Exploiting Sparsity in Signal Acquisition, Separation and Processing

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Joint work with
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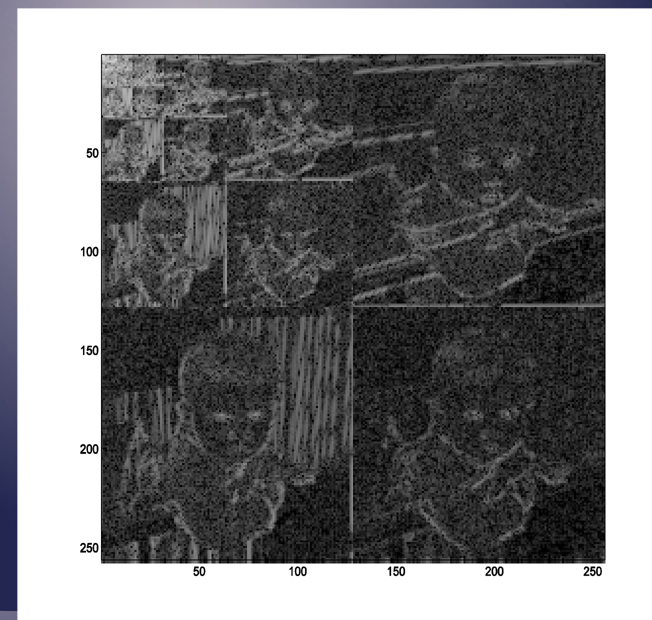
Why Sparsity?

"TOM" image



Wavelet Domain

Sparsity indicates that the underlying dimension of data $\ll N$

Sparse Representations in Inverse Problems

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Sparsity & ill-posed Inverse problems

Linear Inverse Problems generally take the form:

