# University Defence Research Collaboration (UDRC) Signal Processing in a Networked Battlespace

## L_WP5: Low Complexity Algorithms and Efficient Implementation

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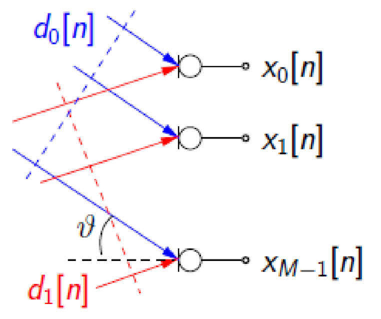
*Aim: To develop novel paradigms and implementation strategies for a range of complex signal processing algorithms operating in a networked environment. Support all WPs in development of efficient methods and hardware implementations. Scientifically Possible → Technically Feasible*

**L_WP5.1 Low Complexity Algorithms**

*Lower dimensional representation of data can lead to significant cost reduction. To exploit both data dependent and independent techniques (e.g. freq. domain, sub-band or subspace-based processing) and demonstrate low-cost algorithms.*

**Data Reduction through Polynomial Eigenvalue Decomposition (PEVD)**

- Parahermitian polynomial matrices occur in sensor array problems (useful for calculating correlations of a data vector **x**[n] when proper time delays rather than phase shifts must be considered, e.g. Broadband beamforming).
- Eigenvalue Decomposition offers a powerful tool to factorise Hermitian matrices and thus reveal subspace decompositions useful for compression.
- Define Space-time Covariance Matrix:




$$\mathbf{x}_n = \begin{bmatrix} x_0[n] \\ x_1[n] \\ \vdots \\ x_{M-1}[n] \end{bmatrix} = \sum_l \mathbf{s}(\vartheta_l)[n] * d_l[n] + \mathbf{v}[n]$$

$$\mathbf{R}[\tau] = \mathcal{E}\{\mathbf{x}_n \mathbf{x}_{n-\tau}^{\mathrm{H}}\}$$

Define Cross Spectral Density (CSD) Matrix (z-transform): $\boldsymbol{R}(z) = \sum_\tau \mathbf{R}[\tau] z^{-\tau}$

Parahermitian Operator: $\tilde{\boldsymbol{R}}(z) = \boldsymbol{R}^{\mathrm{H}}(z^{-1})$

Polynomial EVD defined as: $\boldsymbol{R}(z) \approx \boldsymbol{H}(z)\boldsymbol{\Gamma}(z)\tilde{\boldsymbol{H}}(z)$

where ***H***(z) is paraunitary, **Γ**(z) is diagonal $\quad \boldsymbol{H}(z)\tilde{\boldsymbol{H}}(z) = \tilde{\boldsymbol{H}}(z)\boldsymbol{H}(z) = \boldsymbol{I}$

**Sequential Best Rotation Algorithm (SBR2 )**

SBR2 from [1] iteratively diagonalises ***R***(z) , where at each step the maximum off-diagonal element must be identified and its energy transferred onto the diagonal. SBR2 is a generalisation of Jacobi eigenvalue decomposition algorithm for polynomial matrices.

To identify maximum off-diagonal element : $\{k^{(i)}, \tau^{(i)}\} = \arg\max_{k,\tau} \|\hat{\mathbf{s}}_k^{(i-1)}[\tau]\|_\infty$

With modified column vector: $\hat{\mathbf{s}}_k^{(i)}[\tau] \in \mathbb{C}^{M-1}$

Energy transfer performed by an elementary paraunitary transformation:

Step 1: Delay onto lag-zero matrix : $\boldsymbol{S}^{(i)\prime}(z) = \tilde{\boldsymbol{\Lambda}}^{(i)}(z)\boldsymbol{S}^{(i-1)}(z)\boldsymbol{\Lambda}^{(i)}(z)\,, \quad i = 1 \ldots I$

Where $\boldsymbol{\Lambda}^{(i)} = \mathrm{diag}\{\underbrace{1 \;\ldots\; 1}_{k^{(i)}-1}\; z^{-\tau^{(i)}}\; \underbrace{1 \;\ldots\; 1}_{M-k^{(i)}}\}$ , $\boldsymbol{S}^{(0)}(z) = \boldsymbol{R}(z)$

Step 2: Elimination through Jacobi rotation: $\boldsymbol{S}^{(i)}(z) = \mathbf{Q}^{(i)\mathrm{H}}\boldsymbol{S}^{(i)\prime}(z)\mathbf{Q}^{(i)}$

Jacobi rotation applied to only two rows and columns $S^{(i)}[z]$, defined by column and row indices of max off-diag element. Energy transferred to diagonal of $S^{(i)}[0]$, with more to higher elements to promote (but not guarantee spectral majorisation).

