Detecting LFM Parameters in Joint Communications and Radar Frequency Bands

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***Abstract*—As a significant radar waveform, linear frequency modulation (LFM) is widely used in both military and civilian applications. Recently, there has been increasing interest in using the same radio spectrum to enable the radar and communication signals to coexist. This paper explores methods capable of estimating LFM parameters, particularly in the presence of co-channel orthogonal frequency division multiplexing (OFDM) signals. In particular, this paper applies the discrete chirp Fourier transform (DCFT) to this specific scenario and also compares this approach to the idea of reusing an OFDM receiver to estimate the LFM parameters. Through simulations, we demonstrate the use of the Hough transform to confirm that these can be identified to a high degree of accuracy. In addition, we discuss how this "reuse" approach opens up new applications for storing waveforms of interest for later data analysis.**

I. Introduction

Nowadays, as the number of mobile terminal users increases and diversified sensors are applied in various scenarios, a great amount of information is usually disposed of in traditional signal processing to reduce complexity and improve efficiency in radio frequency (RF) receivers. However, such information could contain latent features of interest and be worth storing or recovering for further processing; for example, for detecting abnormal waveforms or other signal processing tasks. In the meantime, in these various scenarios, spectrum resources grow scarce and different waveforms or different sensors are more likely to interfere with each other. Thus, robust signal detection in the presence of co-channel interference becomes an important and imminent problem to be solved.

The two most common radio frequency receivers in defence are communication and radar. Many researchers are now focussed on communication and radar systems working simultaneously to improve spectral efficiency. For example, [1] discusses two scenarios under the topic of communication and radar spectrum sharing (CRSS): radar-communication coexistence (RCC) and dual-functional radar-communication (DFRC). With respect to DFRC, [2] and [3] present orthogonal frequency division multiplexing (OFDM) as a promising candidate that is used in multiple scenarios, notably for autonomous vehicles. Also, [4] introduces the use of the cyclic prefix (CP) in OFDM system to combat channel effects by circular convolution; this inherent ability to overcome channel effects makes OFDM a common modulation scheme widely used in communication systems.

Reference [5] introduces the linear frequency modulation (LFM) waveform as one kind of continuous waveform (CW) that is frequently used in defence and civilian applications. In addition, by utilising a combined communication and radar waveform, [6] proposes chirp modulation to achieve radar and communication functions simultaneously and [7] discusses modulating a LFM signal by adjusting the data with orthogonal sequences in an integrated radar and communication scenario.

Thus, CRSS is the application scenario selected for study in this paper, where OFDM and LFM waveforms represent the communication waveform and the radar waveform, respectively. In the CRSS scenario with LFM and OFDM waveforms, one waveform could be seen as the interference to the other. Much of the research to date focuses on the application and the performance of CRSS. However, in this paper the question to be studied is how to detect LFM waveforms in the presence of co-channel OFDM signals. The receiver may then choose to store such waveforms for later processing. With respect to CRSS scenarios, other techniques, such as compressive sensing (CS), have already been applied in [8] to improve the communication symbol-error-rate performance when uncoordinated radar waveforms are also present. At a receiver, the received waveforms may contain information which is useful for signal processing and may be worth storing. Based on this point, CS [9] is also a promising technique for analysing or storing large amounts of data and can be applied on detection, classification, estimation, and filtering problems. Recently, it has been shown that CS methods are also able to compress data and store results for further processing [10]. Specifically, [10] utilised an information-theoretic projection design approach to control the input-output mutual information terms for two independent sources, which is extremely suitable for CRSS scenarios. Thus CS is a significant technique which will be investigated and applied to CRSS in future research.