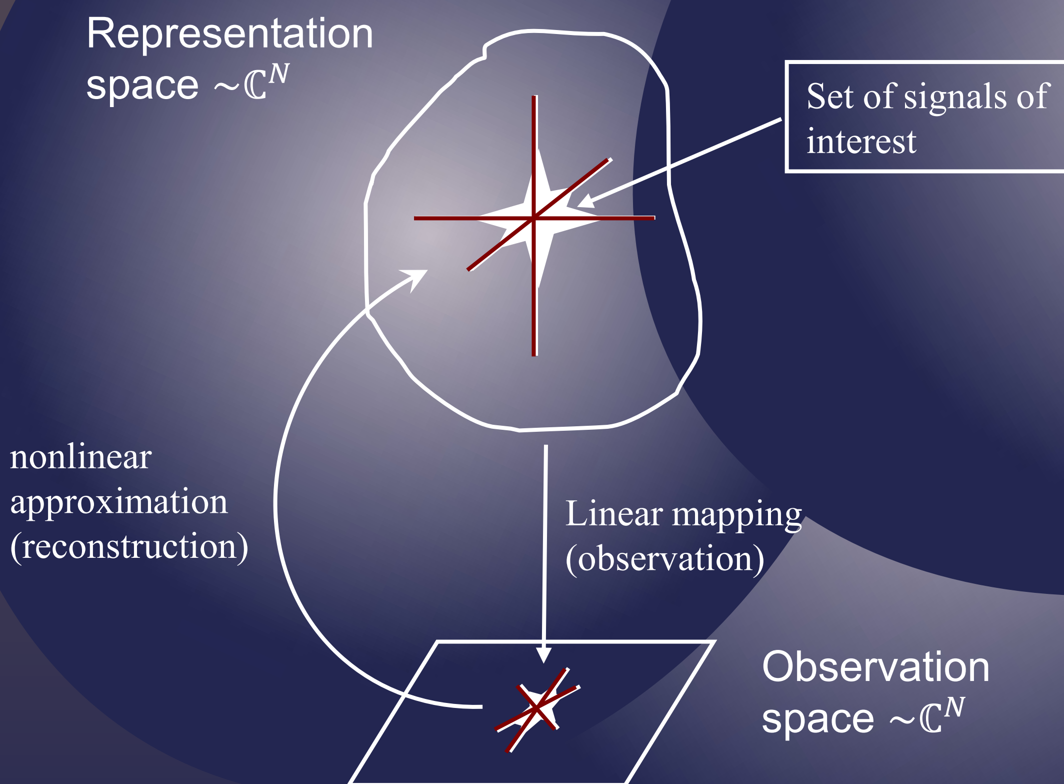
$$Ax = y$$

with $x \in \mathbb{C}^N, y \in \mathbb{C}^m$. If $m < N$ then the problem is ill-posed. i.e. there are an infinity of solutions.

Kruskal Rank

If x is K-sparse problem is still well posed if for all index sets $|T| \leq 2K$ the submatrices $A_T \in \mathbb{C}^{m \times 2K}$ are full rank… $\mathrm{krank}(A) \geq 2K$

Recovering Sparse Representations



In order to recover a sparse representation the mapping must be invertible on the sparse set (an embedding)

For the solution to be stable we need a little bit more: restricted isometry property (RIP) … a low distortion embedding

Practical Reconstruction algorithms

Sparse recovery - combinatorial search:

$$x^* = \min_x \|x\|_0 \text{ such that } \|y - Ax\|_2 \leq \epsilon$$

But this problem is combinatorial and NP-hard. However there are practical solutions with guaranteed performance under RIP

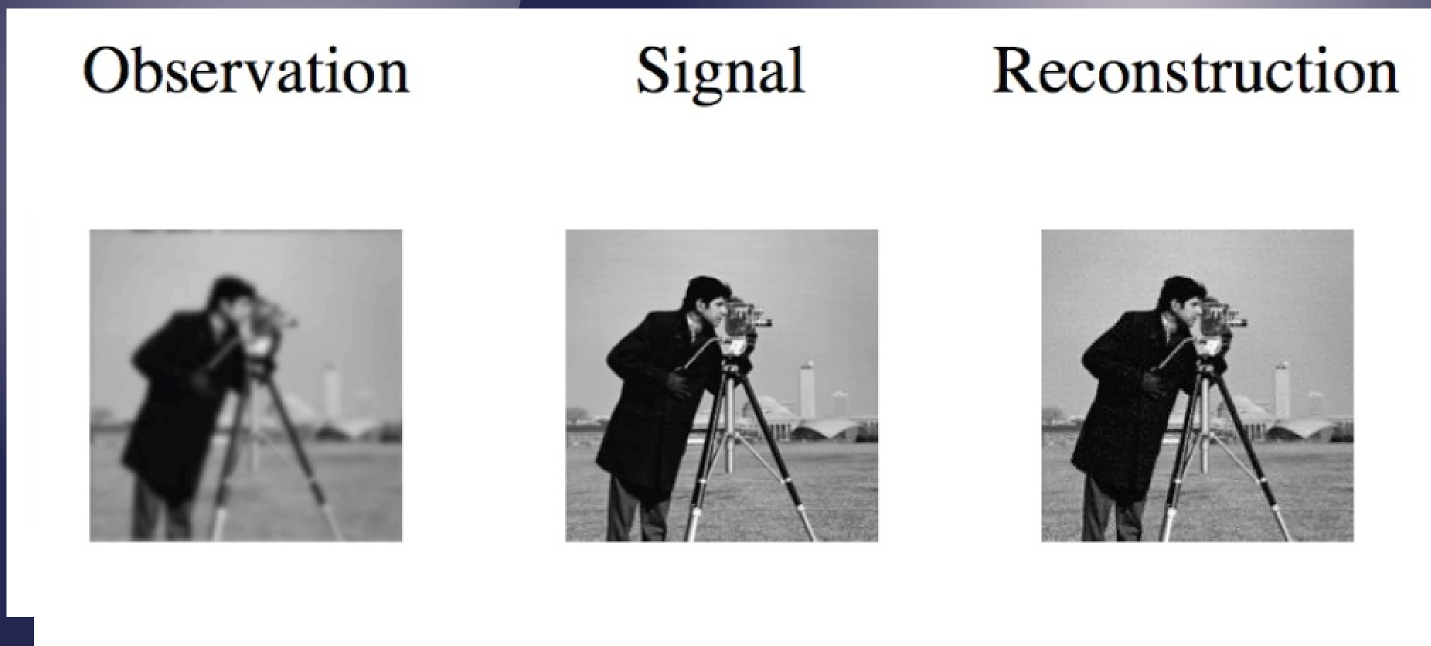
Convex relaxation – solve l_1 optimization e.g.

