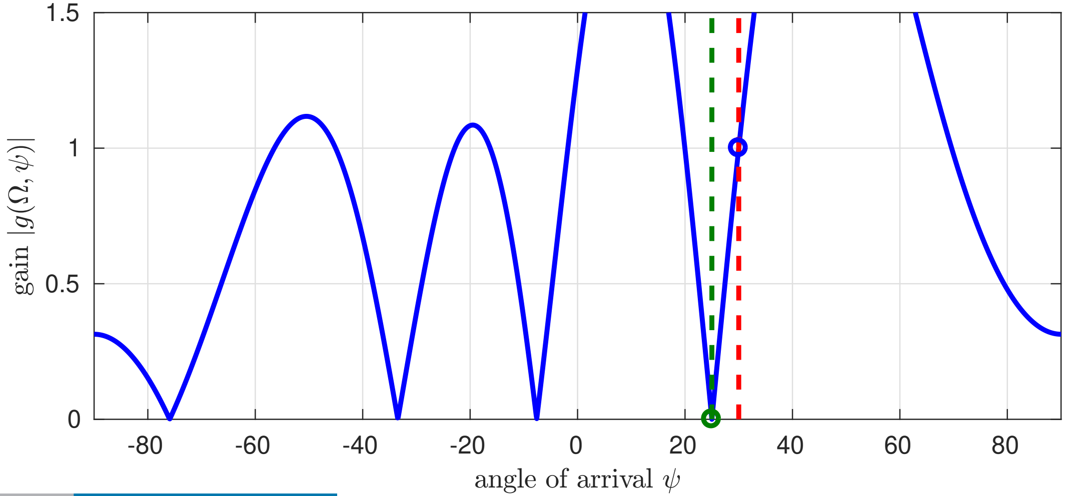
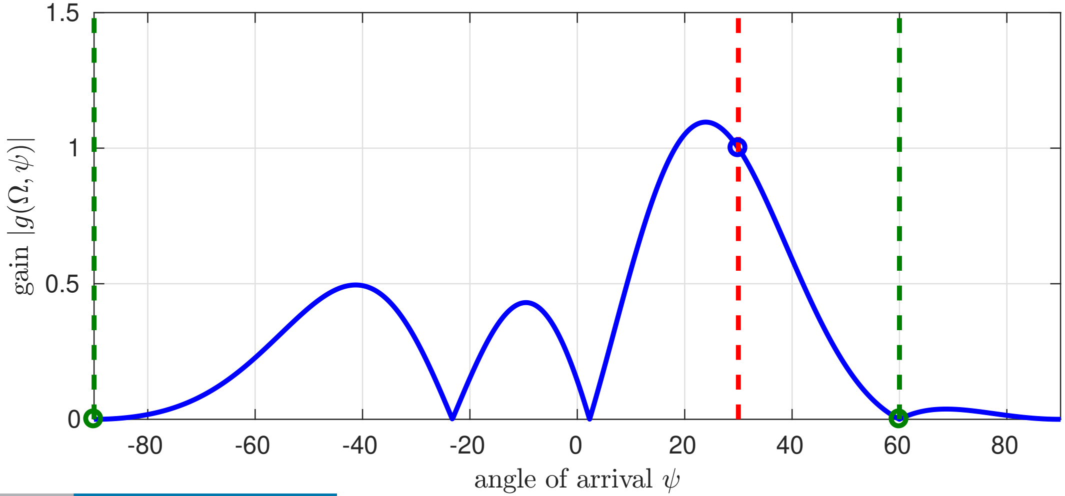


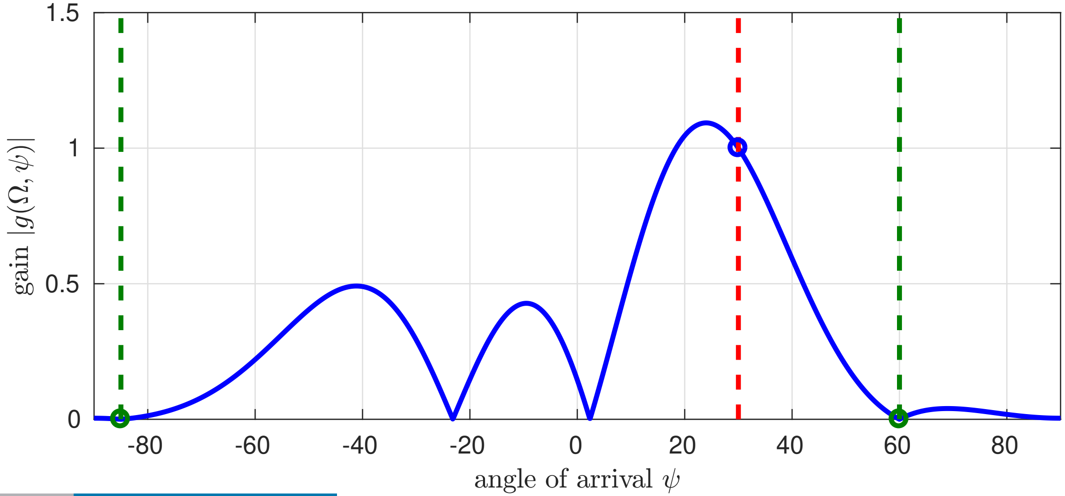
Beamforming Example — Variable Interferer II

- $M = 5$ sensors, SOI $\theta_0 = 30^\circ$, one fixed and one variable interferer:



Beamforming Example — Variable Interferer II

- $M = 5$ sensors, SOI $\theta_0 = 30°$, one fixed and one variable interferer:



Beamforming Example — Variable Interferer II

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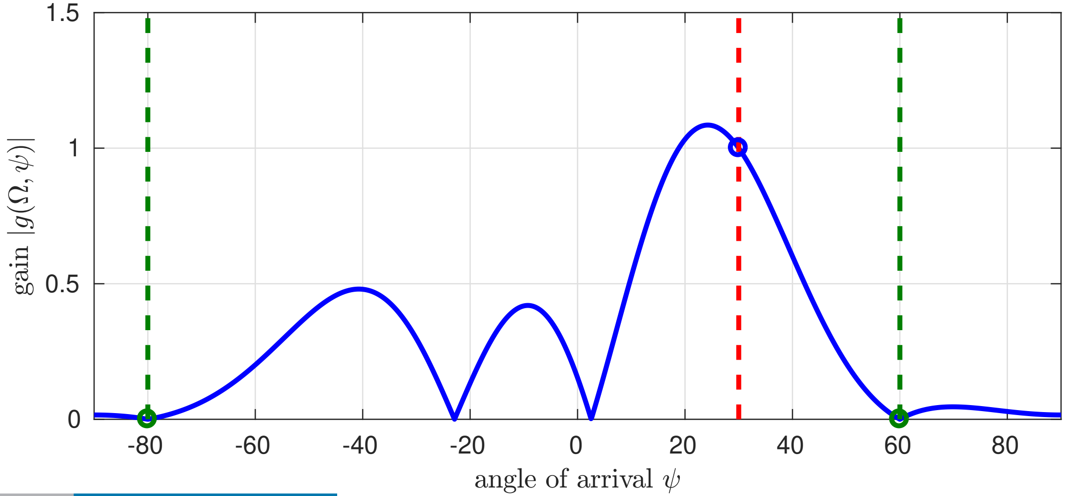

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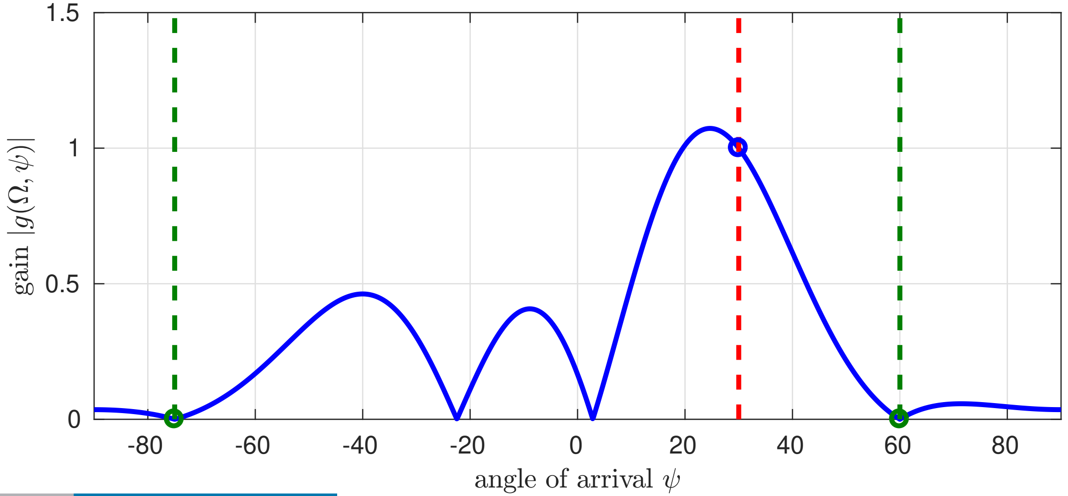

Beamforming Example — Variable Interferer II

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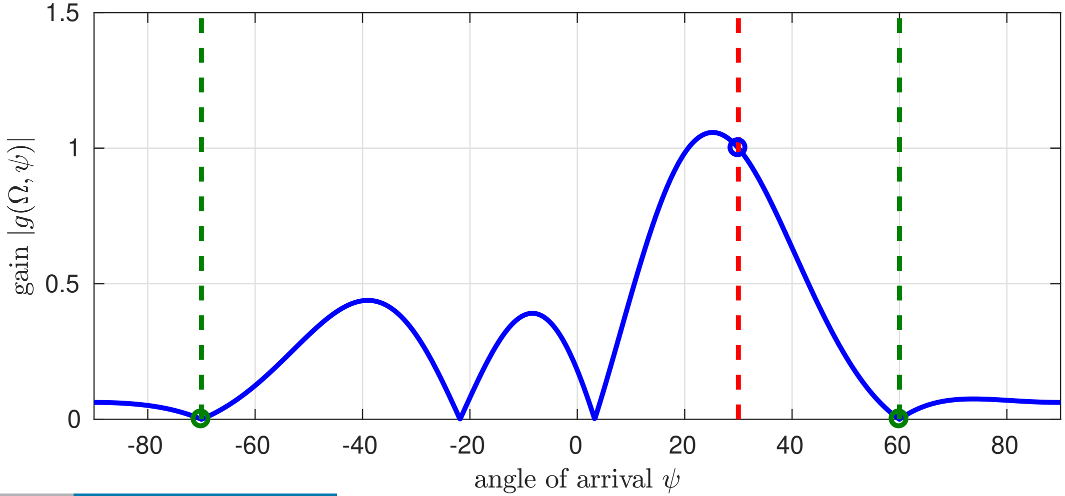

Beamforming Example — Variable Interferer II

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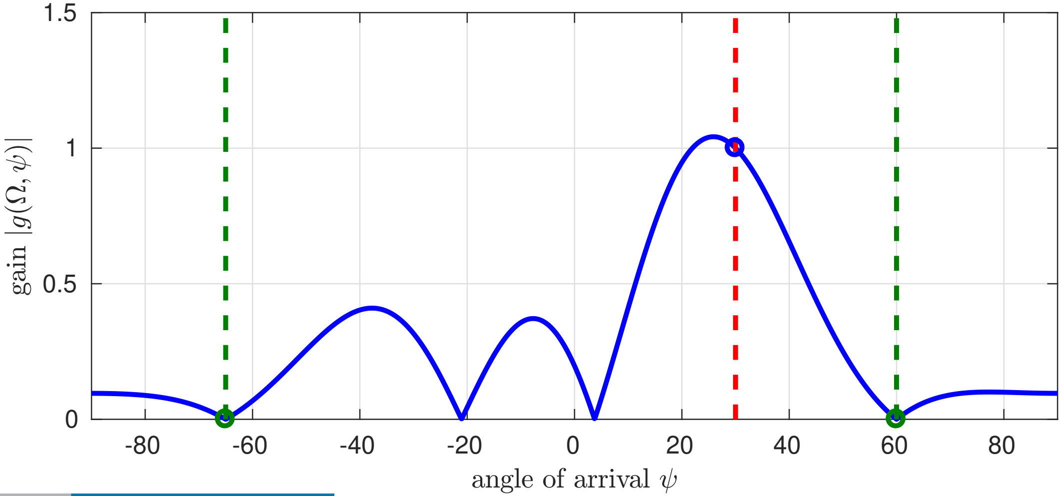

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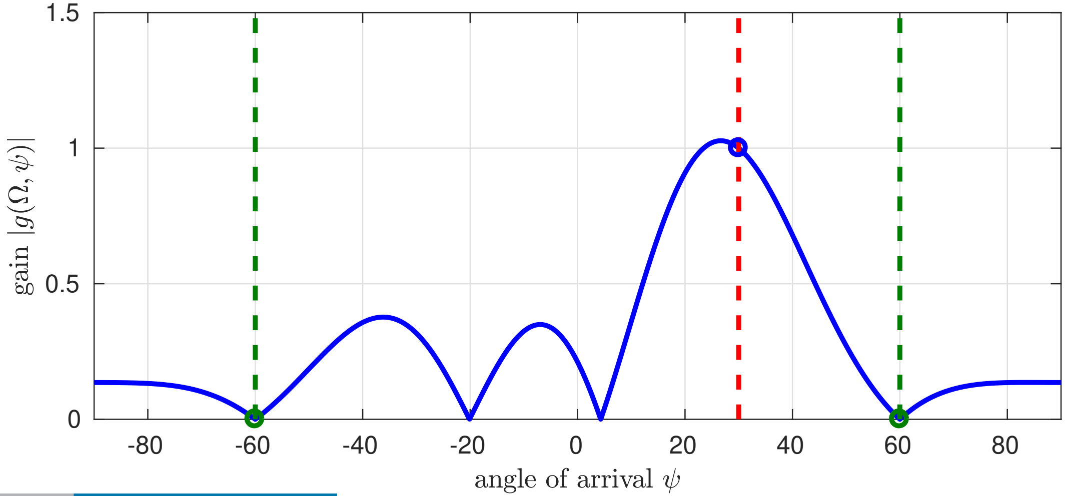

Beamforming Example — Variable Interferer II

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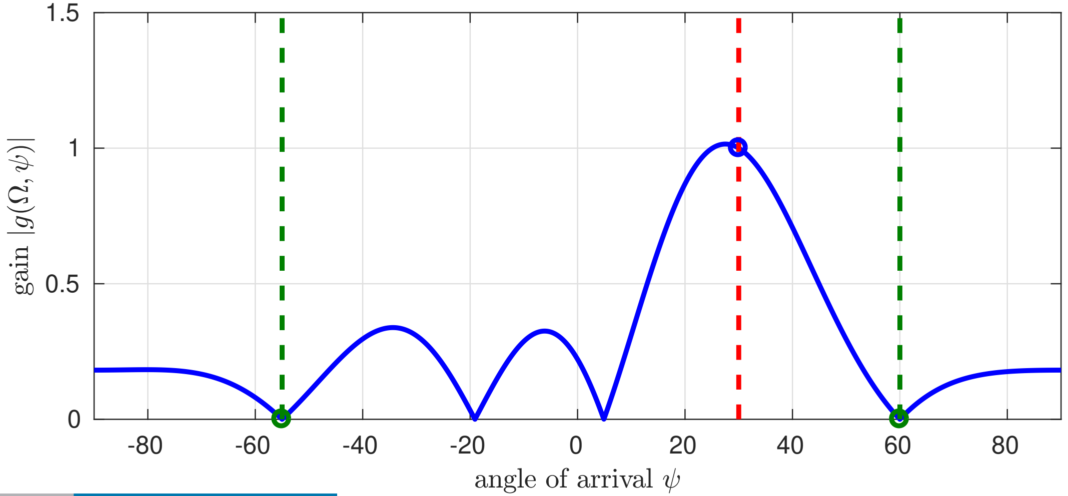

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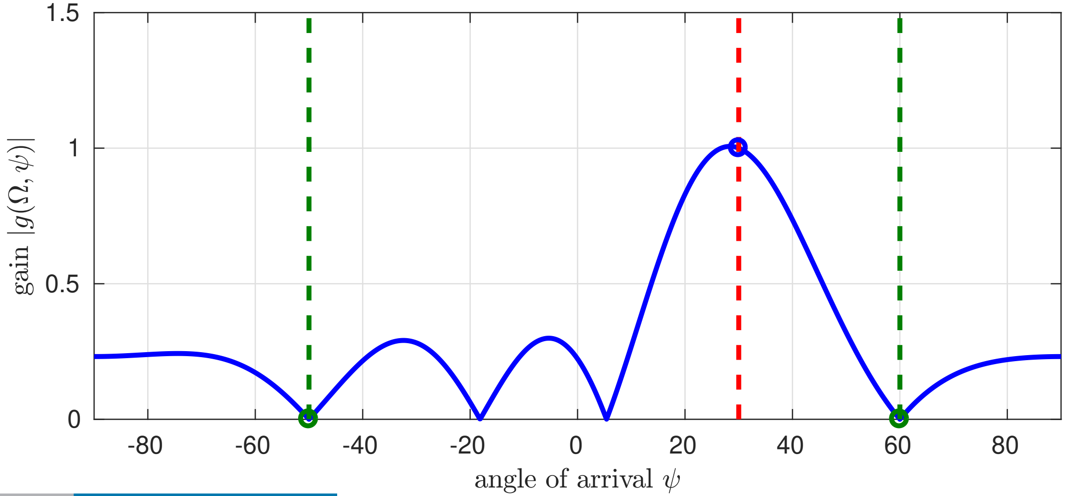

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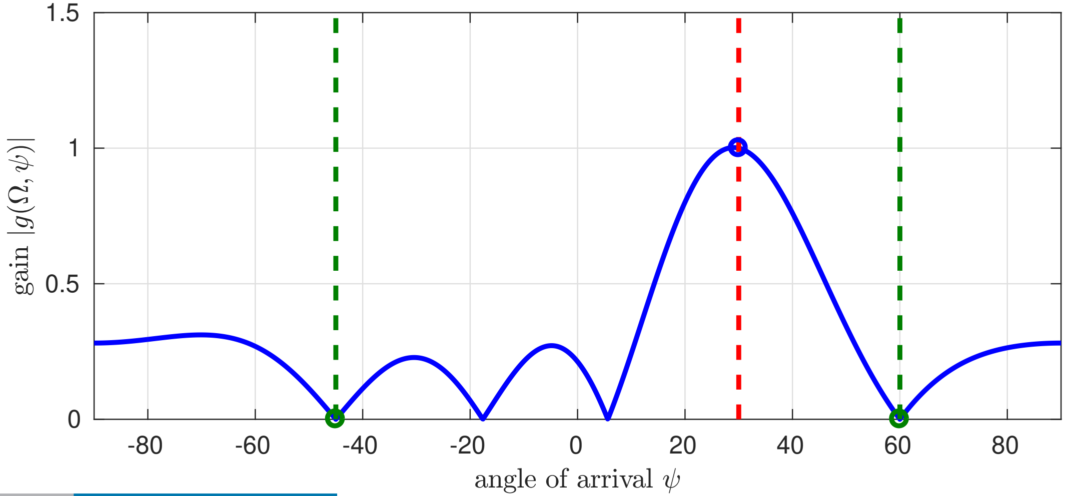

