Distributed Learning Gaussian Process Tracking with Uncertainty Quantification (Part of SIGNeTs Project)

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Outline

- Distributed learning Gaussian process tracking
- Case study: Tracking over wireless sensor networks
- Multi-class Gaussian process classification based on acoustic-seismic classification identification data set
- Summary

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Background Knowledge

- Model-free tracking approach: Learning the model dynamics directly from data assuming temporal correlation in the target trajectory

Input (time step) → Unknown function → Output (object state)

- At time step t, the Gaussian process (GP)-based motion tracker is written as:

$$x = f^x(t), \; f^x(t) \sim \mathrm{GP}^x(m^x(t), k^x(t, t')),$$
$$z^x = x + \epsilon^x, \; \epsilon^x \sim \mathcal{N}(0, \sigma_x^2),$$

Gaussian Process Regression

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- The GP regression equations (predicted mean and variance) at an input location t_* can be written as

$$\mu_* = m(t_*) + K_*^\intercal \Sigma^{-1}(\mathbf{z} - m(t_*)),$$
$$\sigma_*^2 = k(t_*, t_*) - K_*^\intercal \Sigma^{-1} K_*,$$

- Scalability issue: High computational complexity $O(N^3)$ due to kernel matrix inversion
- Distributed GP : Divide-and-conquer and product of expert

Distributed Gaussian Process-based Tracking (DGPT)

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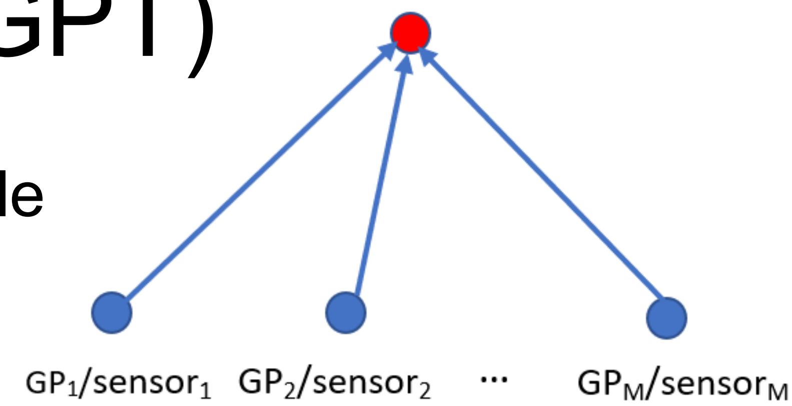


- Multiple local GPs instead of a single global GP

Prediction aggregation method:

– Product of experts (PoE)

$$p(f_*^x \mid t_*, \mathcal{D}) = \prod_{i=1}^{M} p_i(f_*^x \mid t_*, \mathcal{D}^{(i)})$$

$$\mu_*^{\text{PoE}} = (\sigma_*^{\text{PoE}})^2 \sum_{i=1}^{M} \sigma_i^{-2}(t_*)\mu_i(t_*),$$

$$(\sigma_*^{\text{PoE}})^{-2} = \sum_{i=1}^{M} \sigma_i^{-2}(t_*).$$

– Bayesian committee machine (BCM)

$$p(f_*^x \mid t_*, \mathcal{D}) = \frac{\prod_{i=1}^{M} p_i(f_*^x \mid t_*, \mathcal{D}^{(i)})}{p^{M-1}(f_*^x \mid t_*)}$$

$$\mu_*^{\text{BCM}} = (\sigma_*^{\text{BCM}})^2 \sum_{i=1}^{M} \sigma_i^{-2}(t_*)\mu_i(t_*),$$

$$(\sigma_*^{\text{BCM}})^{-2} = \sum_{i=1}^{M} \sigma_i^{-2}(t_*) + (1-M)\sigma_{**}^{-2}.$$

Hyperparameter Online Learning

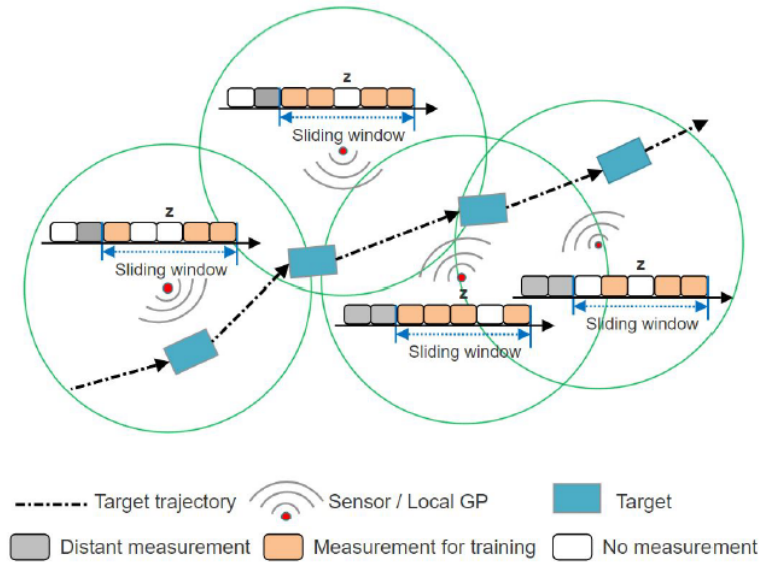


Fig. 1: A distributed point tracking system with 4 sensors. The length of sliding window in this example is 5 time steps

