[C3] Widely Linear Adaptive Processing of Noncircular Complex Signals

Theme: Detection, Localisation & Tracking Theme

PI: Danilo Mandic, Imperial College

Researchers: Dahir Dini

1 Project Objectives

- This research will provide a novel theoretical framework and enhanced practical solutions for adaptive processing of noncircular complex signals.
- Standard solutions inherently assume second order circularity of signal distributions, and are therefore inadequate when the signals are observed through nonlinear sensors or as mixtures of sources, when the noise model is not doubly white, and when high resolution and enhanced separability are paramount. This will be achieved based on recent fundamental developments in the statistics of complex variables, called augmented complex statistics and widely linear modeling.
- The fundamental novelty in this work is the design of statistical signal processing techniques for the detection, estimation, and quantification of the degree of noncircularity of real world signals. This information will be used in conjunction with widely linear adaptive signal processing techniques, to enable optimal processing of complex signals with both circular and noncircular distributions.

2 Research Approach

2.1 2nd Order Statistics

- Statistical signal processing algorithms for complex valued signals are generally *not simple extensions* of real valued signals
- The second order statistical properties of a complex vector $\mathbf{x} = \mathbf{x}_r + \jmath\mathbf{x}_i$ is normally characterised by its covariance $\mathbf{R}_\mathbf{x} = E\{\mathbf{x}\mathbf{x}^H\}$
- However, this only holds for the special class of complex signals known as second order *circular* or *proper*, that is, those with rotation invariant probability distributions, that is $\mathcal{P}[\mathbf{x}] = \mathcal{P}[\mathbf{x}e^{\jmath\theta}]$ for $\theta \in [0, 2\pi]$
- Most real world processes are noncircular and require a second moment function known as the pseudocovariance $\mathbf{P}_\mathbf{x} = E\{\mathbf{x}\mathbf{x}^T\}$ to be taken into account

2.2 Widely Linear Modeling

Consider the MSE estimator of a signal y in terms of another observation $\mathbf{x}$

$$\hat{y} = E[y|\mathbf{x}] \quad \longrightarrow \quad \hat{y}_l = \mathbf{a}^T\mathbf{x}$$

where y and $\mathbf{x}$ are zero mean and jointly normal. In standard MSE in the complex domain: $\hat{y} = \mathbf{a}^H\mathbf{x}$, however

$$\hat{y}_r = E[y_r|\mathbf{x}_r, \mathbf{x}_i] \quad \& \quad \hat{y}_i = E[y_i|\mathbf{x}_r, \mathbf{x}_i] \quad \longrightarrow \quad \hat{y} = E[y_r|\mathbf{x}_r, \mathbf{x}_i] + \jmath E[y_i|\mathbf{x}_r, \mathbf{x}_i]$$

Upon employing the identities $\mathbf{x}_r = (\mathbf{x} + \mathbf{x}^*)/2$ and $\mathbf{x}_i = (\mathbf{x} - \mathbf{x}^*)/2\jmath$

$$\hat{y} = E[y_r|\mathbf{x}, \mathbf{x}^*] + \jmath E[y_i|\mathbf{x}, \mathbf{x}^*]$$

we thus arrive at the **widely linear** estimator for general (noncircular) complex signals

$$\hat{y}_{wl} = \mathbf{h}^T\mathbf{x} + \mathbf{g}^T\mathbf{x}^*$$

The full second order information is contained in the covariance of the augmented vector $\mathbf{x}^a = [\mathbf{x}^T, \mathbf{x}^H]^T$

$$\mathbf{R}^a_\mathbf{x} = E\{\mathbf{x}^a\mathbf{x}^{aH}\} = \begin{bmatrix} \mathbf{R}_\mathbf{x} & \mathbf{P}_\mathbf{x} \\ \mathbf{P}^*_\mathbf{x} & \mathbf{R}^*_\mathbf{x} \end{bmatrix}$$