$$x^* = \min_x \|x\|_1 \text{such that } \|y - Ax\|_2 \leq \epsilon$$

or greedy solutions – combine least squares minimization with hard subset selection, e.g. (orthogonal) Matching Pursuit, Iterative Hard Thresholding, etc.

Sparsity & ill-posed Inverse problems

Sparse signal models help in a number of signal processing tasks such as…



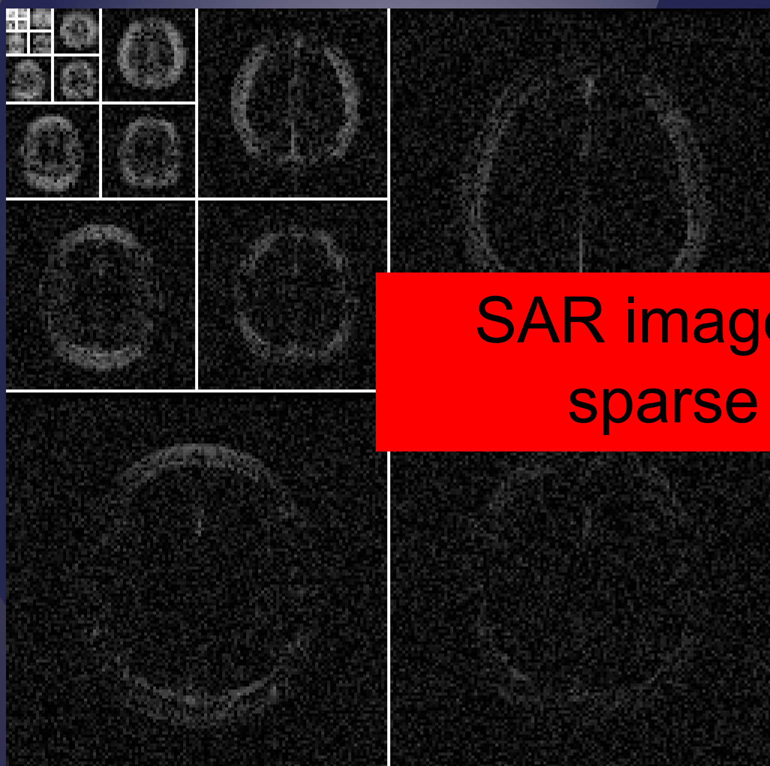
Missing Data Recovery

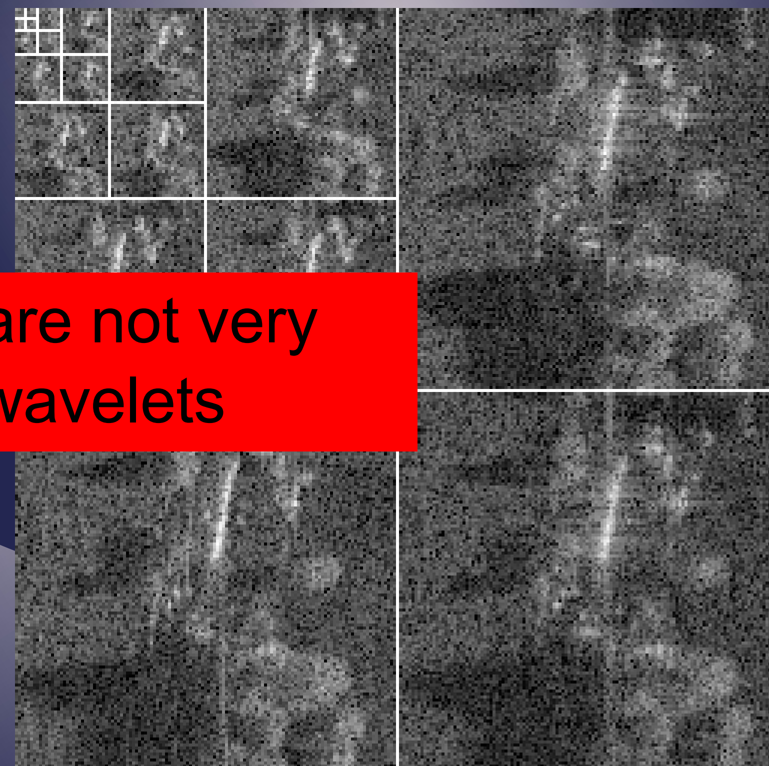
Image De-blurring

Sparsity in Synthetic Aperture Radar

Sparsity model?

Unlike other Fourier based CS applications, e.g. MRI...