Beamforming Example — Variable Interferer II

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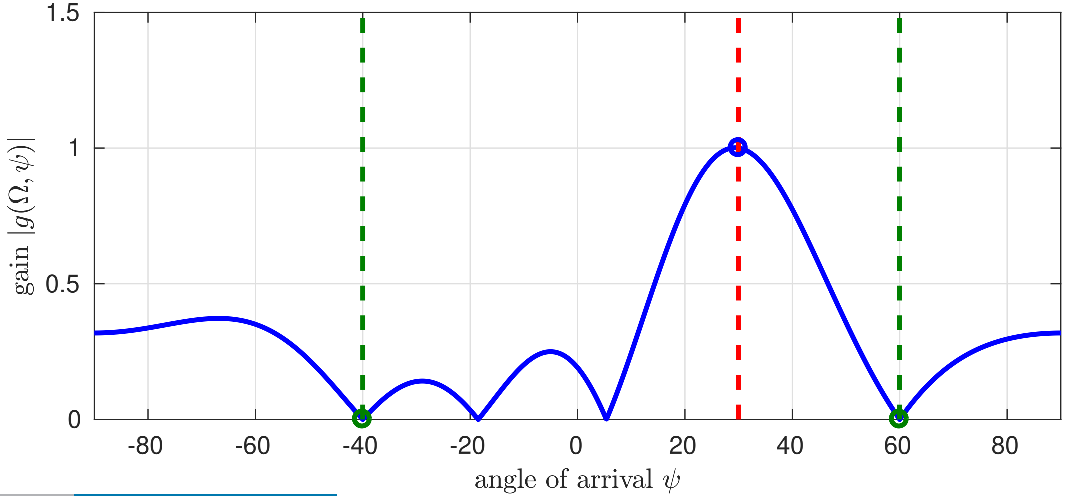

Beamforming Example — Variable Interferer II

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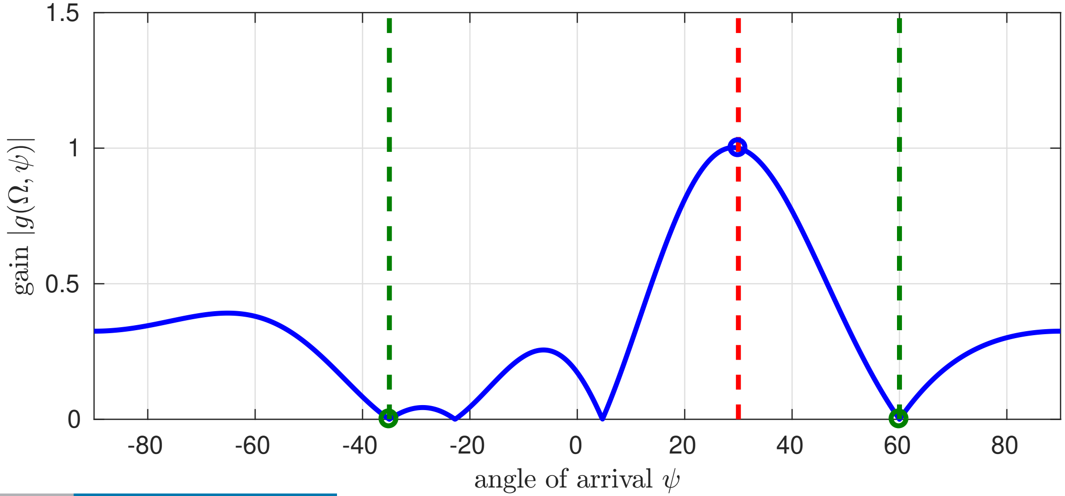

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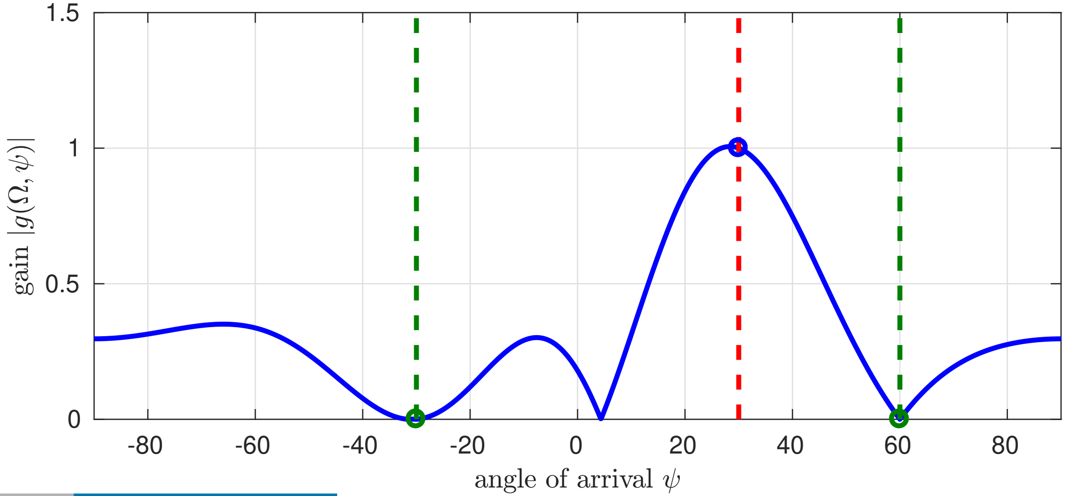

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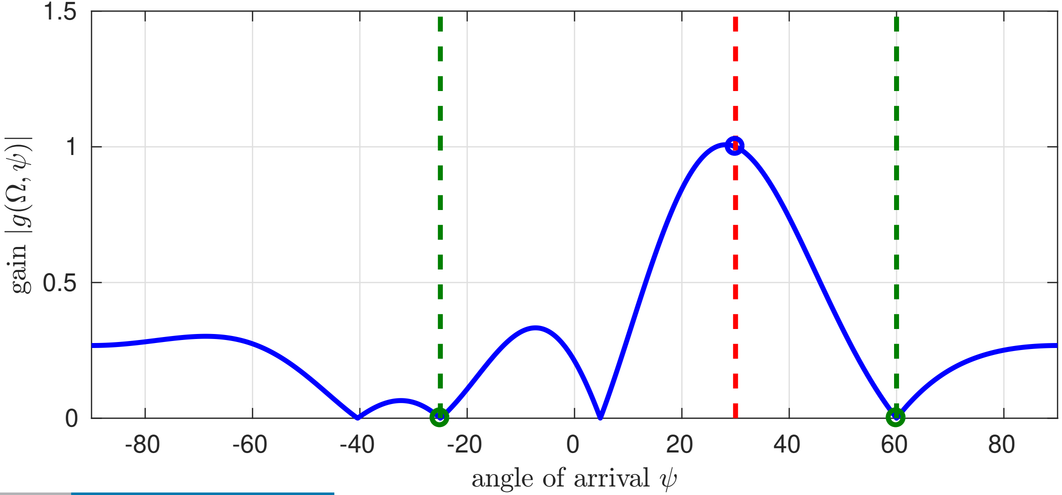

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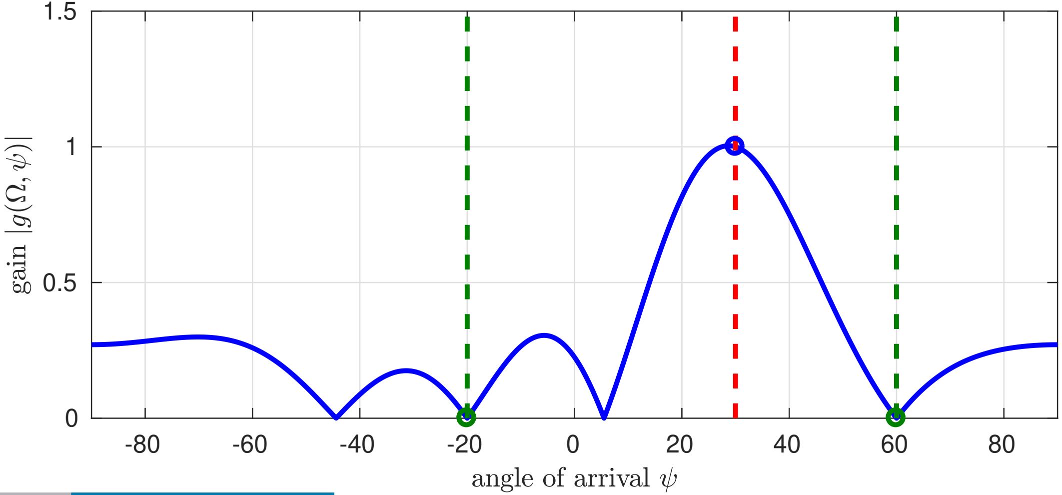

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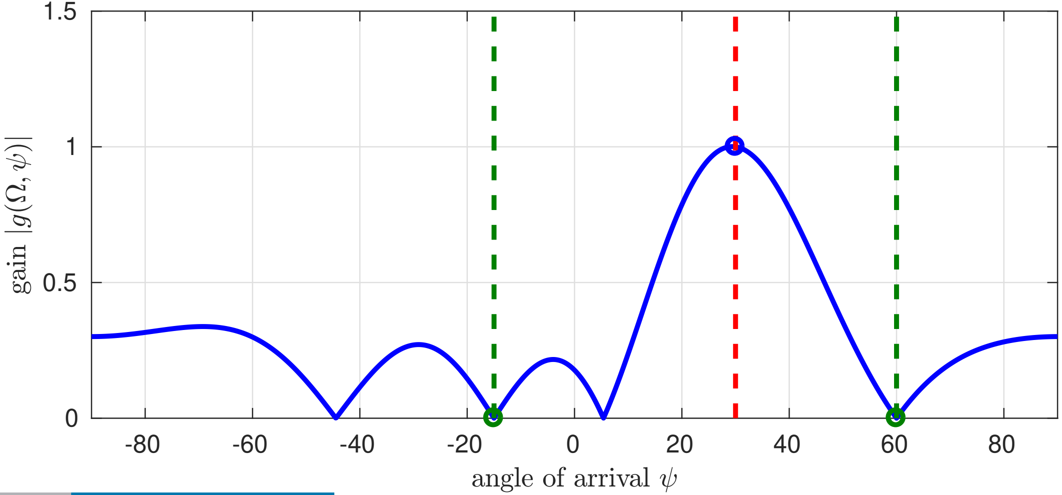

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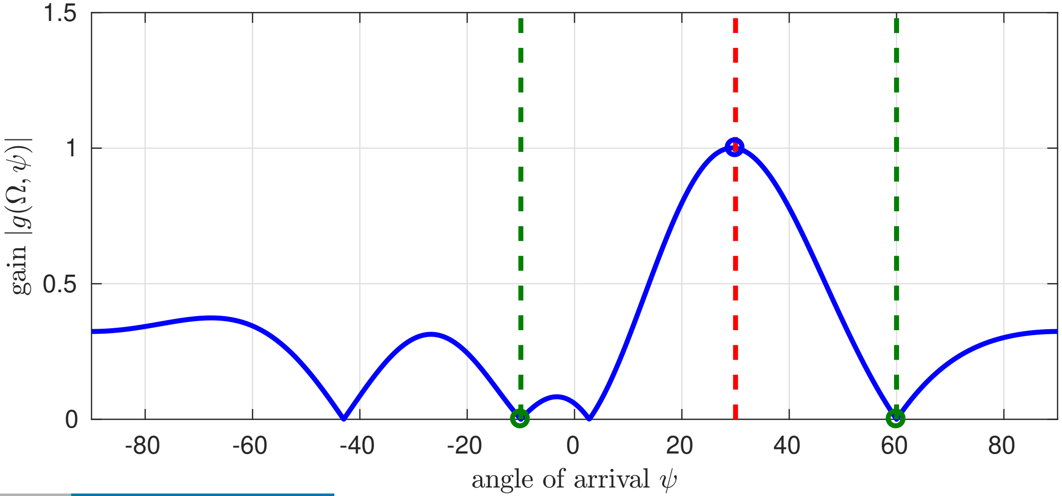

Beamforming Example — Variable Interferer II

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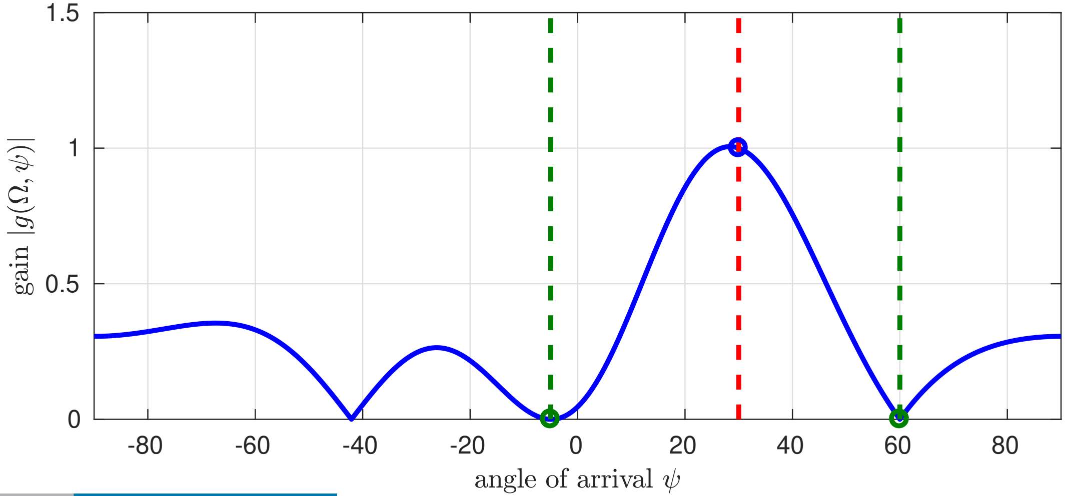

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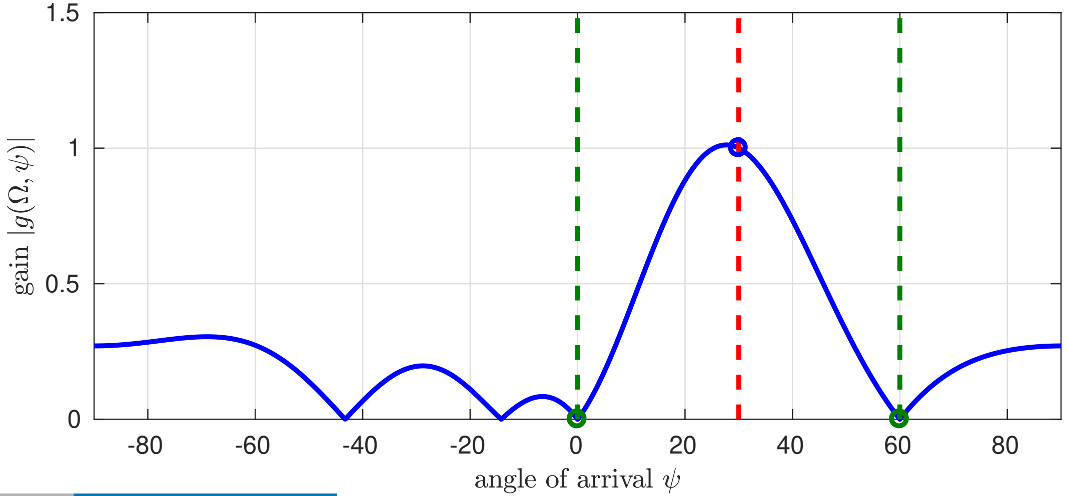

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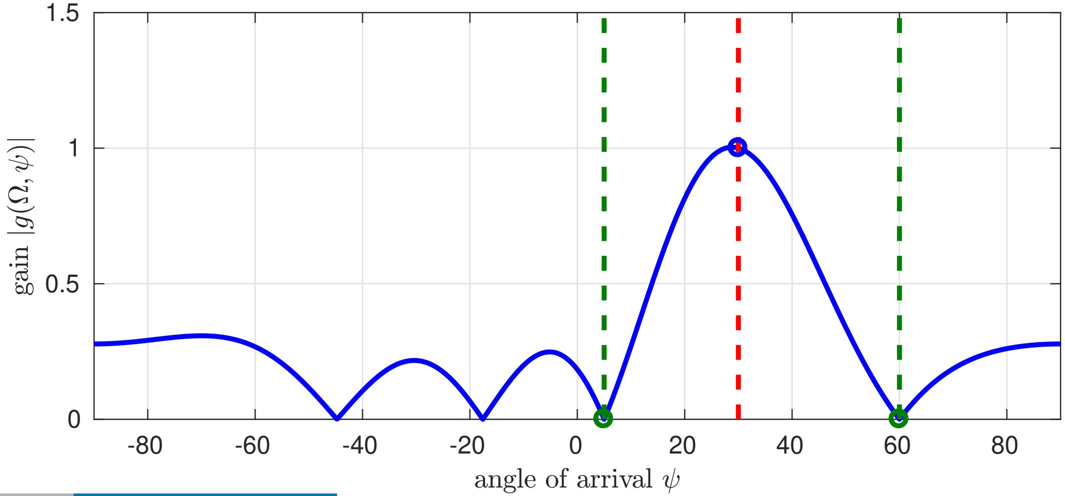

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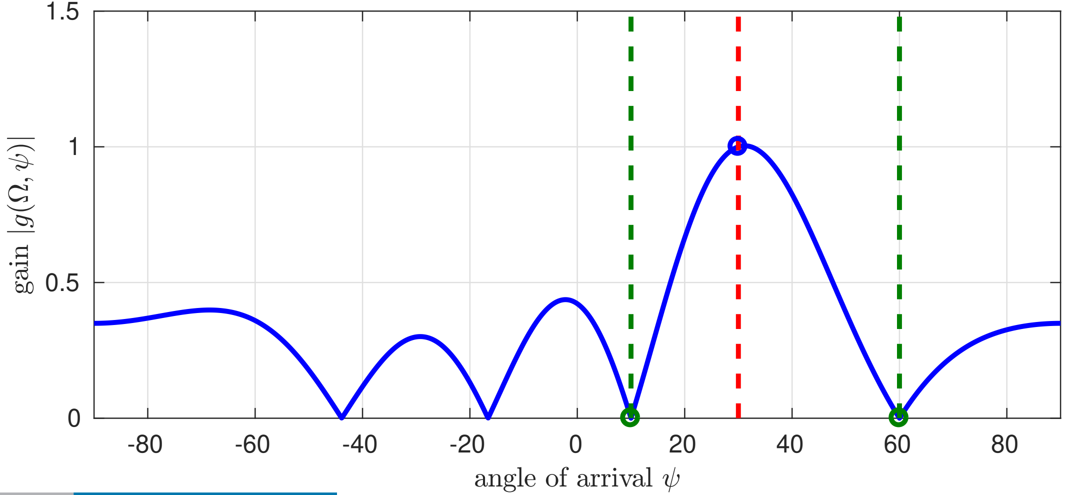

