Compressive Sensing Applications and Demonstrations: Synthetic Aperture Radar

Shaun I. Kelly
The University of Edinburgh

Outline

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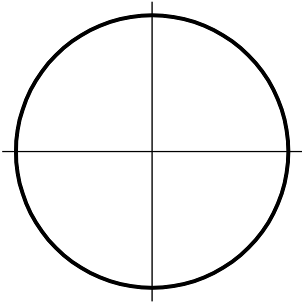
SAR Data Acquisition



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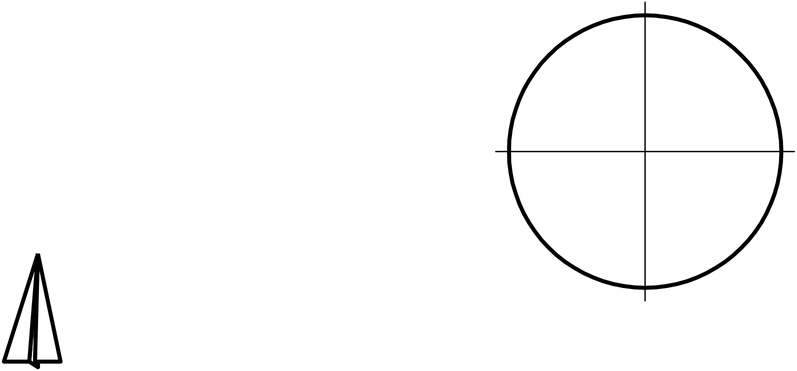

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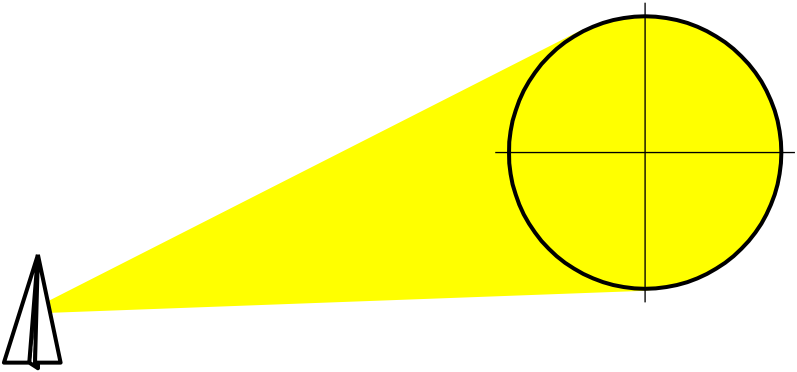

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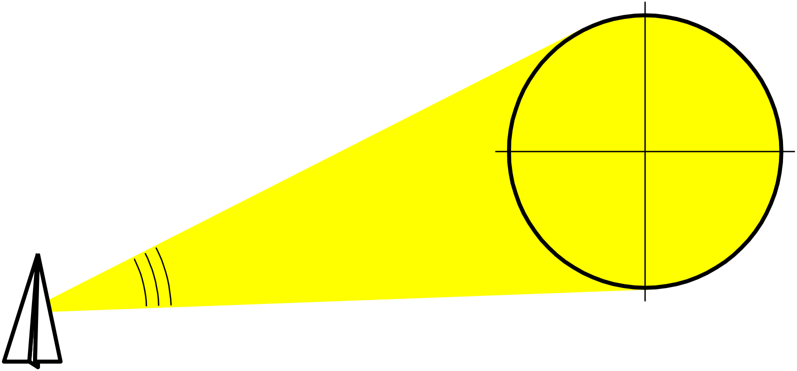

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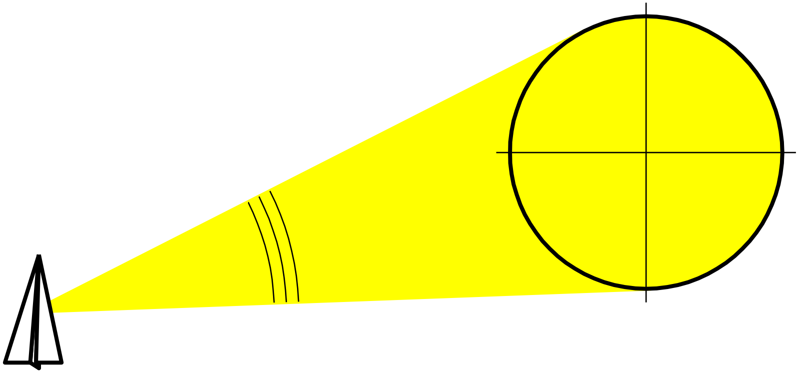

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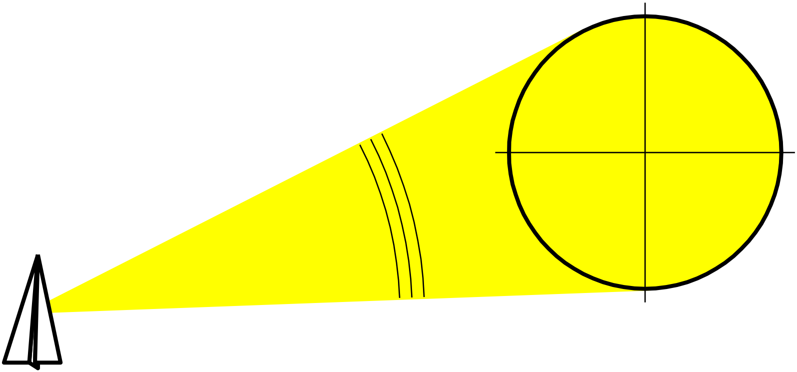

