A MULTIPLE TARGET TRACKING STRATEGY USING MOVING HORIZON ESTIMATION APPROACH

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ABSTRACT

Tracking multiple road users is playing a significant role in autonomous vehicles and advanced driver assistance systems. Different from Multiple Target Tracking (MTT) in aerospace, the motion of the ground vehicles is likely constrained by their operational environment such as road and terrain. This information could be taken as additional domain knowledge and exploited in the development of tracking algorithms so as to enhance tracking quality and continuity.This paper proposes a new MTT strategy, Multiple Hypothesis Tracking using Moving Horizon Estimation approach (MHE-MHT), for tracking ground vehicles aided by road width constraints. In this strategy, tracking association ambiguity is handled by MHT algorithms which are proved as a preferred data association method for solving the data association problem arising in MTT. Unlike most of the MTT strategies, which solve target state estimation using Kalman filter (and its derivations), we propose a new solution using the moving horizon estimation (MHE) concept. By applying optimization based MHE, not only nonlinear dynamic systems but additional state constraints in target tracking problems such as road width can be naturally handled. The proposed MHE-MHT algorithm is demonstrated by a ground vehicle tracking scenario with an unknown and time varying number of targets observed in clutter environments. Using the optimal subpattern assignment metric, numerical results are presented to show the advantages of the constrained MHE-MHT structure by comparing it with the Kalman filter based MHT.

Keywords: Multiple target tracking, Multiple hypothesis tracking, Moving horizon estimation, Inequality constraints, Autonomous vehicles

INTRODUCTION

Multiple target tracking (MTT) is an important research topic in automated vehicle field. Although a number of MTT algorithms have been developed, e.g. [1], it is still a quite challenging task to implement MTT in realistic situations, especially when suffering from low visibility of sensors, high clutter and high target density. One promising approach that has drawn a great deal of attention recently is to improve the performance of tracking algorithms by utilizing trajectory and other constraints/knowledge imposed from environments including available road maps. It has become a consensus that prior nonstandard information such as target speed constraints, road network and terrain information can be exploited in the tracker to reduce estimaiton error and provide better tracking accuracy [2]. For instance, a vehicle travelling on a road is expected to move within the road boundaries and follow its speed limitation. In other words, the performance of tracking systems is often limited if ignoring or not taking use of this additional source of informaiton. Even for the cases of low signal quality with high clutter density, the incorporation fo such constaint information is sufficient enough to get a relatively good tracking performance [11].

A. Constrained state estimation

One effective approach of solving the road constrained MTT is to incorporate the constraint-related information into a standard filter algorithm (state estimation process) as state constraints. For most MTT structures, Kalman filtering and its variations are commonly used to estimate the state of a target based on its state process and measurement models. However, when the road state constraints cannot fit easily into the structure of a Kalman filter, they are often ignored or dealt with heuristically [3Although constrained Kalman filter methods are relatively easy in implementation, these methods have several disadvantages even for basic linear and equality constraints [3]. Recently, some other methods, for example, see [7], [8], [9], [10], are also developed based on optimization and truncation approaches. The majority of filters proposed to solve the constrained estimation problems focus on linear (in)equality or nonlinear equality constraints. A little research has been conducted on nonlinear inequality

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constraints so far. However, (non)linear inequality constraints have played an important role for most tracking scenarios in ground vehicle tracking problems,, e.g. roundabout boundary.

More specifically, Rao et al. [10] have proposed a constrained state estimation for nonlinear discrete-time systems. It is based on a moving horizon concept based state estimation known as moving horizon estimation (MHE). The basic strategy of MHE in determining the optimal state estimation is to reformulate the estimation problem as an optimisation problem using a fixed-size estimation window. This method has been widely used in chemical engineering. Other applications include hybrid system, distributed, network system, large-scale system and so on. However, the implementation of moving horizon approach based estimation method in target tracking is still relatively an uncharted area. Advantages for using MHE to solve target tracking state estimation could be significant. Since the method is optimization based, road constraints or similar in target tracking problems can be naturally handled by MHE as additional (non)linear and/or (in)equality constraints on linear or nonlinear systems under consideration. In addition to state constraints, MHE is also able to incorporate constraints on the state process and/or observation noises. In vehicle tracking, such constraints are typically used to model bounded disturbance or truncated distribution/density representing the influence of the operation environment on vehicle movement such as vehicle acceleration and deceleration.

Another advantage of using MHE as a state estimation method in target tracking is that it always considers a window of N latest measurements. Such feature is very meaningful in target tracking problems especially when targets are occluded by each other/static obstacles which leads to no reliable measurement at specific time step/steps. MHE utilizes the measurements in a receding horizon window could reduce the effect of unreliable measurements such as in the above situation in state estimation. Simulation results in [4] show that MHE achieves the smallest estimation error for nonlinear systems and nonlinear constraints. Theoretically, for a linear system without constraints and with a quadratic cost, MHE reduces to Kalman filtering algorithms when an infinite horizon window is considered.

B. Multiple target tracking problem

The problem of estimating the position of moving targets, also known as MTT, has become an important part in autonomous vehicles and advanced driver assistance systems. Knowledge about the state of moving objects can be taken as powerful information to improve the level of autonomy for vehicles. MTT techniques are required in a number of automotive applications including Advanced Driver Assistance Systems (ADAS), Collision Avoidance Systems, and Vehicle-automation Systems. Such systems can incorporate functions such as adaptive cruise control, lane keeping, precise manoeuvring, pedestrian detection and so on [12] aiming for achieving an improved collision avoidance behaviour and safe road driving even in populated environments. By using state-of-the-art on-board sensors such like radar, lidar, GPS and camera vision systems together with accurate global and local maps, different levels of automation could be achieved in automotive applications, from individual autonomous functionalities to fully automated vehicles.