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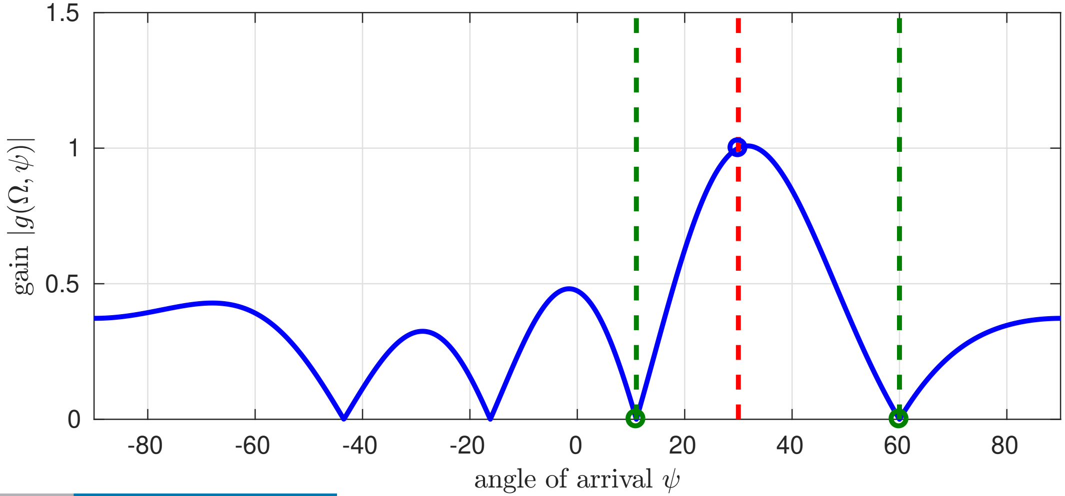

Beamforming Example — Variable Interferer II

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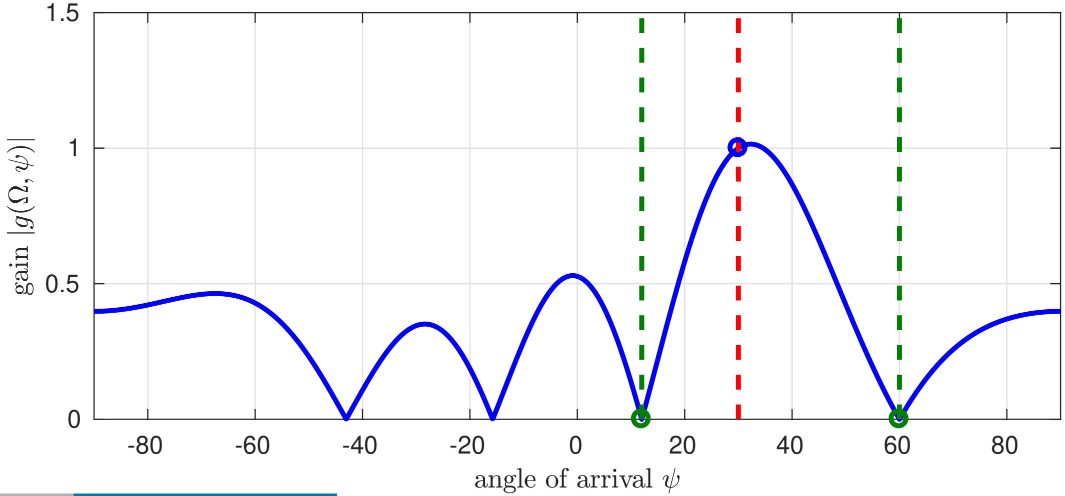
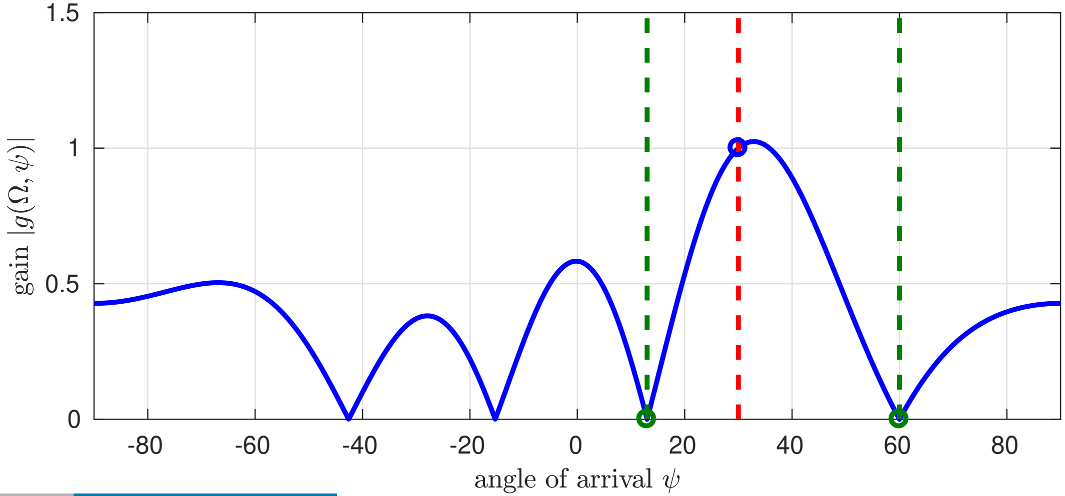


Beamforming Example — Variable Interferer II

- $M = 5$ sensors, SOI $\theta_0 = 30^\circ$, one fixed and one variable interferer:



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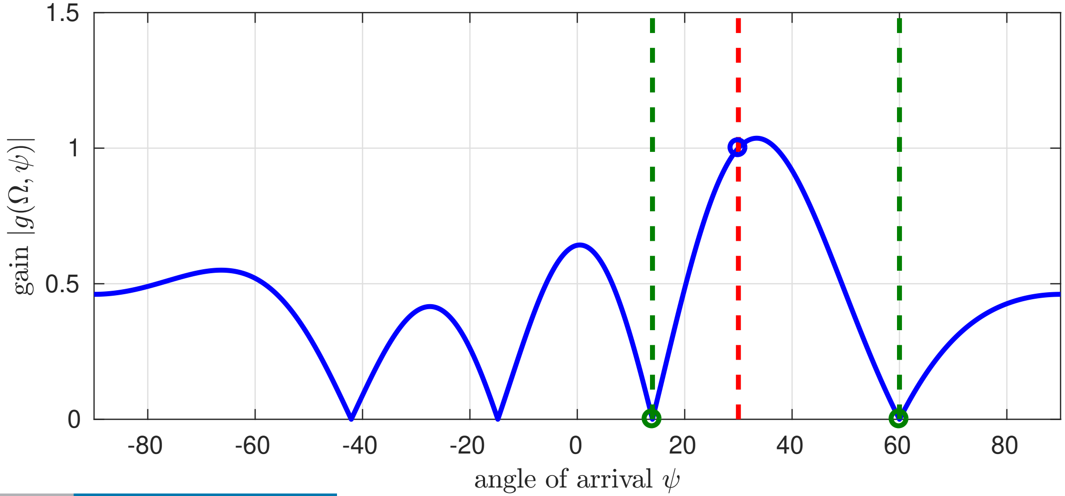

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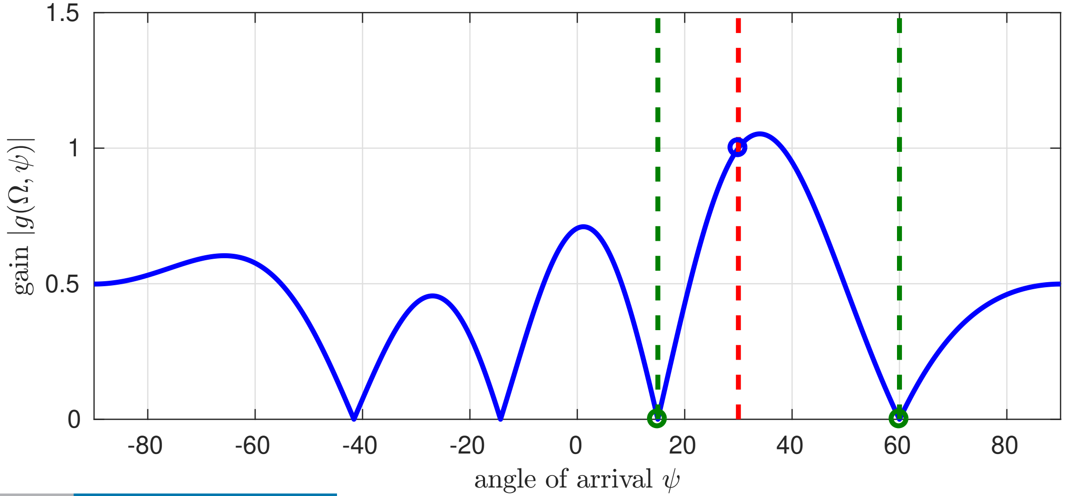

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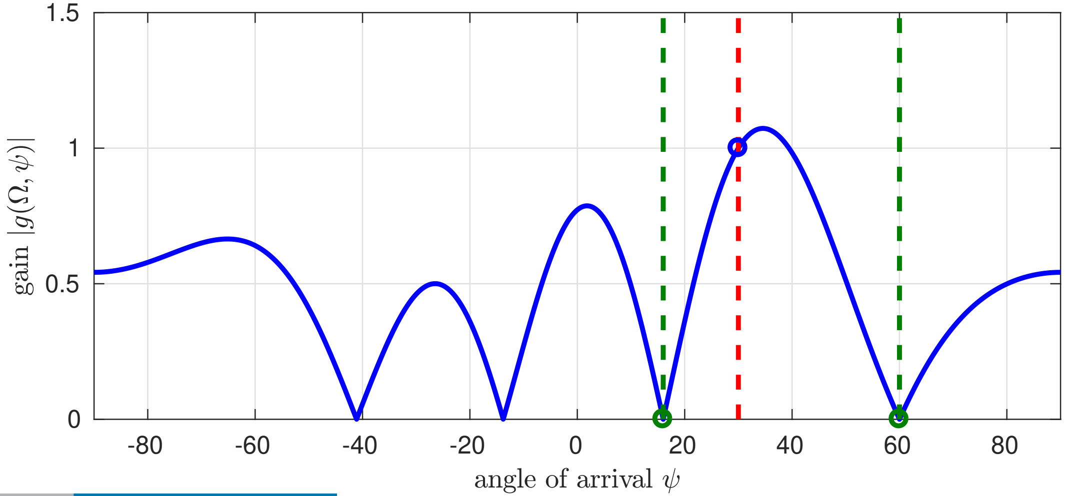

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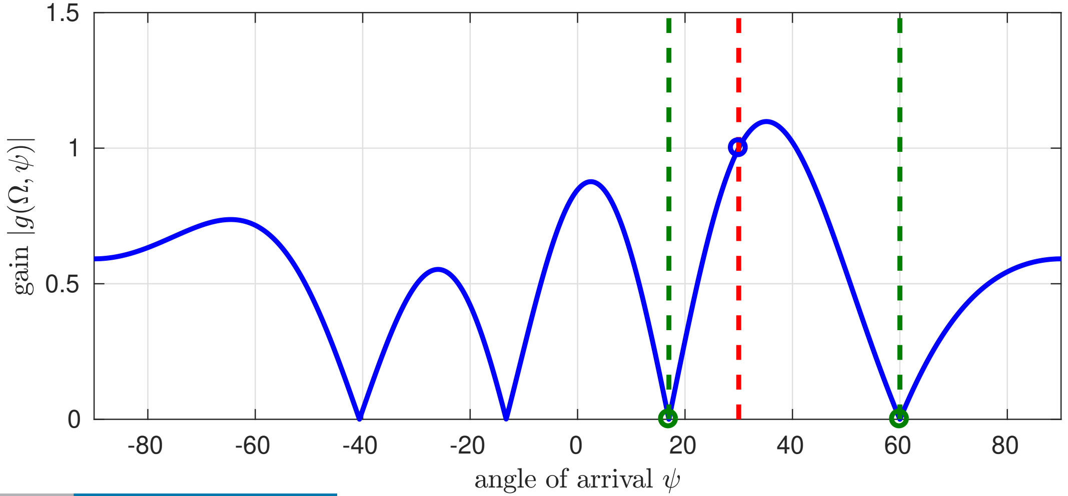

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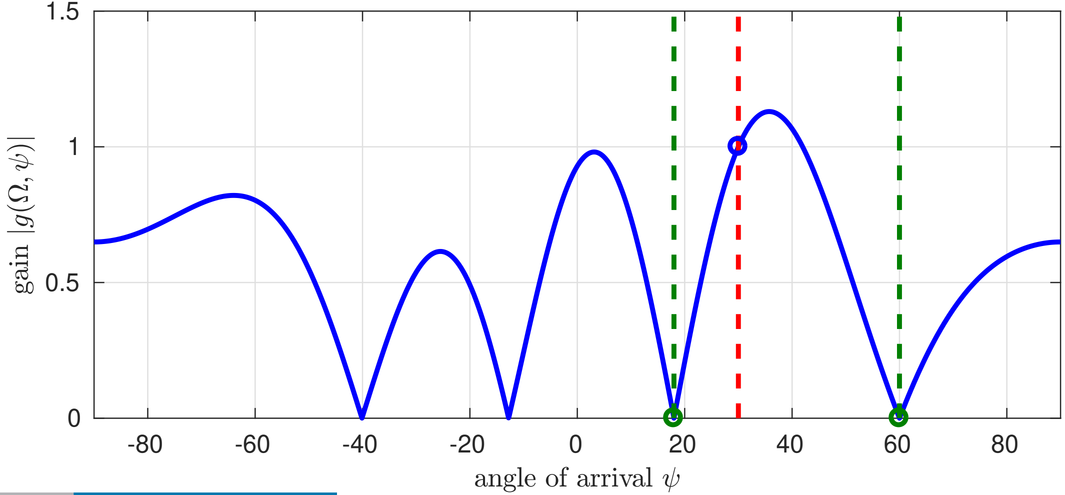

Beamforming Example — Variable Interferer II

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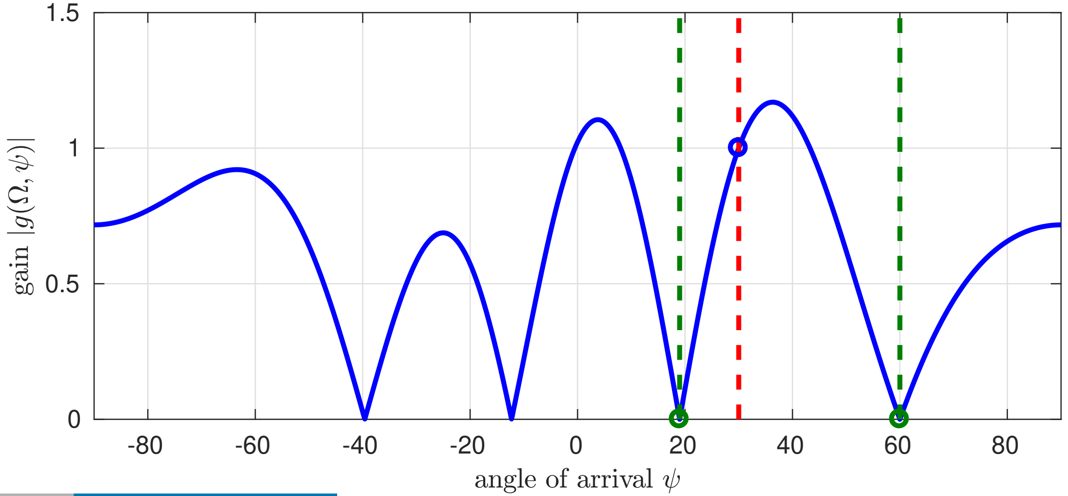

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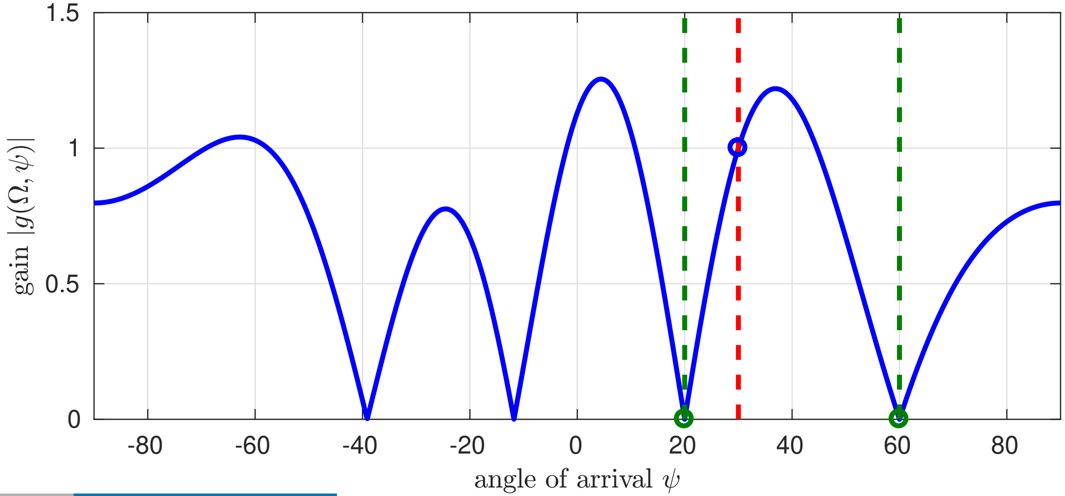

Beamforming Example — Variable Interferer II

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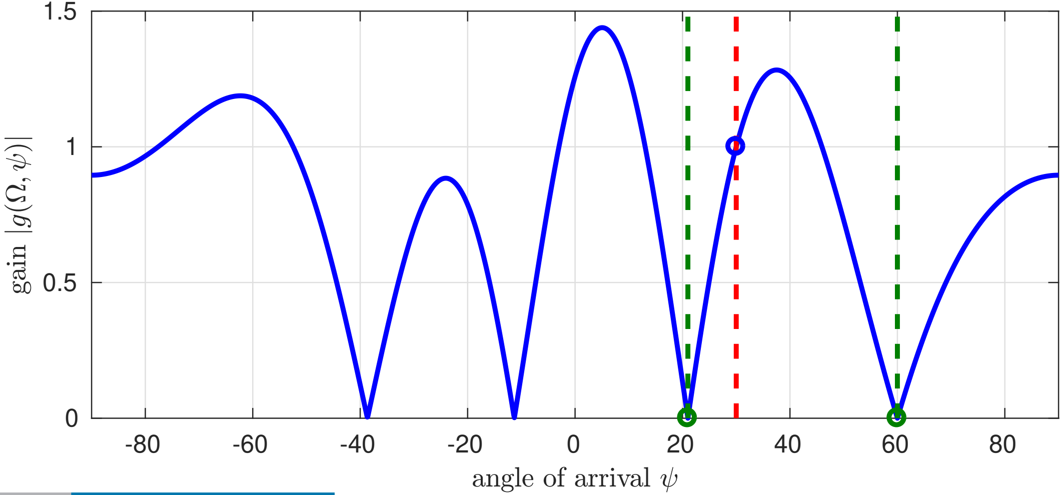

Beamforming Example — Variable Interferer II

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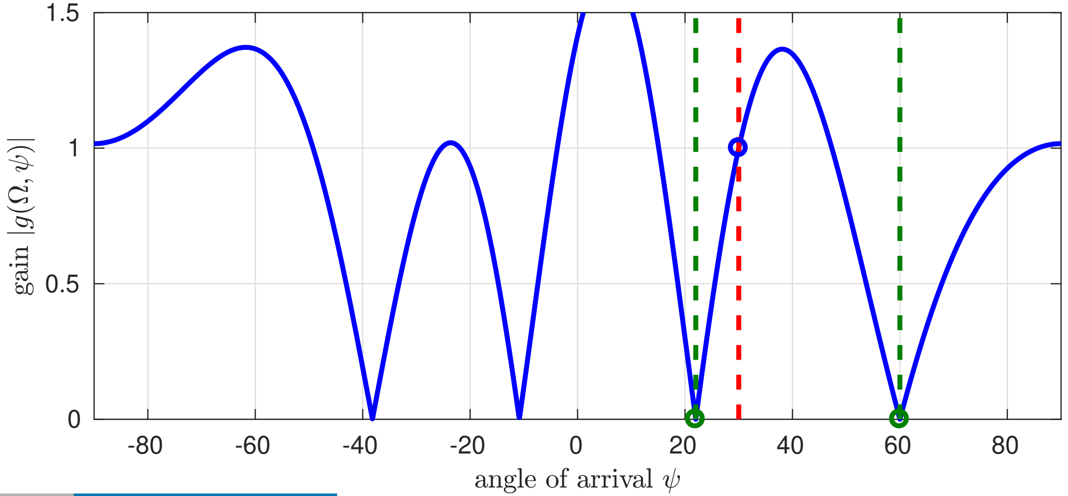