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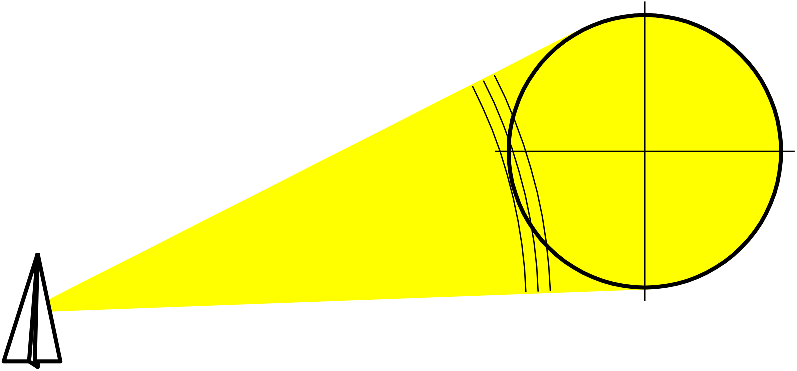

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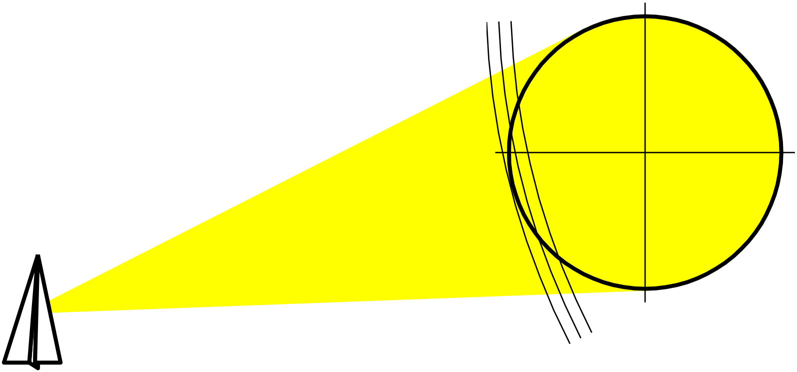

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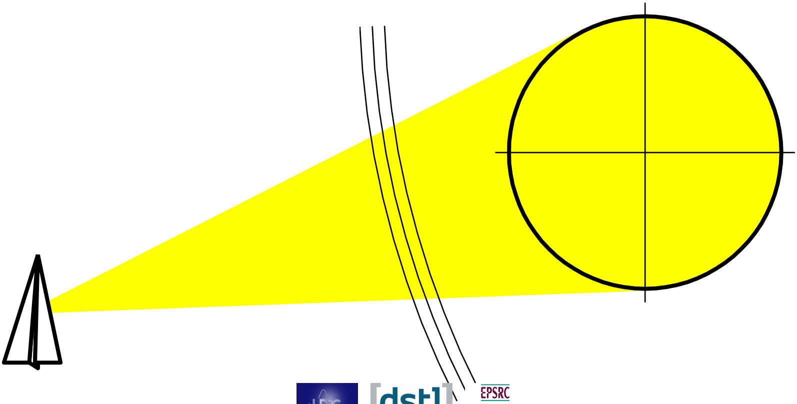

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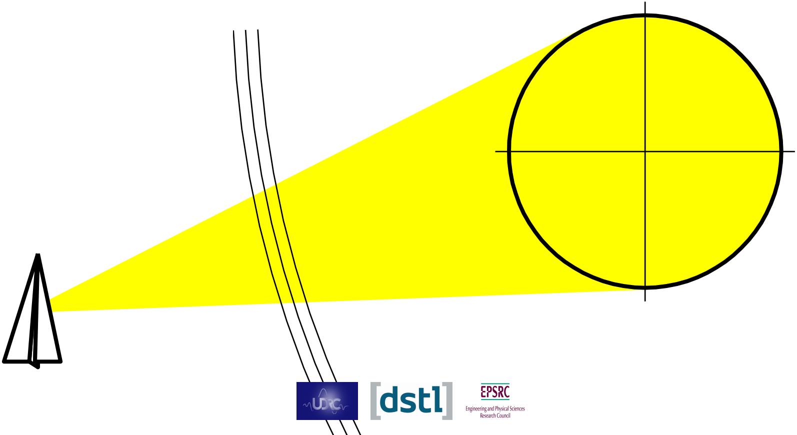

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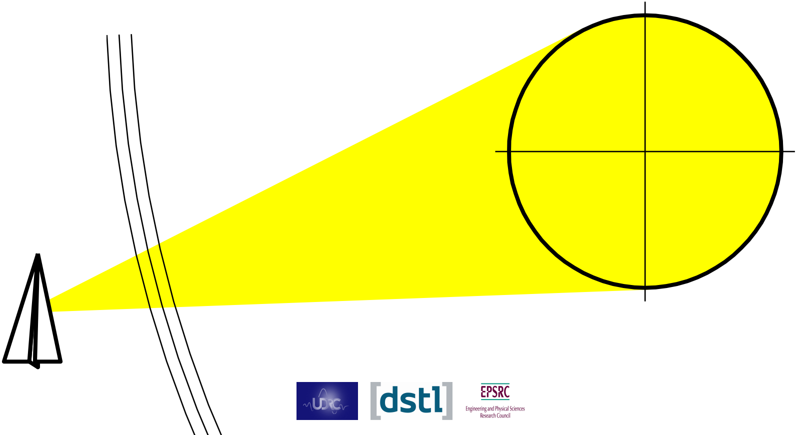

