Multi-Polarization SAR Change Detection with Invariant Decision Rules

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***Abstract*—This paper deals with coherent multi-polarization SAR change detection assuming the availability of reference and test images collected from N multiple polarimetric channels. At the design stage, the change detection problem is formulated as a binary hypothesis testing problem and the principle of invariance is used to come up with decision rules sharing the Constant False Alarm Rate (CFAR) property. A class of sub-optimum invariant receivers, which also includes the Generalized Likelihood Ratio Test (GLRT), is considered. Detection maps on real high resolution SAR data are computed showing the effectiveness of the considered invariant decision structures.**

I. Introduction

A technical challenge in SAR signal processing is change detection, namely the capability to identify temporal changes within a given scene [1], [2] starting from a pair of co-registered images representing the area of interest. Two main approaches, known as incoherent and coherent, have been proposed in open literature to process the image pair. The former attempts to detect changes in the mean power level of a given scene exploiting only the intensity information from the available images (thus neglecting phase information [3]): differencing and ratioing are well-known techniques in this context [4].

Starting from the multi-polarization data model developed in [5] and [6], we propose a new framework for change detection based on the theory of invariance in hypothesis testing problems [7], [8]. This is a viable mean to force some desired properties to a decision statistic at the design stage and has already been successfully applied in some different radar detection problems [9]–[11]. Otherwise stated, the principle of invariance allows to focus on decision rules which exhibit some natural symmetries implying important practical properties such as the Constant False Alarm Rate (CFAR) behavior. Besides, the use of invariance leads to a data reduction because all invariant tests can be expressed in terms of a statistic, called maximal invariant, which organizes the original data into equivalence classes. Also the parameter space is usually compressed after reduction by invariance and the dependence on the original set of parameters become embodied into a maximal invariant in the parameter space (induced maximal invariant).

The paper is organized as follows. In Section II, we deal with the formulation of the multi-polarization SAR change detection problem. In Section III the maximal invariant for the SAR change detection problem is defined. The sub-optimum invariant detectors are introduced in Section IV, and in Section V we assess the performance of the introduced invariant tests on real multi-polarization SAR images. Finally, in Section VI, we draw conclusions and outline some possible future research tracks.

II. Problem Formulation

A multipolarization SAR sensor measures for each pixel of the image under test $N \in \{2, 3\}$ complex returns, collected from different polarimetric channels (for instance HH and VV for $N = 2$; HH, VV, and HV with reference to $N = 3$). The N returns from the same pixel are stacked to form the vector $\boldsymbol{X}(l, m)$, where $l = 1, \ldots, L$ and $m = 1, \ldots, M$ (L and M represent the vertical and horizontal size of the image, respectively). Therefore, the sensor provides a 3-D data stack $\boldsymbol{X}$ of size $M \times L \times N$ which is referred to in the following as a datacube and is illustrated in Figure 1.

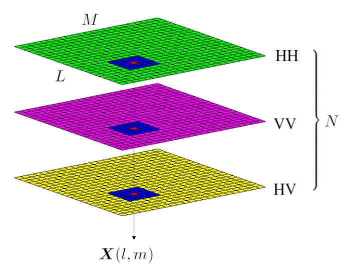

Fig. 1: Construction of the datacube.

For SAR change detection applications, we suppose that two datacubes $\boldsymbol{X}$ (reference data) and $\boldsymbol{Y}$ (test data) of the same geographic area are available; they are collected from two different sensor passes and are accurately pixel aligned (co-registrated). We focus on the problem of detecting the presence of possible changes in a rectangular neighborhood $\mathcal{A}$, with size $K = W_1 \times W_2 \geq N$, of a given pixel. To this end, we denote by $\boldsymbol{R}_X$ ($\boldsymbol{R}_Y$) the matrix whose columns are the vectors of the polarimetric returns from the pixels of $\boldsymbol{X}$ ($\boldsymbol{Y}$) which fall in the region $\mathcal{A}$ and $\boldsymbol{S}_X = \boldsymbol{R}_X \boldsymbol{R}_X^\dagger$ ($\boldsymbol{S}_Y = \boldsymbol{R}_Y \boldsymbol{R}_Y^\dagger$). where $(\cdot)^\dagger$ represents the conjugate transpose;