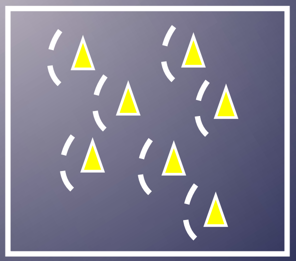


SAR images are not very sparse in wavelets

SAR image statistics

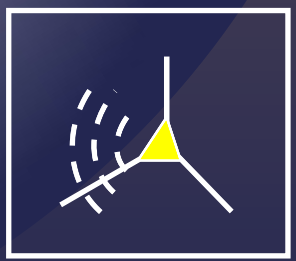
SAR images composed of two main components:

1. Speckle dominated images due to multiple random reflectors in a single range cell - not compressible.



range cell

2. Coherent reflectors whose intensity can be ~10^3 larger than incoherent reflections - compressible in pixel domain.



range cell

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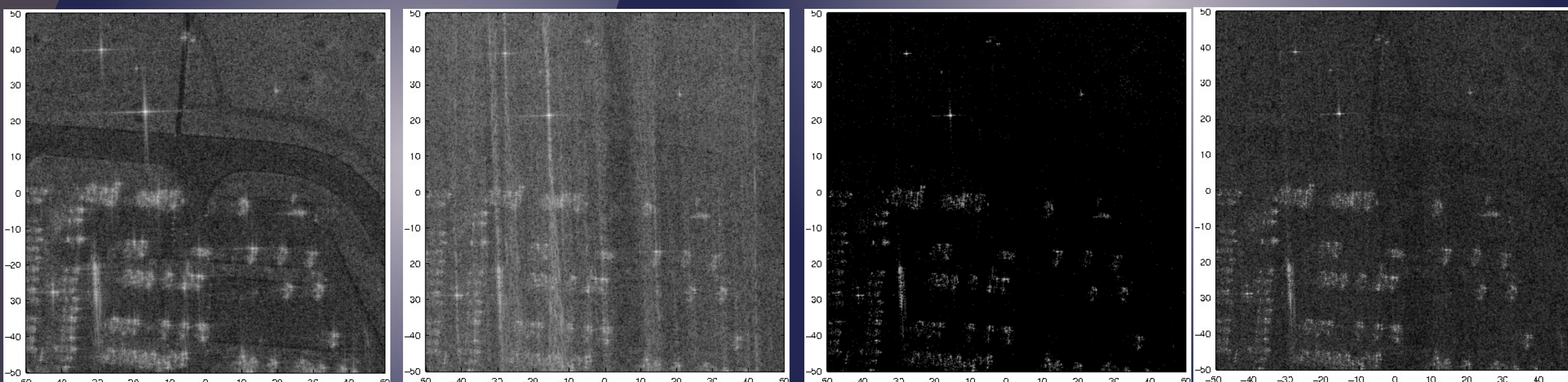
CS SAR reconstruction from limited data

