Transmit Beamforming Design for Two-Dimensional Phased-MIMO Radar with Fully-Overlapped Subarrays

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***Abstract*—We investigate a subaperturing technique for two-dimensional (2D) transmit arrays within the context of multiple-input multiple-output (MIMO) radar. Specifically, we investigate the performance of transmit beamforming using fully overlapped subarrays of a 2D transmit array. As reported for linear array of antennas, this 2D transmit array exploits the advantages of the MIMO radar technology without sacrificing the coherent processing gain at the transmit side provided by the phased-array concept. In order to achieve high coherent processing gain, a weight vector should be designed for each subarray to steer the transmit beam in certain 2D sector in space. This is achieved by solving a convex optimization problem that minimizes the difference between a desired transmit beampattern and the actual beampattern produced by the 2D array of antennas, under a constraint in terms of uniform power allocation across the transmit antennas.**

I. Introduction

The innovative concept of MIMO radar has been the focus of intensive research within the academic community and practitioners all over the world in the last decade [1], [2], [3]. In contrast to a phased-array radar which transmits scaled versions of a single waveform, the MIMO radar system transmits via its antennas multiple probing signals that may be chosen to be either correlated or uncorrelated. This waveform diversity enables superior capabilities when compared to a phased-array radar [1]. There are two basic regimes of operation considered in the literature. In the first regime, the transmit and receive array elements are broadly spaced. This kind of concept is known as MIMO radar with widely separated antennas. In the second regime, the transmit and receive array elements are closely spaced, known as MIMO radar with collocated antennas.

Exploiting the concept of widely separated antennas [3], it is possible to capture the spatial diversity of the target's radar cross section (RCS), improve the target detection performance, enhance the ability to combat signal scintillation, and estimate accurately the parameters of rapidly moving targets. Moving on to the concept of MIMO radar with colocated antennas [2], this paradigm has been shown to offer higher resolution, higher sensitivity to detect slowly moving targets, better parameter identifiability, and direct applicability of adaptive array techniques.

It is known that the aforementioned advantages of MIMO radar are achieved at the cost of losing the transmit coherent processing gain offered by the phased-array radar [5]. The innovative idea of combining the benefits of phased-array and MIMO radars has been reported recently in the literature [5], [9]. In [5], the breakthrough concept of fully overlapped subaperturing of a Uniform Linear Array (ULA) has been introduced. As proved in [5], the partition of the transmit array into a number of subarrays that are allowed to overlap overcomes the loss of transmit coherent gain and jointly exploits the advantages of the phased-array and MIMO radars.

The transmit beamforming design in MIMO radar with colocated transmit arrays has been extensively investigated in the literature [4]-[8]. In particular, most of the work is focused on a one dimensional ULA. It has been shown in [6] and [8] that convex optimization techniques can solve efficiently the problem of transmit beamforming in a ULA. The extension of this to two-dimensional transmit beamforming optimization is presented in [4].

In this paper, we investigate transmit beamforming design for phased-MIMO radar with fully overlapped 2D transmit subarrays. Each subarray is programed to coherently transmit a waveform which is orthogonal to the waveforms transmitted by other subarrays. In order to achieve high coherent processing gain, a weight vector should be designed for each subarray to steer the transmit beam to a specific 2D sector in space, determined by a desired transmit beampattern. To accomplish this, we solve an optimization problem that minimizes the difference between the desired transmit beampattern and the actual beampattern obtained by the array of antennas under the constraint of uniform power allocation across the transmit antennas. It is possible to add more constraints, such as minimum sidelobe level and uniform power distribution over each subarray. As the optimization problem in its original form is non-convex, it has been converted to convex form using semidefinite relaxation techniques. The simulation results highlight the advantage of the 2D subaperturing technique for MIMO radars.

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II. SYSTEM MODEL

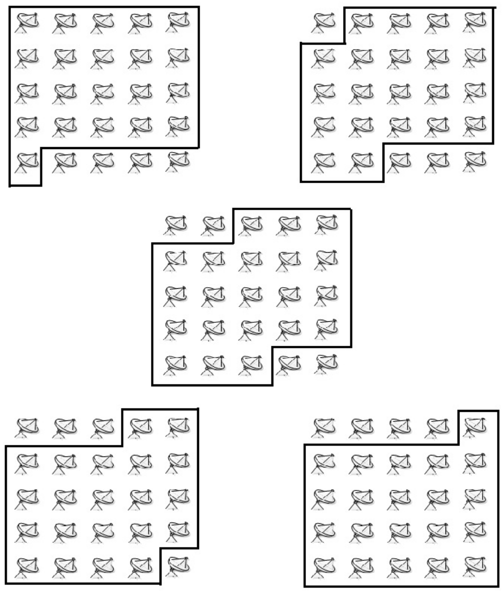


Fig. 1: Fully overlapped subaperturing of a 5×5 uniform rectangular array(URA) when K=5.