The matrices $\boldsymbol{R}_X$ and $\boldsymbol{R}_Y$ are modeled as statistically independent random matrices. Moreover, the columns of $\boldsymbol{R}_x$

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($\boldsymbol{R}_Y$) are assumed statistically independent and identically distributed random vectors drawn from a complex circular zero-mean Gaussian distribution with positive definite covariance matrix $\boldsymbol{\Sigma}_X$ ($\boldsymbol{\Sigma}_Y$). Under the aforementioned settings, the change detection problem in the region $\mathcal{A}$ can be formulated in terms of the following binary hypothesis test

$$\begin{cases} H_0 : \boldsymbol{\Sigma}_X = \boldsymbol{\Sigma}_Y \\ H_1 : \boldsymbol{\Sigma}_X \neq \boldsymbol{\Sigma}_Y \end{cases} \tag{1}$$

where the null hypothesis H_0 of change absence is tested versus the alternative[1] H_1.

III. Maximal Invariant

It is not difficult to prove that our testing problem is invariant under the group of transformations G acting on the sufficient statistic $(\boldsymbol{S}_X, \boldsymbol{S}_Y)$ as:

$$G = \Big\{ g : \boldsymbol{S}_X \to \boldsymbol{B}\boldsymbol{S}_X\boldsymbol{B}^\dagger , \quad \boldsymbol{S}_Y \to \boldsymbol{B}\boldsymbol{S}_Y\boldsymbol{B}^\dagger , \\ \boldsymbol{B} \in \mathcal{GL}(N) \Big\} . \tag{2}$$

where $\mathcal{GL}(N)$ is the General Linear group of degree N over the field of complex numbers, representing the set of $N \times N$ non-singular matrices together with the operation of ordinary matrix multiplication.

The invariance property induces a partition of the data space into orbits (or equivalence classes) where, over each orbit, every point is related to every other through a transformation which is a member of the group G. Any statistic that identifies different orbits in a one-to-one way significantly reduces the total amount of data necessary for solving the hypothesis testing problem and constitutes the compressed data set to be used in the design of any invariant detector. These kind of statistics are called maximal invariants since they are constant over each orbit (invariance) while assuming different values on different orbits (maximality).

Formally, a statistic $\mathbf{T}(\boldsymbol{S}_X, \boldsymbol{S}_Y)$ is said to be a maximal invariant with respect to the group of transformations G if and only if

- Invariance:
  $\mathbf{T}(\boldsymbol{S}_X, \boldsymbol{S}_Y) = \mathbf{T}[g(\boldsymbol{S}_X, \boldsymbol{S}_Y)]$, $\forall g \in G$.
- Maximality:
  $\mathbf{T}(\boldsymbol{S}_{X_1}, \boldsymbol{S}_{Y_1}) = \mathbf{T}(\boldsymbol{S}_{X_2}, \boldsymbol{S}_{Y_2})$ implies that $\exists\, g \in G$ such that $(\boldsymbol{S}_{X_2}, \boldsymbol{S}_{Y_2}) = g(\boldsymbol{S}_{X_1}, \boldsymbol{S}_{Y_1})$.

Notice that there are many maximal invariant statistics, but they are equivalent in that yield statistically equivalent detectors. Moreover, all invariant tests can be expressed as a function of the maximal invariant statistic [7], [13], which for the problem of interest is provided by the following

Proposition 1: A maximal invariant statistic for problem (1) with respect to the group of transformations (2) is the N-dimensional vector of the eigenvalues $\lambda_1, \ldots, \lambda_N$ of

$$\boldsymbol{S}_X \boldsymbol{S}_Y^{-1} . \tag{3}$$

[1]This testing problem is also well known in statistical literature with reference to real observations [7, Ch. 8].

Interestingly the principle of invariance realizes a significant data reduction: the maximal invariant statistic is a real N-dimensional vector whereas the original sufficient statistic is composed of the two $N \times N$ Grammian matrices $\boldsymbol{S}_X$ and $\boldsymbol{S}_Y$.