Several approaches for MTT have been developed over the last decades, overviews can be found in Pulford [13] and Christophe [14]. Basically, these methods can be divided into two categories – the data association based 'classic' methods and the more recent finite set statistics (FISST) based approaches. The data association based methods are largely based on probability, stochastic processes and estimation theory. Existing methods include Nearest Neighbour Standard Filter (NNSF) [15], Global Nearest Neighbour (GNN) approach [16], Joint Probabilistic Data Association (JPDA) [17] and Multiple Hypothesis tracking (MHT) algorithm [18]. Among them, MHE is a more complex approach that considers data association across multiple scans and multiple hypotheses. In other words, MHT algorithm attempts to keep all possible association hypotheses over multiple frames of data. This results in an exponentially growing number of hypotheses and thus a NP-hard problem. Cox [19] in 1997 developed an efficient implementation by using polynomial time optimization algorithm to find the k-best solutions to an assignment problem along with pruning and merging techniques to reduce the number of low probability hypotheses. MHT essentially keeps a set of multiple hypotheses and thus the assignment ambiguity will be resolved in future when subsequent new observations are arrived. In this case, hard decisions are not made until they need to be with the fact of using more information, rather than just the current data frame, thus possible error association could be corrected when more evidences are updated. Such features along with the dramatic increases in computational capabilities have made MHT a preferred data association method for modern systems [20].

Until very recent, a new concept has been introduced in MTT area - the random finite set statistics (FISST) [27]. While the conventional MTT methods try to solve the problem explicitly by expending single target tracking with data association capabilities, the number of targets is also considered as a random variable (random set) and explicit data association are avoided in FISST. The innovation of FISST is to model both the system and measurement as

random finite sets (RFSs) and directly apply the Bayes recursion to these set-valued random variables and thus solving the data association problem implicitly. In contrast to explicit data association methods, conventional probability-mass functions are replaced by belief-mass functions. Probability hypothesis density filter (PHD) [28] and multi-target multi-Bernoulli (MeMBer) [29] filter proposed by Mahler have successfully implemented the FISST concept into MTT.

The objective of this paper is to derive an efficient strategy for road-constrained MTT. The main contribution of this work is twofold: 1) a constrained MHE algorithm is proposed to solve the state estimation problem arising in road maps assisted target tracking. Since MHE is an optimization based method, it provides a natural way to handle nonlinear systems and incorporate various inequality constraints that may be difficult to be dealt with in other state estimation algorithms. 2) The work is further extended from single target tracking into MTT. A new MTT strategy for tracking multiple ground vehicles, namely MHE-MHT, is proposed, where moving horizon concept is combined with MHT to incorporate various road and other environment information. In this combined strategy, tracking association ambiguity is handled by MHT algorithms that have been proved as a preferred data association method while constrained state estimation is solved by MHE.

The paper is organized as follows. After presenting the introduction in road map constrained MTT, MHE based single target tracking is proposed for incorporating the road and other possible constraints. This work is further extended to MTT by combing with MHT in the following section In order to verify the efficiency of the proposed algorithms, simulation results of multiple target tracking with inequality road width constraints are presented . Finally, this paper ends with conclusions.

MHE BASED TARGET TRACKING WITH ROAD CONSTRAINT

In the operation of automated vehicles, it is necessary to track all the nearby road users to make sure the safety of the vehicles and other road users. Tracking road users is in fact a constrained estimation problem as the objects of interest must be on the road. In this section, both the road constrained state estimation problem and MHE based target tracking are described.

A. System specification

Consider the movement of objects of interest described by the discrete system:

$$x_{k+1} = f(x_k) + \omega_k \qquad (1)$$

and the observation equation:

$$y_k = h(x_k) + v_k \qquad (2)$$

where the time point k takes integer values, $f: \mathbb{R}^n \to \mathbb{R}^n$ is the nonlinear system function and $h: \mathbb{R}^n \to \mathbb{R}^m$ is the nonlinear measurement model. $x_k \in \mathbb{R}^n$ is the state vector, $y_k \in \mathbb{R}^m$ is the vector of available measurements. The vectors $\omega_k \in \mathbb{R}^n$ and $v_k \in \mathbb{R}^m$ are Gaussian noises of the process and the measurement described by independent pdfs $p(\omega_k) = N(0, Q)$ and $p(v_k) = N(0, R)$, respectively, where Q and R are covariance matrices. It is commonly assumed that the initial pdf of the state vector is known as a Gaussian pdf $p(x_0) = N(\tilde{x}_0, \tilde{P}_0)$. Let F_k, G_k and H_k be the Jacobian matrices with respect to x_k, ω_k and measurement states, respectively.. Then the system described in (1) and (2) is now equivalent to a linear system.

B. Target tracking road width constraints

As discussed in Introduction, ground targets are constrained when moving along a road network. Thus the knowledge of terrain database and road maps can be used as constraints and incorporated into the tracking algorithm. In most existing techniques, the road map constraints target motion in a one-dimensional physical space [30] (by ignoring the road width) and incorporate them as equality constraints. This is fairly good approach when an observer is far away from the moving objects such as in the scenario of unmanned aircraft tracking a ground vehicle. However in automated vehicles, only objects in proximity are of interests. The road width is comparable to the measurement accuracy (high accuracy sensors such as lidar). In this paper, road network information is considered as road width inequality constraints and the target motion is restricted by these physical constraints in both straight and curved segments.