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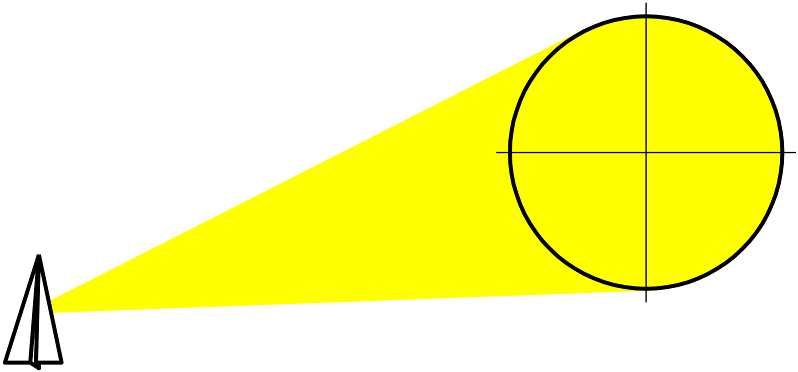

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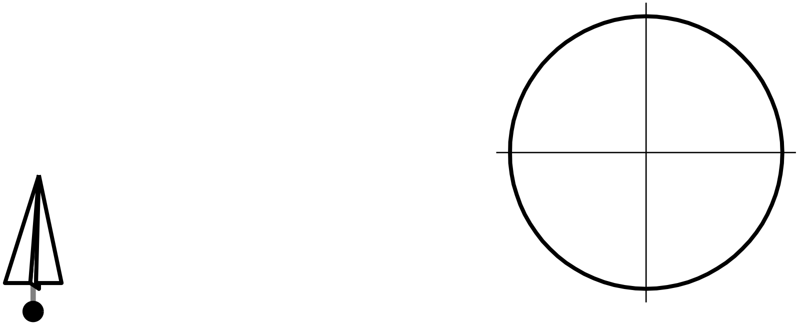

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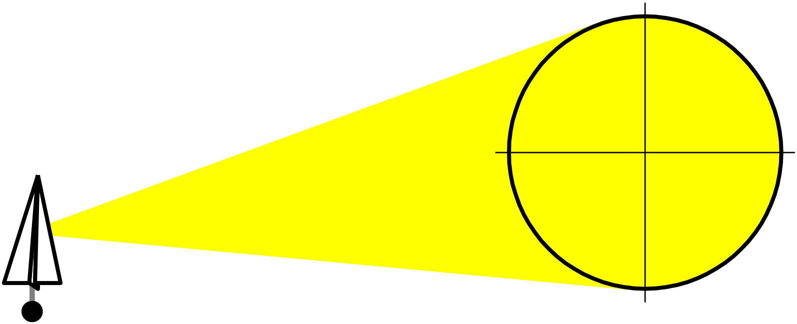

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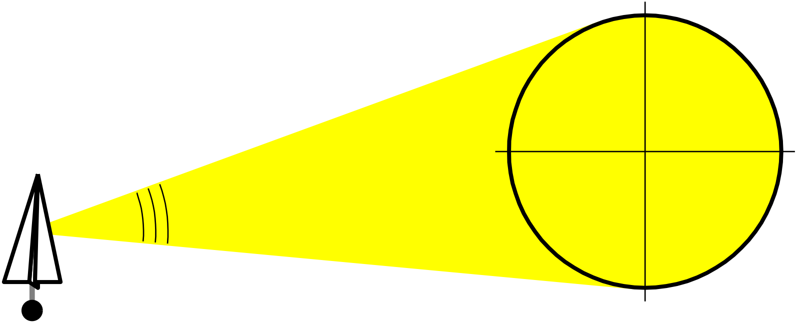

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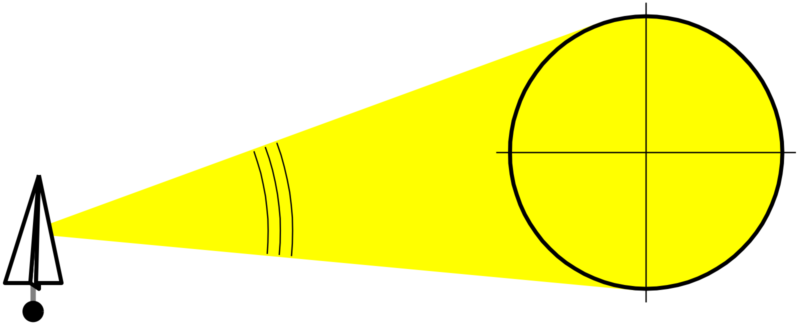

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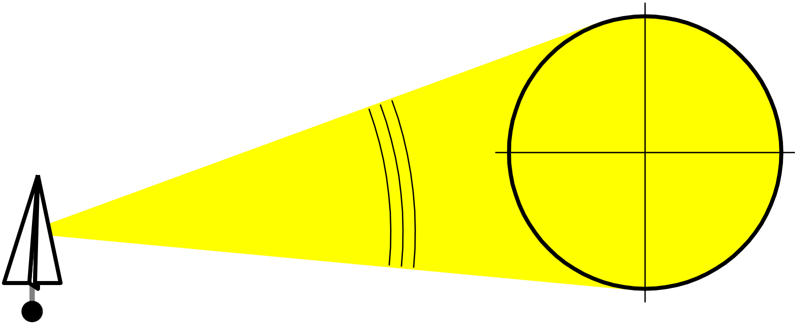

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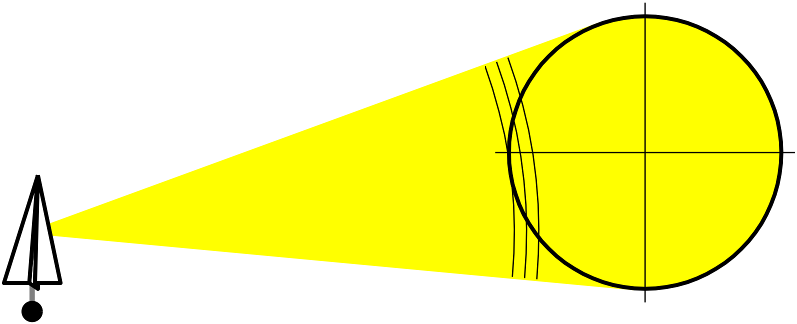