After reduction by invariance, the parameter space is also partitioned into orbits and the relevant parameters are embodied into any induced maximal invariant, namely any function of the parameters that is constant over each orbit of the parameter space but assumes different values over different orbits. For the case at hand, an induced maximal invariant is composed of the eigenvalues $\boldsymbol{\delta} = [\delta_1, \ldots, \delta_N]^T$ of the matrix:

$$\boldsymbol{\Sigma}_X \boldsymbol{\Sigma}_Y^{-1} \tag{4}$$

We explicitly observe that in the reduced parameter space the partition corresponding to the two composite hypotheses of the test (1) is $\Xi_0 = \{\mathbf{1}_N\}$, relative to $\boldsymbol{\Sigma}_X = \boldsymbol{\Sigma}_Y$, and $\Xi_1 = \overline{\{\mathbf{1}_N\}}$, relative to $\boldsymbol{\Sigma}_X \neq \boldsymbol{\Sigma}_Y$, where $\overline{\{\mathbf{1}_N\}}$ is the set of the N-dimensional column vectors with at least one entry different from 1. The structure of Ξ_0, which now corresponds to a simple H_0 hypothesis, clearly shows that all invariant receivers that process a maximal invariant statistic through a transformation independent of $\delta_1, \ldots, \delta_N$, achieve the CFAR property.

IV. Design sub-optimum Invariant Detectors

It can be demonstrated that the Most Powerful Invariant (MPI) test for the considered problem cannot be implemented as it requires the knowledge of the induced maximal invariant. This implies also the lack of an Uniformly Most Powerful Invariant (UMPI) test, suggesting to investigate invariant decision rules based upon different strategies. However, there is no criterion for choosing *a priori* a receiver instead of another. An intuitive rule to select invariant tests for our problem could be based on the following asymptotic observation. For large values of K, the eigenvalues of $\boldsymbol{S}_X^{-1}\boldsymbol{S}_Y$ tend to δ_i, $i = 1, \ldots, N$; hence decision rules

$$h(\lambda_1, \ldots, \lambda_N) \underset{H_0}{\overset{H_1}{\gtrless}} T , \tag{5}$$

1) are very effective to discriminate deviations $\delta_i \gg 1$, when $h(\cdot, \ldots, \cdot)$ is an increasing function of the arguments. However they perform poor when δ_i are smaller than 1.
2) are very effective to discriminate deviations $\delta_i \ll 1$, when $h(\cdot, \ldots, \cdot)$ is a decreasing function of the arguments. Nevertheless they perform poor when δ_i is greater than 1.
3) in principle could achieve good detection levels for both $\delta_i \gg 1$ and $\delta_i \ll 1$, when $h(\cdot, \ldots, \cdot)$ complies with $h\left(\frac{1}{\lambda_1}, \ldots, \frac{1}{\lambda_N}\right) = h(\lambda_1, \ldots, \lambda_N)$.

On the other hand one cannot analyze all possible reasonable detectors; so in the following, we focus on six decision rules, which, based on extensive numerical analysis, are seen to achieve satisfactory detection performance.

1) GLRT: This approach is equivalent to replacing the unknown parameters in the likelihood ratio with their maximum likelihood estimates, under each hypothesis [14]. Interestingly, under very mild technical assumptions, the GLRT is invariant [15]. For the present problem it has been proposed in [5], [6][2] and is given by

$$\frac{\max_{\boldsymbol{\Sigma}_X}\max_{\boldsymbol{\Sigma}_Y} f_{\boldsymbol{S}_X,\boldsymbol{S}_Y}(\boldsymbol{S}_X,\boldsymbol{S}_Y|H_1,\boldsymbol{\Sigma}_X,\boldsymbol{\Sigma}_Y)}{\max_{\boldsymbol{\Sigma}_X} f_{\boldsymbol{S}_X,\boldsymbol{S}_Y}(\boldsymbol{S}_X,\boldsymbol{S}_Y|H_0,\boldsymbol{\Sigma}_X)} \underset{H_0}{\overset{H_1}{\gtrless}} T_1\,, \tag{6}$$

with T_1 the detection threshold. After optimizations and monotonic transformations, (19) can be shown statistically equivalent to

$$\prod_{i=1}^{N}\frac{(1+\lambda_i)^2}{\lambda_i} \underset{H_0}{\overset{H_1}{\gtrless}} T_1\,, \tag{7}$$

where the same symbol T_1 has been used to denote the modified detection threshold.