Linear state inequality constraints Suppose that at each time step k, x_k is subject to the following linear inequality constraint:

$$a_k \leq C_k(x_k) \leq b_k \qquad (3)$$

where $C_k: \mathbb{R}^n \to \mathbb{R}^c$, $a_k, b_k \in \mathbb{R}^c$, and the inequality $\leq$ holds for all elements of the vectors and $a_k \neq b_k, \forall k$. C_k is a known $c \times n$ matrix, a_k and b_k are the known vectors each with a dimension of $c \times 1$ representing the lower and upper road boundary individually, c is the number of constraints, n is the number of states, and $c \leq n$. C_k is supposed to be of full rank. For target tracking with straight (linear) road width constraint shown in Figure 1, Eq (3) is expressed as:

$$\begin{bmatrix} -I \\ I \end{bmatrix} * T_{g,l}(x_k) \leq \begin{bmatrix} -ub \\ lb \end{bmatrix} \qquad (4)$$

where $T_{g,l}$ is known as the transformation matrix representing the rotation from the global coordinate to the road network local coordinate (with orientation along and orthogonal to the road) by rotation angle θ.
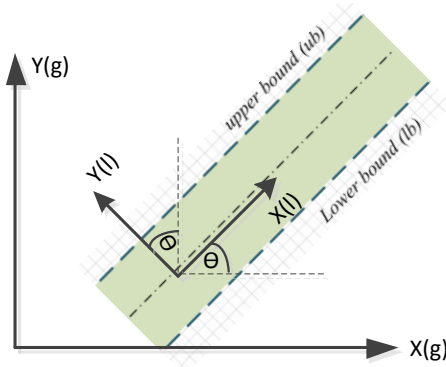

Figure 1. Straight road width linear constraint

Nonlinear state inequality constraints In the same fashion as the linear road width constraint shown in (3), a circular or curved road segment shown in Figure 2 can be represented as a nonlinear inequality constraint as

$$r_1 \leq \sqrt{{x_{1,k}}^2 + {x_{2,k}}^2} \leq r_2 \qquad (5)$$

The road is defined by two arcs with radii r_1 and r_2 representing the lower/upper road boundary, with the center at the origin of the Cartesian coordinate system. At each time step k, target position state $x_{1,k}$ and $x_{2,k}$ are subject to the following nonlinear inequality constraint
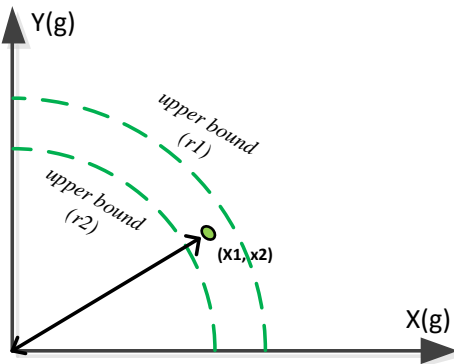

Figure 2. Curved road width nonlinear constraint

C. Moving horizon estimation with constraints

MHE is an optimization approach based state estimation method that can take into account the constraint during estimation process and provide a constrained estimate directly. Essentially, MHE follows Bayes rule which maximizes the probability density function of the past states given the measurements in a fixed length of horizon . Considering a horizon length of N past time steps, the joint conditional density is then given by:

$$p(X_N|Y_N) \propto p(Y_N|X_N)\, p(X_N|Y_{0:k-N-1}), \qquad (6)$$

where $p(X_N|Y_{0:k-N-1}) = p(x_{k-N}, \dots, x_{k-1}|y_0, \dots, y_{k-N-1})$, is the *a priori* state density given the measurements before the horizon; $p(Y_N|X_N) = p(y_{k-N}, \dots, y_{k-1}|x_{k-N}, \dots, x_{k-1})$ is the joint measurement likelihood function. Assuming that X_N is a first order Markovian chain, the *a posteriori* joint conditional density of (6) is:

$$p(X_N|Y_N) = c \prod_{j=k-N}^{k-1} p(y_j|x_j) \prod_{j=k-N}^{k-1} p(x_{j+1}|x_j) p(x_{k-N}|Y_{0:k-N-1}), \qquad (7)$$

where c is the constant and $p(y_j|x_j)$ is the likelihood function for each measurement within the horizon. $p(x_{j+1}|x_j)$ is the state transition probability density function and $p(x_{k-N}|Y_{0:k-N-1})$ is the *a priori* density of the initial state of the horizon. For system (1) and (2), the state transition pdf is defined as $p(x_{k+1} - f(x_k))$:

$$p(x_{k+1}|x_k) = p(\omega_k) = p(x_{k+1} - \hat{x}_{k+1}) = p(x_{k+1} - f(x_k)), \qquad (8)$$

where ω_k is the system process noise defined by $N(0, Q)$, and the likelihood function is defined by $p(y_k - h(x_k))$

$$p(y_k|x_k) = p(v_k) = p(y_k - \hat{y}_k) = p(y_k - h(x_k)), \qquad (9)$$

where v_k is the measurement noise of $N(0, R)$. Now by applying negative logarithm to joint density (7), we obtain the MHE cost function for system (1)-(2) which is a quadratic programming (optimization) problem:

