

# Vibrating Micro-Doppler signature extraction from SAR data using Singular Value Decomposition.

Carmine Clemente, John J. Soraghan

Centre for Excellence in Signal and Image Processing, University of Strathclyde, United Kingdom

## Abstract

The effect of target micro-motions on the Synthetic Aperture Radar signal results in micro-Doppler target signatures. These micro-Doppler signatures can be a useful source of information for target classification. However the presence of stationary target and noise in the scene makes the extraction of the micro-Doppler signature a difficult challenge. In this paper a micro-Doppler signature extraction method based on the Singular Value Decomposition is applied to extract the target micro-Doppler features from the received SAR signal. The effectiveness of the proposed approach is confirmed through simulations.

## 1 Introduction

A common problem in SAR systems is the detection and the characterization of moving targets within a scene. If the target exhibits a regular and straight motion pattern during the image acquisition then a ground moving target indicator (GMTI) is the technique that is normally used. However characterizing target movement is generally different from the constant or linear motion observed in the GMTI. Vibrations and rotations of the target facilitates further characterization of a specific target. The capability to identify and classify these kind of motions aids in more effective and accurate target classification. For example it has been demonstrated that vibrating micro-Doppler signatures allow us to distinguish a diesel engine of a bus from a gas turbine engine of a tank [1, 3].

In [11] the micro-Doppler signature for vibrating targets in a monostatic SAR system was analyzed and tested with an X-band system, while in [10] a millimeter Wave (mmW) system was used to analyze the micro-Doppler for vibrating and rotating SAR. In both cases time-frequency analysis was performed using the Wigner Ville Distribution exploiting the properties of the Cohen's family of transforms [4]. However these analysis were performed in a very high signal-to-clutter ratio (SCR) environments in order to validate the micro-Doppler model. In an operative system the micro-Doppler signal will normally be embed in low SCR and the challenge becomes the extraction of this feature from the received signal.

Various approaches have been reported for this extraction problem including, [9] wherein a chirplet based extraction method was presented, in [12] the Radon and wavelet transform were used to extract the features from ISAR data, for both these methods the computational overload resulted to be very high. In [14] the Along Track Interfer-

ometry (ATI) was applied to extract the micro-Doppler signature removing the clutter signal, however this approach requires a specific system desing.

In [6] Singular Spectrum Analysis (SSA) was applied to the ISAR signal to minimise the effect of micro-motions in order to improve the image quality. In this paper a technique based on SSA is developed and applied for micro-Doppler extraction from SAR data that operates in the case of high SCR. The reminder of the paper is organised as follows. In section II the basics of micro-Doppler effect in monostatic SAR is presented. Section III presents the SSA approach while section IV presents the novel application of SSA for the effective extraction of micro-Doppler signatures from received SAR echoes. Section V presents results obtained through simulations that confirms the usefulness of the extraction technique. Section VI concludes the paper.

## 2 Micro Doppler effect in SAR

The micro Doppler effect in SAR was theoretically studied in [2], while in [11] and [10] the effect was analyzed using real experiments. Mechanical vibrations or rotations of radar targets induce a superimposed phase modulations in the received SAR signal. In the case of vibrations, analyzing the micro-motions in the range direction, the phase modulation induced on the SAR received signal is [10]:

$$\Phi_{MD}(\eta) = \frac{4\pi}{\lambda_c} A_v \cos(w_v \eta) \quad (1)$$

where  $\eta$  represents the slow time,  $\lambda_c$  the carrier frequency,  $A_v$  and  $w_v$  the vibrating amplitude and pulsation respectively. From the derivative of (1) the induced micro-

Doppler frequency results:

$$f_{MD}(\eta) = \frac{-2A_v w_v}{\lambda_c} \sin(w_v \eta) \quad (2)$$

From (2) the induced micro-Doppler frequency depends on the system working frequency and from the vibrating amplitude and frequency.

Thus the received SAR signal from a vibrating target after range compression results to be:

$$s_{rc}(\tau, \eta) = p_r \left( \tau - \frac{R(\eta)}{c} \right) w_{az}(\eta) \exp \left\{ -j \frac{4\pi f_0 R(\eta)}{c} \right\} \exp \{ -j 2\pi f_{MD} \eta \} \quad (3)$$

where  $c$  is the speed of light,  $R(\eta)$  is the slant range function,  $\tau$  is the fast time,  $p_r(\cdot)$  is the range envelope and  $w_{az}(\cdot)$  is the antenna pattern. For a stationary target the equation describing the signal model is the same in (3) without the second exponential. The entire received signal comprises the superposition of the contribution of the echoes from all the cells in the same range gate. When the vibrating target is embedded in an environment with strong scatterers the extraction of the micro-Doppler feature from the signal becomes mandatory to be exploited for target recognition purposes and for the formation of micro-Doppler databases.