Interestingly, the GLRT complies with condition 3 that was given at the beginning of the section, namely we expect the GLRT capable to achieve good detection levels both when $\delta_i \gg 1$ and $\delta_i \ll 1$.

2) Arithmetic and Armonic Mean Based Detectors: These decision rules are respectively given by

$$\sum_{i=1}^{N}\lambda_i \underset{H_0}{\overset{H_1}{\gtrless}} T_2\,, \tag{8}$$

$$\sum_{i=1}^{N}\frac{1}{\lambda_i} \underset{H_0}{\overset{H_1}{\gtrless}} T_3\,, \tag{9}$$

where T_2 and T_3 are the detection thresholds.

The former complies with condition 1 whereas the latter with condition 2. As a consequence (8) is suitable to detect deviations $\delta_i \gg 1$ while (9) $\delta_i \ll 1$.

From (8) and (9) it is also possible to construct another decision rule satisfying condition 3 merely summing the decision statistics, i.e.

$$\sum_{i=1}^{N}\left(\frac{1}{\lambda_i}+\lambda_i\right) \underset{H_0}{\overset{H_1}{\gtrless}} T_4\,, \tag{10}$$

with T_4 the detection threshold.

3) Maximum and Minimum Eigenvalue Based Detectors: These tests are respectively based on the following comparisons

$$\lambda_1+\frac{1}{\lambda_N} \underset{H_0}{\overset{H_1}{\gtrless}} T_5\,, \tag{11}$$

[2]With reference to real observations it is derived in [7, Ch. 8].

$$\max\left(\lambda_1\,,\,\frac{1}{\lambda_N}\right) \underset{H_0}{\overset{H_1}{\gtrless}} T_6\,, \tag{12}$$

with T_5 and T_6 the decision thresholds. An intuitive explanation to the decision rules is based on the following arguments: the former term, λ_1, dominates for large deviations $\delta_i \gg 1$, whereas, the latter term, $\frac{1}{\lambda_N}$, if $\delta_i \ll 1$. Hence, (11) and (12) are supposed to perform well both for $\delta_i \gg 1$ and $\delta_i \ll 1$.

V. Testing on Real Data

In this section the performance analysis of the algorithms proposed in Section IV is presented. The analysis is performed using real X-band data, the dataset used is the Coherent Change Detection Challenge dataset acquired by the Air Force Research Laboratory (AFRL) [12], the data contains passes acquired with three polarizations (HH, VV and HV).

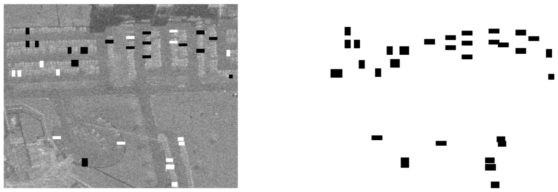
For our analysis we focus on two acquisitions from the entire dataset, unfortunately the ground truths of the data is not available (e.g. the actual changes between two different acquisitions), so the selection of two passes providing the opportunity to generate a sufficiently accurate ground truth was required. For this reason the best candidates result to be two passes: the acquisition named “FP0124” is used as reference pass, while the acquisition “FP0121” is used as test pass. The selected area of interest is a sub-image of 1000×1000 pixels (i.e., $L = M = 1000$) and is composed of several parking lots which are occupied by numerous parked, (i.e., stationary) vehicles. For this particular scenario the changes between the reference and test images (denoted by $\boldsymbol{X}$ and $\boldsymbol{Y}$ respectively), occurred during the time interval between the two acquisitions can be distinguished in two cases:

- a vehicle is present in $\boldsymbol{X}$ but is not present in $\boldsymbol{Y}$, this case is referred as departure;
- a vehicle is not present in $\boldsymbol{X}$ but is present in $\boldsymbol{Y}$, this event is referred as arrival.