$$\phi_T^* = \min_{x_0, \{\omega_k\}_{k=0}^{T-1}} \phi_T(x_0, \{\omega_k\}) = \arg \min_{z(,\{\omega_k\}_{k=T-N}^{T-1})} \sum_{k=T-N}^{T-1} \|\omega_k\|_{Q^{-1}}^2 + \|v_k\|_{R^{-1}}^2 + \Gamma_{T-N}(z), \qquad (10)$$

where $\|\alpha\|_A^2 = \alpha^T A \alpha$ for quadratic form. $x_k := x(k; z, \{\omega_j\}_{j=T-N}^{k-1})$ denotes the solution of (10) for system (1),(2) at time k with initial state z and process noise $\{\omega_j\}_{j=T-N}^{k-1}$ in horizon length. $\Gamma_{T-N}(z)$ is referred to as arrival cost which plays an important role in summarizing the effect of the past measurements $\{y_k\}_{k=0}^{T-N-1}$ as *a priori* information on the state x_{T-N} ($\Gamma_{T-N}(z) = -\log(p(x_{k-N}|Y_{0:k-N-1}))$). However, the initialization of MHE with the best choice of the arrival cost term is an open issue. In this paper, the arrival cost is approximated using the EKF with the following form:

$$\Gamma_{T-N}(z) \approx (z - \bar{x}_{T-N}^{mh})' P_{T-N}^{-1} (z - \bar{x}_{T-N}^{mh}), \qquad (11)$$

where $\bar{x}_{T-N}^{mh}$ is the optimal estimate at time *T-N* generated in (10) given measurements from time 0 to *T-N-1*, the covariance matrix P_{T-N} is an estimate of the covariance of $\bar{x}_{T-N}^{mh}$ calculated by EKF. Typically any nonlinear filter capable of propagating the conditional mean and covariance could be used to compute the arrival cost in MHE such as unscented Kalmen filters, particle filters and cell filters.

Since MHE is an optimization framework based state estimation algorithm, the physical road width constraints discussed above could be easily imposed on the MHE state variables.

MHE BASED MULTIPLE HYPOTHESIS TRACKING (MHE-MHT)

In this section, we first review the original MHT algorithm described by Reid [18] and Cox [19]. Then the formation of MHE-MHT structure is set forth explicitly.

A. Multiple hypothesis tracking structure

The original MHT algorithm is a deferred decision logic which forms alternative association hypotheses in order to deal with observation to track assignment uncertainties. According to Reid's paper, the hypothesis based MHT keeps the past hypotheses in the memory between consecutive time steps. MHT has the advantage of being able to deal with track creation, confirmation, occlusion and deletion in a probabilistically consistent way. The original MHT framework contains three main processes: hypothesis generation, probability calculation and hypothesis reduction. When a new measurement is received, observations that fall within the gate region set a possible measurement to track assignment thus an existing hypothesis is extended to a set of new hypotheses by considering all possible tracks to measurements assignments. Several assumptions are made when generating hypothesis:

Assumption 1

(i) Each hypothesis contains a set of compatible observation to track assignments,
(ii) Assignments are defined as 'compatible' if they have no measurements in common which means in each Hypothesis, each measurement can only update with one of the existing tracks.

B. MHE-MHT framework

In Figure 3, we present the flow diagram of MHE-MHT algorithm. Let $Y_k = \{y_i^k\}_{i=1}^{m_k}$ denote the set of m_k measurements received at time k. Each of the measurement has three possible hypotheses:
- The measurement starts a new target
- The measurement is a false alarm
- The measurement belongs to an existing target

1) Gate Check: First the distance between the predicted priori target and the current measurements is calculated known as measurement prediction error/innovation. The prediction of target position is done by KF prediction update and the distance is defined as the Mahalanobis distance:

$$\left(y_m^k - \hat{y}_{k/k-1}\right)^T S_{k/k-1}\left(y_m^k - \hat{y}_{k/k-1}\right) \leq Gating, \qquad (12)$$

where y_m^k is the measurement m at time k, $\hat{y}_{k/k-1}$ is the predicted target position and $S_{k/k-1}$ is the covariance of innovation vector , $S^{-1}_{k/k-1} = H\,P_{(k)}H + R$ both are calculated by KF. $Gating$ is a matrix of binary values which indicates maximum possible distance between measurement and targets. Only the measurements inside the gate are considered for assignment. Later, these statistical differences are used in data association.

2) Data association: MHE-MHT implements the same data association process as the Reids algorithm[18] which has been explained above. The assignment matrix is generated to represent all possible target-to-measurement associations. Then each new hypothesis contains a set of potential target-to-measurement assignments, leading to an exhaustive approach of enumerating all the possible assignment combinations. To solve this problem, the Murty's algorithm [19] is used to find the k-best assignment/new hypotheses generated from each parent hypothesis. To further reduce the computational cost, a merging algorithm is also implemented in to prevent hypotheses from being considered if the ratio of their probability to the best hypothesis becomes too small.