In the proposed model, we consider a radar system that has a uniform rectangular array (URA) at the transmit side, which consists of $M_t \times N_t$ antennas, where M_t is the number of antennas in each column and N_t is the number of antennas in each row of the planar transmit array. The adjacent antenna elements at each column are assumed to be equidistant with spacing d_m and at each row also equidistant with spacing d_n. The main idea behind our system model is to partition the transmit 2D array into K subarrays ($1 \leq K \leq M_t \times N_t$) which are fully overlapped. An example of the fully overlapped subaperturing of a 5×5 URA into 5 subarrays is shown in Fig. 1. As described in Fig. 1, each subarray consists of $M_t \times N_t - K + 1$ antennas. In order to achieve this subaperturing, we introduce the selection matrix $\mathbf{Z}_k$, which is an $M_t \times N_t$ matrix containing 0 and 1 entries. When the $(mn)^{th}$ entry equals 1 then the $(mn)^{th}$ element of the 2D array belongs to the k^{th} subarray, while 0 means that the element does not belong to the k^{th} subarray. As a result, the matrix $\mathbf{Z}_k$ defines the k^{th} subarray. The $M_t N_t \times 1$ steering vector associated with the k^{th} subarray can be denoted as:

$$\mathbf{a}_k(\theta, \phi) = vec(\mathbf{Z}_k \odot [\mathbf{u}(\theta, \phi)\mathbf{v}^T(\theta, \phi)]) \tag{1}$$

where $vec(\cdot)$ is the operator that stacks the columns of a matrix in one column vector, $\odot$ denotes the Hadamard product, $(\cdot)^T$ denotes the transpose, θ and ϕ denote the elevation and azimuth angles respectively. The vectors $\mathbf{u}(\theta, \phi) \in C^{M_t \times 1}$ and $\mathbf{v}(\theta, \phi) \in C^{N_t \times 1}$ are written as:

$$\mathbf{u}(\theta, \phi) = [1, e^{j2\pi d_m sin(\theta) cos(\phi)}, \ldots, e^{j2\pi (M_t - 1) d_m sin(\theta) cos(\phi)}]^T$$

$$\mathbf{v}(\theta, \phi) = [1, e^{j2\pi d_n sin(\theta) sin(\phi)}, \ldots, e^{j2\pi (N_t - 1) d_n sin(\theta) sin(\phi)}]^T$$

The k^{th} subarray of the transmit URA emits the k^{th} element of the predesigned independent waveform vector $\boldsymbol{\psi}(t) = [\psi_1(t), \ldots, \psi_K(t)]^T$ of size $K \times 1$, which satisfies the orthogonality condition $\int_{T_0} \boldsymbol{\psi}(t)\boldsymbol{\psi}^H(t) = \mathbf{I}_K$, where T_0 is the radar pulse width, t refers to the time index within the radar pulse, $\mathbf{I}_K$ is the $K \times K$ identity matrix, and $(\cdot)^H$ denotes the Hermitian transpose.

The aim is to focus the energy of the transmit array into a 2D spatial sector defined by $\Theta = [\theta_1 \quad \theta_2]$ in the elevation domain and $\Phi = [\phi_1 \quad \phi_2]$ in the azimuth domain. Therefore, we form K transmit beams, each of them is steered by the corresponding subarray. Then each of the orthogonal waveforms ψ_k is radiated over one beam. The complex envelope of the signals at the output of the k^{th} subarray can be designed by $\mathbf{s}_k(t) = \mathbf{w}_k \psi_k(t)$, where $\mathbf{w}_k \in C^{M_t N_t \times 1}$ is the transmit weight vector, used to form the k^{th} transmit beam. The power of the probing signal emitted by the k^{th} subarray can be modeled as

$$P_k(\theta, \phi) = \mathbf{a}_k^H(\theta, \phi) E\{\mathbf{s}_k(t)\mathbf{s}_k^H(t)\}\mathbf{a}_k(\theta, \phi)$$

$$= \mathbf{a}_k^H(\theta, \phi)\mathbf{w}_k\mathbf{w}_k^H\mathbf{a}_k(\theta, \phi) \tag{2}$$

The array transmit beampattern is hence defined as

$$P(\theta, \phi) = \sum_{k=1}^{K} \mathbf{a}_k^H(\theta, \phi)\mathbf{w}_k\mathbf{w}_k^H\mathbf{a}_k(\theta, \phi) \tag{3}$$

The equation (3) of the total transmit power defines the array transmit beampattern.