- GP hyperparameters online update

$$\log p(\mathbf{z}|X,\theta) = -\frac{1}{2}\mathbf{z}^\intercal \Sigma^{-1}\mathbf{z} - \frac{1}{2}\log|\Sigma| - \frac{n}{2}\log 2\pi,$$

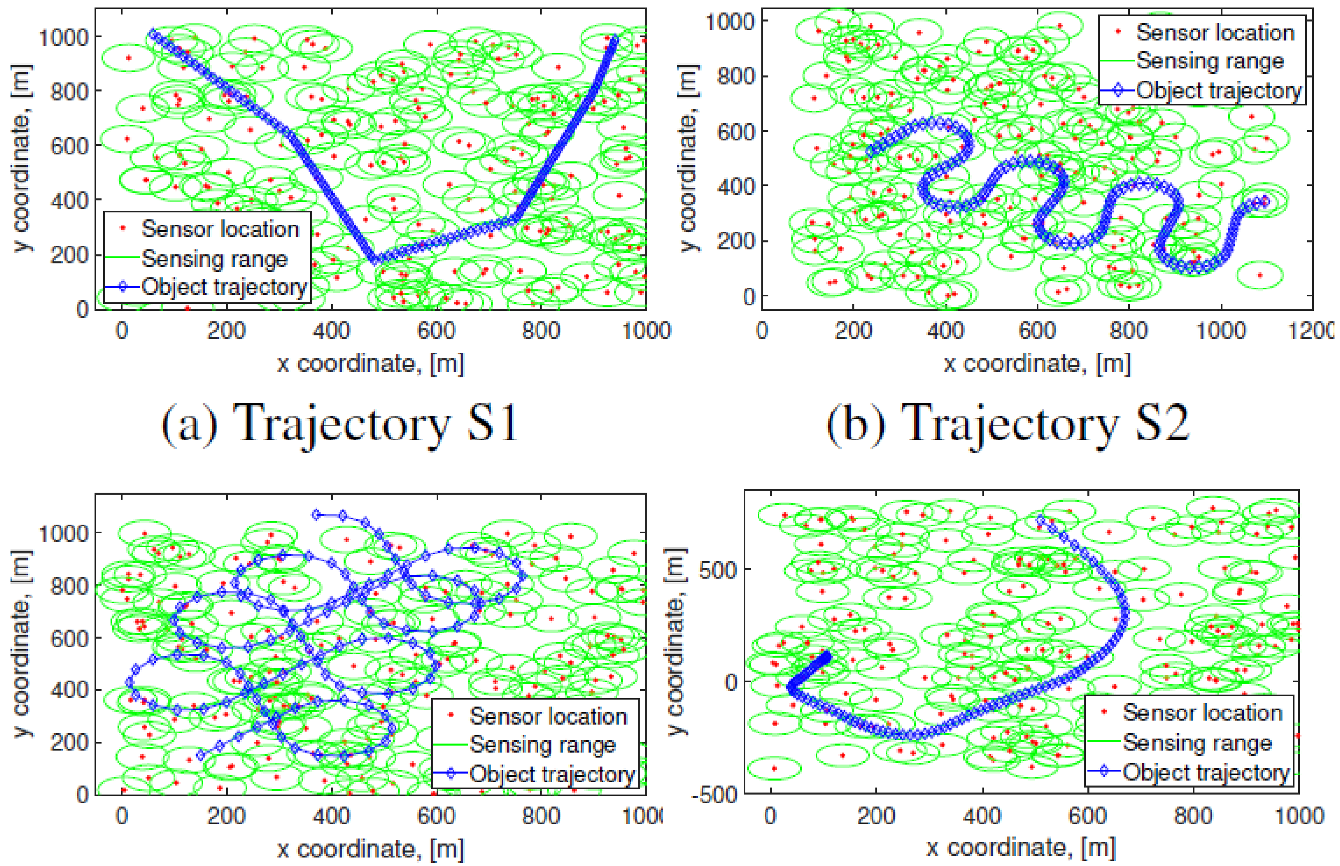
- Sliding window-based measurements
- Maximum likelihood estimation with consistent initial values of hyperparameters

Numerical Results

- Tracking performances under 4 trajectories and 3 noise levels
- Schemes:
  - DGPT-RBCM $(\mathcal{O}(Nn^2))$
  - DGPT-GPoE $(\mathcal{O}(Nn^2))$
  - Standard GP $(\mathcal{O}(N^3))$
  - Sparse GP $\left(\mathcal{O}(NM^2)\right)$
- Performance metric: Normalized root mean squared error (NRMSE), standard deviation of RMSEs over 100 MC runs

Numerical Results

- Testing scenarios



(a) Trajectory S1 (b) Trajectory S2

(c) Trajectory S3 (d) Trajectory S4

Fig. 2: Object trajectories

Numerical Results

- Superior performance is observed when the measurement noise level is relatively low (noise level 1 & 2), competitive performance is observed at high noise level