3) Target Maintenance: For ground target tracking scenarios, vehicles may enter or leave the surveillance field of view during the tracking process. Moreover, occlusion or miss detection is also possible when a vehicle is hidden behind another one. In order to achieve a fully functional tracking algorithm, we implement target maintenance logic in MHE-MHT structure. Basically, there are three possible status for a set of targets in this logic: target initiation, confirmation/deletion and maintenance. The implementation is based on track-oriented approach. The targets present at a time step are a combination of existing targets from the parent tracks and any new targets resulting from the set of measurement associations. For any targets in existence at time k-1, the possible associations at time k:
- Target initiation: If the measurement is associated with a new target and the new target hypothesis appears in the current k-best hypotheses. Add a target *lifetime* index to the target with value 1.
- Target confirmation/deletion: The new target is confirmed only if the detected target appears along the same track over a consecutive iterations of *Ct* times. The *lifetime* index is accumulated by 1whenever the tentative target is detected and will become *Ct* (confirmation threshold) when confirmed. On the contrary, the *lifetime* index for any existing target is reduced by 1 whenever the target is not associated with the current measurement and will be permanently deleted from target list when the *lifetime* is 0.
- Target maintenance: The confirmed target may be temporally occluded or undetected by the sensor.

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For this situation, the track measurement for unassociated targets is updated according to the predicted position of the target last associated states.
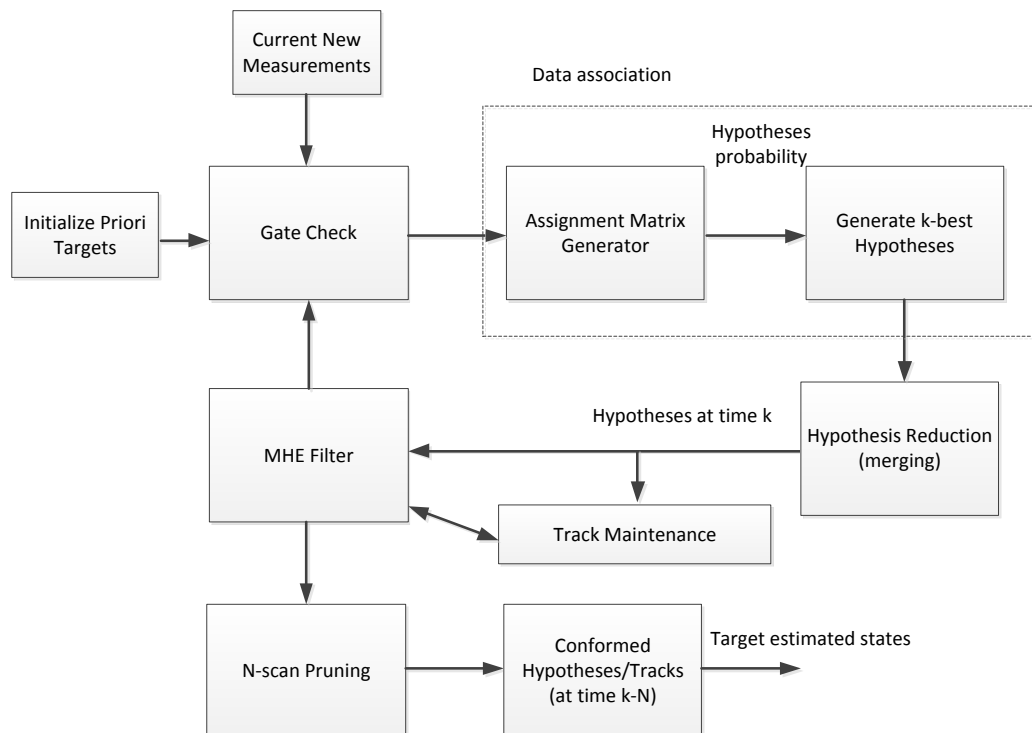

Figure 3. Flow diagram of MHE-MHT algorithm

4) MHE filter: The details about implementing MHE for constrained target tracking have been discussed in previous section in this paper. In this part, the main work will focus on comparing the difference between MHE and KF under the MHT structure. In the original MHT, the 'Filter' process is based on Kalman state estimation including two individual steps: prediction update and measurement update. However, the two steps are combined in MHE and solved directly by optimization solver. In MHE, the state estimation is determined online by solving a finite horizon state estimation problem. To determine new estimate of the target state, the finite horizon of latest measurements are resolved while the problem is solved recursively with only the current step measurement being considered in KF. Assuming that at time k, $x_k := x(k; z, \{\omega_j\}_{j=T-N}^{k-1})$ denotes the solution of MHE optimization function (10) for a linear, time-invariant discrete-time system with initial state z and process noise $\{\omega_j\}_{j=T-N}^{k-1}$ in horizon length N. Then the estimation result is:

$$x(k; z, \{\omega_j\}_{j=T-N}^{k-1}) = F^k z + \sum_{j=0}^{k-1} F^{k-j-1} G \omega_j \ , \qquad (13)$$

and if considering the road linear inequality constraint in (3), an additional MHE state constraint is consideredas

$$a_k \leq H_{C_k} F^k z + H_{C_k} \sum_{j=0}^{k-1} F^{k-j-1} G w_j + \sum_{j=0}^{k-1} v_j \leq b_k, \qquad (14)$$

where F is the linear state transition matrix, and H_{C_k} is the linear constraint matrix. $\{v_j\}_{j=T-N}^{k-1}$ is the estimated measurement noise for N horizon length.