Using the cases defined above, can be visually identified (by flickering the two images) a total of 34 changes between $\boldsymbol{X}$ and $\boldsymbol{Y}$. The obtained ground truth is shown in Figure 2-a, whereas the black regions represent the departures and the white ones indicate the arrivals.

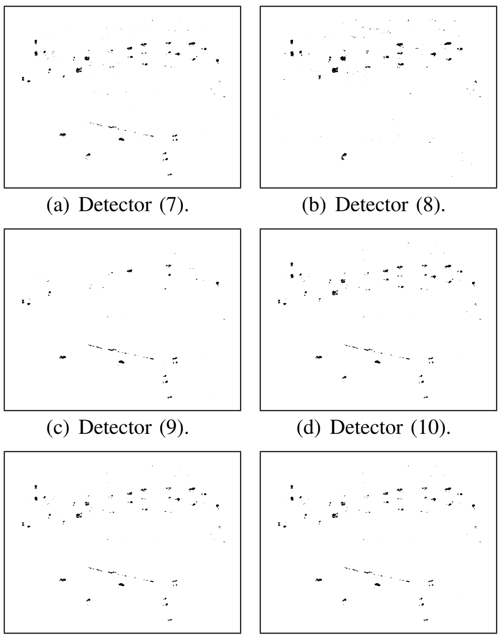
Although the acquisitions were performed during the same day and the images were registered, the returns from a scatterer can contribute differently to neighbour pixels, for example a slightly different aspect angle can produce a different amount of energy spill-over. These relative differences in the imaged data can lead to false alarms in the change detection results. In order to prevent false alarm caused by pixel contamination by target returns, we consider a guard area around each arrival-departure. This allows the definition of an extended ground truth (see Figure 2-b) used in the following to compare the performance of the considered detection algorithms.

In order to assess the performance of the detectors we present both the number of detected changes and the change



(a) Ground truth. (b) Ground truth with guard cells.

Fig. 2: Ground truth superimposed to the reference image and ground truth with the addition of guard cells.



(a) Detector (7). (b) Detector (8).

(c) Detector (9). (d) Detector (10).

(e) Detector (11). (f) Detector (12).

Fig. 3: Detections maps for $W = 5$ and $N = 3$.

detection maps.

In the analysis presented in this section, the thresholds are set to ensure $P_{fa} = 10^{-3}$ in the complement of the extended ground truth area, namely, in the region where no changes occur. Moreover, are considered both the cases of $N = 2$ and $N = 3$ which correspond, respectively, to jointly consider HH and VV polarizations, or to jointly consider all the available channels (HH, VV and HV).

The number of change detections for each detector corresponding to a change present in the extended ground truth are summarized in Table I. As expected, the common trend is that the performance improves by increasing W and N.

The detection maps obtained for the case with $W = 5$ and $N = 3$ for the different detectors are shown in Figure 3. From the detection maps and Table I it can be observed that detectors (7), (10), (11), and (12) achieve a comparable detection performance level. Of particular interest are the tests (8) and (9), from the corresponding detection maps it is easy to recognize that the former identifies the departures, whereas, the latter identifies the arrivals.

		Detector					
W	N	(7)	(8)	(9)	(10)	(11)	(12)
3	2	3389	3757	1984	4051	3955	4040
	3	3802	3868	2314	4408	4247	4415
5	2	5372	5129	2337	5103	4877	4980
	3	6492	5513	2884	5901	5463	5644

TABLE I: Number of correct detections in the extended ground truth.

VI. Conclusions

Multi-polarization SAR change detection has been considered in this paper. The problem has been formulated as a binary hypothesis test and the principle of invariance has been applied to design decision rules exhibiting a special symmetry, which is a sufficient condition to ensure the CFAR property. The effectiveness of the proposed approaches has been tested on real SAR data.

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