Scenario	Approach	Noise Level 1				Noise Level 2				Noise Level 3			
		Predicted NRMSE		Updated NRMSE		Predicted NRMSE		Updated NRMSE		Predicted NRMSE		Updated NRMSE	
		X	Y	X	Y	X	Y	X	Y	X	Y	X	Y
S1	Standard GP	1.74%	1.71%	1.20%	1.14%	1.83%	2.19%	1.36%	1.55%	2.73%	3.50%	2.09%	2.65%
	Sparse GP	2.79%	1.95%	2.25%	1.18%	1.94%	2.00%	1.55%	1.31%	2.43%	2.54%	2.02%	1.91%
	DGPT-RBCM	1.20%	1.37%	0.78%	0.84%	1.79%	1.92%	1.27%	1.30%	3.06%	3.57%	2.45%	2.75%
	DGPT-GPoE	1.22%	1.76%	1.03%	1.04%	1.99%	1.99%	1.82%	1.72%	3.69%	3.52%	3.56%	3.68%
S2	Standard GP	6.36%	4.87%	4.61%	3.72%	5.67%	5.12%	4.01%	3.96%	6.78%	5.50%	5.04%	4.31%
	Sparse GP	4.82%	6.28%	2.70%	5.25%	5.05%	6.03%	3.88%	4.92%	5.50%	5.54%	4.25%	4.41%
	DGPT-RBCM	2.70%	3.58%	1.83%	2.50%	2.93%	7.29%	1.98%	5.82%	3.86%	10.01%	2.89%	8.19%
	DGPT-GPoE	2.40%	5.05%	1.86%	4.15%	2.47%	7.74%	2.23%	7.26%	3.60%	10.01%	3.55%	9.63%
S3	Standard GP	4.75%	5.89%	2.71%	3.81%	5.53%	6.25%	3.19%	4.02%	7.05%	7.24%	4.23%	4.63%
	Sparse GP	5.75%	5.85%	3.48%	4.25%	5.87%	5.87%	3.64%	4.34%	6.31%	6.13%	4.16%	4.67%
	DGPT-RBCM	6.62%	6.44%	3.24%	4.18%	6.69%	6.78%	3.48%	4.29%	7.45%	7.48%	4.07%	4.74%
	DGPT-GPoE	5.41%	5.99%	3.18%	4.47%	5.56%	6.06%	3.41%	4.57%	6.05%	6.40%	3.99%	4.98%
S4	Standard GP	2.21%	1.30%	1.59%	0.90%	3.30%	1.52%	2.58%	1.09%	4.54%	2.17%	3.61%	1.68%
	Sparse GP	3.89%	2.21%	3.59%	2.00%	3.49%	2.27%	3.26%	2.13%	3.52%	2.25%	3.33%	2.21%
	DGPT-RBCM	2.71%	1.82%	1.85%	1.07%	3.73%	2.42%	2.72%	1.49%	5.64%	4.10%	4.58%	2.98%
	DGPT-GPoE	3.33%	1.55%	3.06%	1.31%	4.88%	2.67%	4.61%	2.37%	8.16%	5.03%	7.86%	4.72%

Numerical Results

- Impact of uncertainties on tracking

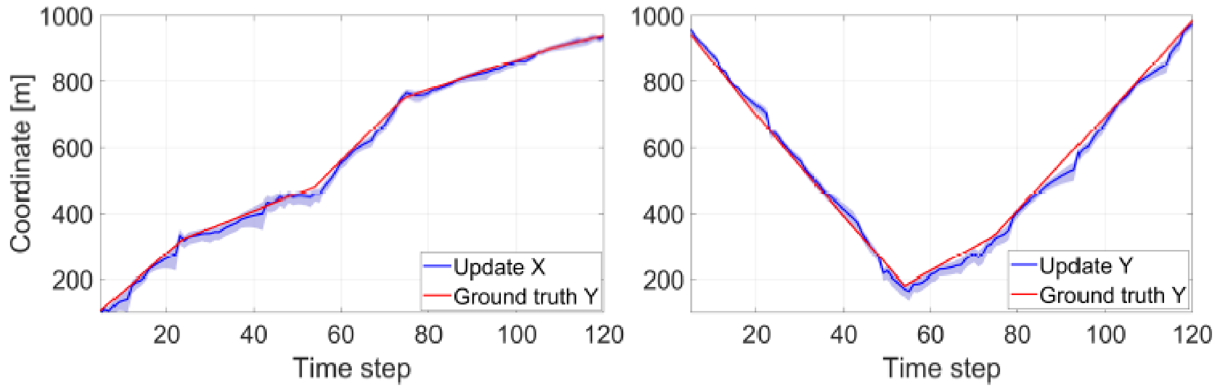


Fig. 3: Tracking uncertainty in both coordinates. The results are collected in S1 under noise level three.