The MSE difference between the linear and widely linear models is given by

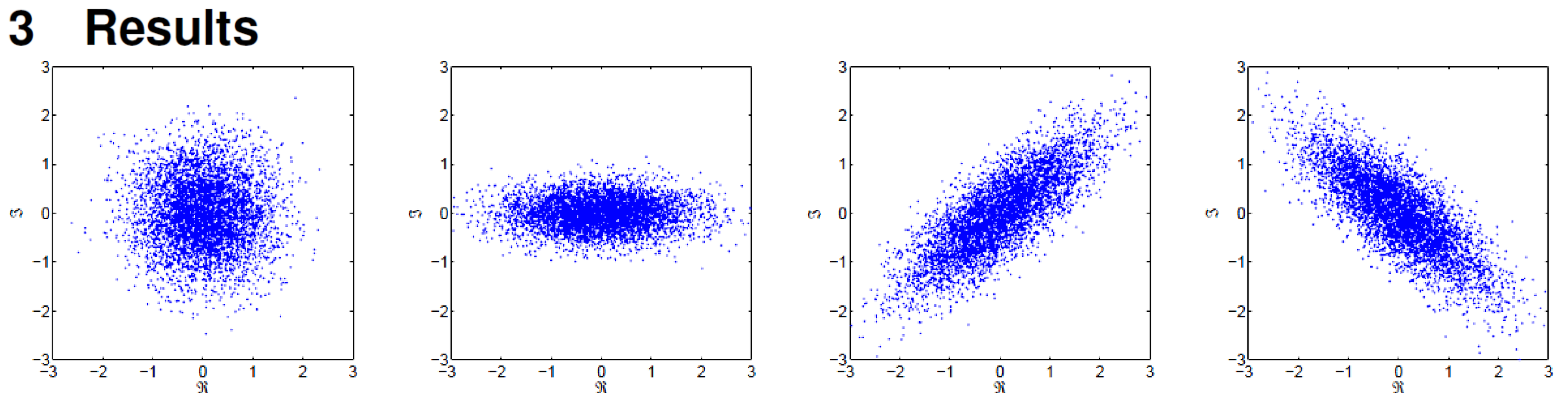
$$\begin{aligned} \delta\varepsilon^2 &= \varepsilon_l^2 - \varepsilon_{wl}^2 \\ &= (\mathbf{P}_{\mathbf{yx}} - \mathbf{R}_{\mathbf{yx}}\mathbf{R}_\mathbf{x}^{-1}\mathbf{P}_\mathbf{x})(\mathbf{R}^*_\mathbf{x} - \mathbf{P}^*_\mathbf{x}\mathbf{R}_\mathbf{x}^{-1}\mathbf{P}_\mathbf{x})^{-1}(\mathbf{P}_{\mathbf{yx}} - \mathbf{R}_{\mathbf{yx}}\mathbf{R}_\mathbf{x}^{-1}\mathbf{P}_\mathbf{x})^H \end{aligned}$$

where

$$\begin{aligned} \mathbf{R}_{\mathbf{yx}} &= E\big\{(\mathbf{y} - E\{\mathbf{y}\})(\mathbf{x} - E\{\mathbf{x}\})^H\big\} \\ \mathbf{P}_{\mathbf{yx}} &= E\big\{(\mathbf{y} - E\{\mathbf{y}\})(\mathbf{x} - E\{\mathbf{x}\})^T\big\} \end{aligned}$$