III. TRANSMIT BEAMFORMING DESIGN

In order to design the 2D transmit beamforming, we derive the optimization problem of minimizing the maximum difference between the desired 2D transmit beampattern and the transmit beampattern of our system given by (3). The constraint of our optimization problem is the uniform power allocation across the transmit antennas. Therefore, similar to the work in [4] for URA without subaperturing, we wish to solve the following optimization problem:

$$\min_{\mathbf{w}_1, \ldots, \mathbf{w}_K} \quad \max_{\theta, \phi} |P_d(\theta, \phi) - \sum_{k=1}^{K} \mathbf{w}_k^H \mathbf{a}_k(\theta, \phi)\mathbf{a}_k^H(\theta, \phi)\mathbf{w}_k| \tag{4}$$

$$s.t. \quad \sum_{k=1}^{K} |\mathbf{W}_{[lk]}|^2 = \frac{E}{M_t N_t - (K-1)}, \quad l = 1, \ldots, M_t N_t \tag{5}$$

where $\mathbf{W} = [\mathbf{w}_1, ..., \mathbf{w}_K] \in C^{M_t N_t \times K}$ is the transmit beampattern weight matrix, $P_d(\theta, \phi)$ is the desired beampattern and E is the total available power. In the constraint (5), we divide the total power with $M_t N_t - (K-1)$ because there are $M_t N_t - (K-1)$ elements in each subarray space. It is possible to have additional constraints for this optimization problem, such as sidelobe control or uniform power distribution over each subarray. This optimization problem is in a non-convex form. Defining a matrix $\mathbf{X}_k = \mathbf{w}_k\mathbf{w}_k^H \in C^{M_tN_t \times M_tN_t}, k = 1, ..., K$, we formulate the optimization problem as:

$$\min_{\mathbf{X}_1,\ldots,\mathbf{X}_K} \quad \max_{\theta,\phi} |P_d(\theta,\phi) - \sum_{k=1}^{K} Tr\{\mathbf{a}_k(\theta,\phi)\mathbf{a}_k^H(\theta,\phi)\mathbf{X}_k\}|$$

$$s.t. \quad \sum_{k=1}^{K} diag\{\mathbf{X}_k\} = \frac{E}{M_tN_t - (K-1)}\mathbf{1}_{M_tN_t\times 1}$$

$$\mathbf{X}_k \succeq 0, \quad k = 1, \ldots, K$$

$$rank(\mathbf{X}_k) = 1, \quad k = 1, \ldots, K \tag{6}$$

where $Tr\{\cdot\}$ denotes the trace of a matrix, $diag\{\cdot\}$ denotes the diagonal of a square matrix, $\mathbf{1}_{M_tN_t}$ defines the $M_tN_t \times 1$ vector of ones, and $rank(...)$ denotes the rank of a matrix. We use $\mathbf{X}_k \succeq 0, \quad k = 1, \ldots, K$ to indicate that $\mathbf{X}_k$ is positive semidefinite. The rank constraint maintains the optimization problem (6) as non-convex. Relaxing the rank constraint (semidefinite relaxation), we recast the optimization problem as follows [10]:

$$\min_{\mathbf{X}_1,\ldots,\mathbf{X}_K} \quad \max_{\theta,\phi} |P_d(\theta,\phi) - \sum_{k=1}^{K} Tr\{\mathbf{a}_k(\theta,\phi)\mathbf{a}_k^H(\theta,\phi)\mathbf{X}_k\}|$$

$$s.t. \quad \sum_{k=1}^{K} diag\{\mathbf{X}_k\} = \frac{E}{M_tN_t - (K-1)}\mathbf{1}_{M_tN_t\times 1}$$