Beamforming Example — Variable Interferer II

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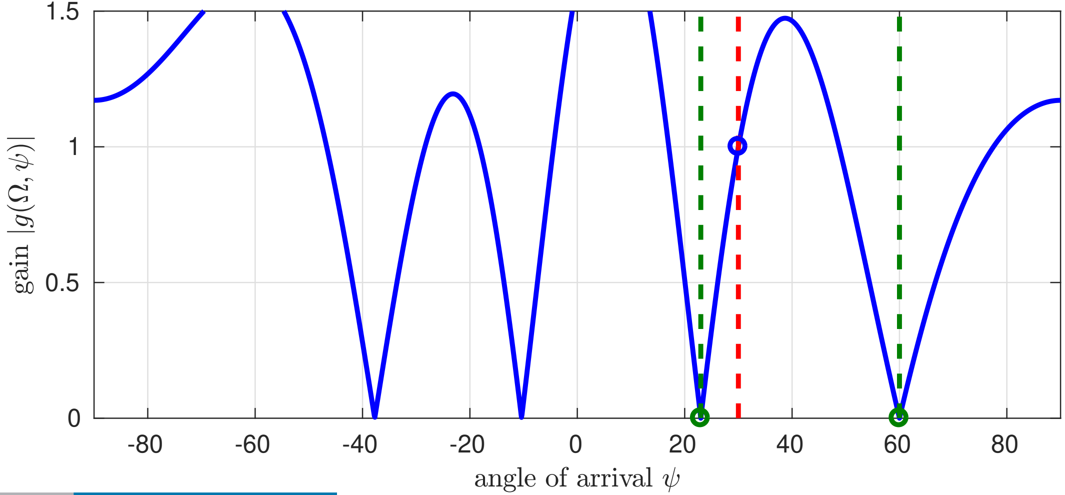

Beamforming Example — Variable Interferer II

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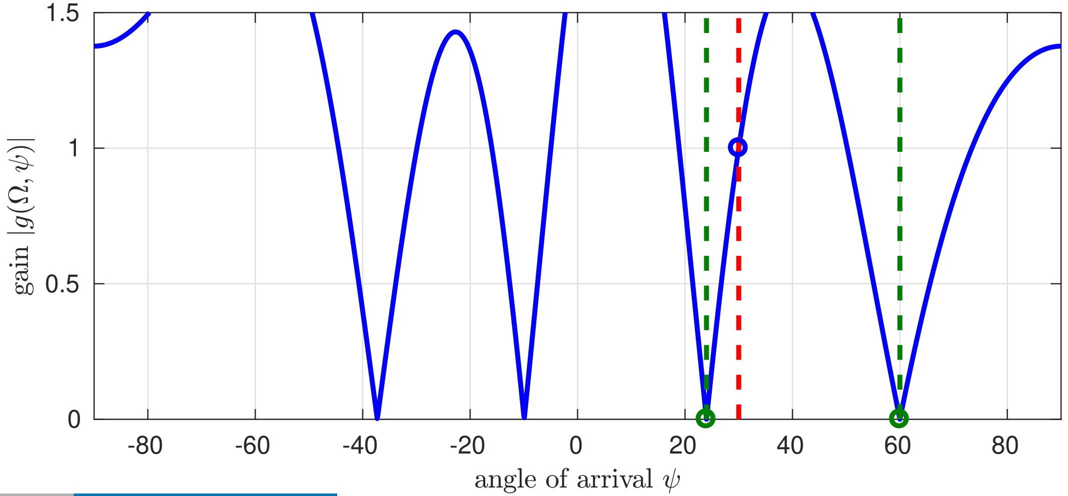

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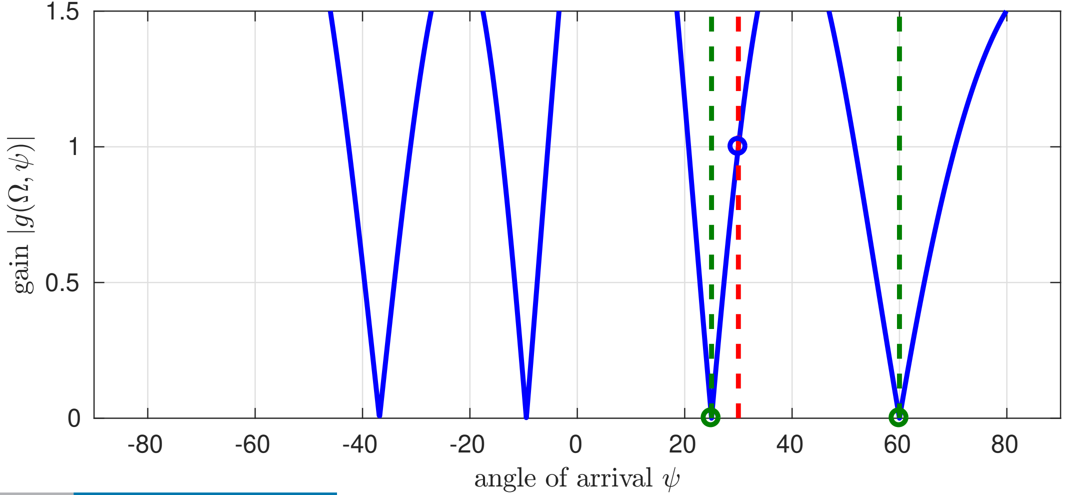
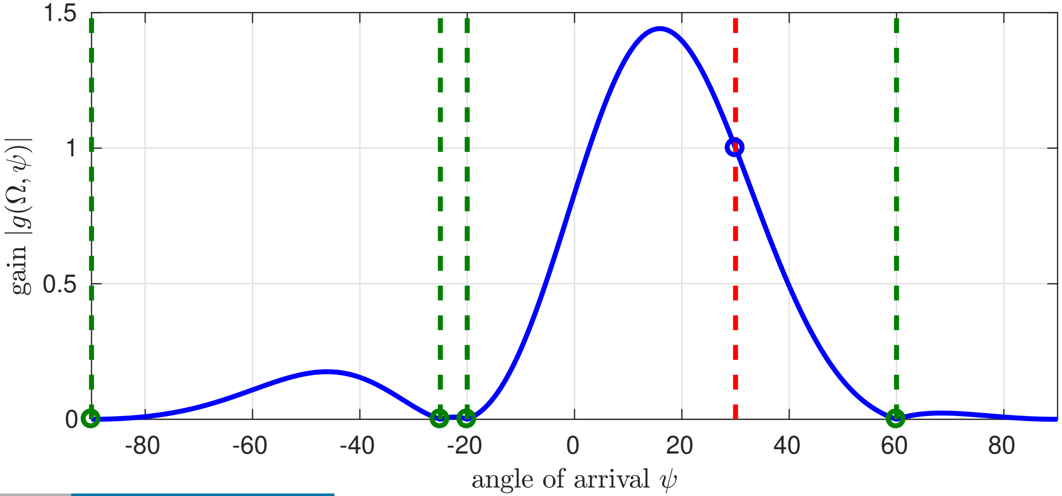


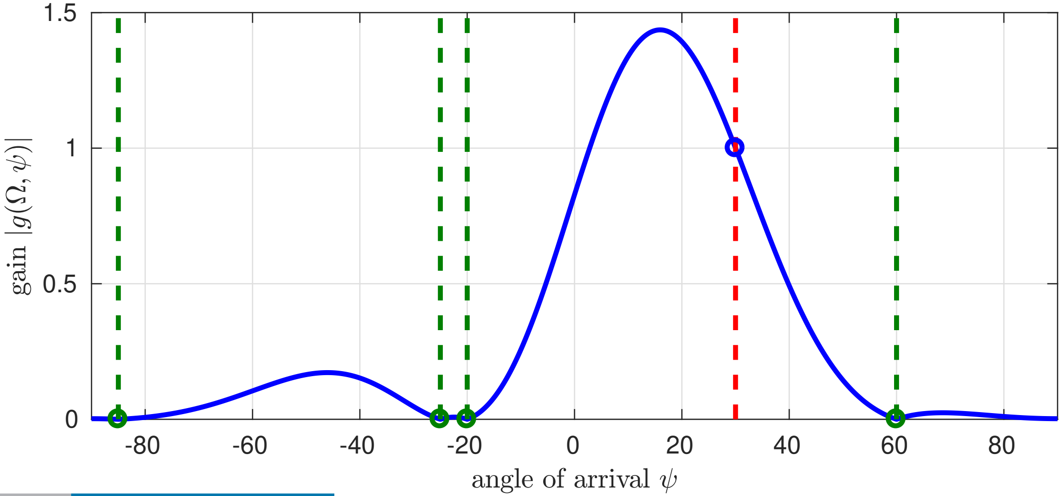
Beamforming Example — Variable Interferer III

- $M = 5$ sensors, SOI $\theta_0 = 30°$, one variable and three fixed interferers:



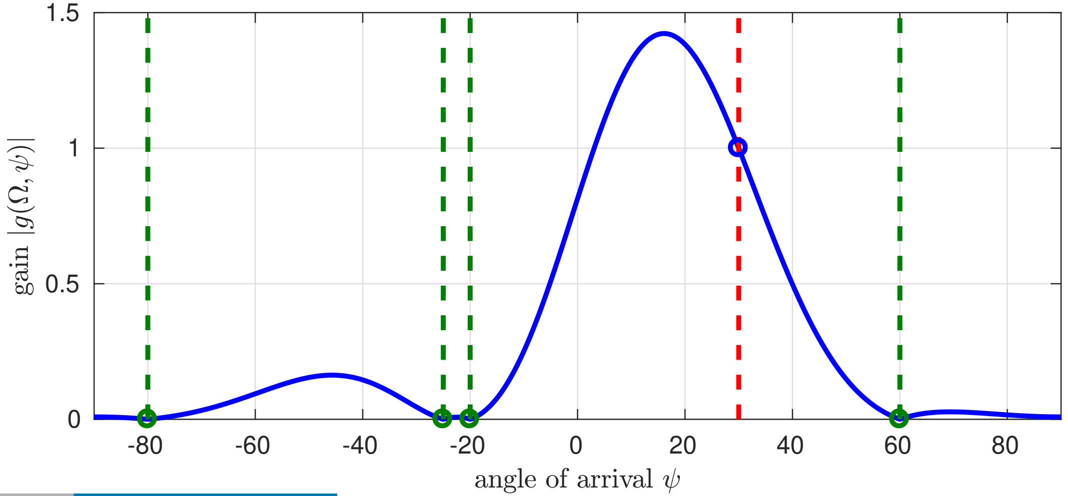
Beamforming Example — Variable Interferer III

- $M = 5$ sensors, SOI $\theta_0 = 30°$, one variable and three fixed interferers:



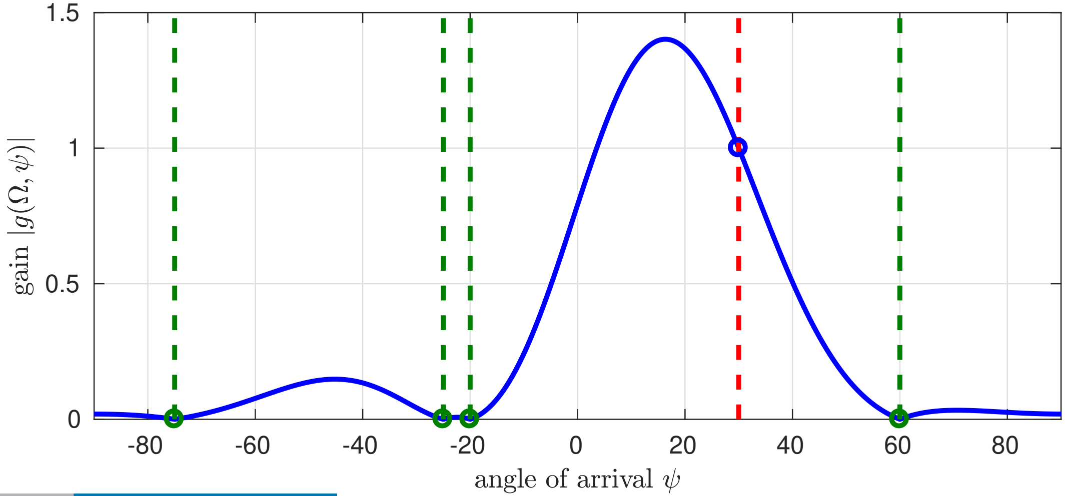
Beamforming Example — Variable Interferer III

- $M = 5$ sensors, SOI $\theta_0 = 30°$, one variable and three fixed interferers:



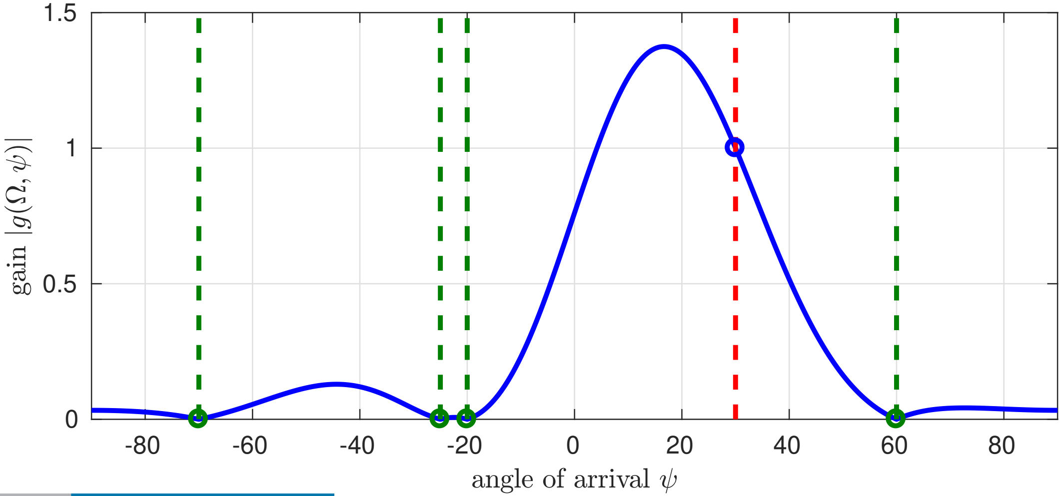
Beamforming Example — Variable Interferer III

- $M = 5$ sensors, SOI $\theta_0 = 30°$, one variable and three fixed interferers:



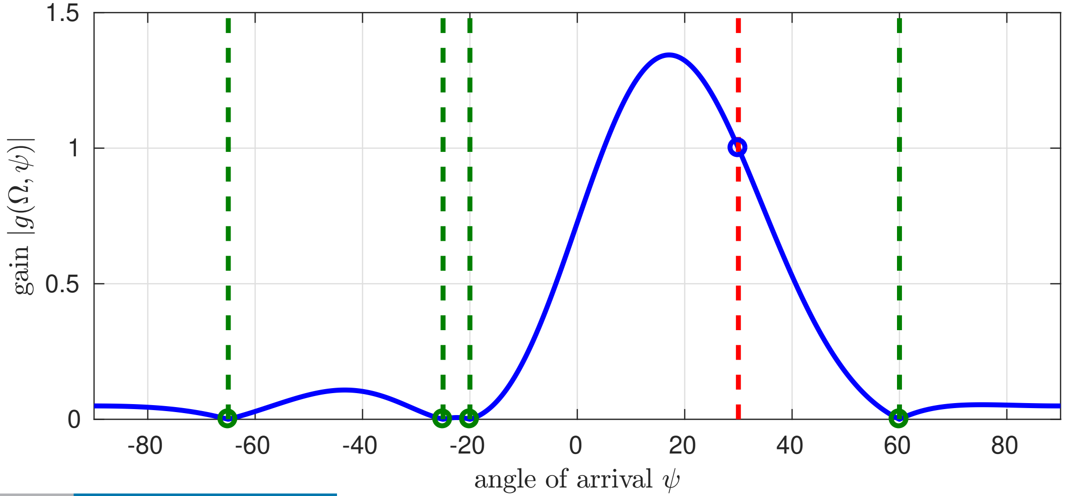
Beamforming Example — Variable Interferer III

- $M = 5$ sensors, SOI $\theta_0 = 30°$, one variable and three fixed interferers:



Beamforming Example — Variable Interferer III

- $M = 5$ sensors, SOI $\theta_0 = 30°$, one variable and three fixed interferers:



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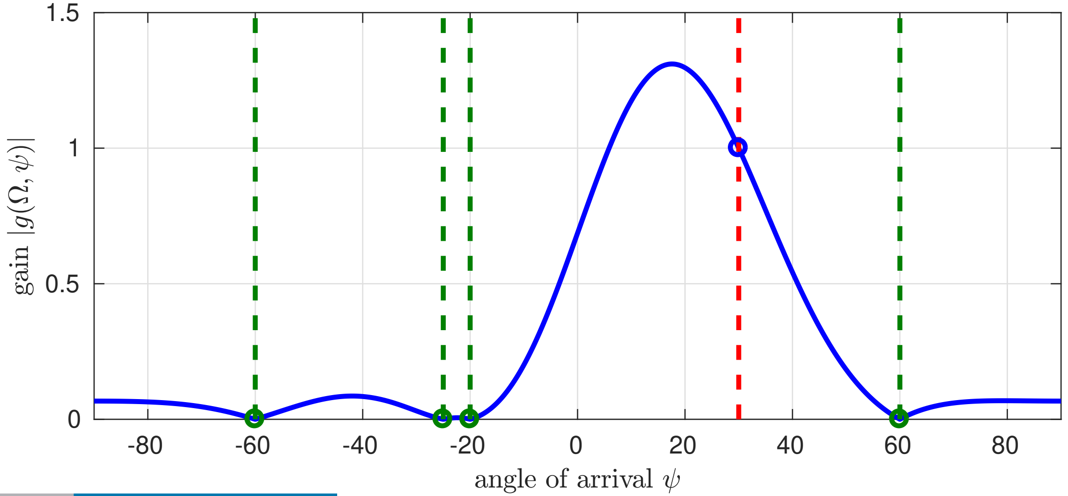

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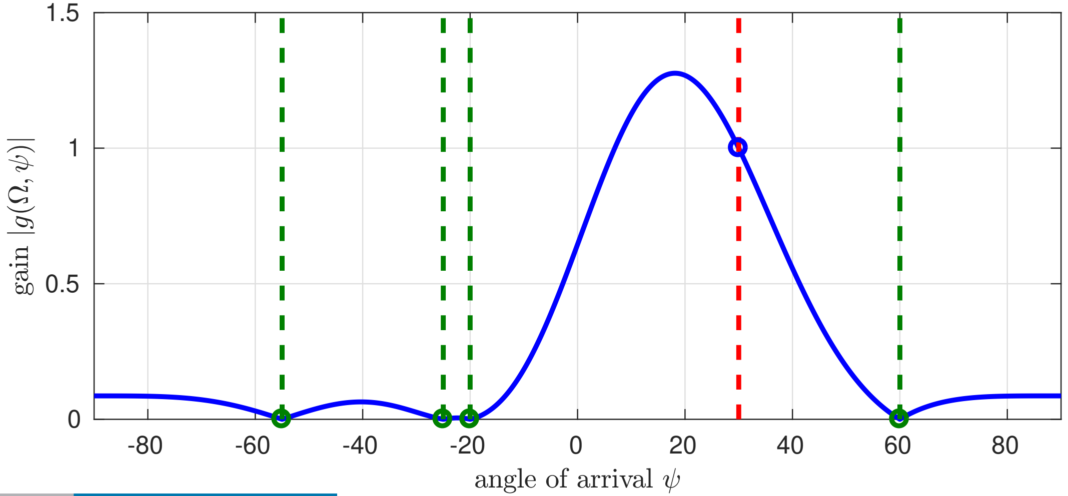