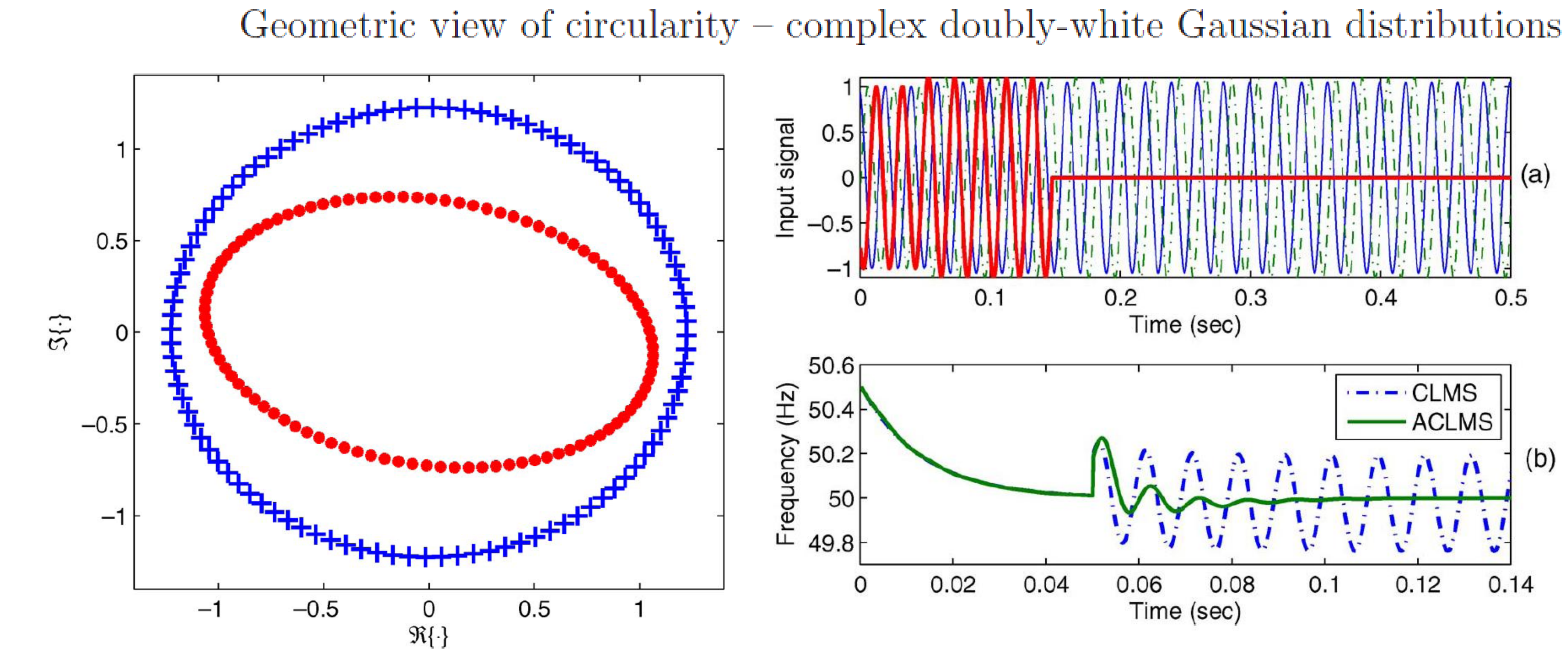
For improper signals, $\delta\varepsilon^2 > 0$, the WL model is superior.