### 3 Singular Spectrum Analysis

Singular Spectrum Analysis (SSA) is a technique used to analyze time series signals based on Singular Value Decomposition (SVD) [7]. The aim of SSA is to decompose the original time series into the sum of a small number of independent and interpretable components such as trends, slowly varying components and noise.

The SSA technique consists of two complementary stages: decomposition and reconstruction. The decomposition firstly uses embedding to decompose the original signal in a trajectory matrix which is then separated into an independent trajectory matrix using SVD. In the reconstruction stage subgroups of the trajectory matrices and diagonal averaging are used to reconstruct the new time series.

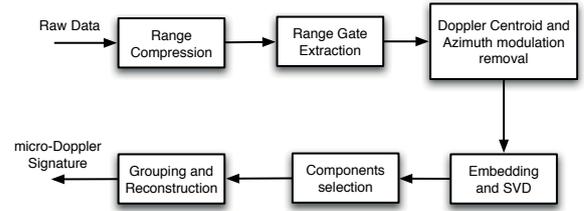
The first step in the analysis is the construction of the trajectory matrix [8]. Consider a time series of length  $N$ ,  $Y_N = (y_1, \dots, y_N)$ , fix the window length  $L \leq N/2$  and defining  $K = N - L + 1$ . The embedding stage builds a multi-dimensional time series  $X_1, \dots, X_K$  starting from  $Y_N$ , where  $X_i$  is populated with  $(y_i, \dots, y_{i+L-1}) \in \mathbf{R}^L$ . The parameter to be selected in this stage is the window length  $L$ , the trajectory matrix  $\mathbf{X} = [X_1, \dots, X_k]$  is a Hankel matrix which means that all the elements on the diagonal  $i + j = \text{const}$  are equal.

SVD is then applied to the matrix  $\mathbf{X}\mathbf{X}^T$ , to form eigenvalues and eigenvectors of  $\mathbf{X}\mathbf{X}^T \mathbf{X}\mathbf{X}^T$  represented with  $\mathbf{X}\mathbf{X}^T = \mathbf{P}\mathbf{\Lambda}\mathbf{P}$  where  $\mathbf{\Lambda} = \text{diag}(\Lambda_1, \Lambda_2, \dots, \Lambda_L)$  is

the diagonal matrix of the eigenvalues of  $\mathbf{X}\mathbf{X}^T$  in non-increasing order and  $\mathbf{P} = (P_1, P_2, \dots, P_L)$  is the corresponding orthogonal matrix of the eigenvectors of  $\mathbf{X}\mathbf{X}^T$ . At this stage a selection of the eigen-vectors, from a selection of a group of  $l$  ( $1 \leq l \leq L$ ) eigen-vectors  $P_{i_1}, P_{i_2}, \dots, P_{i_l}$  from the matrix  $\mathbf{P}$  can be performed. This step corresponds to splitting the elementary matrix  $\mathbf{X}_i$  in groups and summing the matrix within each group. Last step is the reconstruction, the matrix  $\tilde{\mathbf{X}}$  can be computed as an approximation to  $\mathbf{X}$  as  $\|\tilde{x}_{ij}\| = \sum_{k=1}^l P_{i_k} P_{i_k}^T \mathbf{X}$ . The one dimensional series can be obtained by the averaging over the diagonals of  $\tilde{\mathbf{X}}$  containing the contribute of the components corresponding to the  $l$  selected eigen-vectors.

### 4 Singular Spectrum Analysis for the m-D extraction in SAR

The proposed approach to extract the micro-Doppler signature is shown in **Figure 1**.



**Figure 1:** Proposed micro-Doppler extraction process.

Starting from the raw data, the range compression step is performed using a replica of the transmitted signal. From the range compressed data the range gate containing the scatterer is selected. Depending on the analyzed configurations a range cell migration correction step [5] can be performed before the extraction of the range gate.

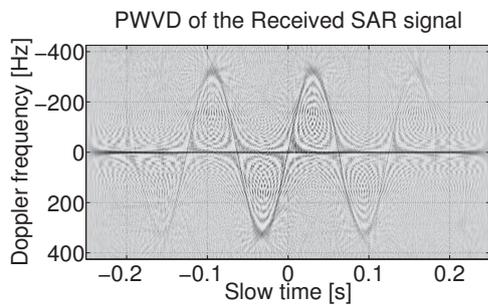
In order to align the signal within the time-frequency plane to obtain a correct visualization and positioning of the time-frequency distribution, the Doppler centroid and the azimuth frequency slope is removed from the signal before the three steps of the SSA. After the reconstruction the resulting micro-Doppler signature can be visualized and used for target classification through the computation of the Pseudo Wigner Ville Distribution (PWVD) [4].