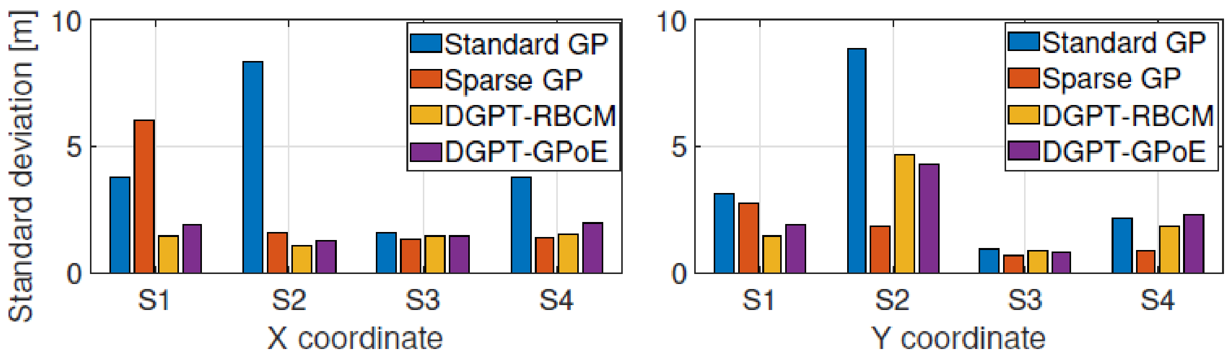
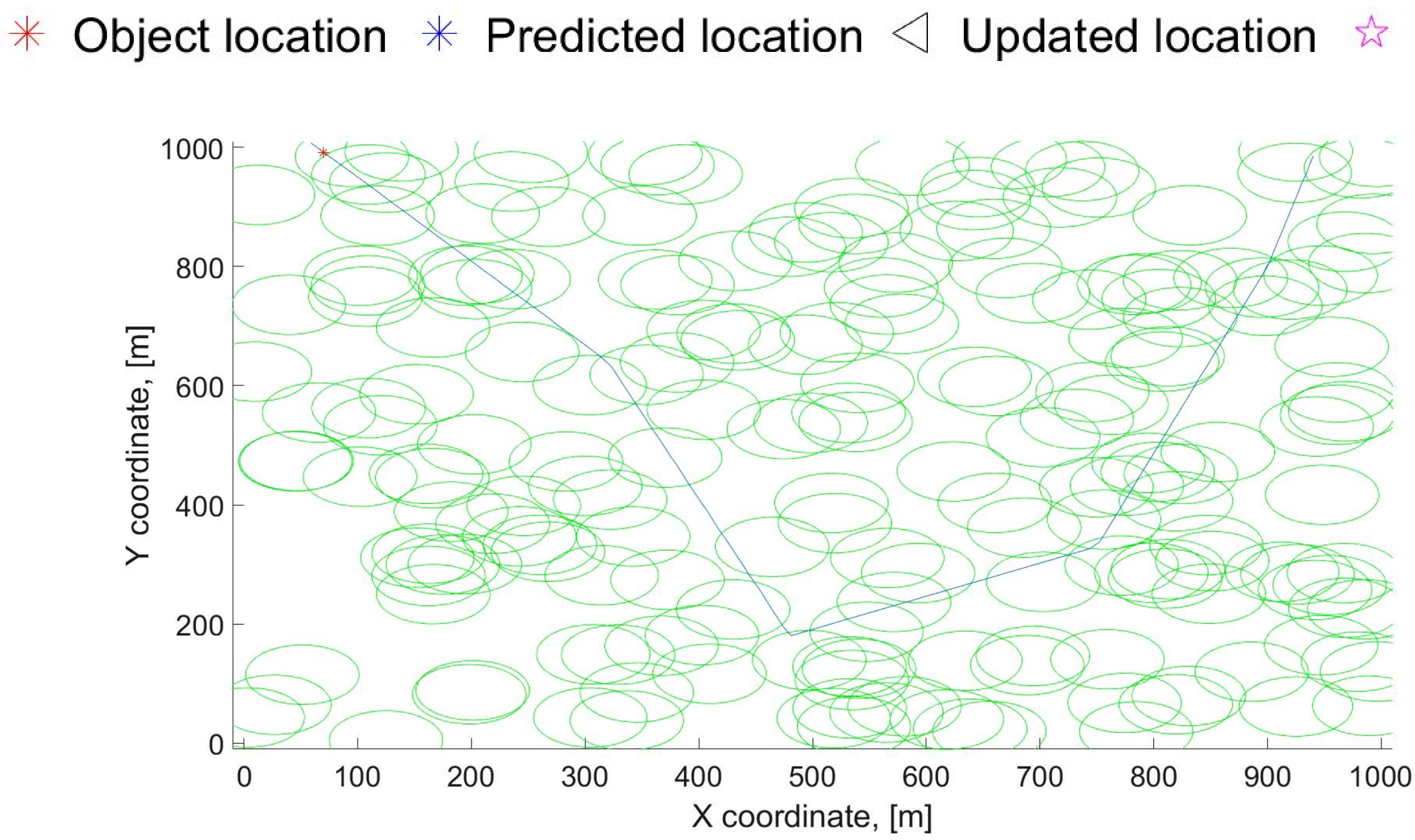


Fig. 4: Standard deviation of RMSE in both coordinates. The results are collected under noise level one.