Combine delay and Jacobi rotation for paraunitary matrix: $\boldsymbol{H}(z) = \prod_{i=1}^{I} \mathbf{Q}^{(i)}\boldsymbol{\Lambda}^{(i)}(z)$

**Sequential Matrix Diagonalisation (SMD), Maximum Element ME-SMD**

SMD algorithms [2] differ from SBR2 in that they **clear all off-diagonal elements** of the zero lag matrix $S^{(i)}[0]$ at every step.

- Initialisation EVD step required to remove instantaneous correlations: $\mathbf{S}^{(0)}[0] = \mathbf{Q}^{(0)\mathrm{H}}\mathbf{R}[0]\mathbf{Q}^{(0)}$
- SMD identifies $k^{(i)}$th column containing max off-diag energy by replacing $L_\infty$ –norm with $L_2$-norm
- ME-SMD searches for the column containing the max off-diag element in the same way as SBR2 but performs SMD-type complete diagonalisation (ME-SMD a ~hybrid of SBR2 and SMD).
- SMD Algorithms transfer more energy per step → Faster convergence (in same # iterations)
- Greater level of diagonalisation is possible, Lower-order Paraunitary Filterbank requirements
- Higher Computational Complexity

**Multiple Shift (MSME-SMD)**

New MSME-SMD development [3] is targeted at improving the convergence of SMD. MSME-SMD uses a set of reduced search spaces to ensure (M-1) columns are shifted onto the zero lag at each iteration.

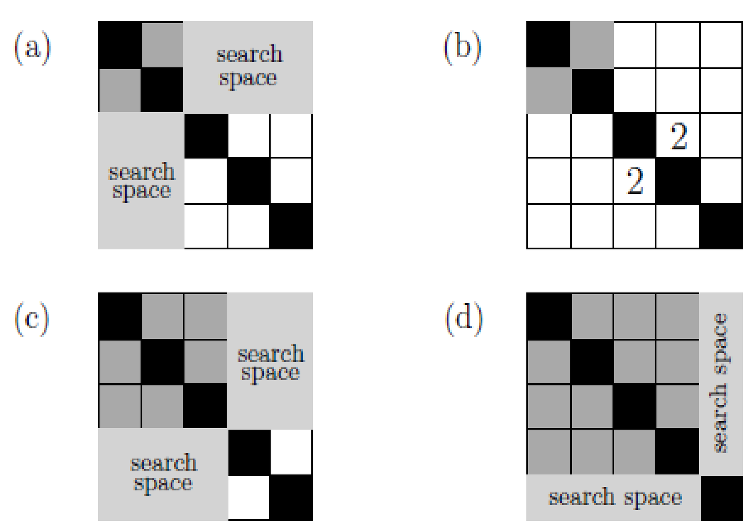
New shift and permutation steps in delay matrix:

$$\boldsymbol{\Lambda}^{(i)} = \mathrm{diag}\{1 \;\; z^{-\tau^{(i,1)}} \;\; \ldots \;\; z^{-\tau^{(i,M-1)}}\}\,\mathbf{P}^{(i)}$$

Diagonalisation Measure (for performance comp.):