The filtering process would be similar to KF if measurements are always detected and updated with the target, However, a problem arises when miss detection happens among a horizon of measurements, since there is no individual predict update process in MHE and the estimation problem is solved by an optimization solver. In the MHE-MHT algorithm, the missing target measurement is presumed as one step predicted state calculated by KF: $x_k = F x_{k-1}$ and thus the estimated process noise ω_j and measurement noise v_j for time k is taken as null. This

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assumption is equivalent the one used in KF-MHT for missed detection which treats the non-available posterior measurement updated estimate as the prior predicted state. The proof is shown below:

- For Kalman filter at time k
  Prediction Update: $\hat{x}_{k|k-1} = F\hat{x}_{k-1|k-1}$
  Measurement Update: $\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(y_k - \hat{y}_{k|k-1})$
- If at time k, measurement y_k is missing, then the predicted state $\hat{x}_{k|k-1}$ is taken as estimated state $\hat{x}_{k|k}$, In other words, the measurement update step is rejected
- So $K_k(y_k - \hat{y}_{k|k-1}) = 0$, and thus $y_k = \hat{y}_{k|k-1}$, where $\hat{y}_{k|k-1}$ is predicted target $\hat{y}_{k|k-1} = HF\hat{x}_{k-1|k-1}$
- In this case, for system :
  $x_{k+1} = Fx_k + \omega_k$
  $y_k = Hx_k + v_k$
- So $y_k = \hat{y}_{k|k-1} = HFx_k$ and thus ω_k and v_k are null

Correspondingly, the high level logic for MHE-MHT target maintenance is shown below in Table 1:

Table 1.

High level logic for MHE-MHT target maintenance

-- At time k, for nExistedTarg number of existing target in a hypothesis
For k=1: nExistedTarg
  (Case one: permanent deleted targets)
  If LifePoint == 0
    Continue; (the target is permanently deleted/already disappeared)
  End
  (Case two: target maintenance—target updating with measurement or temporally miss detection)
  If Targ≠asso (Target not associated with current measurement)
    LifePoint=LifePoint-1;
      If LifePoint>0
        Implement KF prediction for MHE estimation
      End
  Else (Target associated with current measurement)
    Implement MHE update;
      If LifePoint<MaxLifePoint
        LifePoint= LifePoint+1;
      End
  End
  (Case three: target initialization)
 For k=1: nNewTarg (measurement is associated to a new target)
    Use current measurement as initial position;
    LifePoint=0;
End

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5) N-scan pruning: The key principle of the MHT method is that difficult data association decisions are deferred until more data are received, which could be achieved by using N-scan pruning. The structure provides a convenient mechanism for implementing deferred decision logic and for presenting a coherent output from the MHT. The continued growth of the tracks is also controlled by N-scan pruning technique by keeping only the N previous scans in the trees. The hypotheses with low probability are deleted after N-scan pruning. The survive target after pruning process are predicted using the new measurements obtained and reformed into new hypotheses. In MHE-MHT the number of N scans is chosen as the same value for horizon length in MHE. As a result, the association uncertainty at time k-N is resolved by the hypotheses given at time k and meanwhile the estimation process considers all measurements within the last N scans.

SIMULATION and RESULTS

In this section, the proposed algorithm is evaluated by means of two examples. The first example is aimed at illustration of handling nonlinear inequality road constraint using a MHE based approach, using a single target circular road tracking scenario. The second one which is inspired by [28] is a complex multiple target tracking scenario incorporating road inequality constraints for an intersection scenario.

A. Target tracking with nonlinear road inequality constraints

In the first example, we follow the previous study of [8] in 2012 to set up the test scenario. A moving vehicle on a circular road section is considered as shown in Figure 4. The road is defined by two boundaries with two arcs of $r1=96m$ and $r2=100m$, respectively, centered at the origin of a Cartesian coordinate system. The vehicle dynamics is described by a white noise acceleration motion model.

$$x_{k+1} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} T^2/2 & 0 \\ 0 & T^2/2 \\ T & 0 \\ 0 & T \end{bmatrix} \omega_k \qquad (15)$$

where the state vector $x_k = [x_{1,k}, x_{2,k}, \dot{x}_{1,k}, \dot{x}_{1,k}]^T$ consists of the vehicle position and velocity in x and y directions, $T = 1$ is the sampling interval, and ω_k is a two-dimensional Gaussian process noise with zero mean and covariance matrix $Q = eye(2)$. The initial state of the vehicle is $x_0 = [98,0,0,10]^T$. The vehicle is supposed to move for $k = 1, ..., K$ with $K = 20$.

The vehicle is tracked by range and bearing sensors modelled as:

$$z_k = \begin{bmatrix} \sqrt{x_{1,k}^2 + x_{2,k}^2} \\ arctan\left(\frac{x_{2,k}}{x_{1,k}}\right) \end{bmatrix} + v_k \qquad (16)$$

where v_k is a two-dimensional Gaussian zero-mean measurement noise with a diagonal covariance matrix $R = diag\{8, 10^{-3}\}$. Given the road boundaries, the state inequality constraint is shown in (5): $r_1 \leq \sqrt{x_{1,k}^2 + x_{2,k}^2} \leq r_2$.
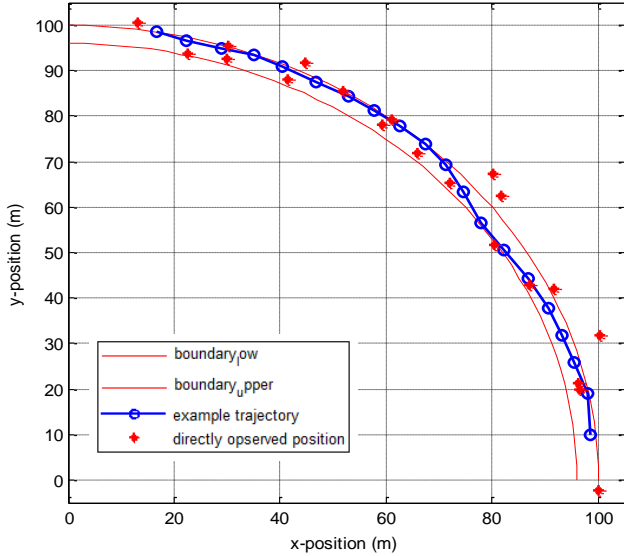