To detect the LFM parameters, there are several methods proposed in the literature. Reference [11] proposes joint estimation of the phase, frequency, and frequency rate based on the application of least squares to the unwrapped phase of the signal. In [12], an estimation algorithm based on a simple iterative approach is proposed whose main characteristics include accuracy, reduction in error propagation effects, and operation over a wide range of phase parameter values. In [13] and [14], methods are proposed to estimate chirp parameters or achieve chirp detection based on fast Fourier transform (FFT) techniques. In addition, a novel algorithm for the parameter estimation of multicomponent chirp signals in complicated noise environments is proposed in [15]. Research in [16]

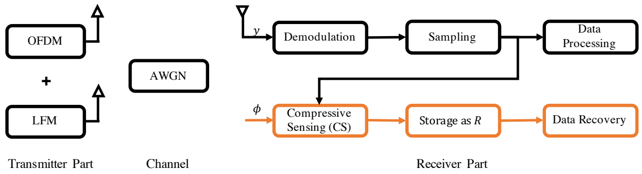


Fig. 1. Joint OFDM and LFM System.

studies the application of the discrete chirp Fourier transform (DCFT) for LFM waveform parameter estimation and [17] discusses the maximum chirplet transform. The latter is a simplification of the chirplet transform, where an iterative detection followed by window subtraction is employed to avoid re-computation of the spectrum.

This paper will focus on one specific application for LFM estimation, where a combined co-channel OFDM and LFM signal model is considered. Specifically, most OFDM receivers contain an efficient FFT implementation. The main contribution of our paper is to show that we are able to estimate the LFM parameters by reusing an existing OFDM receiver directly in order to reduce hardware complexity. We also compare this approach with the more standard DCFT technique from [16]. This work is the starting point for an exploration into the use of general CS techniques for the analysis of two independent signal source, combined communications and radar signals.

The layout of this paper is as follows: Section II introduces the system diagram and the system model of OFDM and LFM transmitting simultaneously. Section III explains how the DCFT can be used to estimate the LFM parameters in the system model. Section IV proposes an alternative approach to detect LFM parameters by reusing an OFDM receiver in combination with the Hough transform [18]. Section V discusses the two methods in Section III and Section IV and also introduces future research topics.

II. System Model

In this section, we provide an overview of the system model utilised throughout the paper. The model is constructed from three main components — a transmitter subsystem, a channel, and a receiver subsystem — and is illustrated in Fig. 1. Here, the black blocks are standard signal processing elements, and the orange blocks are the objective of future research.

Assuming there are M_s subcarriers in the OFDM waveform, N_c symbols for each symbol block, and the length of CP is N_{cp} symbols, the complex-valued time domain OFDM signal $x(t)$ considered in this paper is defined as in [4]

$$x(t) = \sum_{k=0}^{M_s-1} X_k e^{j2\pi kt/T}, \qquad 0 < t < T + T_g, \tag{1}$$

where X_k is the data symbol modulated via quadrature phase shift keying (QPSK) modulation, and the frequency interval for each adjacent subcarrier is $\Delta W = 1/T$, while T is symbol time. Equation (1) is usually implemented using the inverse discrete Fourier transform (IDFT).

Furthermore, for the radar waveform, assuming f_l is the linear frequency and f_c is the carrier frequency, we define the instantaneous frequency of an LFM waveform at time t as $f_{\text{LFM}}(t)$, which is expressed as:

$$f_{\text{LFM}}(t) = f_l t + f_c. \tag{2}$$

Thus, the corresponding complex analogue form of LFM is

$$f(t) = e^{j(\beta_0 t^2 + \alpha_0 t)}. \tag{3}$$

The derivative of the phase term in (3) is the instantaneous frequency of LFM as in (2), therefore

$$\beta_0 = \pi f_l, \qquad \alpha_0 = 2\pi f_c. \tag{4}$$

Thus, (2) can be rewritten as

$$f_{\text{LFM}}(t) = \frac{\beta_0}{\pi} t + \frac{\alpha_0}{2\pi}. \tag{5}$$

At the receiver part, the received signal is the $(N_c+N_{cp})\times 1$ vector $\mathbf{y}$ as follows:

$$\mathbf{y} = \mathbf{y_{ofdm}} + \mathbf{y_{lfm}} + \mathbf{w}, \tag{6}$$

where $\mathbf{y_{ofdm}}$ is the $(N_c + N_{cp}) \times 1$ OFDM data, $\mathbf{y_{lfm}}$ is the $(N_c+N_{cp})\times 1$ LFM data, and $\mathbf{w}$ is the $(N_c+N_{cp})\times 1$ additive white Gaussian noise (AWGN).