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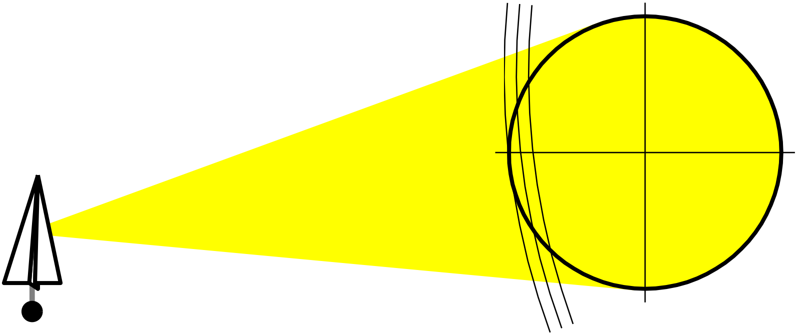

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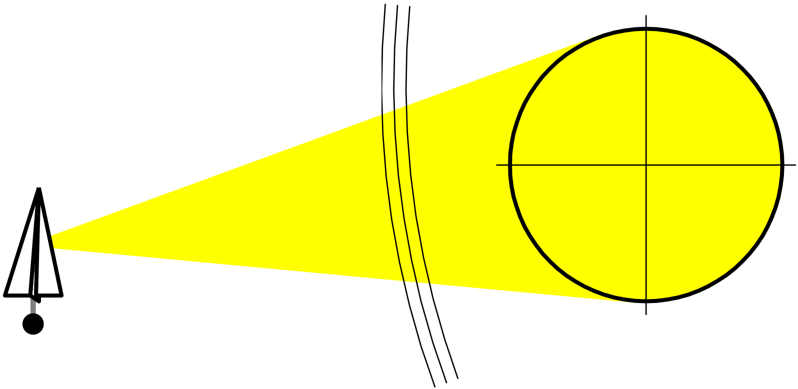

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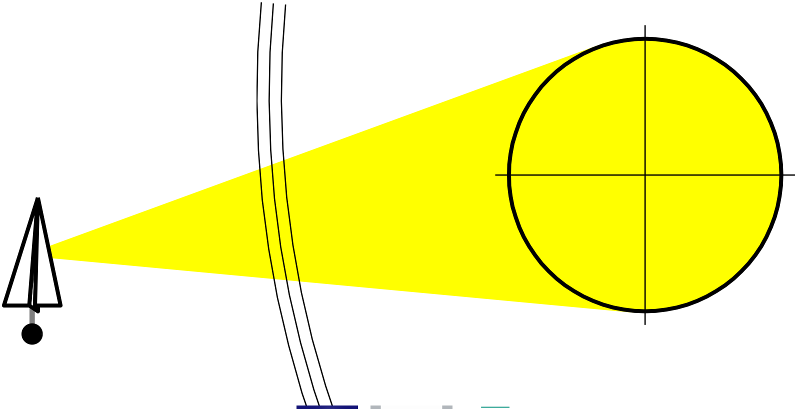

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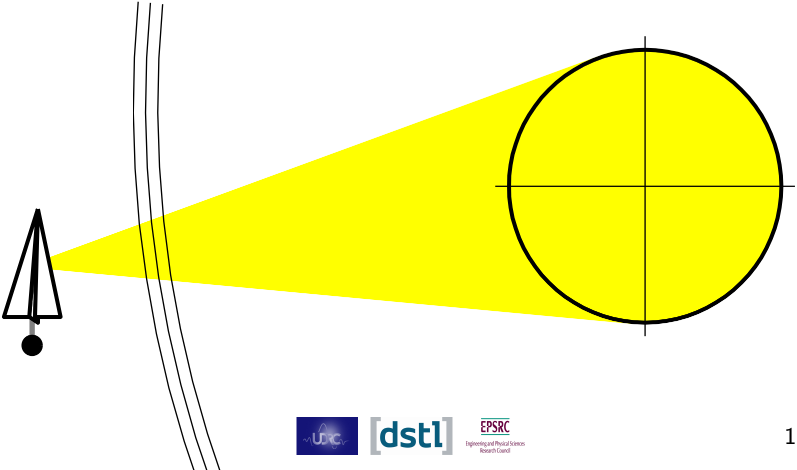

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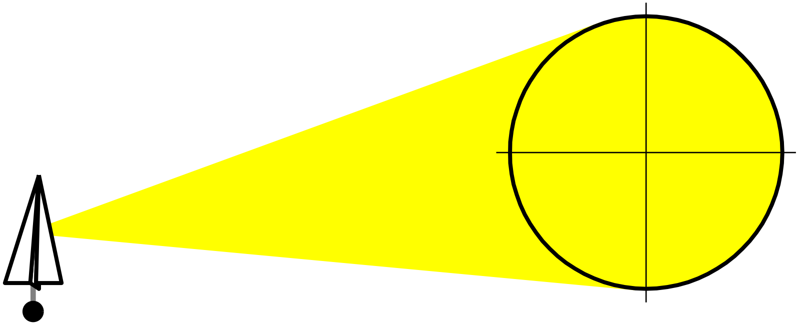

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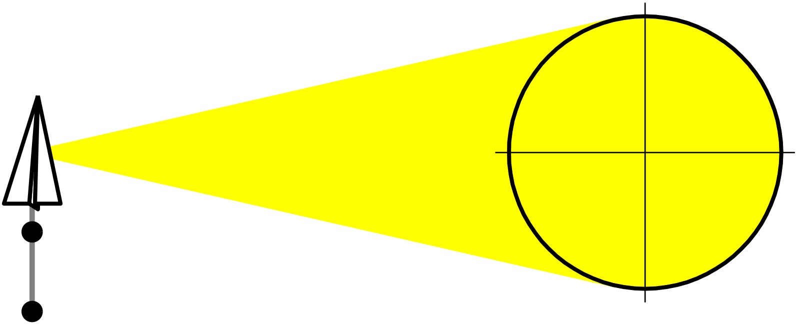