Compressed sensing can only extract the coherent points in the image:



fully sampled image | back projection | sparse CS | mixed l_1/l_2

$\underbrace{\text{back projection} \quad \text{sparse CS} \quad \text{mixed } l_1/l_2}_{\text{50\% Nyquist}}$

A mixed l_1/l_2 solution

$$x^* = \min \|x\|_1 \text{ such that } \|y - Ax\|_2 \leq \lambda$$

$$\hat{x} = A^\dagger(y - Ax^*)$$

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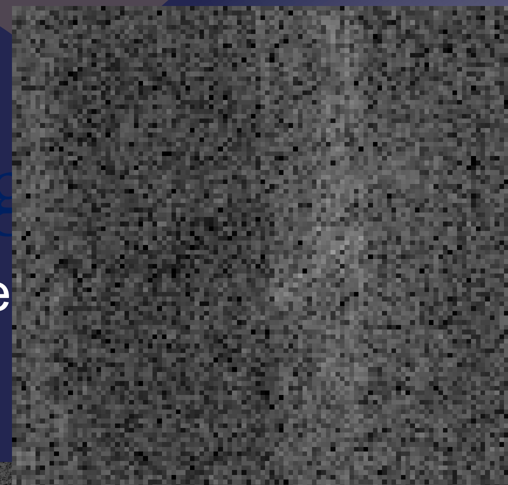
Compressive targ… o

Target's coherent points are pre…

fully sampled reference with tank



back projection @25% Nyquist

CS reconstruction @25% Nyquist (coherent only)

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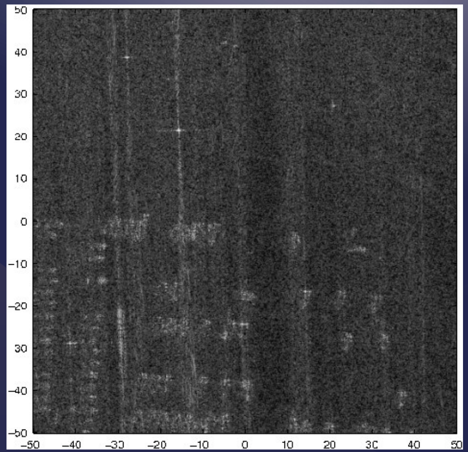
SAR image auto focus

An added complication is estimating the propagation delay for each radar return. This introduces a phase error. Traditional auto focus techniques (e.g. Phase Gradient Autofocus) indirectly use sparsity.

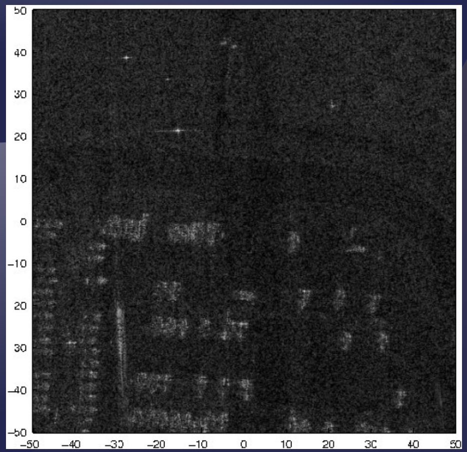
Here we can be explicit:

$$\{\theta, x^*\} = \min_{\{\phi, x\}} \|x\|_1 + \gamma \left\| y - \mathrm{diag}(e^{i\phi}) A x \right\|_2^2 \leq \lambda$$