The data processing block after sampling and demodulation in Fig. 1 can be a LFM receiver or an OFDM receiver. For the orange blocks, CS is considered as a data analysing process to store data, which is different to the normal receivers. These processing blocks will be discussed further in Section V of the paper.

III. Discrete Chirp Fourier Transform

Research in [16] studies using the N-point DCFT to estimate chirp parameters and this paper also presents theoretical analyses on the selection of N. In this section, we will briefly introduce the DCFT method, apply it to OFDM and LFM combined signals, and present the simulation results.

The DCFT is similar to the discrete Fourier transform (DFT), which is efficiently applied in OFDM signal reception via the FFT. The N-point DFT is defined as

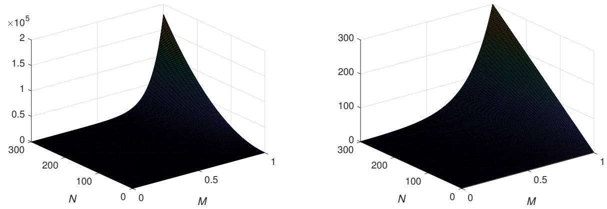
$$X[k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] W_N^{nk}, \qquad k = 0, 1, \ldots, N-1, \tag{7}$$

where $W_N = \exp(-2\pi j/N)$. Similarly, the expression for the N-point DCFT method is presented as

$$X[l,k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] W_N^{ln^2+kn}, l, k = 0, 1, \ldots, N-1. \tag{8}$$

When applying the DCFT to equation (3) to estimate LFM parameters, [16] proposed the following equation when the sampling rate is $t = n/N^{1/3}$;

$$f[n,l,k] = e^{j\left(\beta_0 \frac{n^2}{N^{\frac{2}{3}}} + \alpha_0 \frac{n}{N^{\frac{1}{3}}}\right)} = W_N^{-(ln^2+kn)}. \tag{9}$$



(a) (b)

Fig. 2. Frequency estimation ranges: (a) $\tilde{f}_l$ and (b) $\tilde{f}_c$ based on N and M.

Based on (3) and (9), the relationship between l, k and β_0, α_0 are as follows:

$$l = \frac{\beta_0}{2\pi} N^{\frac{1}{3}} \qquad k = \frac{\alpha_0}{2\pi} N^{\frac{2}{3}}. \tag{10}$$

Thus, we are able to determine the estimated l as $\tilde{l}$ and the estimated k as $\tilde{k}$ by (8), then the estimated β_0 as $\tilde{\beta_0}$ and the estimated α_0 as $\tilde{\alpha_0}$ are also able to be deduced from equation (10). Furthermore, [16] also discusses limitations on the selection of N, which should be an odd value to avoid multiple peaks appearing in the DCFT spectrum. Even values of N are typically preferred for efficient implementations of the DFT. Thus, being constrained to odd values of N during computation of the DCFT might inhibit low-complexity solutions. The estimation range of an N-point DCFT under this sampling rate, when $\tilde{l} \leq N$ and $\tilde{k} \leq N$ can be characterised by

$$\tilde{f}_l \leq 2N^{\frac{2}{3}} \qquad \tilde{f}_c \leq N^{\frac{1}{3}}. \tag{11}$$