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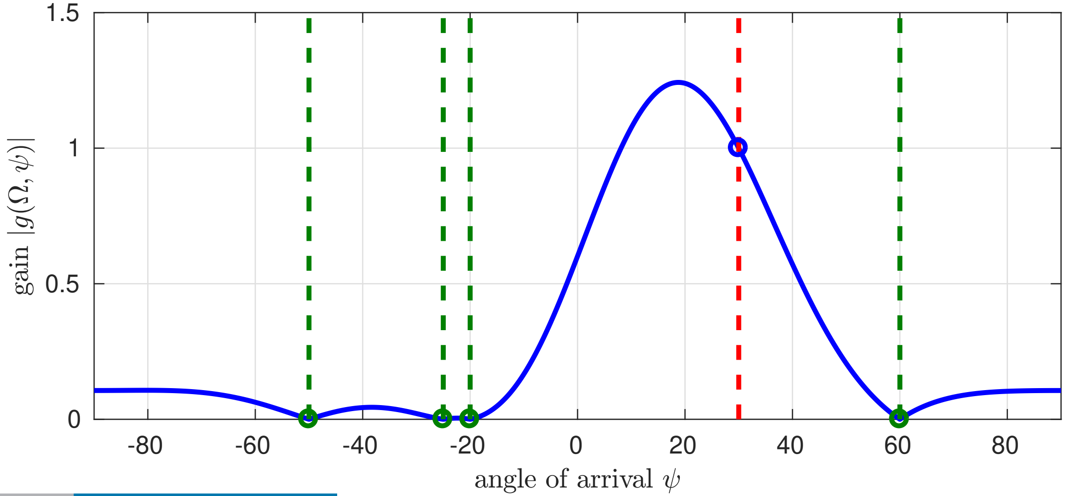

Beamforming Example — Variable Interferer III

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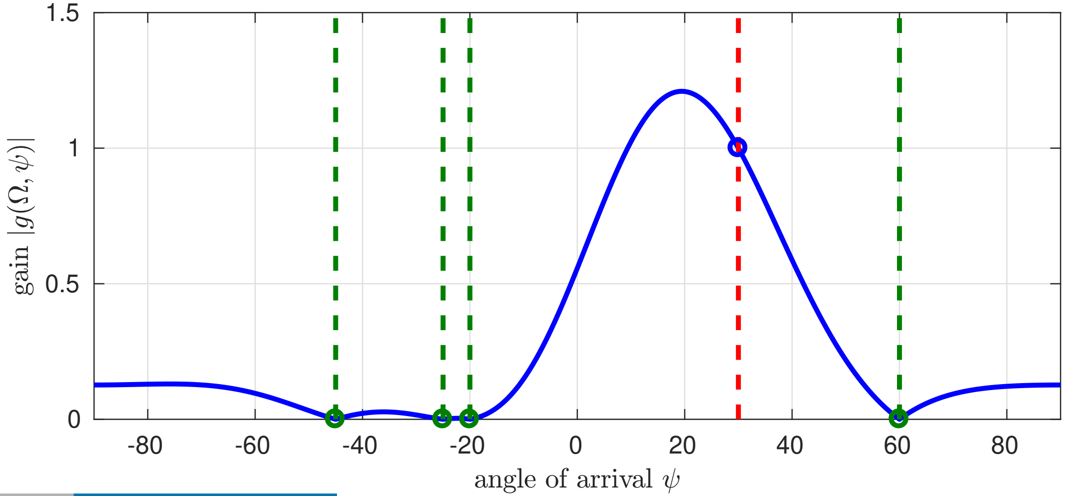
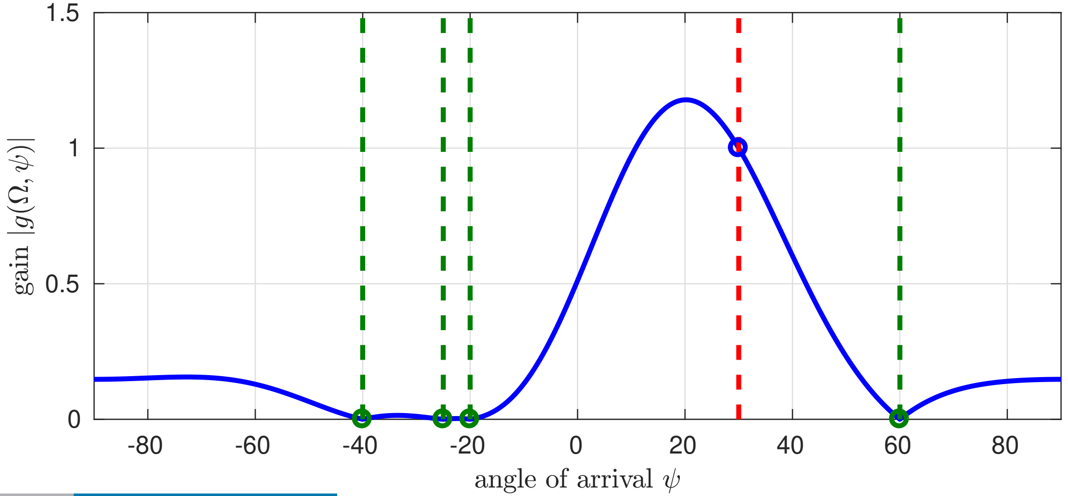


Beamforming Example — Variable Interferer III

- $M = 5$ sensors, SOI $\theta_0 = 30°$, one variable and three fixed interferers:



Beamforming Example — Variable Interferer III

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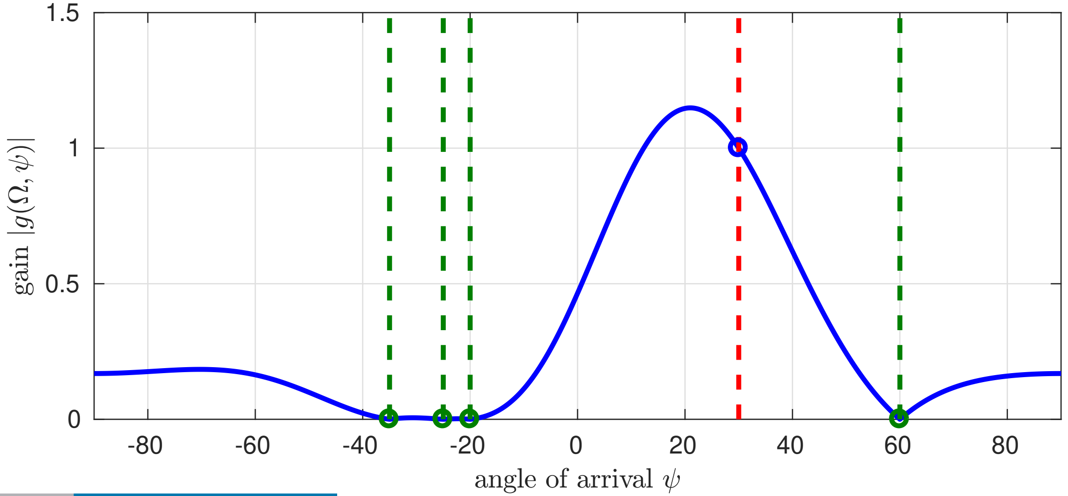
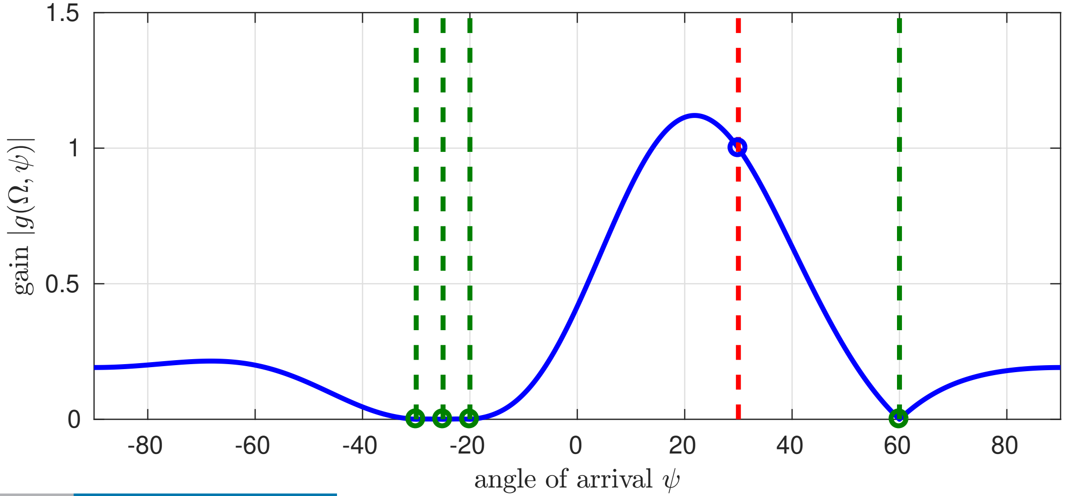


Beamforming Example — Variable Interferer III

- $M = 5$ sensors, SOI $\theta_0 = 30°$, one variable and three fixed interferers:



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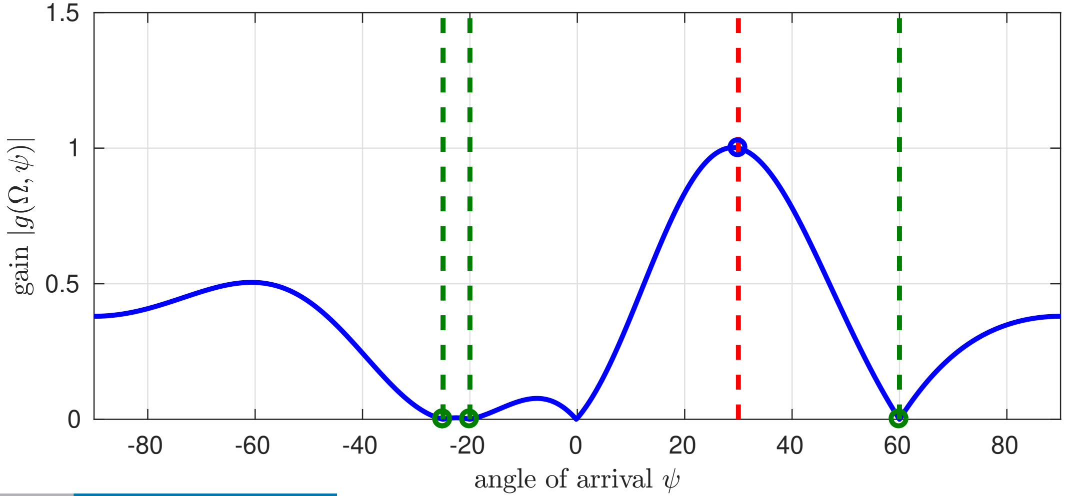

Beamforming Example — Variable Interferer III

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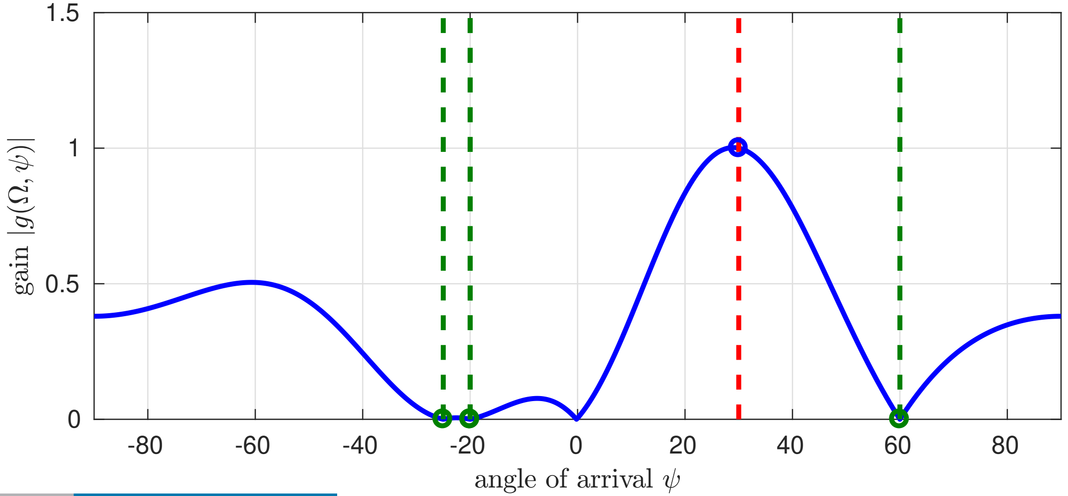

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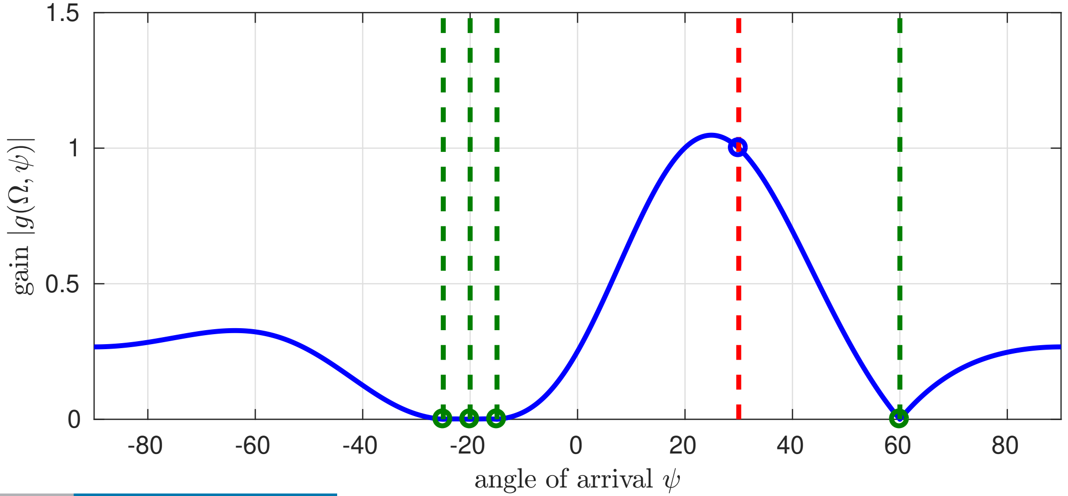

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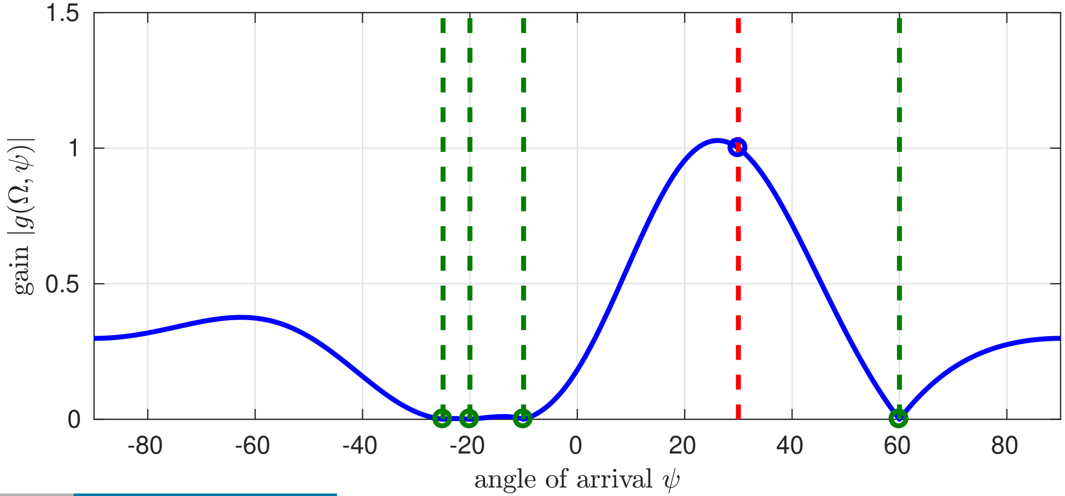

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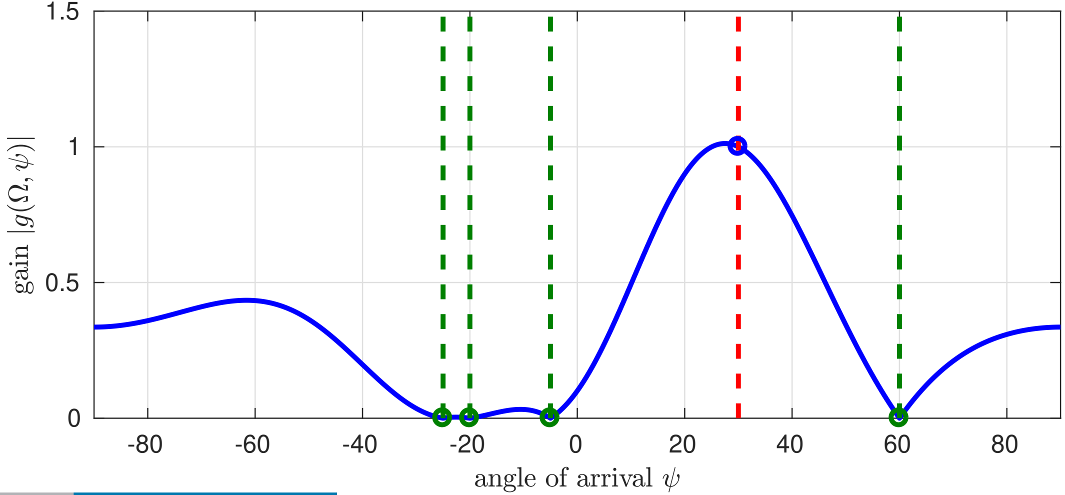

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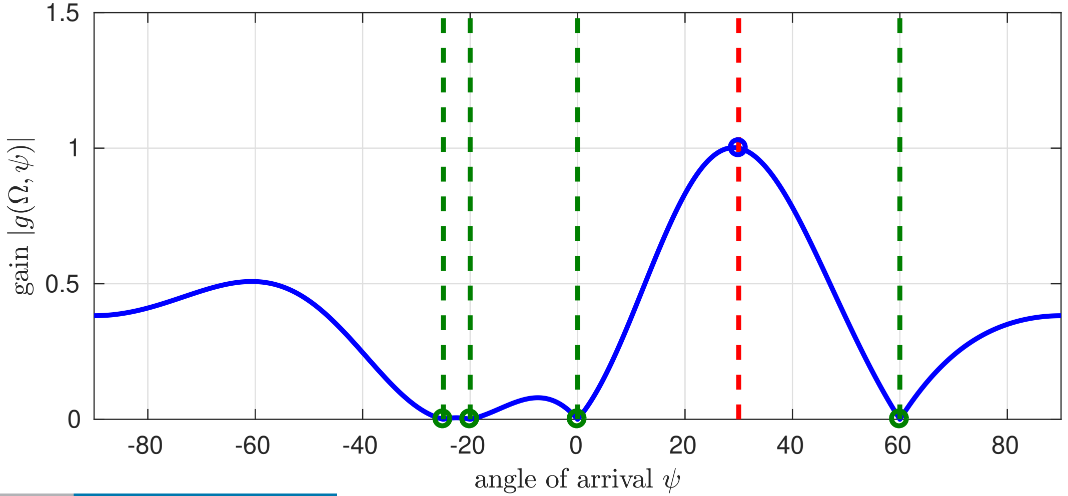