$$\mathbf{X}_k \succeq 0, \quad k = 1, \ldots, K \tag{7}$$

After the rank relaxation, the optimization problem (7) is convex and it is solved using semidefinite programming (SDP). The next step is to extract the transmit weight vectors from the optimal solution of the optimization problem (7), denoted as $\mathbf{X}_k^*$, for $k = 1, \ldots, K$. There are two cases for deriving the optimal weight vectors $\mathbf{w}_k$. If the rank of $\mathbf{X}_k^*$ is one, which is the ideal case, the optimal $\mathbf{w}_k$ is obtained straightforwardly as the eigenvector of $\mathbf{X}_k^*$, corresponding to the principal eigenvalue, multiplied by the square root of the principal eigenvalue. On the other hand, it is still possible the rank of $\mathbf{X}_k^*$ is greater than one. In this case we need to use randomization techniques to derive the optimal transmit weight vectors [4].

The following randomization technique is applied. Initially, we define the eigenvalue decomposition of $\mathbf{X}_k^*$ as $\mathbf{X}_k^* = \mathbf{U}_k\mathbf{L}_k\mathbf{U}_k^H$. Then we produce Λ random vectors $\mathbf{r}_k^\lambda$, $\lambda = 1, ..., \Lambda$, with elements uniformly distributed on the unit circle of the complex plane, providing us with Λ candidate transmit weight vectors as $\mathbf{w}_k^\lambda = \mathbf{U}_k\mathbf{L}_k^{(1/2)}\mathbf{r}_k^\lambda$. Then, we choose the optimal weight vector $\mathbf{w}_{opt,k}$, as the one which minimizes the objective function of the optimization problem (7). Finally, we normalize the optimal weight vector $\mathbf{w}_{opt,k}$ as:

$$\mathbf{w}_{norm,k} = \mathbf{w}_{opt,k}\frac{||\mathbf{X}_k||_F}{||\mathbf{w}_{opt,k}\mathbf{w}_{opt,k}^H||_F} \tag{8}$$

where $||\cdot||_F$ denotes the Frobenius norm. Using the transmit weight vectors derived in (8) we design the transmit beampattern for our systems.

IV. Simulation Results

In this section, we present the simulation results of the proposed design model. We assume a 5×5 transmit URA with half-wavelength spacing between adjacent antennas ($d_m = d_n = \lambda/2$, where λ is the wavelength). In the first example, the transmit array is divided into 5 subarrays which are fully overlapped as described in Fig.1. Each subarray consists of 21 antennas. The desired beampattern has a mainlobe defined by the 2D sector $\Theta = [-40^o, -20^o]$ in the elevation domain and $\Phi = [50^o, 85^o]$ in the azimuth domain. We also incorporate a transition zone defined by $\Theta = [-50^o, -40^o]\bigcup[-20^0, -10^o]$ and $\Phi = [40^o, 50^o]\bigcup[85^o, 95^o]$. Any error that occurs in this region is ignored in the beamforming design. The 2D transmit beampattern is obtained by solving the optimization problem (7) and it is shown in Fig.2. It is obvious that the power allocation of the transmit beampattern is concentrated in the desired 2D sector. Moreover, the sidelobe levels are very low and do not extend to the whole 2D space.

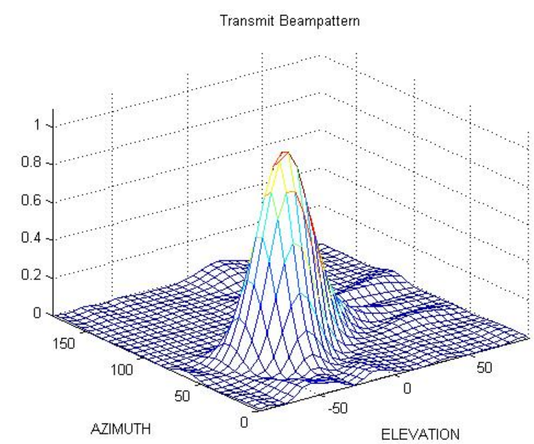


Fig. 2: Transmit beampattern in the case of k=5 subarrays.