When modifying the sampling rate to $t = n \times 1/N^M$, where M is a value in the interval of $(0, 1]$, equation (3) can be rewritten as

$$\begin{aligned} f[n,l,k] &= e^{j\left(\beta_0 \frac{n^2}{N^{2M}} - \alpha_0 \frac{n}{N^M}\right)} \\ &= W_N^{-(ln^2+kn)}. \end{aligned} \tag{12}$$

Thus, the relationship between $\tilde{l}$, $\tilde{k}$ and $\tilde{\beta_0}$, $\tilde{\alpha_0}$ under M sampling rate are as follows:

$$\tilde{l} = \frac{\tilde{\beta_0}}{2\pi N^{2M-1}}, \qquad \tilde{k} = \frac{\tilde{\alpha_0}}{2\pi N^{M-1}}. \tag{13}$$

In the meanwhile, the corresponding estimation range of $\tilde{f}_l$ and $\tilde{f}_c$ are as follows:

$$\tilde{f}_l \leq 2N^{2M} \qquad \tilde{f}_c \leq N^M. \tag{14}$$

Based on (14), Fig. 2(a) and Fig. 2(b) present the estimation range of $\tilde{f}_l$ and $\tilde{f}_c$ depending on different values of N and M, respectively. From Fig. 2, we observe that the estimation ranges of $\tilde{f}_l$ and $\tilde{f}_c$ increase with N and M simultaneously. In both plots, the maximum estimation range occurs when both N and M are maximised. In addition, computational complexity is increasing more expensively with the increasing of the value of N.

TABLE I
OFDM PARAMETERS

Name	Value
Length of CP	16
FFT length	64
Symbol Period	0.08 s
Bandwidth	1k Hz
Modulation	QPSK
Total Subcarriers	52 (Freq Index -26 to $+26$)
DC Subcarrier	Null (0 subcarrier)
Signal to Noise Ratio (SNR)	0 dB

TABLE II
LFM PARAMETERS

Name	Value
Start Frequency	0 Hz
End Frequency	500 Hz
Repetition Period	10 s

Considering the application scenario and referring to the OFDM standard parameters of [19], we set the OFDM parameters and the LFM parameters for this paper as shown in Table I and in Table II, respectively. From the parameters in Table II, the frequency settings for LFM in the joint radar and communication system are as follows: $f_l = 50\,\mathrm{Hz}$ and $f_c = 0\,\mathrm{Hz}$. Furthermore, in the received signal (6), the SNR for the OFDM signal with respect to the AWGN is $0\,\mathrm{dB}$, while the transmission power of the OFDM signal is equal to the transmission power of the LFM signal.

After obtaining the receiver signal (6), we transform the data using the DCFT with a sampling rate of $t = n \times 1/N^M$, where $N = 25$ and $M = 1$. Then we acquire $f[n, l, k]$ for different l and k as shown in Fig. 3. This figure indicates that the peak of the magnitude of the DCFT is 27.9; the corresponding coordinates $(0, 1)$ are the most suitable for the frequency estimation. By placing $\tilde{l} = 1$ and $\tilde{k} = 0$ into (4) and (13), we estimate the LFM parameters as: $\tilde{f}_l = 50\,\mathrm{Hz}$ and $\tilde{f}_c = 0\,\mathrm{Hz}$, respectively.

IV. Reusing the OFDM Receiver for LFM Parameter Estimation

In [13] and [14], FFT-based methods to estimate the chirp parameters and to achieve chirp detection are proposed, respectively, and [20] discusses the communication performance of an OFDM receiver while receiving OFDM and LFM combined signals.