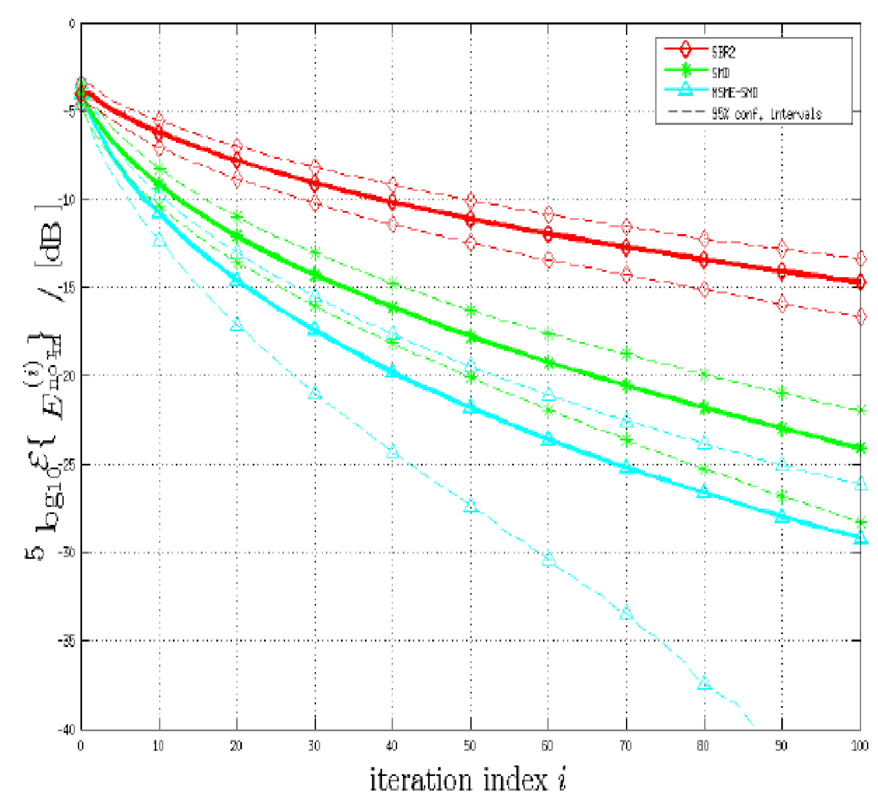
$$E_{\mathrm{norm}}^{(i)} = \frac{\sum_\tau \sum_{k=1}^{M} \|\hat{\mathbf{s}}_k^{(i)}[\tau]\|_2^2}{\sum_\tau \|\mathbf{R}[\tau]\|_{\mathrm{F}}^2}$$

- MSME-SMD algorithms outperform SBR2 & SMD in terms Convergence and Spectral majorisation , see figs 1 to 3.
- SMD retains advantage in implementation cost.