3 Results

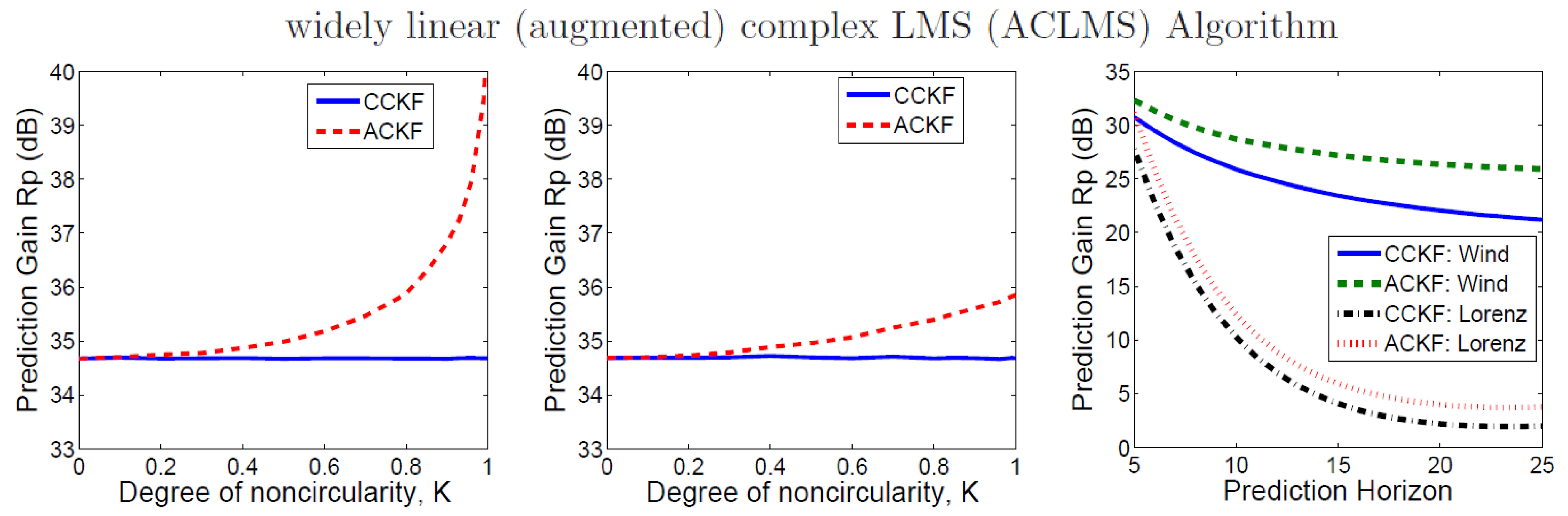


(a) Circular (b) Uncorrelated noncircular (c) Correlated noncircular (d) Correlated noncircular

Geometric view of circularity – complex doubly-white Gaussian distributions

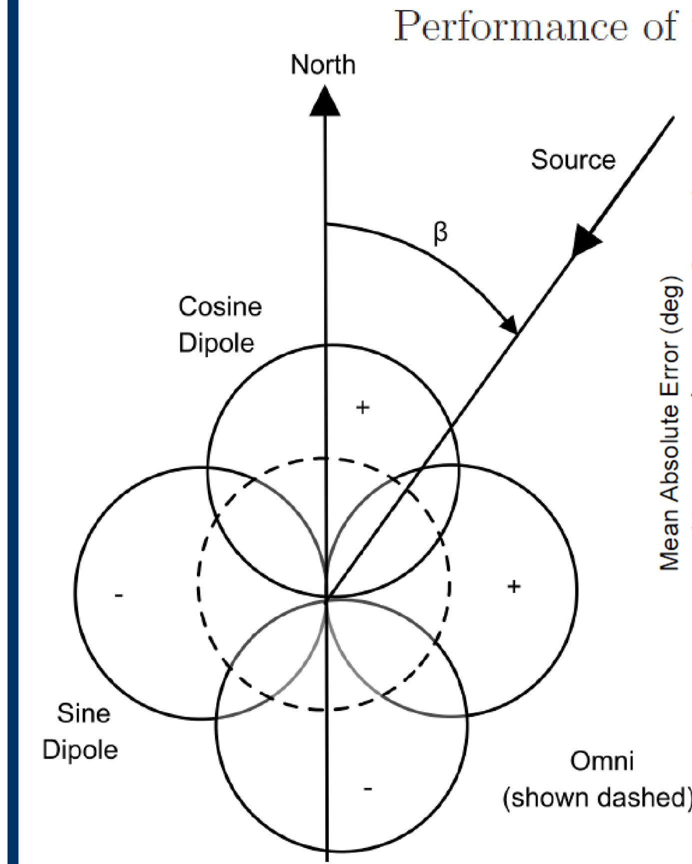


Frequency estimation in unbalanced Three-phase power systems using the widely linear (augmented) complex LMS (ACLMS) Algorithm