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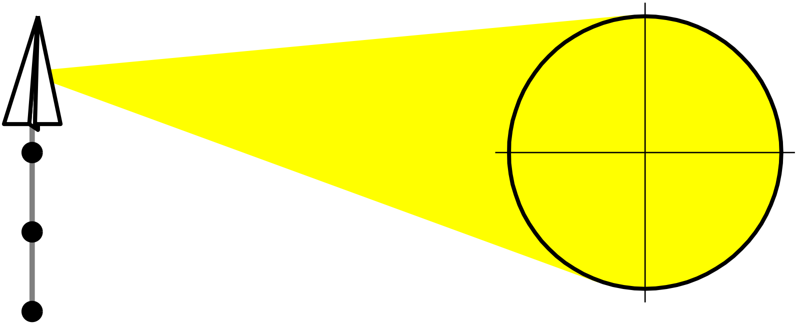

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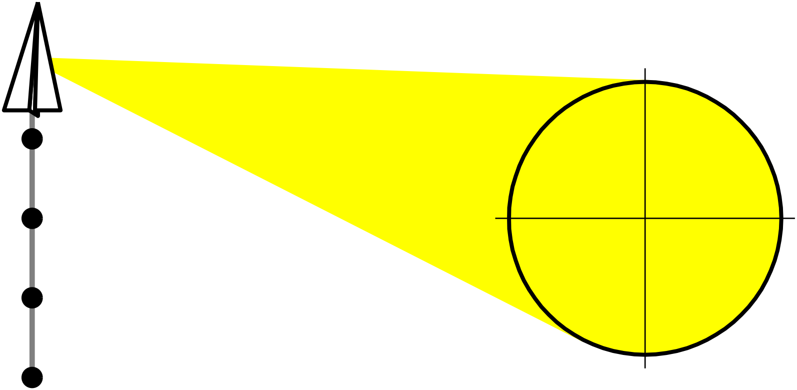

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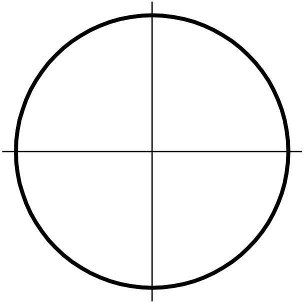
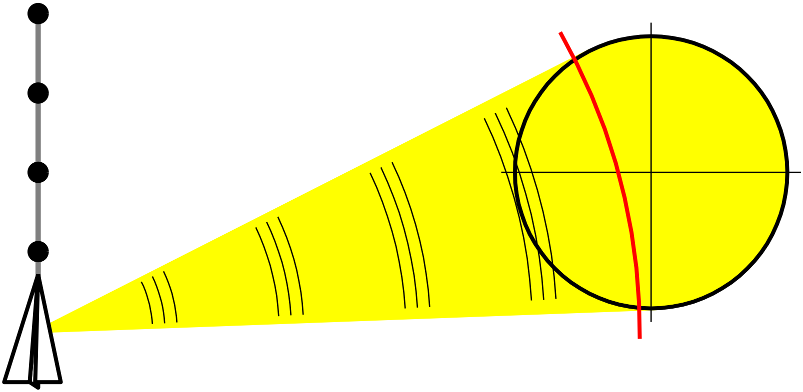


SAR System Model



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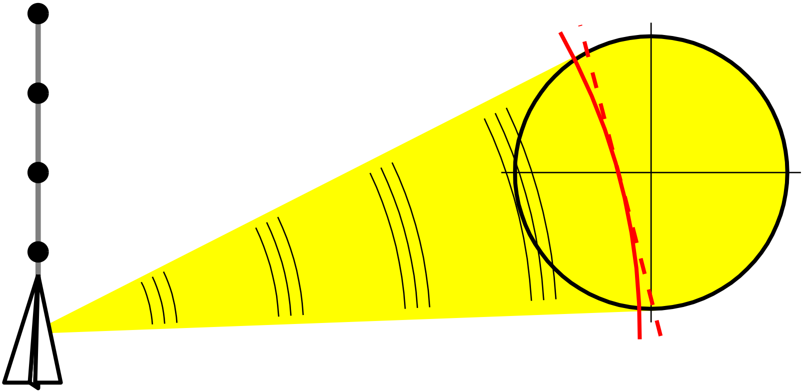
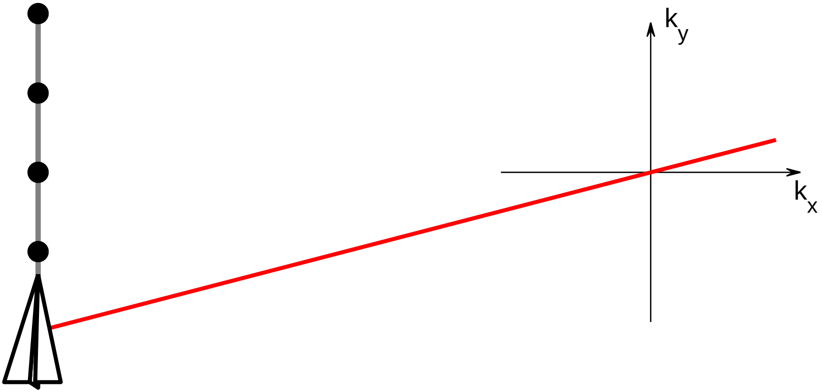