One main feature of the OFDM receiver is that it can easily implement a frequency transform through the use of the FFT [4]. In this section, we propose an alternative estimation approach by reusing the OFDM receiver to estimate the LFM parameters in the OFDM-LFM combined signal, which is defined in (6). Compared with the DCFT method applied in the LFM receiver of Fig. 1, this method directly applies the

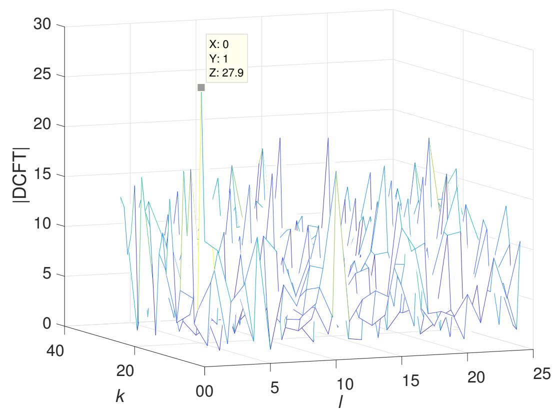


Fig. 3. Magnitude of DCFT spectrum

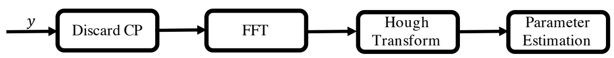


Fig. 4. Data processing diagram for output of OFDM receiver.

OFDM receiver as the first data processing step in Fig. 1 to estimate the LFM parameters.

Fig. 4 introduces the main data processing steps applied to the combined signal in (6) as follows:

1) Discard CP: We dispose of the CP in each symbol period at the receiver part. In our simulations, when removing the CP, 20% of the symbol samples are removed for each symbol period, thus we use zero-padding to extend the remaining samples over $0.064\,\mathrm{s}$ into one full symbol period $0.08\,\mathrm{s}$.

2) FFT: Apply FFT to the data after removing the CP, which is inherently utilised in the OFDM receiver.

3) Hough Transform: In this step, our work builds on ideas published in [21], which studied creating images of communications signals for analysis. Furthermore, [22] applies the Hough transform to multi-component LFM signals to estimate the LFM parameters. Therefore, we apply the Hough transform [18], which is defined as follows:

$$\rho = x\cos\theta + y\sin\theta, \tag{15}$$

where x, y is the coordinates under Cartesian coordinate system, ρ represents the distance between the origin and the given line, and θ denotes the angle between the X-axis and the given line.

4) Parameter Estimation: Fit the ρ and θ from the Hough transform into equation (2) to estimate the LFM parameters.

Step *1)* and Step *2)* are two regular signal processing functions in an OFDM receiver. Step *3)* and Step *4)* are two additional procedures to detect the LFM parameters and can be easily applied after the traditional OFDM receiver.

Based on the OFDM and LFM parameters in Table I and Table II and the data process shown in Fig. 4, we obtain the spectrum of the processed OFDM-LFM signal as shown in Fig. 5. Here, the peak of the spectrum of the combined signal is caused by the LFM waveform. Therefore, in order to estimate the LFM parameters by the Hough transform, we need to perform the following steps. Firstly, we convert Fig. 5 into a 2D black and white image by selecting $30\,\mathrm{dB}$ as a threshold as shown in Fig. 6(a). Then we apply the Hough transform to calculate the gradient of the white line as shown in Fig. 6(b) and subsequently estimate the LFM parameters.

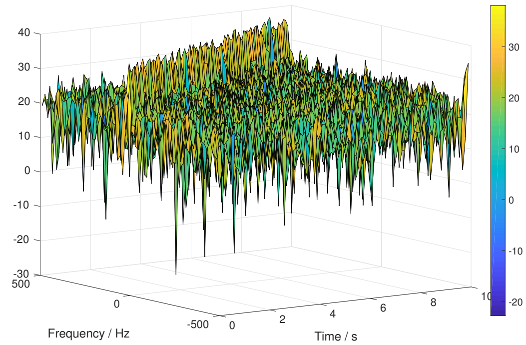
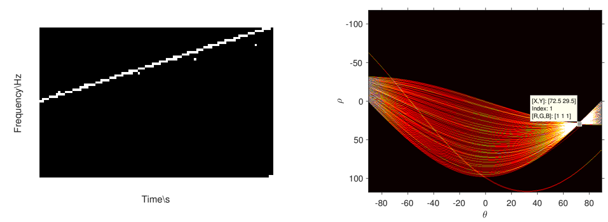


Fig. 5. Combined OFDM and LFM spectrum.