Distributed Point Tracking in Clutter



Numerical Results

Table I: Updated NRMSEs for S1

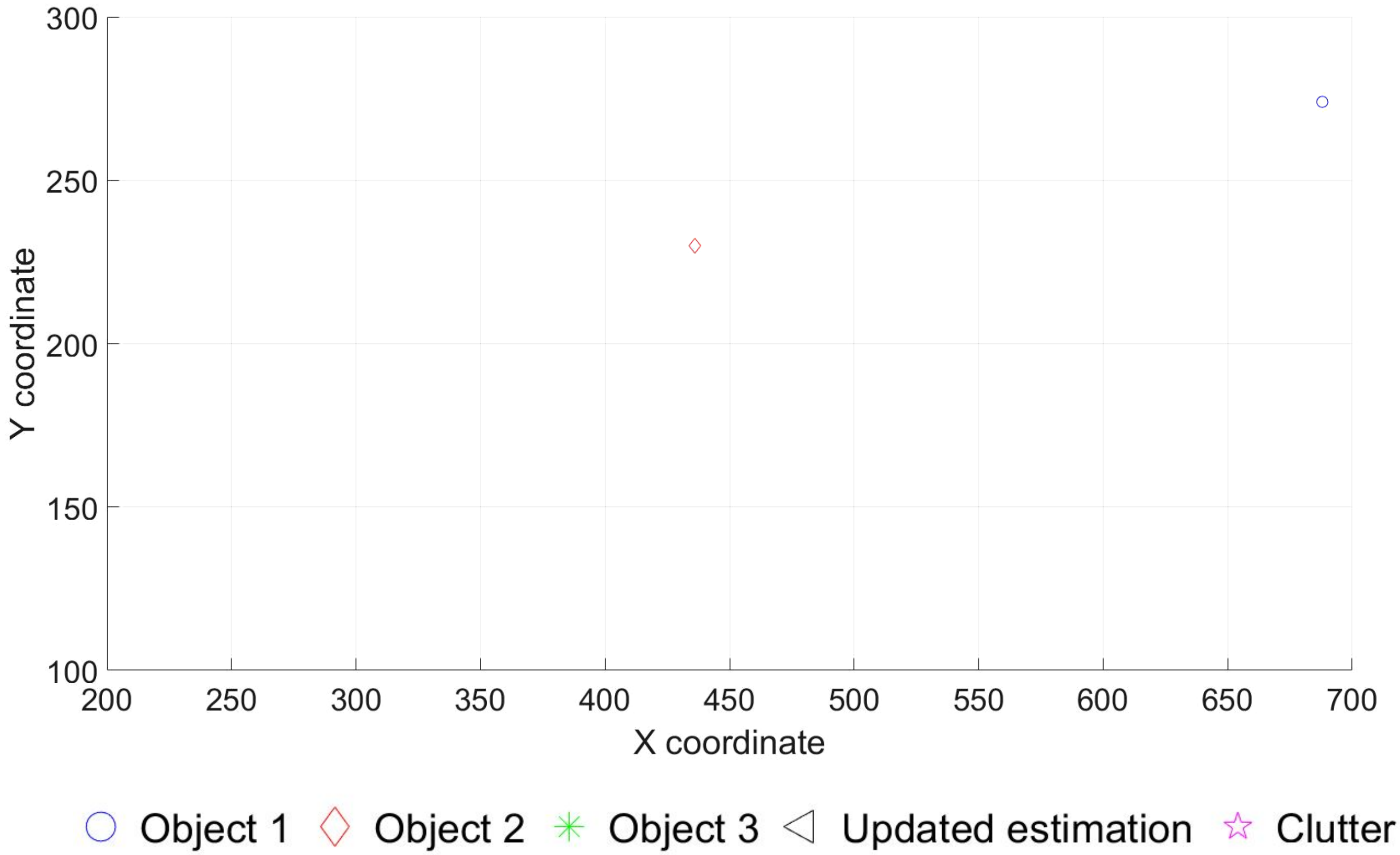
Noise level	Clutter rate	Approach					
		Standard GP		DGPT-RBCM		DGPT-GPoE	
		X	Y	X	Y	X	Y
1	0.0001	1.82%	2.46%	0.78%	0.98%	0.95%	1.13%
	0.0003	2.93%	3.16%	1.46%	1.48%	1.72%	1.64%
	0.0006	4.71%	5.03%	4.76%	3.82%	5.08%	4.05%
2	0.0001	1.79%	2.64%	0.94%	1.05%	1.17%	1.21%
	0.0003	2.76%	3.63%	1.69%	1.63%	2.00%	1.80%
	0.0006	4.74%	5.31%	5.96%	4.27%	6.29%	4.58%
3	0.0001	1.78%	2.63%	1.14%	1.17%	1.46 %	1.39%
	0.0003	2.74%	3.47%	1.90%	1.83%	2.29%	2.04%
	0.0006	4.80%	5.30%	6.19%	4.68%	6.53%	5.05%