SAR System Model — Spatial Fourier Domain



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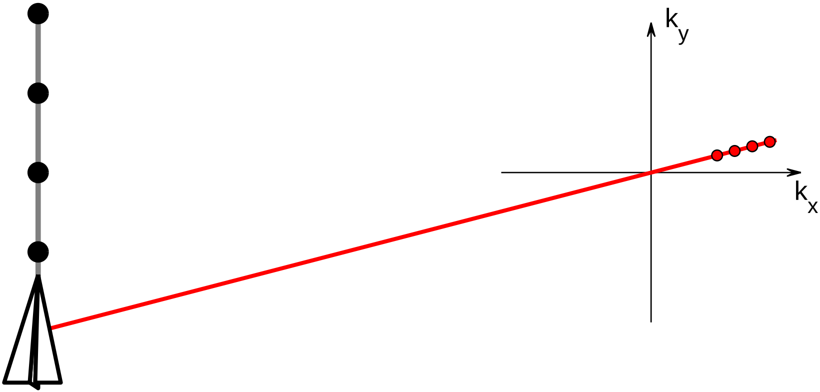

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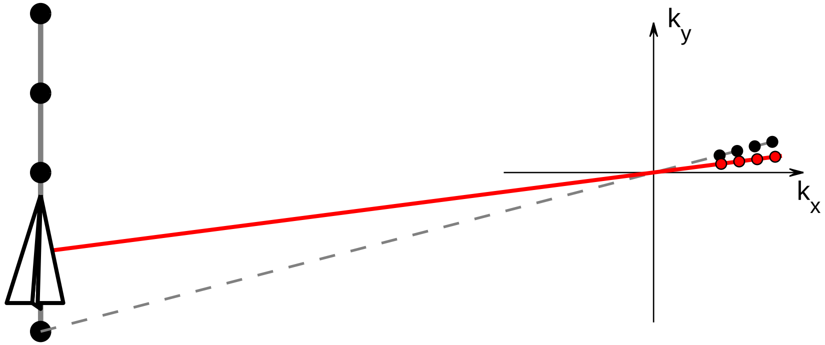

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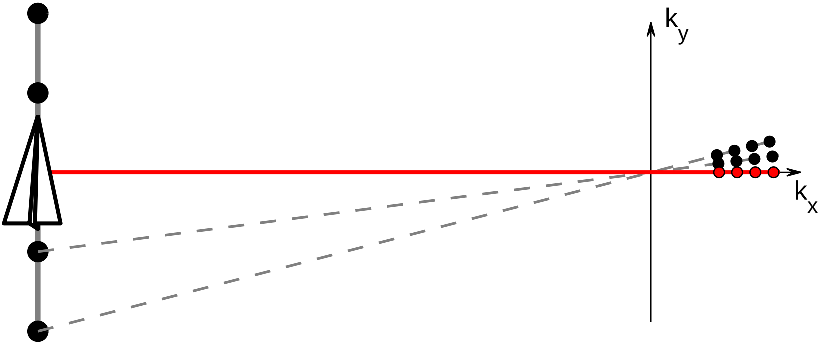

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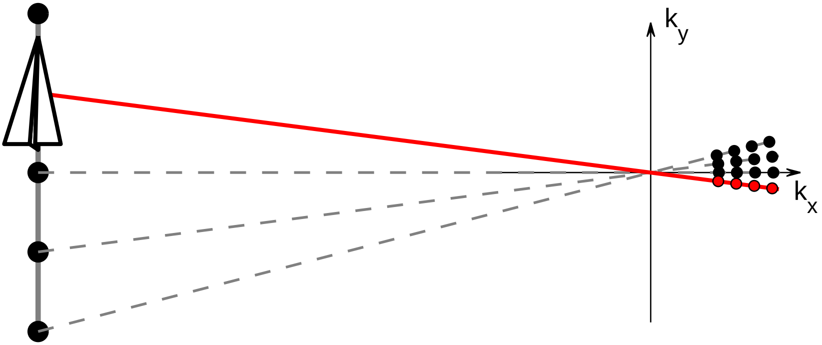

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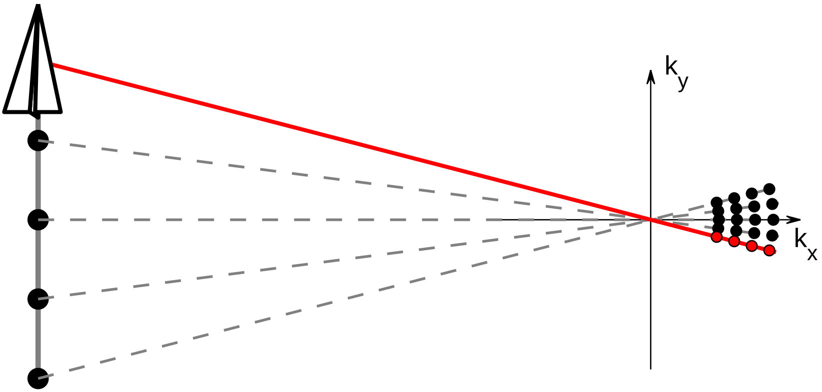