(a) (b)

Fig. 6. Hough transform processing. (a) Black and white image obtained after thresholding of Fig. 5 (b) Result of Hough transform applied to black and white image.

In Fig. 6(b), the coordinate of the peak of the Hough transform is $(72.5, 29.5)$. To express the Hough transform result in a more intuitive manner, we convert Fig. 6(b) into a 3D plot as shown in Fig. 7.

From Fig. 7 with the high resolution zoom shown in Fig. 6(b), we are able to identify parameter values of $\theta = 72.5°$ and $\rho = 29.5$. When considering the characteristics of the LFM waveform, which possesses a frequency that increases with time, we fit the data into (15). Thus, the expression of white line in Fig. 6(a) is as follows:

$$y = 0.3153x + 30.9316. \tag{16}$$

In the meantime, regarding the scale of the axes in Fig. 5, we set $y = 0.08 f_{\mathrm{LFM}}(t)$ and $x = 12.5t$ in (16). Furthermore, the line has a y-intercept of 30.9 in (16); this is approximately equal to the frequency bin at which "DC" (0 Hz) is situated, such that

$$f_{\mathrm{LFM}}(t) = 49.2656t + 0. \tag{17}$$

Then, regarding (2), we obtain the parameter estimation as follows: $\tilde{f}_l = 49.2656\,\mathrm{Hz}$ and $\tilde{f}_c = 0\,\mathrm{Hz}$. When compared with the ground truth parameter value, $f_l = 50\,\mathrm{Hz}$, the parameter estimation error is 1.47%.

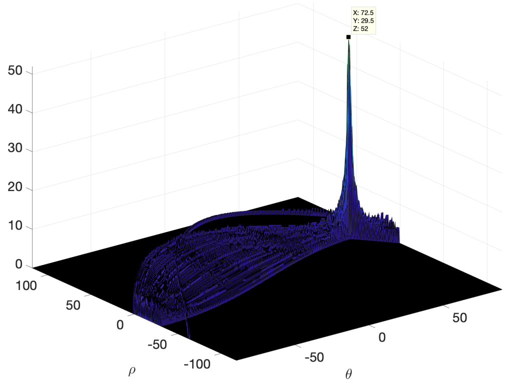


Fig. 7. Peak of Hough Transform of Fig. 6(a)

V. Discussion and Future Work

This paper discusses one alternative way to estimate LFM parameters by reusing an OFDM receiver instead of a traditional LFM receiver. Compared with a traditional LFM estimation method, such as the DCFT, the alternative OFDM receiver approach combined with the Hough transform is a new technique for waveform parameter estimation. Moreover, this alternative method is not restricted by the values of N and M selected in the DCFT, where N must be an odd number and M should be in the interval $(0, 1]$. Furthermore, the estimation range of the alternative method mainly depends on the resolution selected in the Hough transform, however the accuracy of the DCFT depends on both the values of N and M. In addition, the alternative method is able to utilise the OFDM receiver to detect the LFM parameters, which provides the possibility of formulating an RCSS implementation.

In future research, we will investigate the implementation of the orange blocks as shown in Fig. 1 for joint communication and radar scenarios. Since CS techniques are able to reduce the data dimensionality, we will explore how to apply general CS techniques to this CRSS application and how to implement them efficiently for use on low power devices. Exploiting CS, radar and communication data is able to be stored in compact form and can be reconstructed without much distortion. In particular, the CS technique in [10] achieves data recovery from compressed data using a Bayesian inference model and the authors were able to use projection design in a CS scenario to control the input-output mutual information terms for two independent sources, which could possibly be radar and communication data, respectively. Employing the method from [10] will further extend our current research into estimating the LFM parameters by reusing OFDM receivers for the purpose of radar signal detection.

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