Figure 4. Tracking scenario for example 1

The performance of constrained MHE filter was measured using the mean-square error (MSE):

$$MSE = \left(2(K+1)\right)^{-1} \sum_{k=0}^{K} \sum_{i=1}^{2} (x_{1,k} - \hat{x}_{i,k})^2 \qquad (17)$$

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We compare the performance of constrained MHE (cMHE) with different horizon length (N=2 and 8) with some other conventional filters. In [8], Straka compared several conventional filters including the unscented Kalman filter (UKF), divided difference filter (DFF), the Gaussian mixture filter (GMF), constrained particle filter (cPF) and the truncated versions tUKF, tDDF, and tGMF. The results are shown in table.2:

Table 2.

Estimation performance of filters for example 1

	UKF	DDF	GMF	tUKF	tDDF	tGMF	cPF(10^3samples)	cMHE (N=2)	cMHE(N=8)
MSE	7.79	20.27	6.31	4.23	4.90	3.63	4.29	4.46	3.98
Time (s)	0.019	0.027	0.042	3.280	3.458	6.612	9.28	1.09	2.97

It can be seen from Table 2 that the tUKF, tDDF, tGMF outperform their unconstrained conventional filters UKF, DDF and GMF. The cPF provides high quality estimates however at an expense of high computational cost. The proposed constrained MHE in this paper provides reasonable good performance especially when increasing the horizon length. When N=8 the cMHE provides the second best MSE=3.98 among all filters in Table.2 which is slightly worse than tGMF with MSE=3.63 however cMHE provides a much better the computational cost with only half time taken for tGMF by using *fmincon* server in MATLAB.

B. Multiple target tracking for intersecting road scenario

In the second example, we set up a multiple target tracking simulation for interacting scenario. As illustrated in Figure5, the region of interest is [-1000m,1000m] x [-1000m,1000m] with an unknown and time varying number of targets observed in a clutter environment. The vehicle dynamics is described the same as (15) and the state vector $x_k = [x_{1,k}, x_{2,k}, \dot{x}_{1,k}, \dot{x}_{1,k}]^T$ consists of the vehicle position and velocity while the measurement model is defined as a noisy position in x and y directions. $T = 1$ is the sampling interval and the two-dimensional Gaussian process noise has covariance matrix Q of 5 m/s^2 standard deviation. Initially, two targets start moving in the environment with initial state $x_{1,0} = [250,250,0,0]^T$ and $x_{2,0} = [-250, -250,0,0]^T$. The target initial covariance is defined as $P_0 = diag[100,100,25,25]^T$ for both two targets. Each target is detected with a probability of $P_d = 0.98$, and the Gaussian noise based position measurement has a standard deviation of 10m in both directions. The detected measurements are immersed in clutter that can be modeled as a Poisson distribution with clutter density of β_{FA}=12.5 * 10^{-6} over the $4 * 10^6 m^2$ region (i.e., 50 clutter returns over the region of interest). As shown in Figure 5, Target 1 and 2 appear at the same time in different locations, traveling along straight lines and cross each other at K=53s. A new target spawns from Target 1's trajectory at time K=66s. The total simulation time is K=100s. The target trajectories are supposed to be constrained by road boundaries, each with a width of 6 meters using the road inequality constraint in (4). The position estimates are shown in Figure 6, it can be seen that the constrained MHE-MHT algorithm provides accurate tracking performance. Moreover, the algorithm not only tracks Target 1 and 2 but also able to detect and track the spawned Target 3. The lifetime threshold is defined as 4, which means any new target can only be confirmed if successfully detected in 4 sequential time steps. The horizon length used in MHE in chosen as 4 and so as for N-scan pruning. At each time 3-best hypothesis are generated from each parent hypothesis.
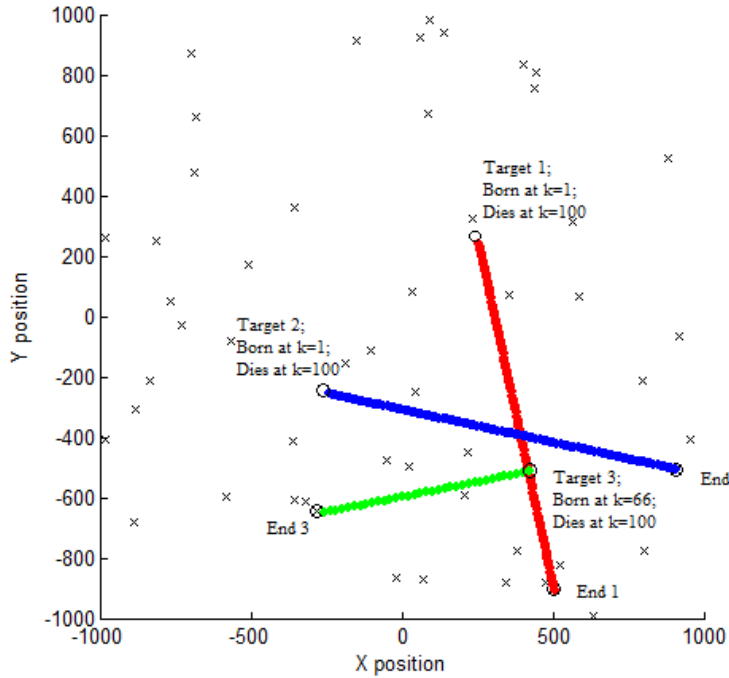
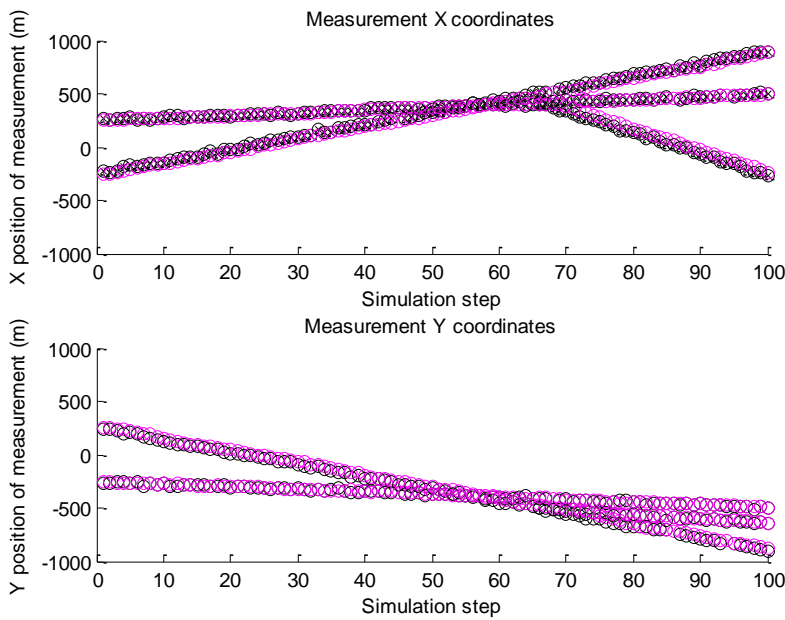*Figure 5. Target trajectories for scenario 2*