Table II: Updated NRMSEs for S2

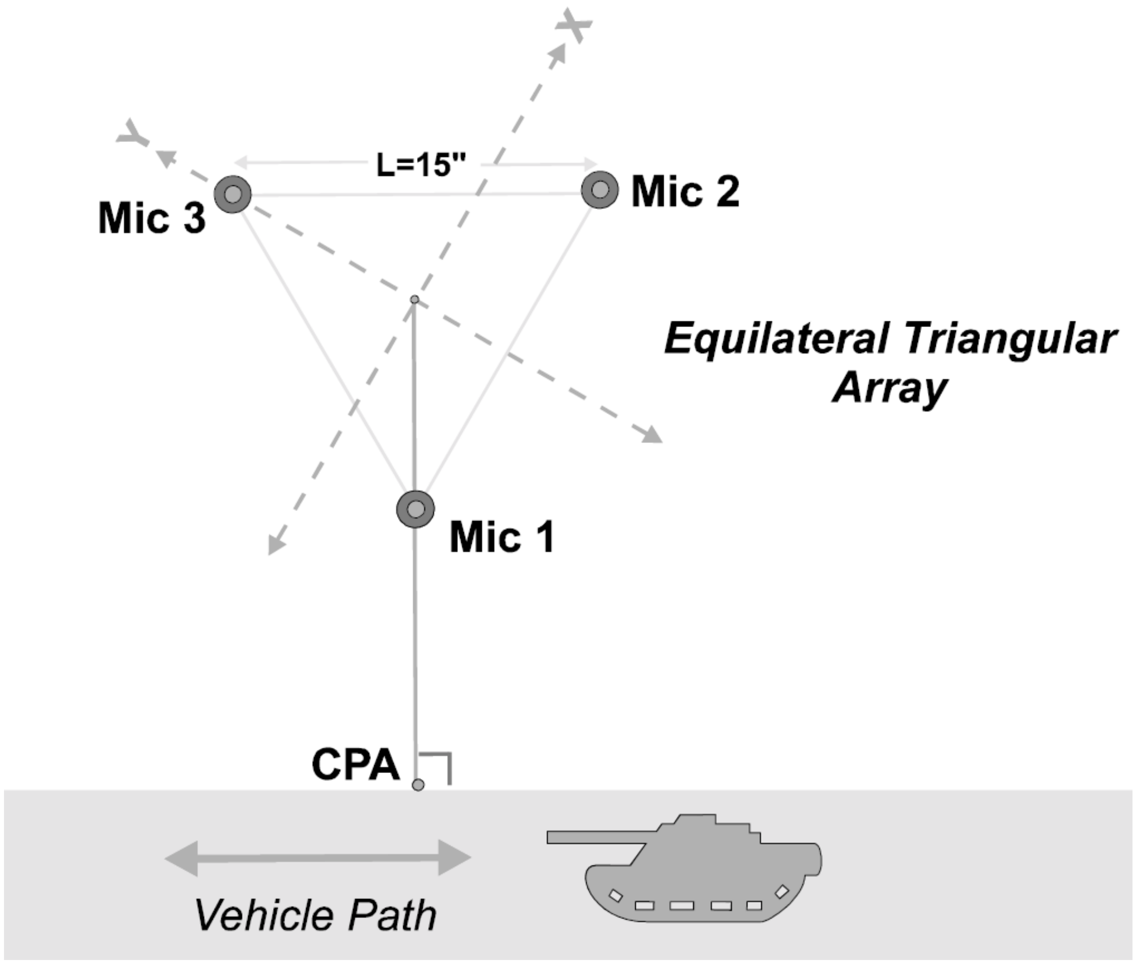
Noise level	Clutter rate	Approach					
		Standard GP		DGPT-RBCM		DGPT-GPoE	
		X	Y	X	Y	X	Y
1	0.0001	5.44%	3.19%	2.46%	2.39%	2.61%	3.55%
	0.0003	6.76%	3.77%	3.28%	3.89%	3.55%	5.13%
	0.0006	8.58%	4.42%	7.00%	10.79%	7.30%	11.31%
2	0.0001	5.44%	3.18%	2.42%	2.70%	2.56%	4.07%
	0.0003	6.83%	3.80%	3.21%	4.20%	3.49%	5.54%
	0.0006	8.58%	4.47%	7.05%	10.59%	7.34%	11.13%
3	0.0001	5.58%	3.24%	2.42%	3.39%	2.61%	4.97%
	0.0003	7.02%	3.90%	3.26%	5.07%	3.55%	6.51%
	0.0006	8.76%	4.49%	7.12%	10.92%	7.42%	11.47%

- Superior performance is observed when the clutter rate is relatively low (0.0001 & 0.0003), competitive performance is observed at high clutter rate

Multi-Target Tracking in Clutter



Acoustic and Seismic Classification Identification Data Set



- Traveling at constant speeds (5km/h-40 km/h) with two directions
- Closest point of approach (CPA) varies from 25m-100m
- 9 different vehicle types
- 4 different test sites
- 2 different sensor systems

Fundamental Frequency Estimation[1],[2]

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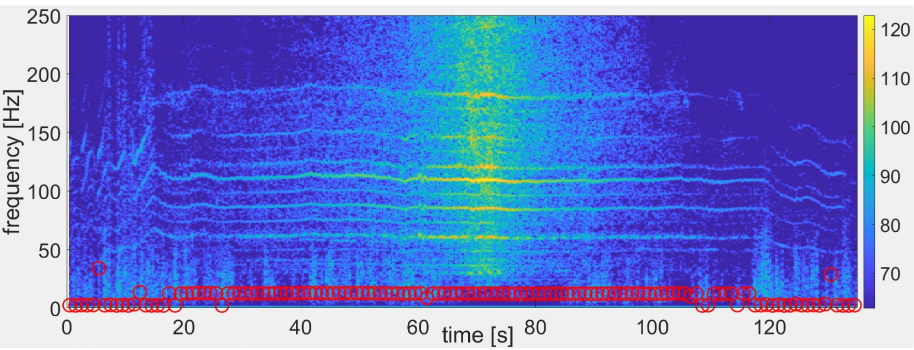

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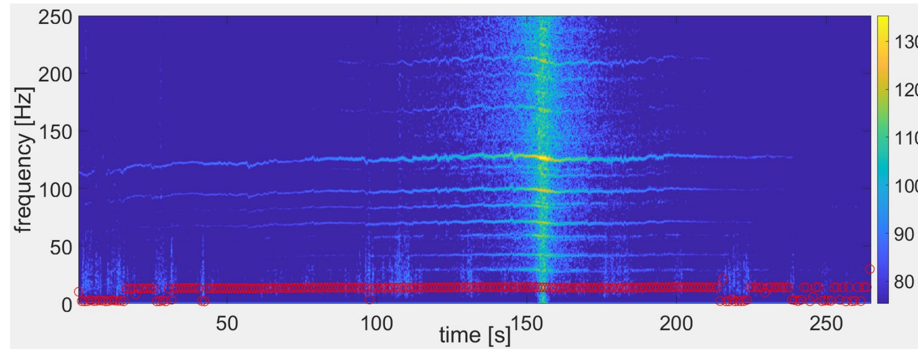