In the second example we assume the same 5×5 transmit URA, but the transmit array is divided into 7 subarrays which are fully overlapped. Each subarray consists of 19 antennas. In this simulation, the 2D sector of interest is defined by $\Theta = [15^o, 55^o]$ in the elevation domain and $\Phi = [110^o, 140^o]$ in the azimuth domain. Furthermore, a transition zone is incorporated and defined by $\Theta = [5^o, 15^o]\bigcup[55^o, 65^o]$ and $\Phi = [100^o, 110^o]\bigcup[140^o, 150^o]$. The resulting 2D transmit beampattern is shown in Fig.3. It is clear from the two figures that in the case of 7 fully overlapped subarrays, the sidelobe levels are even lower than the case of 5 subarrays.

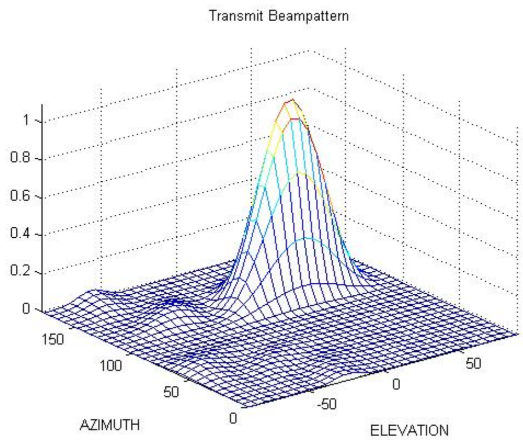


Fig. 3: Transmit beampattern in the case of k=7 subarrays.

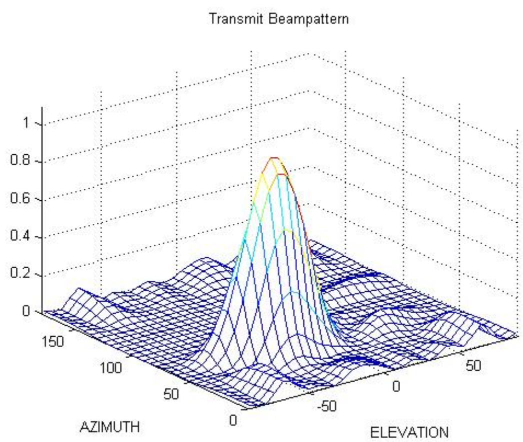
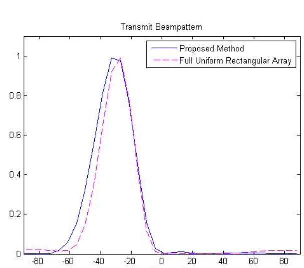
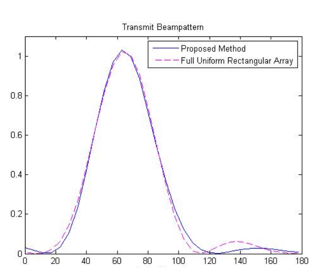


Fig. 4: Transmit beampattern in the case of full URA.

In the third example, our objective is to compare the proposed subaperturing technique with the case when the URA uses all of its elements when transmitting the probing signal. Once again, we assume a 5×5 transmit URA with half-wavelength spacing between adjacent antennas. The 2D sector of interest is defined as in the first example in order to facilitate the comparison. We use 5 transmit beams to synthesize the 2D transmit beampattern. The resulting 2D transmit beampattern is shown in Fig.4. The results in Fig.5 show two cross sections of the transmit beampattern, incorporating both the proposed method and the full URA case. The first cross section is plotted against the elevation angle by keeping the azimuth angle constant at 63^o. Similarly, the second cross section is derived against the azimuth angle by holding the elevation angle constant at -27^o. It is worth noting that the sidelobe levels are clearly lower for the proposed method.



(a) Elevation cross section



(b) Azimuth cross section

Fig. 5: Cross sections of the transmit beampattern at $\phi = 63^o$ and $\theta = -27^o$, respectively.

V. Concluding Remarks

We have investigated a new subaperturing technique for MIMO radars with planar URA at the transmit side. Specificaly, we considered the problem of 2D transmit beamforming design for the MIMO radar with fully overlapped subarrays. The simulation results confirmed that the system transmit beampattern approximates the desired sector of space with high accuracy. Furthermore, the sidelobe levels are very low and are restricted in an area close to the mainlobe, without covering the whole 2D space. Moreover, it is apparent that as the number of subarrays increases the transmit beampattern produces lower sidelobe levels. Finally, a comparison was performed between the proposed method and the case when the transmit side consists of a full URA. It is shown that the concentration of the power within the desired 2D sector is more evident in the proposed method.

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