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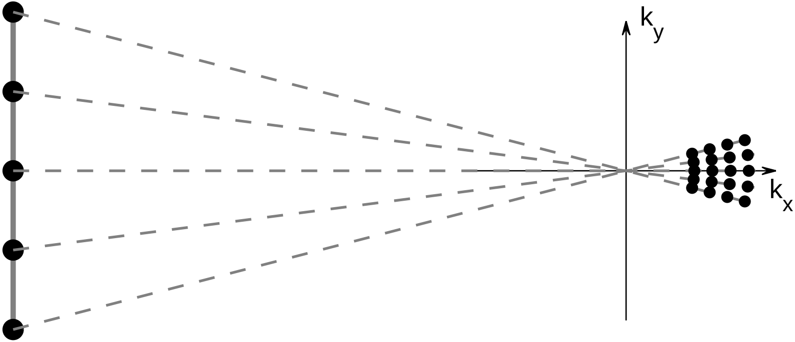
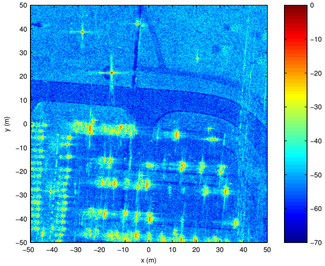


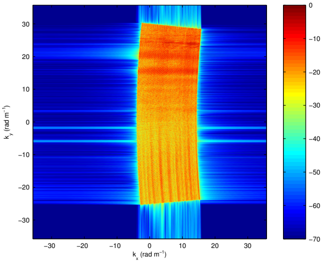
Standard Image Formation

Approximate 2-D Matched Filter

1. Back-projection Algorithm
2. Polar Format Algorithm
3. Range Migration Algorithm
4. Range/Doppler Algorithm

Example SAR Image





Outline

Why is Compressed Sensing Interesting for SAR?

Data Reduction

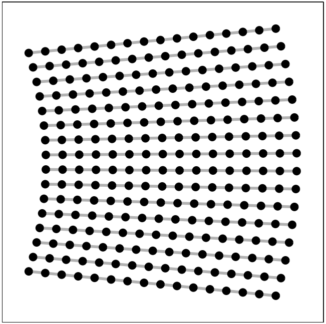
- If the phase history could be sampled at less than the Nyquist rate it would lead to improved storage and transmission capabilities.

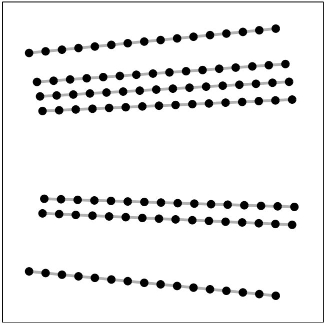
Sensor Constraints

- *Missing radar echoes:* interrupted pulses allow other tasks to be performed (multi-function radar)
- *Missing frequency bands:* the radar band may contain frequencies where there is interference or transmission is not allowed

Aperture Undersampling

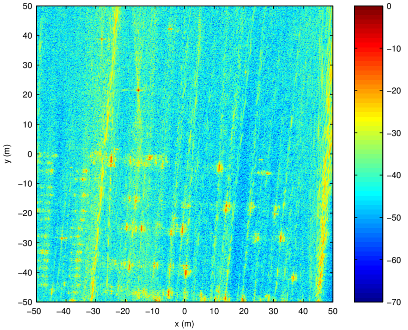
Undersampling of K-space

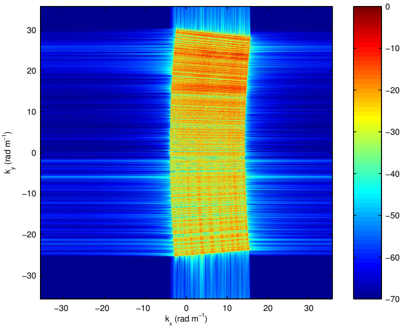




Aperture Undersampling

Standard Image Formation





Compressed Sensing of a SAR Phase History

First Ingredient: Sparsity

- The signal/image $\mathbf{x} \in \mathbb{C}^N$ must be sparse or well approximated by a sparse signal/image (compressible): $\|\mathbf{x}\|_0 = K \ll N$

Will be considered later.

Second Ingredient: "Good" Measurements

- Measurement equation $\mathbf{y} = \mathbf{SFx}(+\mathbf{e})$, with $\mathbf{F}$ the DFT matrix and $\mathbf{S} \in \{0,1\}^{mq \times n}$ a subset selection
- $\mathbf{S}^H\mathbf{S}$ is the projector on the selected subset, $\mathbf{PSF} = (\mathbf{SF})^H(\mathbf{SF})$ is the "point spread function" (more precisely $PSF = \mathbf{F}^{-1}\text{diag}(\mathbf{S}^H\mathbf{S})$ and $\mathbf{PSFx} = PSF * \mathbf{x}$)

An approximate sub-sampling of the k-space!

Third Ingredient: Reconstruction Algorithm

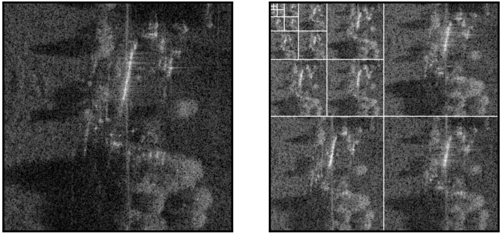
- Optimisation algorithm. ex: constrained ℓ_1 minimisation
- Greedy algorithm. ex: Orthogonal Matching Pursuit (OMP), CoSaMP, Iterative Hard Thresholding (IHT)

Many fast algorithms available if there are fast operators available!

Sparsity of SAR Image

Sparsity in Wavelets?

Image Domain Wavelet Domain



SAR images are not significantly compressible in any basis!

SAR Image Statistics

Interaction of Reflectors in a Range Cell

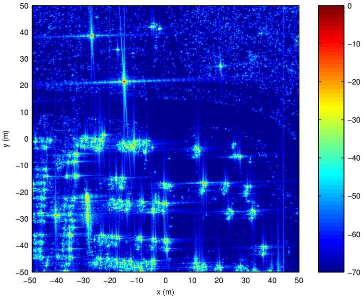
- *Random interference:* Speckle dominates images due to many random reflectors in a range cell inducing multiplicative noise in the reconstructed image - not compressible.
- *Coherent interference:* Coherent reflectors (often targets of interest) whose intensity tend to be much larger than incoherent reflections - compressible in spatial domain.