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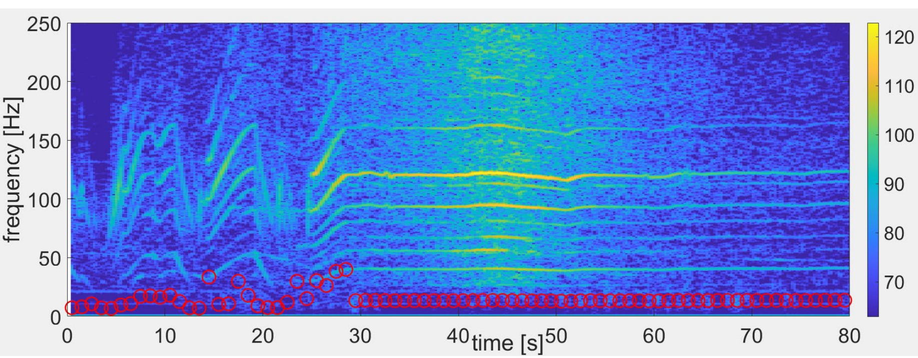

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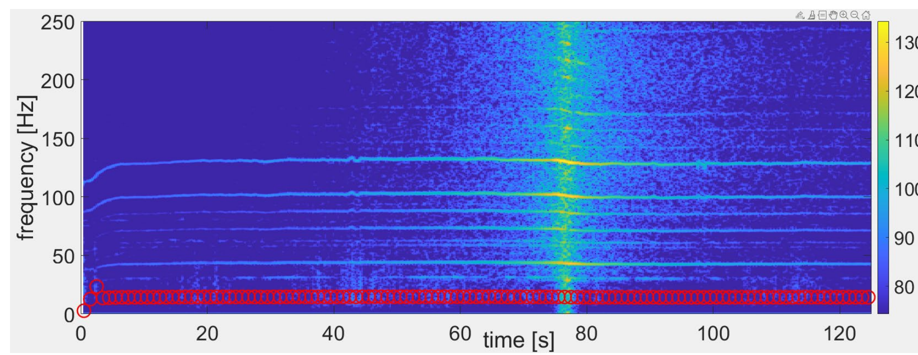
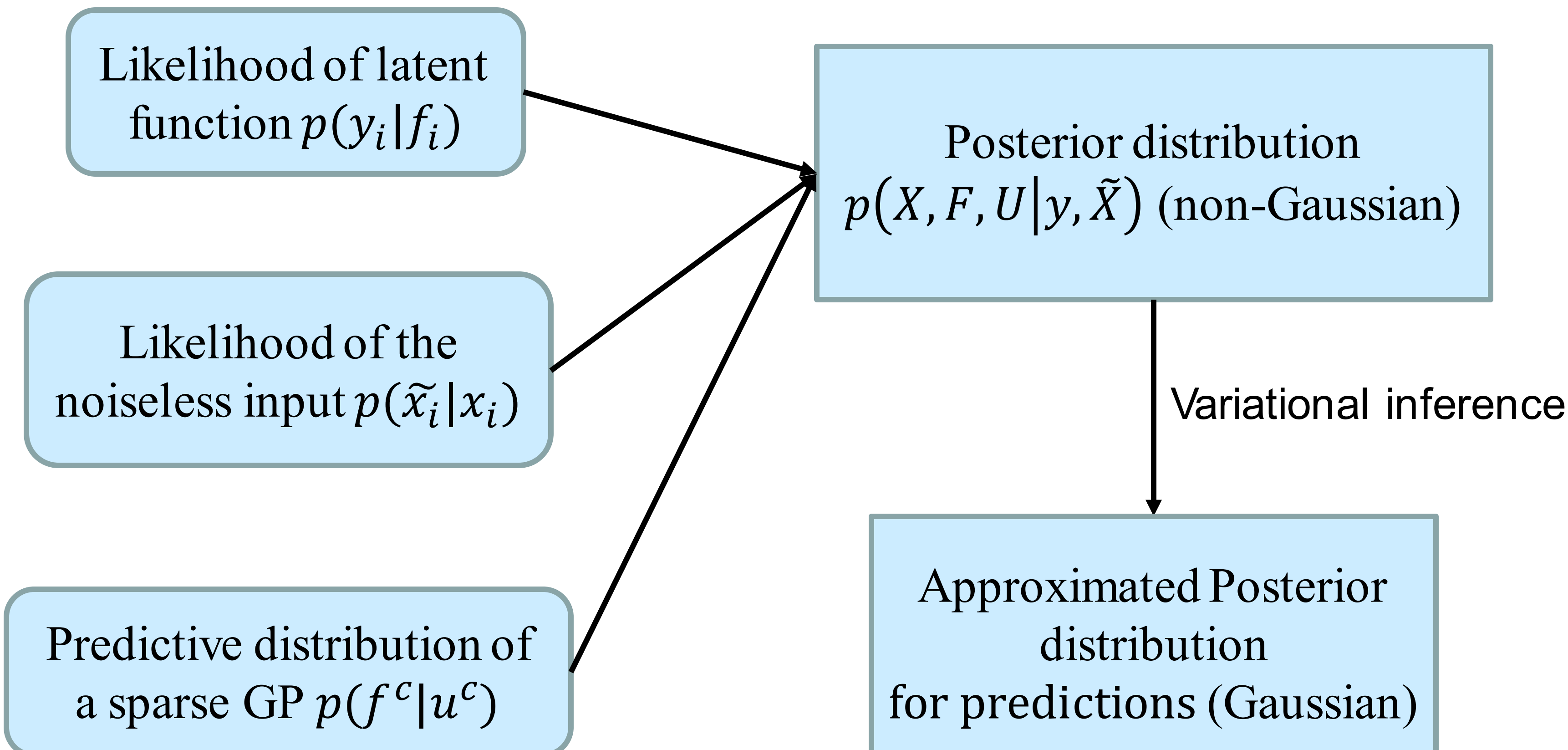