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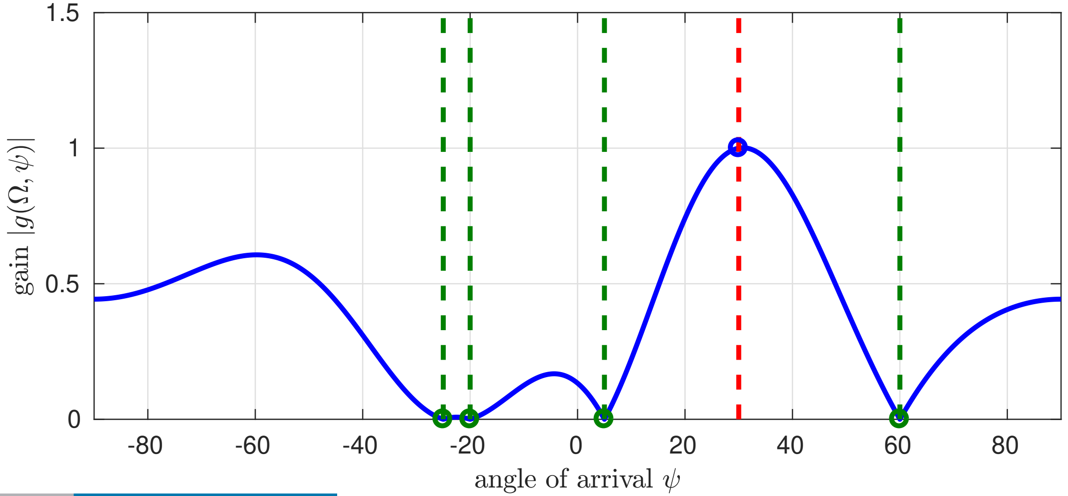

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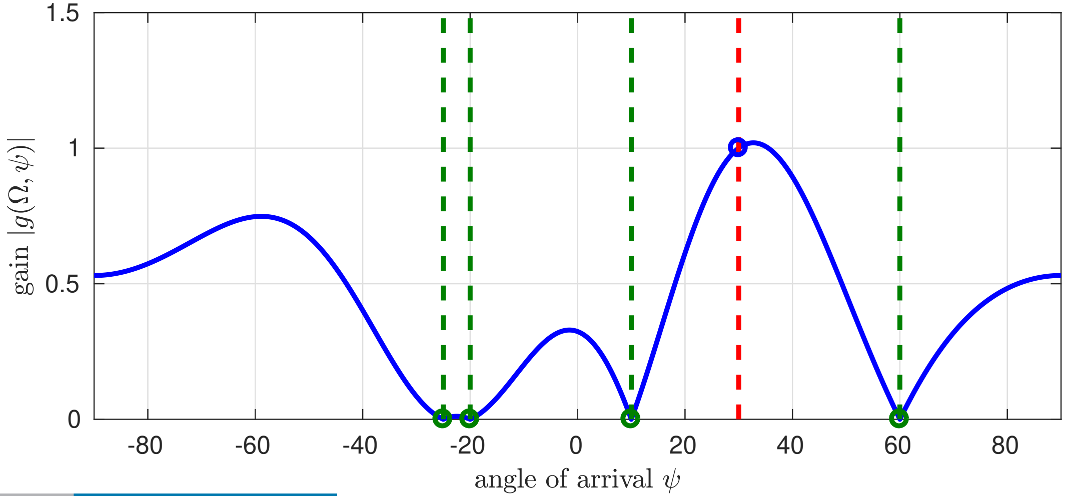

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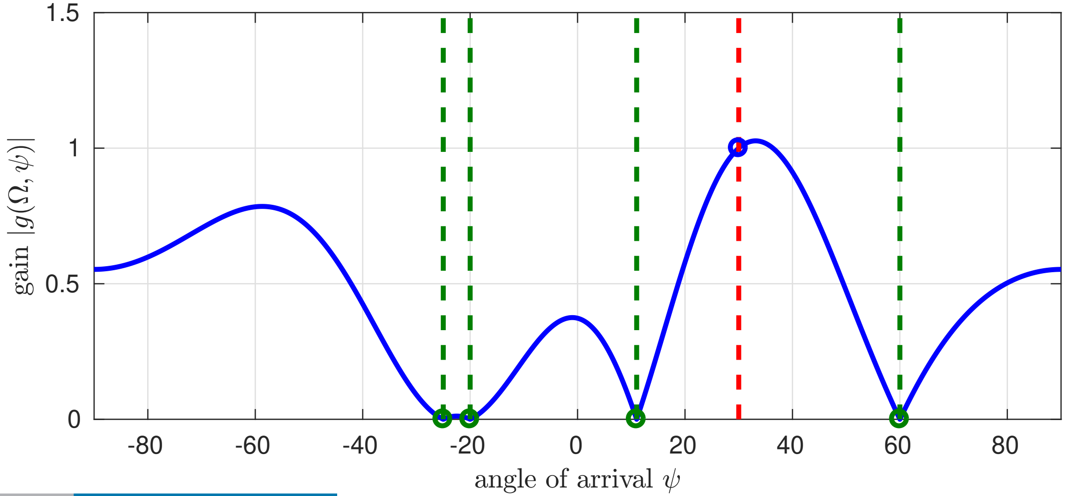

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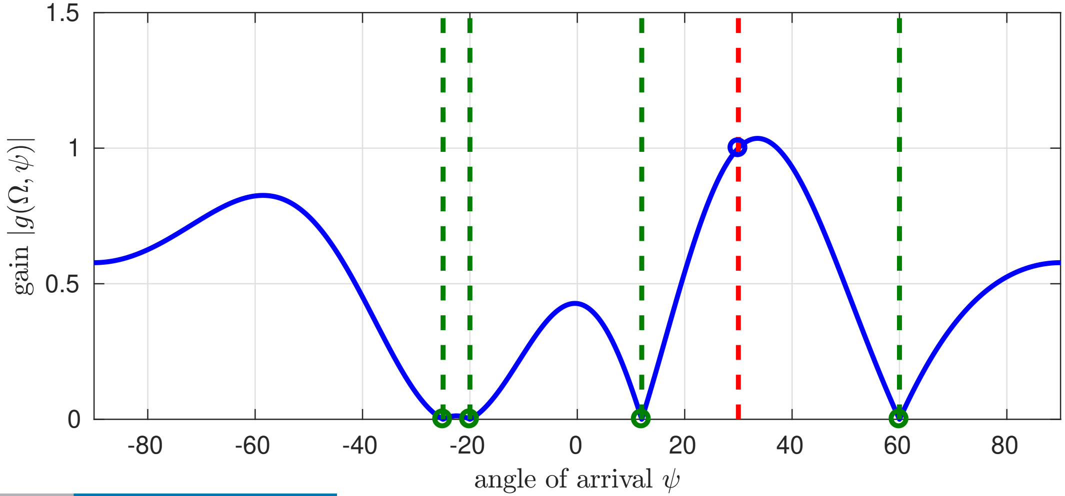

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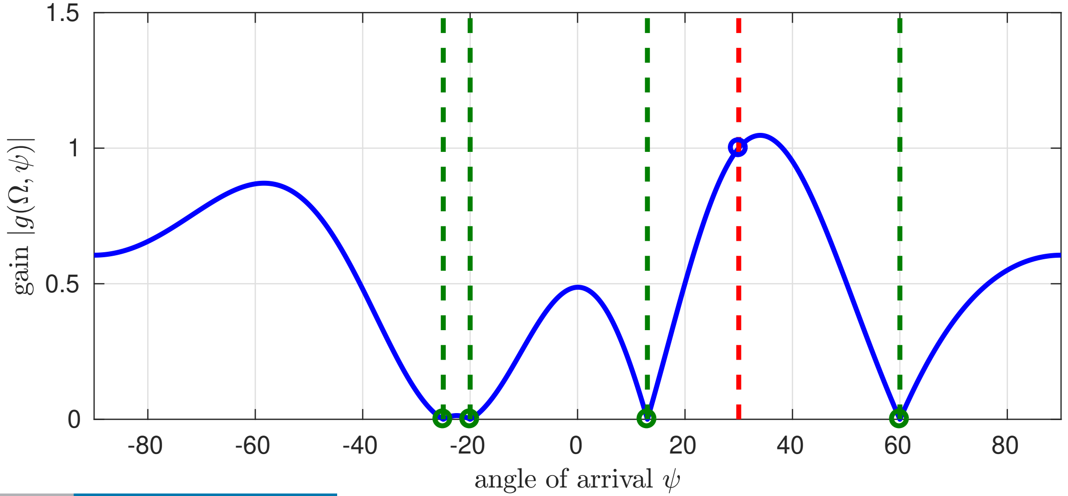

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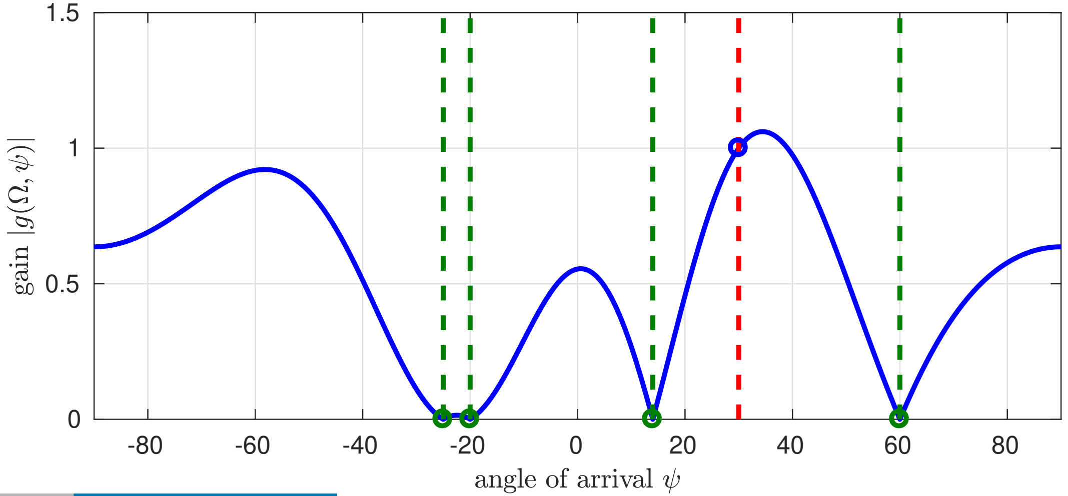

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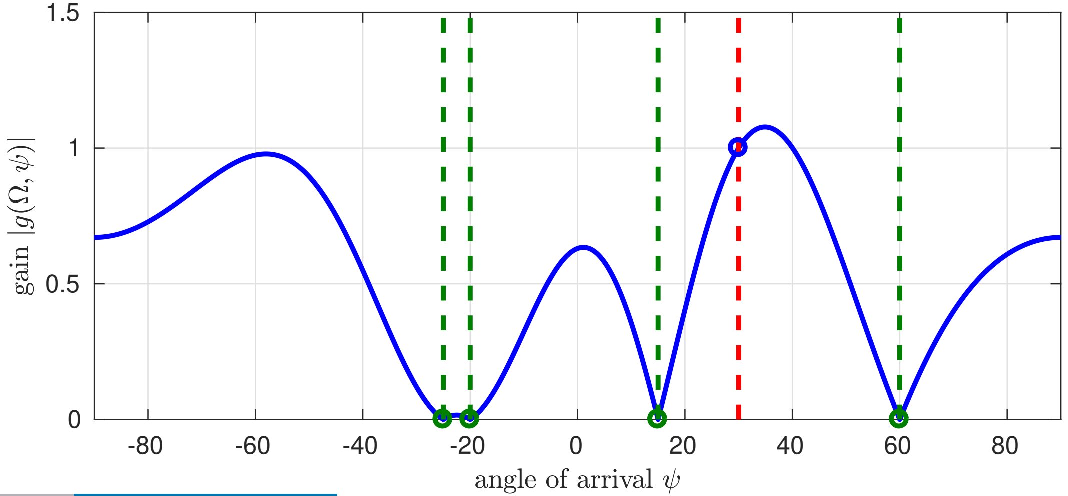

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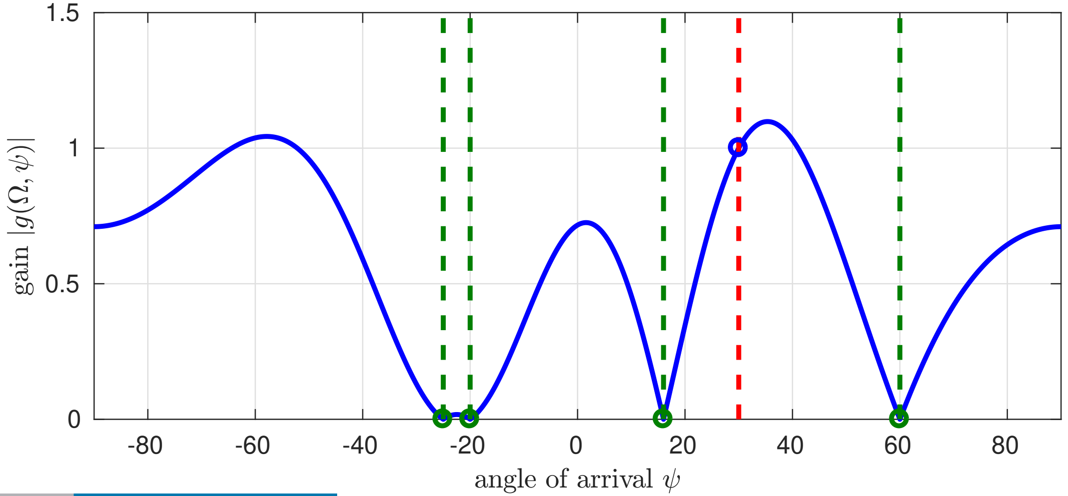

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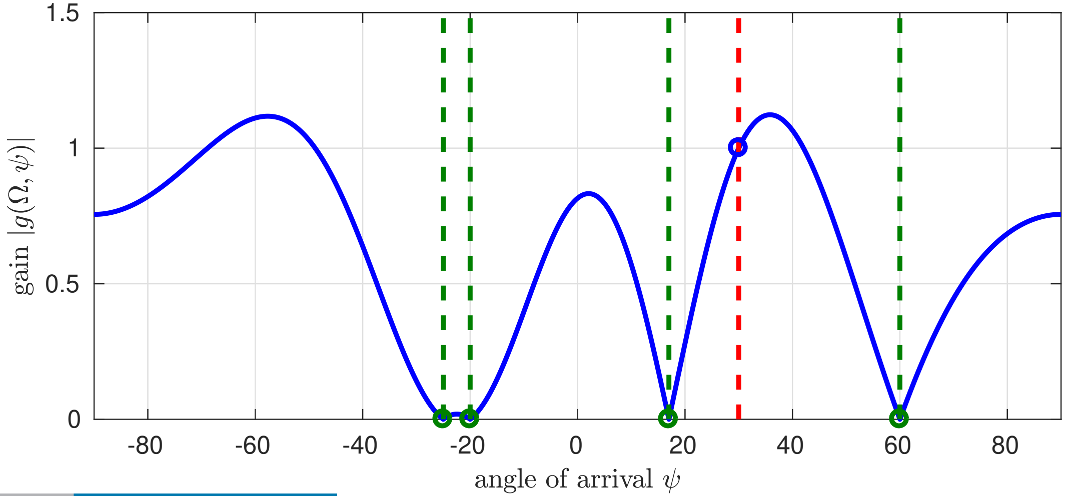

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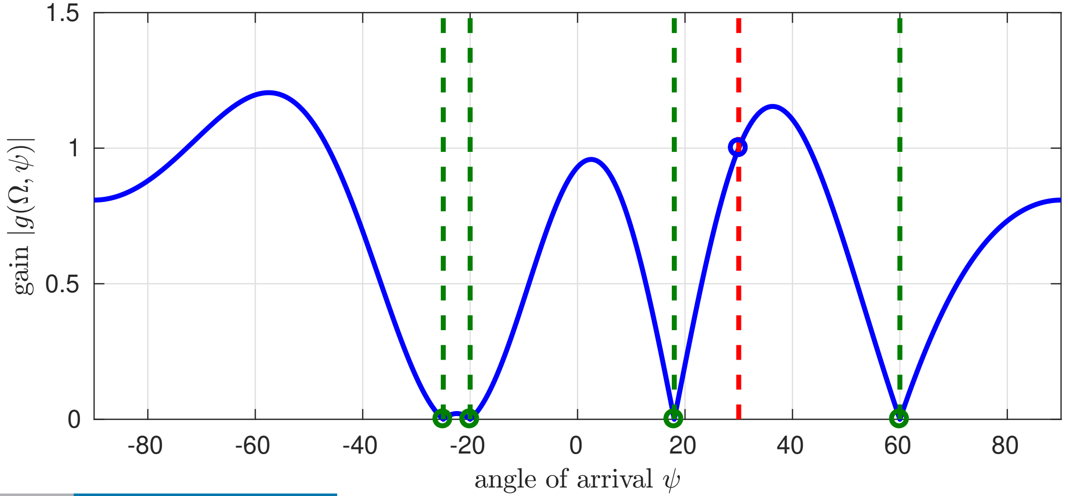

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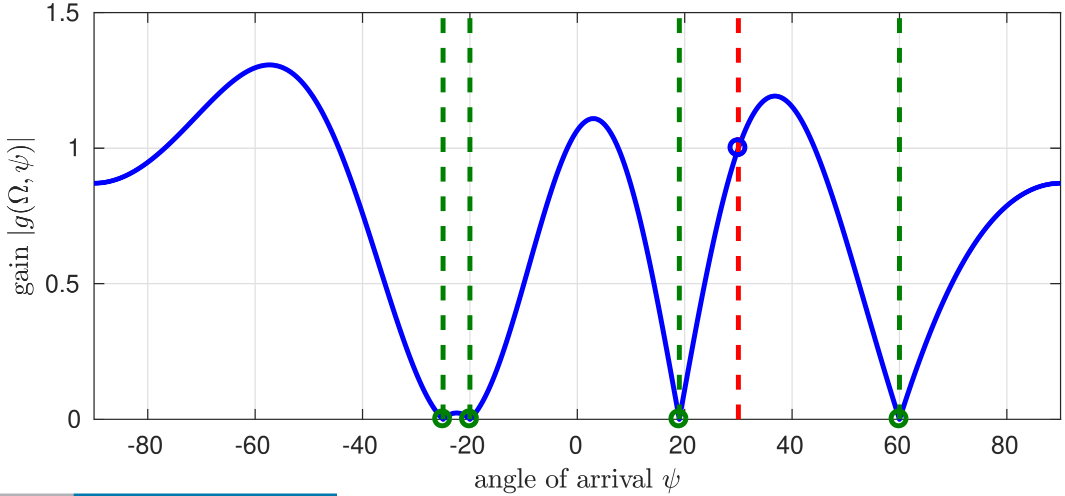