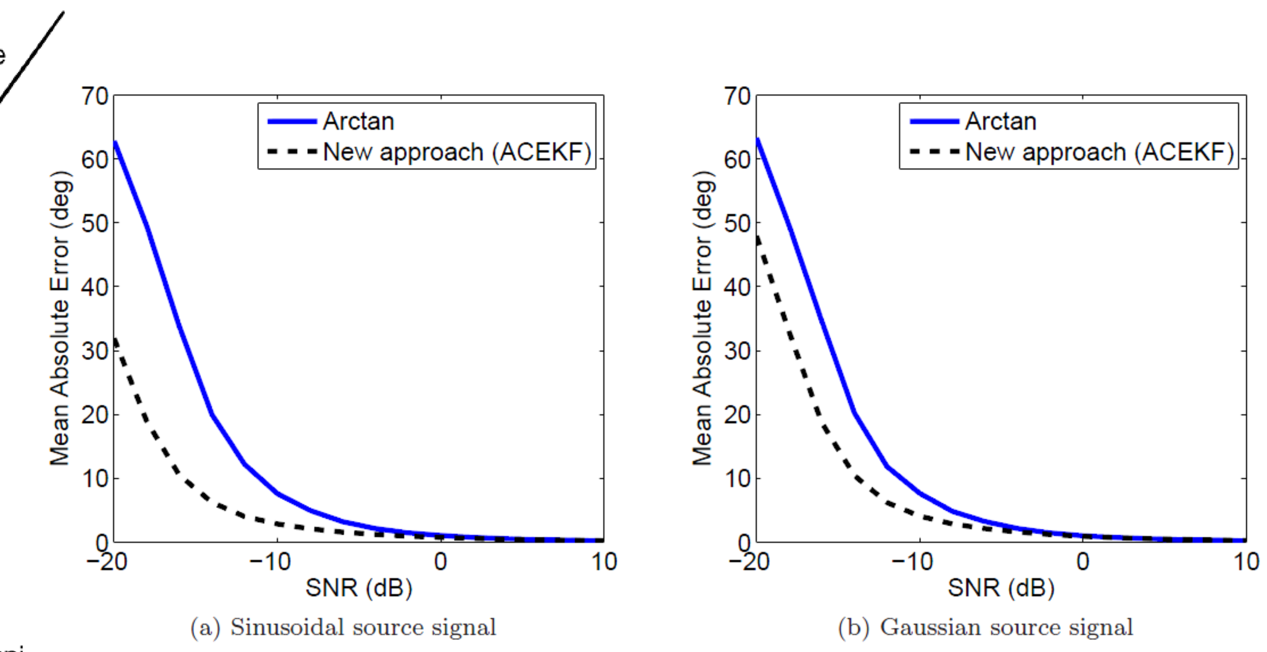


(a) Noncircular state noise (b) Noncircular observation noise (c) Lorenz and Wind prediction

Performance of the augmented complex Kalman filter (ACKF)



Sonobouy sensors (top view)



(a) Sinusoidal source signal (b) Gaussian source signal

Performance comparison between the proposed widely linear approach and the standard arctan estimator

References

[1] D. P. Mandic and V. S. Goh, *Complex Valued Nonlinear Adaptive Filters: Noncircularity, Widely Linear and Neural Models*. Wiley, 2009.

[2] D. H. Dini and D. P. Mandic, "A class of widely linear complex Kalman filters," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 23, pp. 775–786, May 2012.

[3] B. Picinbono and P. Bondon, "Second-order Statistics of Complex Signals," *IEEE Transactions on Signal Processing*, vol. 45, no. 2, pp. 411–420, 1997.

dstl

MINISTRY OF DEFENCE

EPSRC
Engineering and Physical Sciences
Research Council

This work was supported by the Engineering and Physical Sciences Research Council (EPSRC) and the MOD University Defence Research Centre on Signal Processing (UDRC).