In order to select the components the clutter energy should be considered, in a realistic scenario the clutter energy results to be higher than the energy from the target with micro-Doppler meaning that the eigenvalues spectrum exhibits clear separation between the clutter components and the micro-Doppler components.

## 5 Results

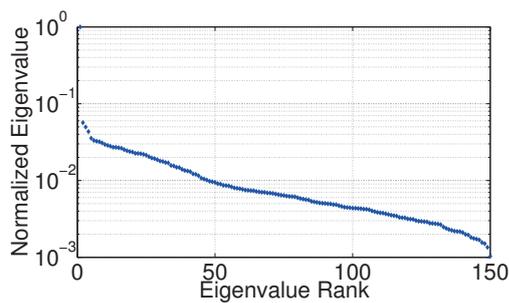
The proposed micro-Doppler extraction method was tested with simulated data. The simulated system is an X band (10 GHz) SAR. In the simulations a K-distributed clutter is assumed. This kind of distribution was shown to fit properly for both monostatic and bistatic ground and sea clutter [13].

Two simulations with different vibrating amplitudes and frequencies were performed all with a SCR less than  $-3$  dBs. The point scatterer model has been used to perform the simulations, in the first simulation a vibrating target with a vibrating amplitude of 10 cm and a vibrating frequency of 8 Hz was placed in the scene centre. The PWVD of the range gate under test is shown in **Figure 2**, it can be seen that the sinusoidal modulation induced from the target vibration is not clearly visible while the clutter is strong and evident in the time-frequency analysis.



**Figure 2:** Pseudo-Wigner Ville Distribution of the analyzed range gate for the first simulated config.

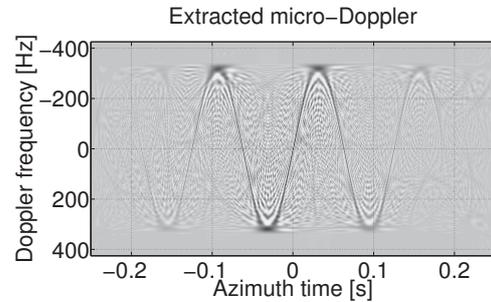
From the eigenvalue spectrum, shown in **Figure 3**, obtained using the SSA it is possible to identify the main component containing the higher amount of energy.



**Figure 3:** Eigenvalue spectrum for the first simulated configuration.

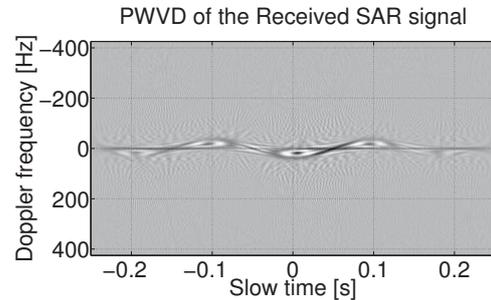
This component contains the contribution of the strong echoes of the stationary clutter signal, while components from 2 to 33 result to contain the micro-Doppler signature of the simulated vibrating target. In **Figure 4** it is shown the obtained reconstructed micro-Doppler signature extracted from the clutter. It can be seen that the resulting

micro-Doppler signature contains all the features describing the vibration. In **Figure 4** are now clearly visible the 4 periods in 0.5 seconds corresponding to the 8 Hz vibration of the simulated point target while it can be measured a maximum micro-Doppler frequency of 325 Hz, this value from the inversion of (2) corresponds to 0.097 m of vibrating amplitude.

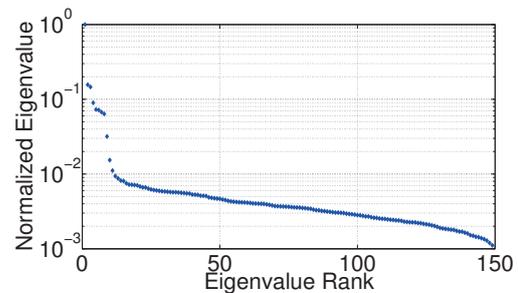


**Figure 4:** Extracted m-D signature for the first simulated config.

The point target for the second case has a vibrating frequency of 5 Hz and 2 cm of vibrating amplitude. The PWVD of the range gate under test is shown in **Figure 5**, it can be seen that the modulation induced from the target vibration is not clearly visible and is actually mixed with the clutter.