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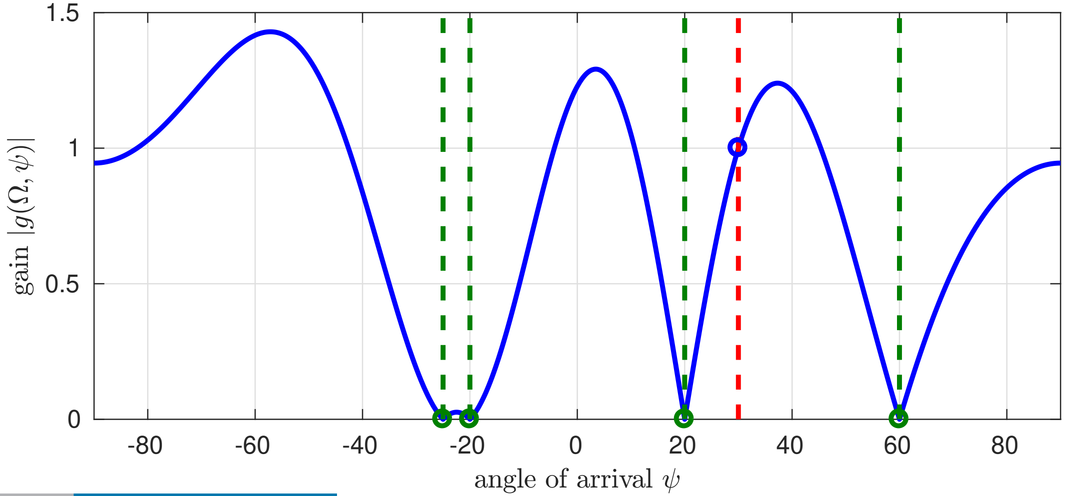

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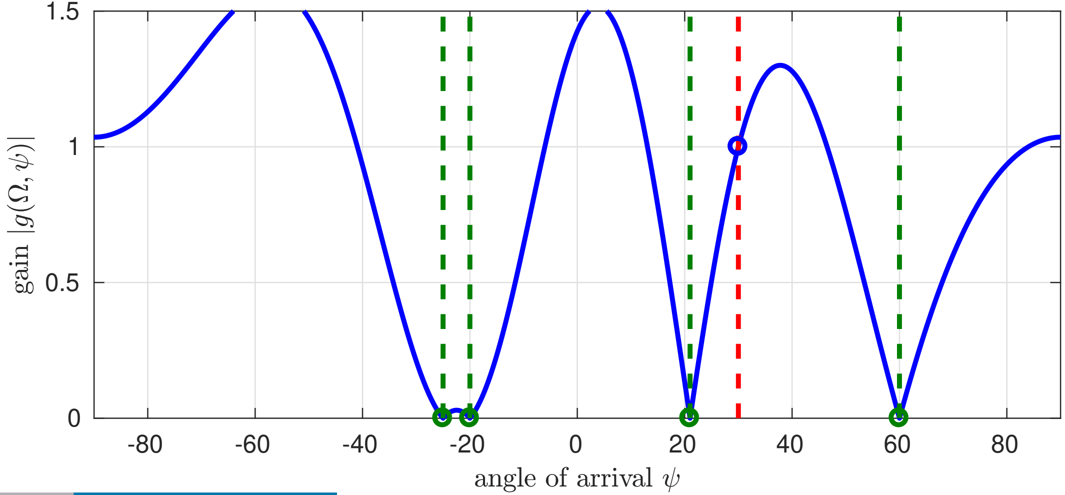

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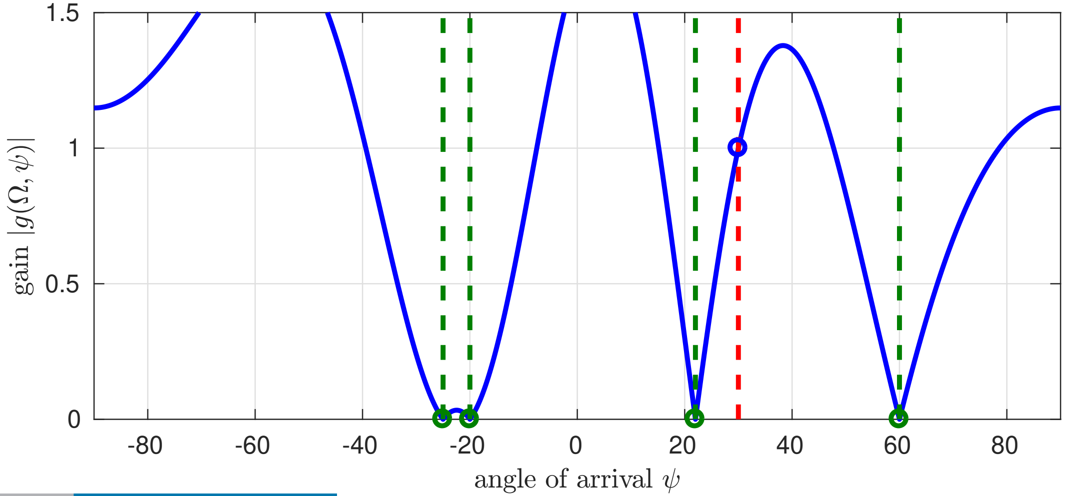

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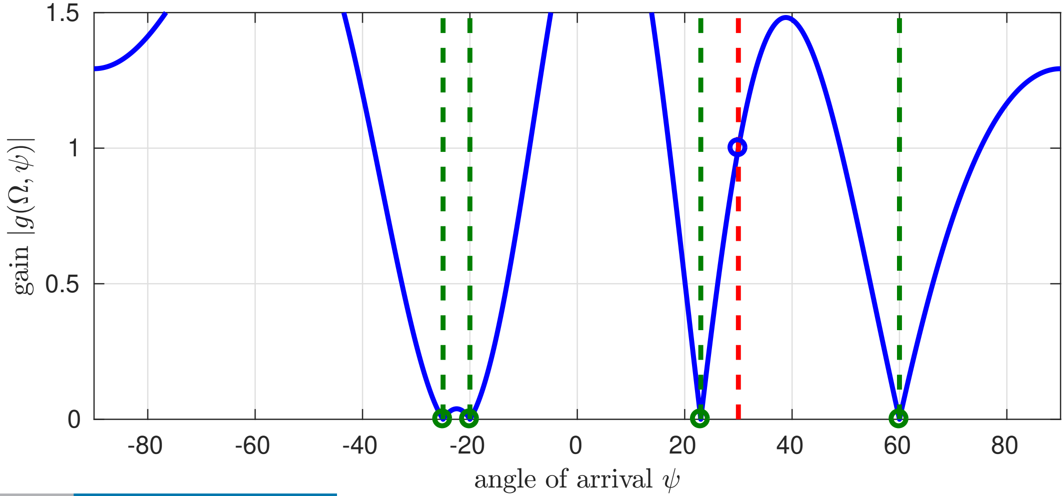

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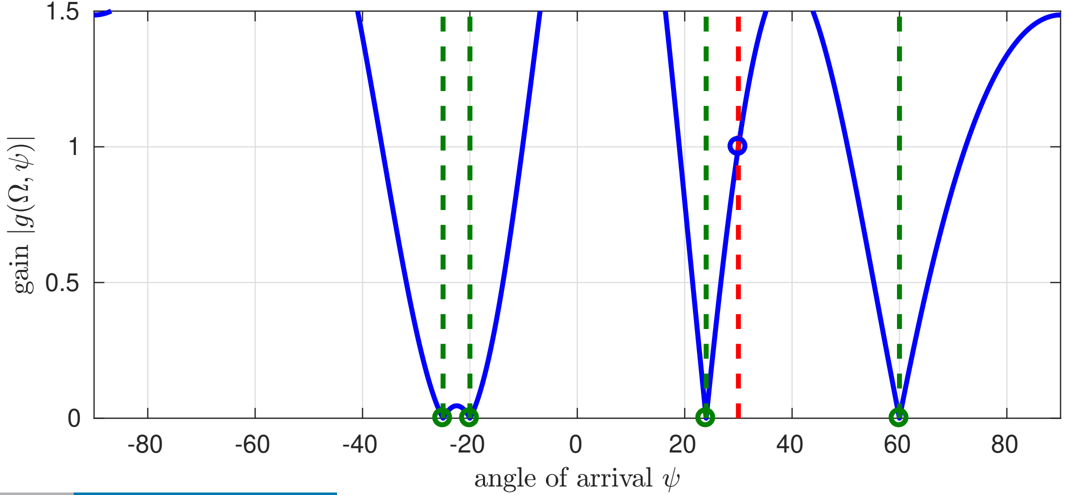

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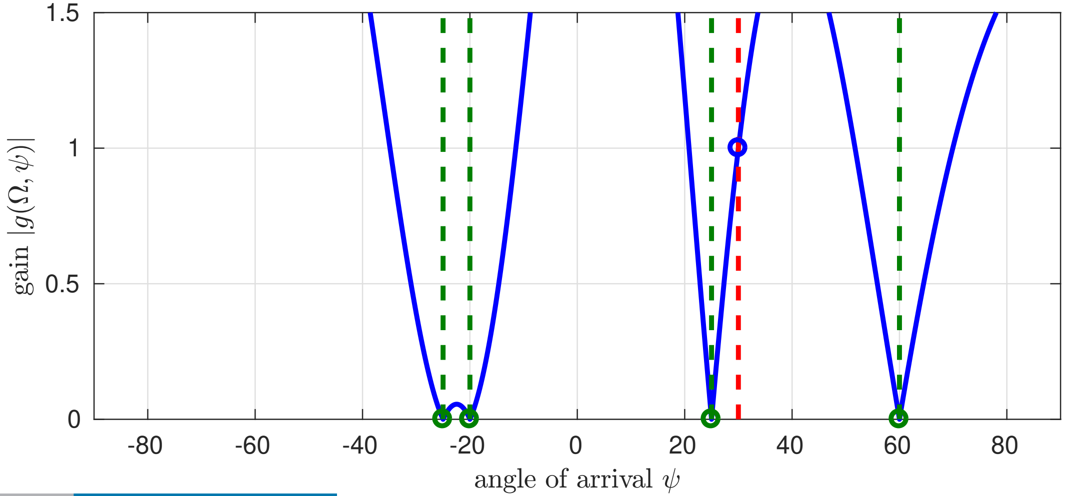
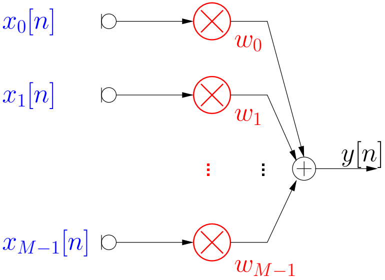


Data Independent Beamforming

- Previous beamformer designs were based on the knowledge of DoA of the signal of interest and of interfering sources;
- remaining degrees of freedom are invested to suppress spatially white noise;
- using the analogy between spatial and temporal processing, classical filter design techniques can be invoked to design arrays with a bandpass-type angular response;
- beamformers based on source parameters (frequency, DoA) rather the actual received waveforms are termed data independent beamformers;
- this is in contrast to statistically optimum beamformers, which take the received signal statistics into account.

1.5 Statistically Optimum Beamforming



- Statistically optimum beamformer minimise e.g. the signal power of the beamformer output, $y[n]$;
- to avoid the trivial solution $\mathbf{w} = \mathbf{0}$, the signal of interest needs to be protected by constraints;
- this results in e.g. the following constrained optimisation problem

$$\min_{\mathbf{w}^*} \mathcal{E}\{|y[n]|^2\} \qquad \text{subject to} \qquad \mathbf{s}_{\Omega,\vartheta}^{\mathrm{H}} \mathbf{w} = 1 \; ;$$

- the solution to this specific statistically optimum beamformer is known as the minimum variance distortionless response (MVDR) [43].

MVDR Beamformer

- Solving the MVDR problem: minimise the power of $y[n] = \mathbf{w}^{\mathrm{H}}\mathbf{x}$ subject to the contraint $\mathbf{w}^{\mathrm{H}}\mathbf{s}_{\Omega_0,\vartheta_0} = 1$;
- Formulation using a Lagrange multiplier λ:

$$\frac{\partial}{\partial \mathbf{w}^*}\left(\mathbf{w}^{\mathrm{H}}\mathcal{E}\left\{\mathbf{x}\mathbf{x}^{\mathrm{H}}\right\}\mathbf{w} - \lambda(\mathbf{w}^{\mathrm{H}}\mathbf{s}_{\Omega_0,\vartheta_0} - 1)\right) = \mathbf{R}_{xx}\mathbf{w} - \lambda\mathbf{s}_{\Omega_0,\vartheta_0} = \mathbf{0}$$

- the solution $\mathbf{w} = \lambda\mathbf{R}_{xx}^{-1}\mathbf{s}_{\Omega_0,\vartheta_0}$ is inserted into the constraint equation to determine λ:

$$\lambda\mathbf{s}_{\Omega_0,\vartheta_0}^{\mathrm{H}}\mathbf{R}_{xx}^{-1}\mathbf{s}_{\Omega_0,\vartheta_0} = 1$$

- therefore

$$\mathbf{w}_{\mathrm{MVDR}} = \left(\mathbf{s}_{\Omega_0,\vartheta_0}^{\mathrm{H}}\mathbf{R}_{xx}^{-1}\mathbf{s}_{\Omega_0,\vartheta_0}\right)^{-1}\mathbf{R}_{xx}^{-1}\mathbf{s}_{\Omega_0,\vartheta_0}$$

- this statistically optimum beamformer has various other names, e.g. Capon beamformer [8, 42].

MVDR Beamformer — Simple Case

- In the case of spatially white noise input, such that

$$\mathbf{R}_{xx} = \sigma_{xx}^2 \mathbf{I} \qquad \longrightarrow \qquad \mathbf{R}_{xx}^{-1} = \sigma_{xx}^{-2} \mathbf{I}$$

  the MVDR solution reduces to

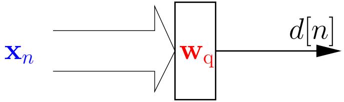
$$\mathbf{w}_{\mathrm{MVDR}} = \frac{\mathbf{s}_{\Omega_0,\vartheta_0}}{\|\mathbf{s}_{\Omega_0,\vartheta_0}\|_2^2} = \frac{\mathbf{s}_{\Omega_0,\vartheta_0}}{M} \quad ;$$

- this is identical to the data independent beamformer in the absence of interference (i.e. no spatially structured noise);

Generalised Sidelobe Canceller (GSC)

- The generalised sidelobe canceller is a specific implementation of the MVDR beamformer; it transforms the constrained MVDR problem into an unconstrained optimisation problem;
- a first guess at the solution is performed by the quiescent beamformer $\mathbf{w}_{\mathrm{q}}$, which is identical to the previously defined data independent beamformer, obtained by inverting the constraint equation

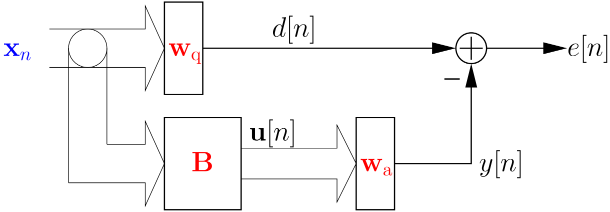
$$\mathbf{C}^{\mathrm{H}}\mathbf{w}_{\mathrm{q}} = \mathbf{f} \qquad \longrightarrow \qquad \mathbf{w}_{\mathrm{q}} = \left(\mathbf{C}^{\mathrm{H}}\right)^{\dagger}\mathbf{f}$$



- the quiescent beamformer eliminates interferers captured by $\mathbf{C}$ and $\mathbf{f}$, but passes the signal of interest, any interferers unaccounted for in the constraints, and spatially distributed noise.

GSC — Idea

- GSE idea: produce array signals that are free of any contribution from the signal of interest, and use the resulting signal vector $\mathbf{u}[n]$ to eliminate remaining interference from the quiescent output:



- the blocking matrix $\mathbf{B}$ eliminates the signal of interest and any interferers captured by the constraints;
- the vector $\mathbf{w}_\mathrm{a}$ will be based on the statistics of $\mathbf{u}[n]$ and $d[n]$ to minimise the beamformer output variance $\mathcal{E}\{|e[n]|^2\}$.

GSC — Blocking Matrix

- In order to project away from the constraints,

$$\mathbf{B} \cdot \mathbf{C} = \mathbf{B} \cdot \left[\mathbf{s}_{\Omega_0,\vartheta_0} \;\; \mathbf{s}_{\Omega_1,\vartheta_1} \;\; \ldots \;\; \mathbf{s}_{\Omega_{r-1},\vartheta_{r-1}}\right] \;=\; \mathbf{0}$$

- assuming that the r constraints are linearly independent, the singular value decomposition of the constraint matrix yields

$$\mathbf{B} \cdot \left[\mathbf{U}_0 \;\; \mathbf{U}_0^{\perp}\right] \left[\begin{array}{ccc|c} \sigma_0 & & & \\ & \ddots & & \mathbf{0} \\ & & \sigma_{r-1} & \\ \hline & \mathbf{0} & & \mathbf{0} \end{array}\right] \cdot \mathbf{V}^{\mathrm{H}} \;=\; \mathbf{0}$$

- the matrix $\mathbf{U}_0^{\perp} \in \mathbb{C}^{M \times (M-r)}$ spans the nullspace of $\mathbf{C}^{\mathrm{H}}$, and

$$\mathbf{B} = (\mathbf{U}_0^{\perp})^{\mathrm{H}} \in \mathbb{C}^{(M-r) \times M}$$

has the required property, as $(\mathbf{U}_0^{\perp})^{\mathrm{H}} \cdot \left[\mathbf{U}_0 \;\; \mathbf{U}_0^{\perp}\right] \boldsymbol{\Sigma} = [\mathbf{0} \;\; \mathbf{I}] \cdot \boldsymbol{\Sigma} = \mathbf{0}$.

GSC — Unconstrained Optimisation

- The beamforming vector $\mathbf{w}_{\mathrm{a}}$ is adjusted to minimise the output power;
- the MMSE or Wiener solution is given by

$$\mathbf{w}_{\mathrm{a}} = \mathbf{R}_{uu}^{-1} \cdot \mathbf{p} = \frac{\mathbf{B}\mathbf{R}_{xx}(\mathbf{C}^{\mathrm{H}})^{\dagger}\mathbf{f}}{\mathbf{B}\mathbf{R}_{xx}\mathbf{B}^{\mathrm{H}}}$$

  with the covariance matrix

$$\mathbf{R}_{uu} = \mathcal{E}\left\{\mathbf{u}[n] \cdot \mathbf{u}^{\mathrm{H}}[n]\right\} = \mathbf{B}\,\mathcal{E}\left\{\mathbf{x}[n] \cdot \mathbf{x}^{\mathrm{H}}[n]\right\}\,\mathbf{B}^{\mathrm{H}} = \mathbf{B}\mathbf{R}_{xx}\mathbf{B}^{\mathrm{H}}$$

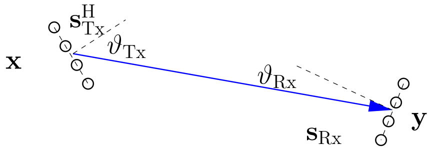
  and the cross-correlation vector

$$\mathbf{p} = \mathcal{E}\{\mathbf{u}[n] \cdot d^{*}[n]\} = \mathbf{B}\mathbf{R}_{xx}\mathbf{w}_{\mathrm{q}}$$

- iterative optimisation schemes, such as the least mean squares (LMS) algorithm [16, 64] may be used to approximate the MMSE solution.