[1]. Nielsen, J. K., Jensen, T. L., Jensen, J. R., Christensen, M. G., & Jensen, S. H. (2017). Fast Fundamental Frequency Estimation: Making a Statistically Efficient Estimator Computationally Efficient. *Signal Processing*, 135, 2017, pp. 188-197.
[2]. J. K. Nielsen, M. G. Christensen, A. T. Cemgil, S. J. Godsill, and S. J. Jensen, "Bayesian Interpolation and Parameter Estimation in a Dynamic Sinusoidal Model," *IEEE Trans. Audio, Speech, Lang. Process.*, vol. 19, no. 7, pp. 1986–1998, 2011.

Multi-Class GP Classification with Noisy Inputs

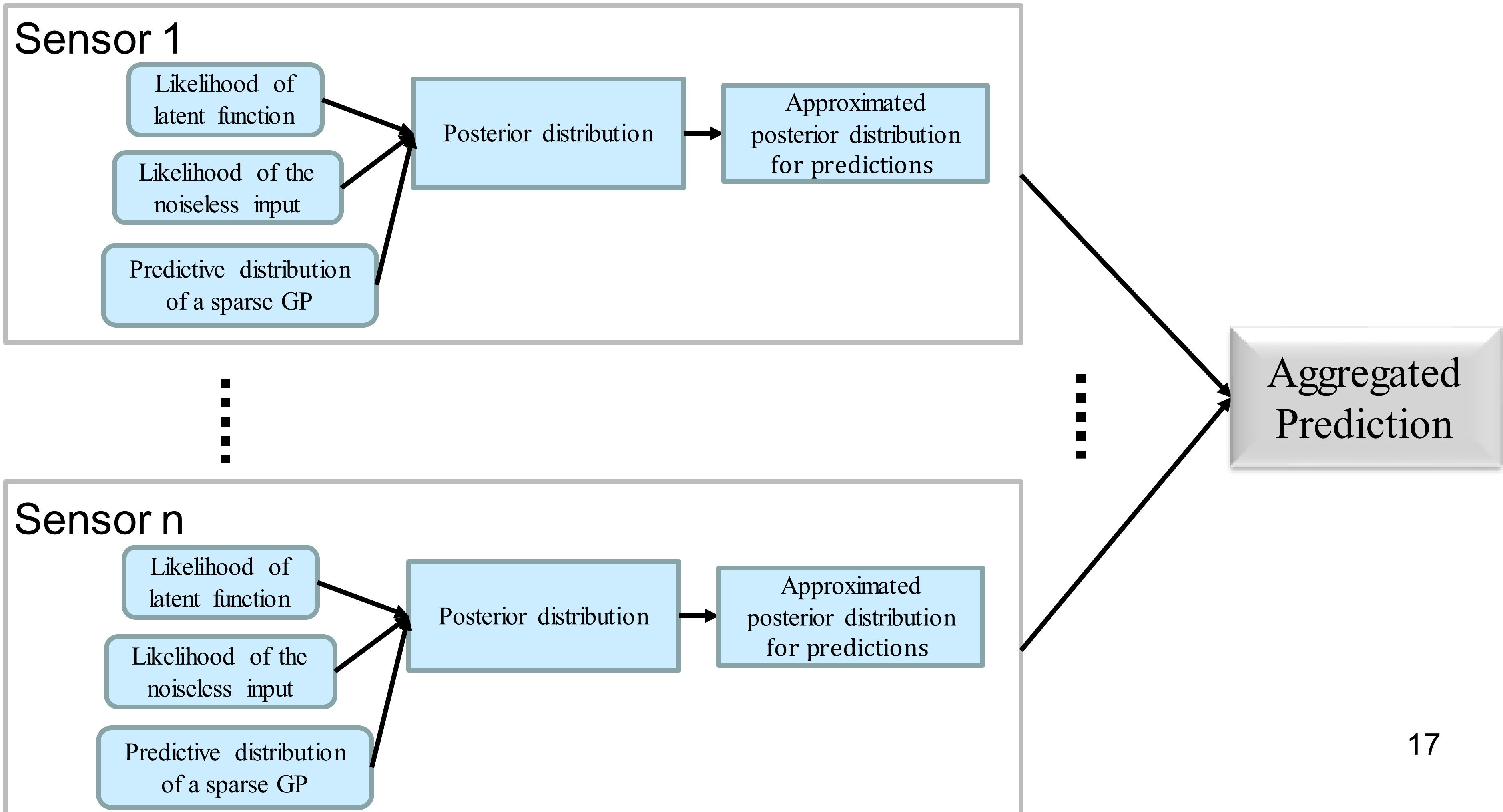
- Labeling rule (for data (label) i, C classes in total)

$$y_i = \arg\max_c \; f^c(\mathbf{x}_i)$$



Distributed Classification based on DGP

- Distributed classification



Summary

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- Model free tracking approach for distributed system
  - Computational complexity
  - Communication cost
- Hyperparameter online learning
- Probabilistic distributed machine learning approach for ground vehicle classification

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Thank you !