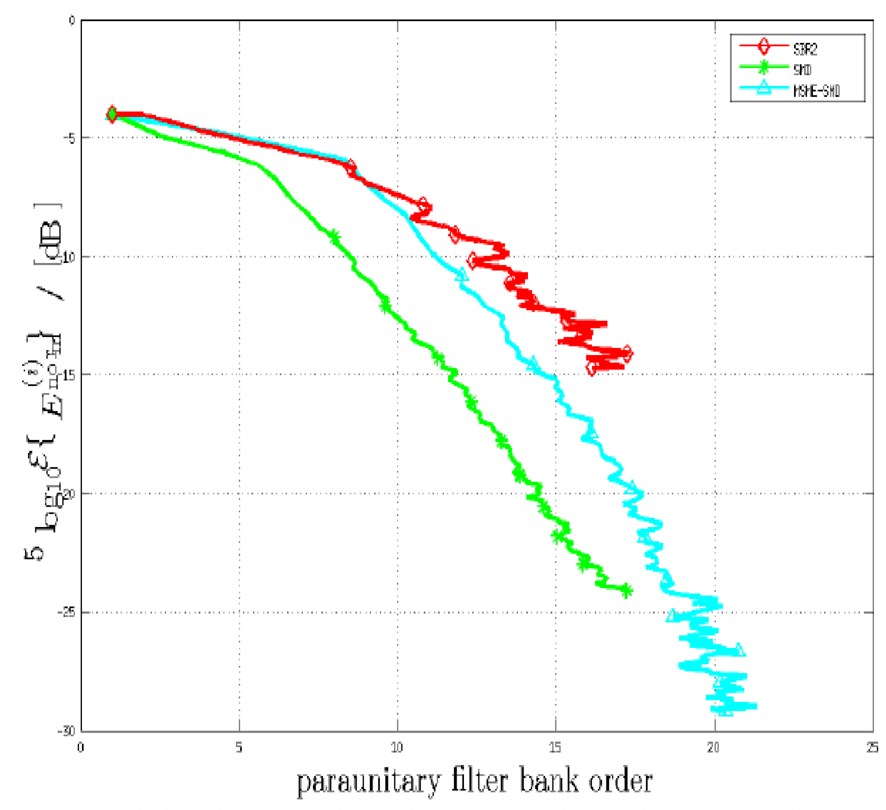



MSME-SMD Search Strategy

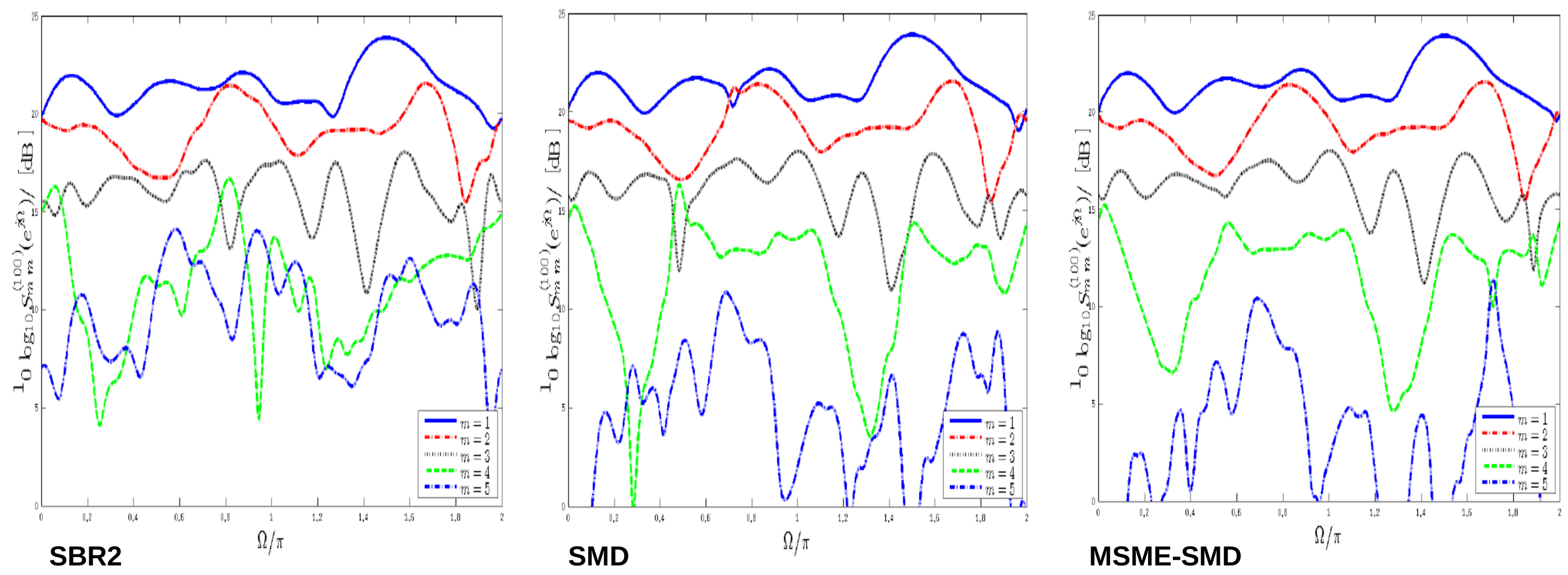
**PEVD Algorithm Performance Comparison**