1.6 Beamforming and MIMO Processing

- Assume a transmission scenario with an M-element transmit (Tx) antenna array and an N-element receive (Rx) array;



- in the absence of scatterers and any attenuation, the farfield transmission from the transmit antenna is characterised by a steering vector $\mathbf{s}_{\mathrm{Tx}}^{\mathrm{H}}$;
- the incoming waveform at the Rx device is described by another steering vector $\mathbf{s}_{\mathrm{Rx}}$;
- the overall MIMO system between a Tx vector $\mathbf{x} \in \mathbb{C}^M$ and an Rx vector $\mathbf{y} \in \mathbb{C}^N$ is described as

$$\mathbf{y} = \mathbf{s}_{\mathrm{Rx}} \cdot \mathbf{s}_{\mathrm{Tx}}^{\mathrm{H}} \cdot \mathbf{x} \quad = \quad \mathbf{H}\mathbf{x}$$

- the MIMO system matrix $\mathbf{H} = \mathbf{s}_{\mathrm{Rx}} \cdot \mathbf{s}_{\mathrm{Tx}}^{\mathrm{H}}$ is rank one only.

MIMO Requirements

- The farfield assumption is convenient for beamforming, but leads to a rank one MIMO system matrix which is incompatible with the desire to extract multiple independent subchannels or with to achieve diversity;
- rich scattering in connection with MIMO usually implies multiple reflections of signals;
- together with a sufficiently large antenna spacing means that the farfield assumption is invalid and the MIMO system matrix is not rank deficient;
- some suggestions of "sufficiently large spacing" imply an antenna element distance of $d > 10\lambda$;
- recall spatial sampling requires $d < \frac{1}{2}\lambda$!

Beamforming with Spatial Aliasing

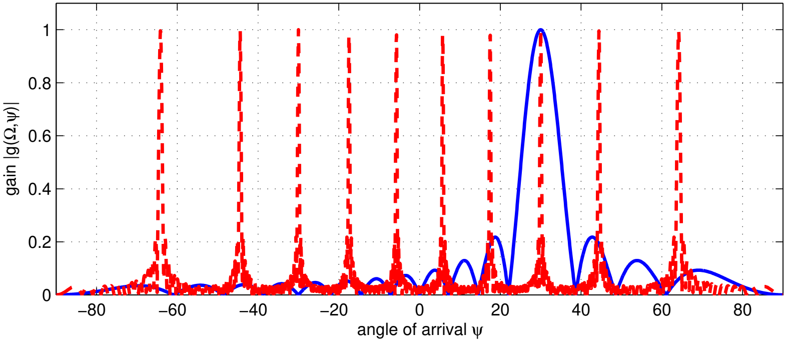
- For a flexible spatial sampling with $d = \alpha\lambda$, $0 < \alpha \in \mathbb{R}$, the steering vector for a waveform with normalised angular frequency Ω and DoA ϑ is

$$\mathbf{y} = e^{j\Omega n} \begin{bmatrix} 1 \\ e^{j2\alpha\Omega\sin(\vartheta)} \\ \vdots \\ e^{j2\alpha(M-1)\Omega\sin(\vartheta)} \end{bmatrix} = \mathbf{s}_{2\alpha\Omega,\vartheta} \cdot e^{j\Omega}$$

- inspecting $\mathbf{s}_{2\alpha\Omega,\vartheta}$ the steering vector is aliased to a different frequency $2\alpha\Omega$;
- although the correct frequency can be retrieved unambigiously from temporal sampling of any array element, at Ω various different angles could provide the same steering vector $\mathbf{s}_{2\alpha\Omega,\vartheta}$;
- the array performs spatial undersampling, resulting in spatial aliasing.

Spatial Undersampling Example

- Beamforming parameters: signal of interest with $\Omega = \frac{\pi}{2}$, direction of arrival $\vartheta = 30°$, $M = 32$ array elements;
- data independent beamformer design with correct spatial sampling $(d = \lambda/2)$ and incorrect spatial sampling $(d = 10\lambda)$:



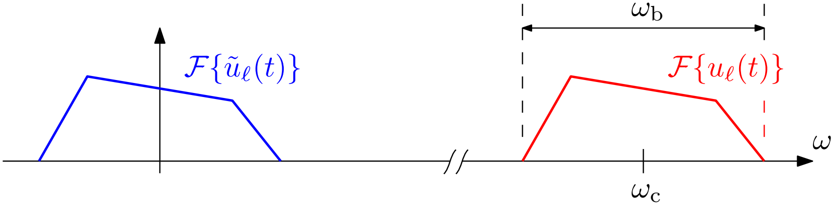
- MIMO systems perform beamforming, but may dissipate energy into aliased directions.

1.7 Narrowband Signals

- We have previously assumed that a narrowband signal is a complex exponential, $\mathrm{e}^{\mathrm{j}\Omega n}$;
- this permitted to characterise the signal received at the array by means of a steering vector that only depends on Ω and the direction of arrival;
- we now relax this restriction: in practice, we deal with bandpass signals of finite bandwidth ω_{b} and centre frequency ω_{c};
- for the lth source:

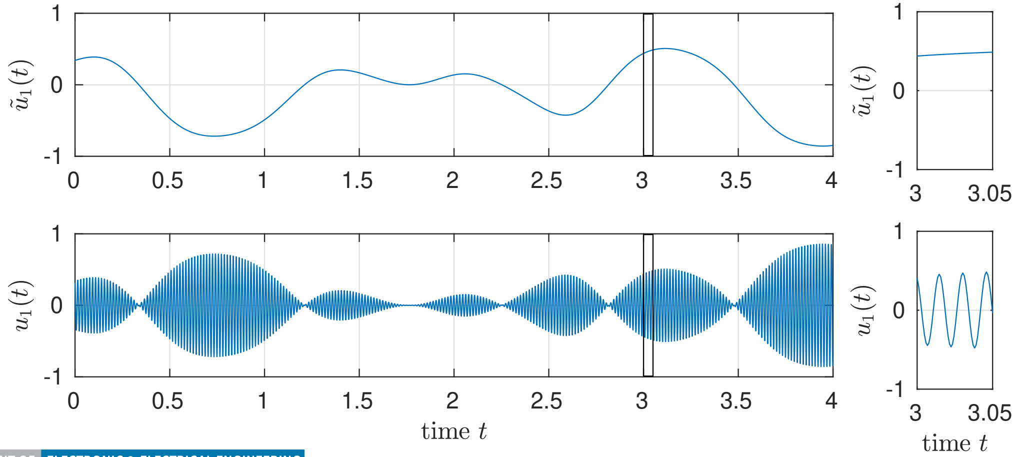
$$u_\ell(t) = \tilde{u}_\mathrm{l}(t) \cdot \mathrm{e}^{\mathrm{j}\omega_c t} \ , \tag{1}$$

where $\tilde{u}_\ell(t)$ is a baseband signal;



Narrowband Assumption

- For a signal to be considered narrowband, the propagation delay across the array must be small w.r.t. any changes in the baseband signal $\tilde{u}_\ell(t)$ (or of the envelope of $u_\ell(t)$);



Received Narrowband Array Signal

- An array receives a single modulated bandpass signal $u_\ell(t)$:

$$\mathbf{x}(t) = \begin{bmatrix} u_\ell(t-\tau_1) \\ \vdots \\ u_\ell(t-\tau_M) \end{bmatrix} = \begin{bmatrix} \tilde{u}_\ell(t-\tau_1) \\ \vdots \\ \tilde{u}_\ell(t-\tau_M) \end{bmatrix} \cdot \begin{bmatrix} \mathrm{e}^{\mathrm{j}\omega_\mathrm{c}(t-\tau_1)} \\ \vdots \\ \mathrm{e}^{\mathrm{j}\omega_\mathrm{cc}(t-\tau_M)} \end{bmatrix} \approx \tilde{u}_\ell(t-\tau_1)\mathrm{e}^{\mathrm{j}\omega_\mathrm{c}-\tau_1}\mathbf{s}_{\vartheta_\ell,\omega_\mathrm{c}} \tag{2}$$

- after sampling: $\mathbf{x}[n] = \tilde{u}_\ell[n] \cdot \mathrm{e}^{\mathrm{j}\omega_\mathrm{c}\tau_1} \cdot \mathbf{s}_{\vartheta_\ell,\Omega_\mathrm{c}}$;
- for the narrowband covariance matrix:

$$\mathbf{R} = \mathcal{E}\{\mathbf{x}[n]\mathbf{x}^\mathrm{H}[n]\} = \mathcal{E}\{\tilde{u}_\ell[n]\tilde{u}_\ell^*[n]\}\,\mathbf{s}_{\vartheta_\ell,\Omega_\mathrm{c}}\mathbf{s}^\mathrm{H}_{\vartheta_\ell,\Omega_\mathrm{c}} = \sigma_\ell^2\mathbf{s}_{\vartheta_\ell,\Omega_\mathrm{c}}\mathbf{s}^\mathrm{H}_{\vartheta_\ell,\Omega_\mathrm{c}} \tag{3}$$

- for L independent source signals, $\mathcal{E}\{\tilde{u}_\ell[n]\tilde{u}_k^*[n]\} = 0$ for $\ell \neq k$; therefore in the noise-free case:

$$\mathbf{R} = \sum_{\ell=1}^{L} \sigma_\ell^2\mathbf{s}_{\vartheta_\ell,\Omega_\mathrm{c}}\mathbf{s}^\mathrm{H}_{\vartheta_\ell,\Omega_\mathrm{c}} \;. \tag{4}$$

Narrowband versus Broadband

How long can a array signal be regarded as narrowband?

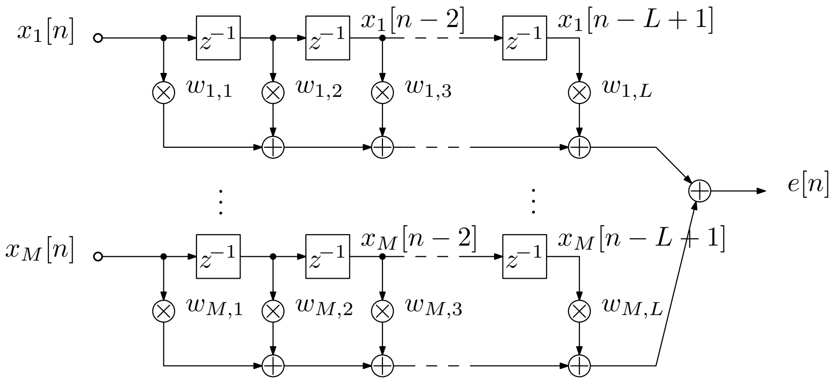
- Compton [9]: signals at opposite ends of the array must not be decorelated;
- some rule of thumb: fractional bandwidth $\omega_{\mathrm{b}}/\omega_{\mathrm{c}} \ll 1$ (typically smaller than 5%);
- these rules are somewhat 'fuzzy'; recall that

$$\mathbf{R} = \sum_{\ell=1}^{L} \sigma_\ell^2 \mathbf{s}_{\vartheta_\ell,\Omega_\mathrm{c}} \mathbf{s}_{\vartheta_\ell,\Omega_\mathrm{c}}^{\mathrm{H}} \; ;$$

- this matrix possesses rank L as long as the steering vectors are linearly independent;
- if the narrowband assumption is no longer satisfied, the approximation in (2) becomes inaccurate, and the rank of $\mathbf{R}$ will increase [65, 66];
- this can also be tied to the array performance [10, 44, 38, 37];
- when must a signal be considered broadband? John McWhirter's "If you need a tap delay line." captures the ambiguity well!

1.8 Broadband MVDR Beamformer

- Each sensor is followed by a tap delay line of dimension L, giving a total of ML coefficients in a vector $\mathbf{v} \in \mathbb{C}^{ML}$ [6]



Broadband MVDR Beamformer Constraints

- A larger input vector $\mathbf{x}_n \in \mathbb{C}^{ML}$ is generated, also including lags;
- the general approach is similar to the narrowband system, minimising the power of $e[n] = \mathbf{v}^{\mathrm{H}}\mathbf{x}_n$;
- however, we require several constraint equations to protect the signal of interest, e.g.

$$\mathbf{C} = [\mathbf{s}(\vartheta_{\mathrm{s}}, \Omega_0), \;\; \mathbf{s}(\vartheta_{\mathrm{s}}, \Omega_1) \; \ldots \; \mathbf{s}(\vartheta_{\mathrm{s}}, \Omega_{L-1})] \tag{5}$$

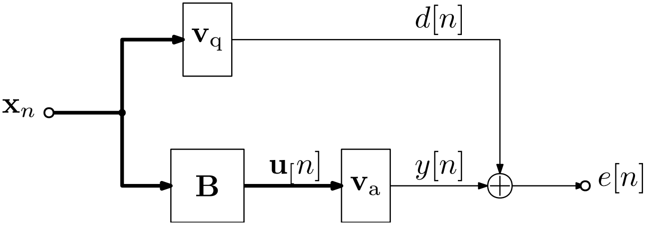
- these L constraints pin down the response to unit gain at L separate points in frequency:

$$\mathbf{C}^{\mathrm{H}}\mathbf{v} = \mathbf{1} \; ; \tag{6}$$

- generally $\mathbf{C} \in \mathbb{C}^{ML \times L}$, but simplifications can be applied if the look direction is towards broadside.

Broadband Generalised Sidelobe Canceller

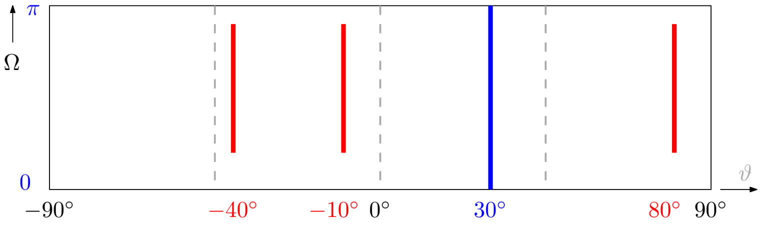
- A quiescent beamformer $\mathbf{v}_\mathrm{q} = \left(\mathbf{C}^\mathrm{H}\right)^\dagger \mathbf{1} \in \mathbb{C}^{ML}$ picks the signal of interest;
- the quiescent beamformer is optimal for AWGN but generally passes structured interference;
- the output of the blocking matrix $\mathbf{B}$ contains interference only, which requires $[\mathbf{BC}]$ to be unitary; hence $\mathbf{B} \in \mathbb{C}^{ML\times(M-1)L}$;
- an adaptive noise canceller $\mathbf{v}_\mathrm{a} \in \mathbb{C}^{(M-1)L}$ aims to remove the residual interference:



- note: all dimensions are determined by $\{M, L\}$.

Broadband Beamformer Example

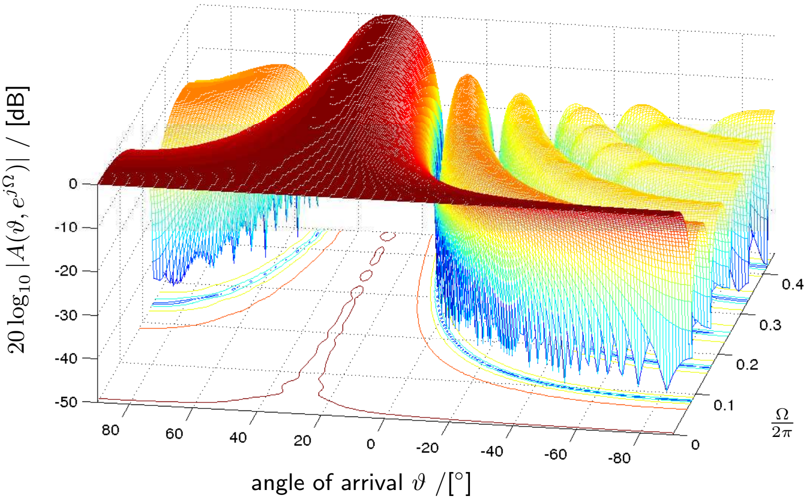
- We assume a signal of interest from $\vartheta = 30^\circ$;
- three interferers with angles $\vartheta_i \in \{-40^\circ, -10^\circ, 80^\circ\}$ active over the frequency range $\Omega = 2\pi \cdot [0.1;\ 0.45]$ at signal to interference ratio of -40 dB;



- $M = 8$ element linear uniform array is also corrupted by spatially and temporally white additive Gaussian noise at 20 dB SNR;
- tap-delay-line length: $L = 150$;
- cost per iteration: approx. 2 MMACs (standard), can be reduced to 10 kMACs when efficiently implemented.

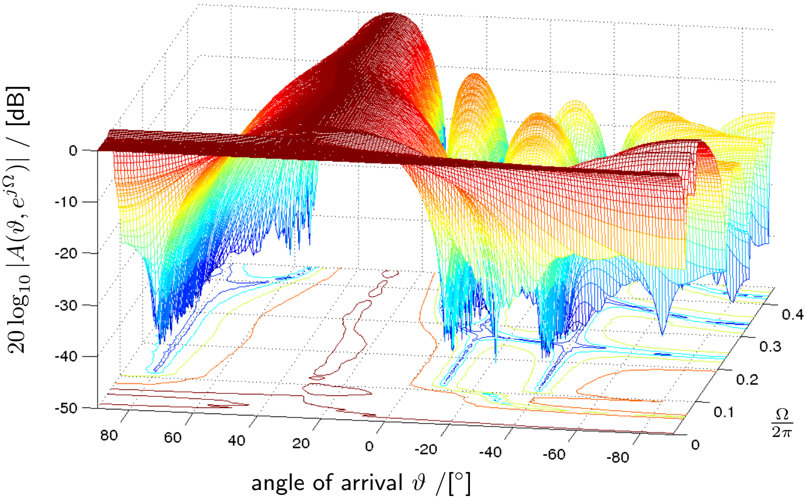
Broadband Quiescent Beamformer

- Directivity pattern of quiescent standard broadband beamformer:



Optimised Broadband Beamformer

- Directivity pattern of the broadband beamformer:



1.9 Summary

- Spatial sampling by an array of sensors (e.g. antenna elements) has been explored;
- the spatial data window of a narrowband source is characterised by the steering vector;
- appropriate data independent beamformers can be designed based on the steering vectors of desired sources and interferers;
- statistically optimum beamformers are based on the signal statistics;
- a specific statistically optimum beamformer, the generalised sidelobe canceller, has been reviewed — it uses signal statistics to improve the performance of a data independent beamformer derived from the constraint equations;
- some similarities and differences between beamforming and MIMO systems have been highlighted;
- broadband beamforming requires the inclusion of tap delay lines.

1.10 Related Broadband Beamforming Work

- General wideband beamforming: [23];
- time domain adaptive broadband beamforming: [6, 7, 15, 18, 27, 35, 43];
- discrete Fourier transform domain processing: [21, 36, 11, 55]
- subband domain beamforming [25, 45, 60, 61, 62, 63, 59, 55];
- frequency-invariant broadband beamforming [22, 26, 27, 49];
- polynomial matrix-based beamforming related work [1, 2, 3, 4, 12, 13, 19, 20, 29, 30, 34, 33, 32, 46, 47, 48, 54, 31] based on polynomial eigenvalued decomposition theory [51, 52, 5] and algorithms [28, 39, 41, 40, 58, 53, 57, 56]

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