Figure 6. Position estimation for MHE-MHT

To further analysis our algorithm, Figure7 shows a comparison between original Kalman filter based MHT and constrained MHE-MHT using the optimal subpattern assignment metric (OSPA) [31] which considers not only the estimation performance but also association accuracy.
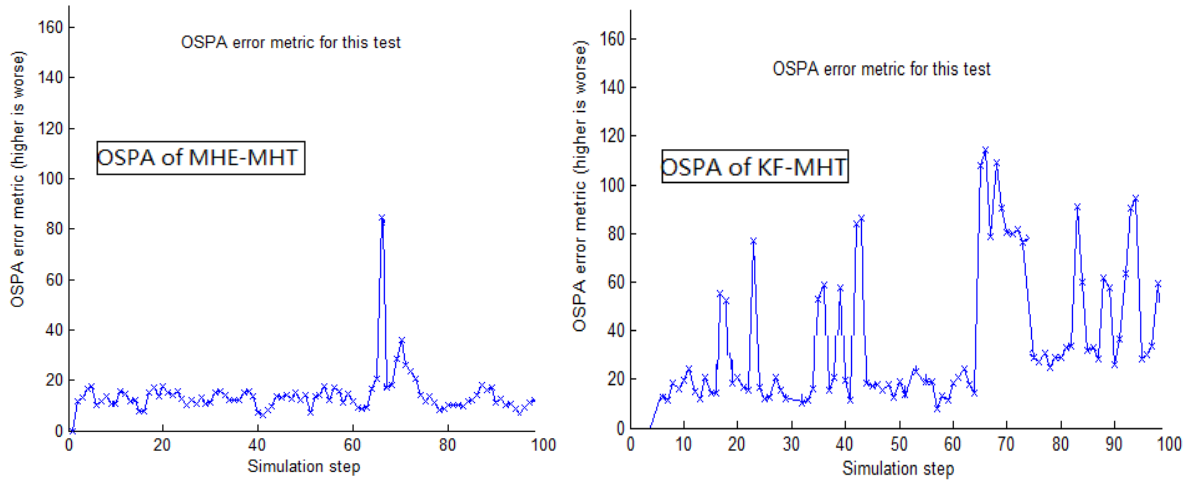
11

Figure 7. OSPA performance for MHE-MHT and KF-MHT algorithm

From the results, it can be seen that the MHE-MHT algorithm performance is more stable than KF-MHT which is concluded by the variation of the OSPA distance over time. This is because of the more accurate state estimation performance for constrained MHE which also affects the accuracy of new target detection and data association. In the original KF-MHT, road width constraint is not considered which makes the predicted target more likely to associate with clutter and thus generate false new targets. At time k=66, the new target appears which makes OSPA increase significantly. However, the faulty association hypotheses are soon discarded in MHE-MHT by the correct one which has higher hypothesis probability.

CONCLUSION

In this paper, we propose a novel MHE-MHT algorithm for constrained multiple target tracking problems. External road information is employed by MHE filters in state estimation process. A target maintenance logic is designed for MHE-MHT algorithm to track multiple targets efficiently and accurately. Initial simulation studies have shown the effectiveness of the proposed algorithm against conventional algorithms.
The future work will focus on incorporating extra domain knowledge in the MHE-MHT structure especially for target interaction problems since the target are considered moving independently in most target tracking algorithms without having interacting behaviors with other targets or physical environment. Experimental research combing real sensor data and digital map information will also be carried out.

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