(1) Convergence Plot of Diagonalisation

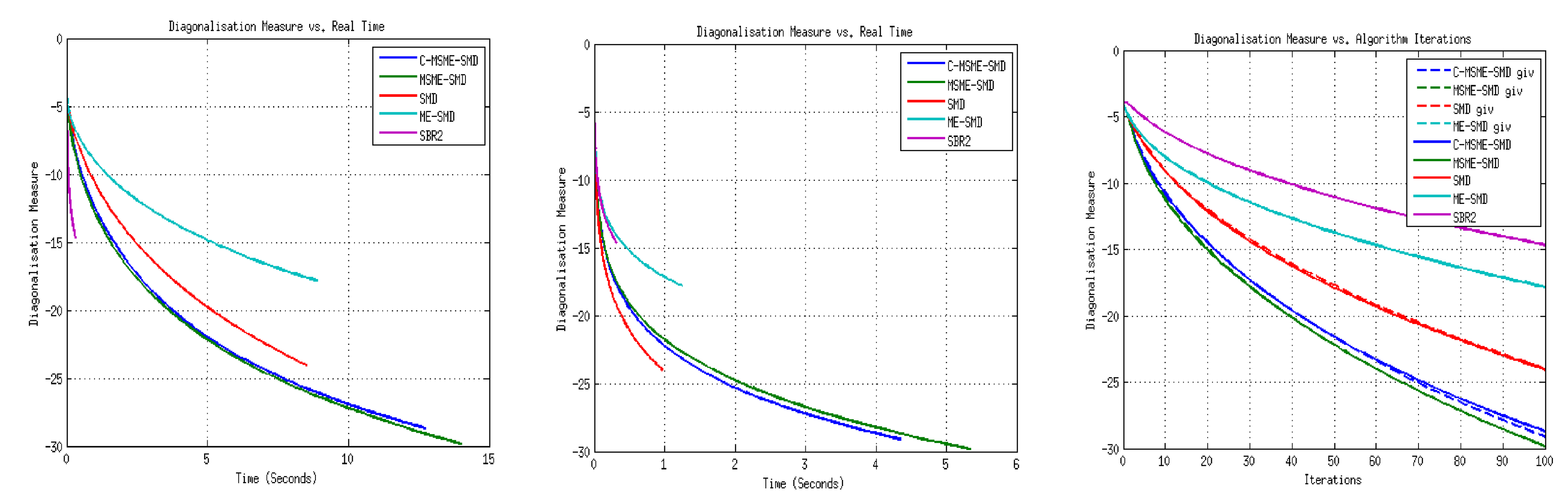



(2) Diagonalisation vs. Filter-bank Order



SBR2 SMD MSME-SMD

(3) PSDs demonstrating approximate Spectral Majorisation




(4) Diagonalisation Measure vs. Execution Time and # Iterations for Exact and Approximate EVDs

**L_WP5.1 Distributed Processing**

*For a networked environment, the efficient organisation of algorithms across a distributed processing platform is to be considered. Statistical signal processing methods (Probabilistic Graphical Models ) are utilised to map algorithms to distributed processors. Target application is Distributed Beamforming*

**Data Reduction through:**

- Approximate Inference techniques: Expectation Propagation, Loopy Belief Propagation, Mean-field approximation, Laplace approximation, etc.
- Approximations to Probability Distributions: Deterministic, scalable methods , e.g. asymptotic approximations, series expansions

**Future Activities**

- Further Develop 'Optimal' multiple-shift PEVD methods
- Establish linkages to Coherent Signal-Subspace Methods
- Further develop 'Hardware Suite', esp. TI Multi-core DSPs
- Complete implementation of SBR2 and ME-SMD algorithms on FPGA hardware, and GPU implementation of SVM and GP Classifier
- Apply GP probabilistic classifier on 'Defence' problem
- Develop Graphical Models (Bayesian Network & Undirected Markov Random Fields) for modelling distributed processing
- Show & Tell Event for 7th -8th April 2014 – at Strathclyde CSG

**References**

[1] J G McWhirter, P Baxter, T Cooper, S Redif and J Foster. An EVD Algorithm for Para-Hermitian Polynomial Matrices. IEEE Trans Signal Processing, Vol 55, No 6 (May 2007).
[2] S. Redif, S. Weiss, and J.G. McWhirter: "Sequential Matrix Diagonalisation Algorithms for Polynomial EVD of Parahermitian Matrices," submitted to IEEE Transactions on Signal Processing, January 2014
[3] J. Corr, K. Thompson, S. Weiss, J.G. McWhirter, S. Redif and I.K. Proudler:
"Maximum Element -- Maximum Energy Sequential Matrix Diagonalisation for Parahermitian Matrices," submitted to IEEE Statistical Signal Processing Workshop, Gold Coast, Australia, June/July 2014

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