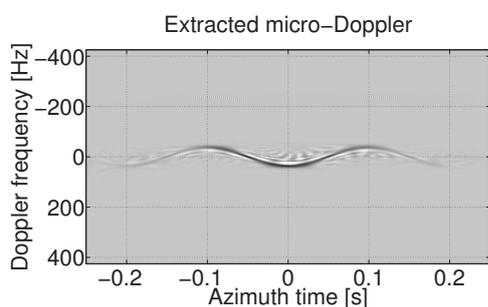


**Figure 5:** Pseudo-Wigner Ville Distribution of the analyzed range gate for the second simulated config.



**Figure 6:** Eigenvalue spectrum for the second simulated configuration.

From the eigenvalue spectrum, shown in **Figure 6**, obtained using the SSA it is possible to identify the main component containing the higher amount of energy. This component contains the contribution of the strong echoes of the stationary clutter signal, while components from 2 to 11 contain the micro-Doppler signature of the simulated vibrating target. In **Figure 7** it is shown that the obtained reconstructed micro-Doppler signature is extracted from the clutter. It can be seen that the resulting micro-Doppler signature contains all the features describing the vibration. In **Figure 7** can be measured 2.5 periods in 0.5 seconds corresponding to the 5 Hz vibration of the simulated point target while a maximum micro-Doppler frequency of 46 Hz corresponding to 0.022 m of vibrating amplitude was measured.



**Figure 7:** Extracted m-D signature for the second simulated config.

## 6 Conclusions

In this paper the effectiveness of the use of the Singular Spectrum Analysis to extract micro-Doppler signatures from SAR signals in clutter was demonstrated. The technique is robust with respect to high signal to clutter ratio providing an useful tool for micro-Doppler analysis in realistic environments.

## Acknowledgment

This work was supported by the Engineering and Physical Research Council (grant N. EP/H012877/1), the MOD University Defence Research Centre in Signal Processing and Selex-Galileo Edinburgh.

## References

[1] V.C. Chen, F. Li, S.-S. Ho, and H. Wechsler. Micro-Doppler effect in radar: phenomenon, model, and simulation study. *Aerospace and Electronic Systems, IEEE Transactions on*, 42(1):2 – 21, 2006.

[2] Victor C. Chen and Hao Ling. *Time-frequency transforms for radar imaging and signal analysis*. Jan 2002.

[3] Victor V. Chen. *Micro-Doppler Effect in Radar*. Artech House, first edition, 2011.

[4] L. Cohen. Time-frequency distributions-a review. *Proceedings of the IEEE*, 77(7):941 –981, July 1989.

[5] I.G. Cumming and F.H. Wong. *Digital Processing of Synthetic Aperture Radar Data: Algorithms and Implementation*. Artech House Publishers, first edition, 2005.

[6] Su Fulin and Jiu Mingyuan. ISAR Imaging of Target with Micro-motion Parts Based on SSA. *Synthetic Aperture Radar (EUSAR), 2010 8th European Conference on*, pages 1 –4, june 2010.

[7] N. et al Golyandina. *Analysis of Time Series Structure: SSA and Related Techniques*. Chapman & Hall/CRC, 2001.

[8] H Hassani. Singular spectrum analysis: methodology and comparison. *mpira.ub.uni-muenchen.de*, Jan 2007.

[9] Junfei Li and Hao Ling. ISAR feature extraction from targets with non-rigid body motion using adaptive chirplet representation. In *Antennas and Propagation Society International Symposium, 2002. IEEE*, volume 4, pages 294 – 297 vol.4, 2002.

[10] M. Ruegg, E. Meier, and D. Nuesch. Vibration and Rotation in Millimeter-Wave SAR. *Geoscience and Remote Sensing, IEEE Transactions on*, 45(2):293 – 304, 2007.

[11] T. Sparr and B. Krane. Micro-Doppler analysis of vibrating targets in SAR. *Radar, Sonar and Navigation, IEE Proceedings -*, 150(4):277–83, 2003.

[12] Ljubisa Stankovic, Igor Djurovic, and Thayanathan Thayaparan. Separation of target rigid body and micro-Doppler effects in ISAR imaging. *Aerospace and Electronic Systems, IEEE Transactions on*, 42(4):1496 –1506, october 2006.

[13] G Yates, A Horne, A Blake, and R Middleton. Bistatic sar image formation. *Radar Sonar and Navigation*, Jan 2006.

[14] Wei Zhang, Chuangming Tong, Qun Zhang, and Xiao Zhang. Micro-Doppler extraction of vibrating target based on dual-channel ATI technique in SAR. In *Geoscience and Remote Sensing (IITA-GRS), 2010 Second IITA International Conference on*, volume 1, pages 422 –425, aug. 2010.