Aperture Undersampling

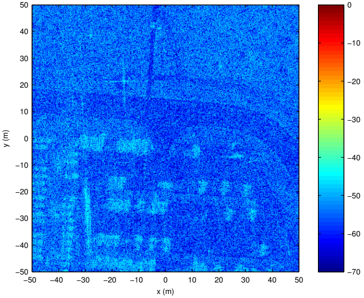
Compressed Sensing Image Formation

$$\arg.\min_{\mathbf{x}_s}. \ \|\mathbf{x}_s\|_1 \ \text{s.t.} \ \|\mathbf{y} - \mathbf{SF}\mathbf{x}_s\|_2 \leq \epsilon$$

$$\arg.\min_{\mathbf{x}_{bg}}. \ \left\|\mathbf{y} - \mathbf{SF}\left(\mathbf{x}_{bg} + \mathbf{x}_s\right)\right\|_2$$



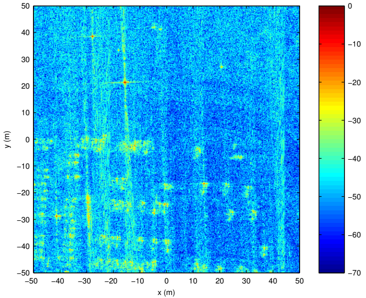
Bright Targets



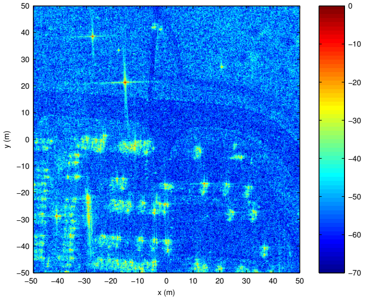
Background Speckle

Aperture Undersampling

Compressed Sensing Image Formation



Standard Image Formation

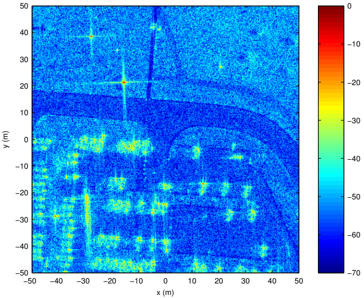


Compressed Sensing Image Formation

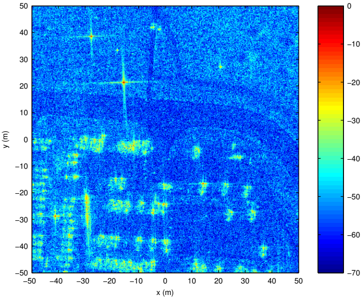
Significant improvement in imaging of bright targets!

Aperture Undersampling

Compressed Sensing Image Formation



Reference Standard Image Formation



Compressed Sensing Image Formation

Degradation in background speckle!

Issues in Compressed Sensing SAR

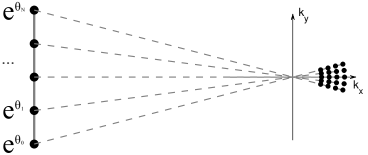
Topics to Consider:

- Fast operators
- *"Sparsity-parameter"* selection
- *"Off-the-grid"* targets
- Deterministic undersampling

Outline

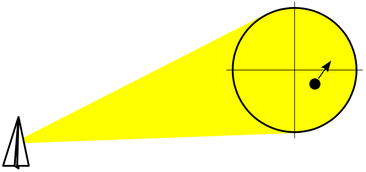
Phase Calibration (Autofocus)

- Motion and propagation errors produce unknown phase errors.
- Each aperture position has a phase error.
- Without a sparsity assumption the problem ill-posed.



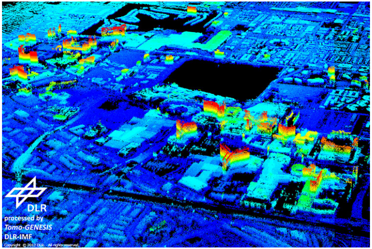
Ground Moving Target Indication

- Targets may have a significant velocity.
- Target velocities can cause target displacement and/or blurring.
- Without a sparsity assumption on the number of moving targets, many antennas and a large amount of aperture oversampling is required.



3-D Imaging

- Multi-pass/2-D aperture is required.
- Meeting the Nyquist sampling rate requirement for a 2-D aperture is typically not possible.
- Scattering occurs at propagation medium boundaries so 3-D images are sparse.



Outline

Compressed Sensing SAR: Example Code

simulation_cs_sar.m

- Main function of example code.
- Range/Doppler algorithm for operators.
- Random aperture undersampling.

simple_sar_simulator.m

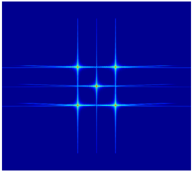
- Mono-static strip-map SAR.
- Returns are delayed and scaled versions of a reference chirp signal.
- Dechirp-on-receive.

fista.m

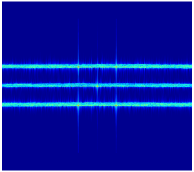
- Implementation of the Fast Iterative Shrinkage-Thresholding Algorithm (Beck et al. 2009).
- Optimal first order method.

Compressed Sensing SAR: Example Code

Results



Reference Standard
Image Formation



Standard
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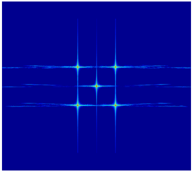

Compressed Sensing
Image Formation

Things to Experiment With

- Use a different solver.
- Change the λ parameter.
- Change the amount of undersampling.
- Use different operators.
- Use a more complicated SAR simulator.
- Add in clutter.

Thank you for your attention!

Questions?