w/o auto focus



with auto focus

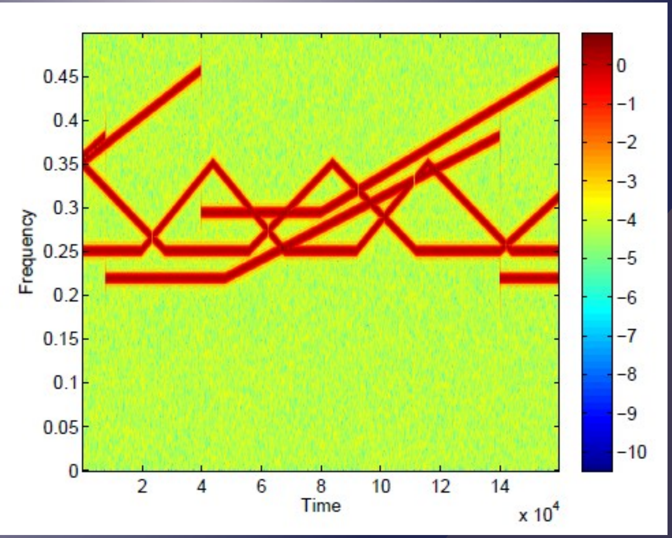


Sparsity for Signal Detection & Separation

Signal Separation in Electronic Surveillance

Aim: detect and separate out target waveforms in Electronic surveillance

- e.g. mixture of multiple FMCW waveforms.
- Need processing to be fast
- Want to exploit sparsity in TF domain (redundant chirplet transform)



$$C[n,k,c] = \sum_{m=-\frac{M-1}{2}}^{m=\frac{M-1}{2}} x[n+m]\phi[m]\, e^{j2\pi\frac{c}{2}m^2} e^{-j2\pi m\frac{k}{K}}$$

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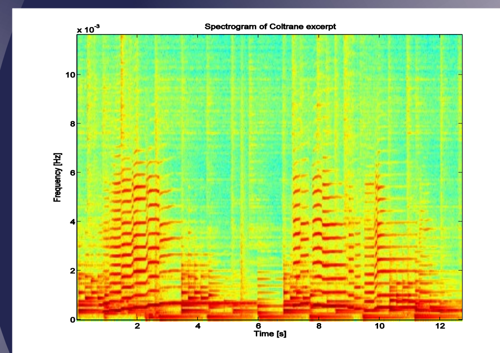
Sparsity & Time-frequency masking

An efficient popular method for source separation in the TF domain is to use TF masking

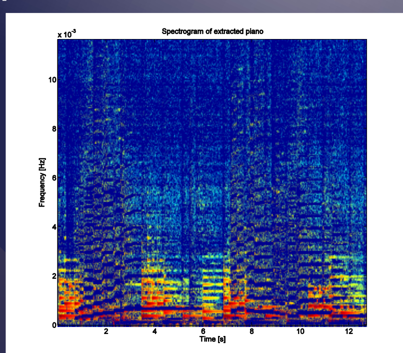
- If signals are TF-sparse each coefficient typically dominated by a single source
- Sources can be reconstructed from groups of associated TF atoms

Stereo audio separation by TF masking

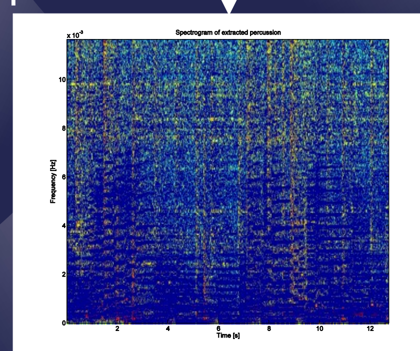
Example of source separation based on TF masking. Sources groups based on direction of arrival.



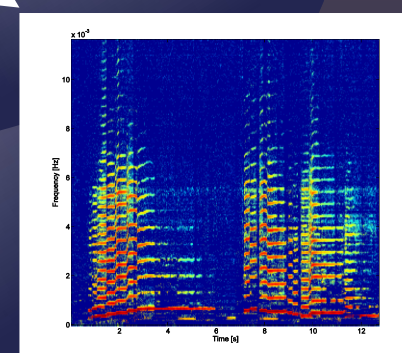
piano



percussion



sax



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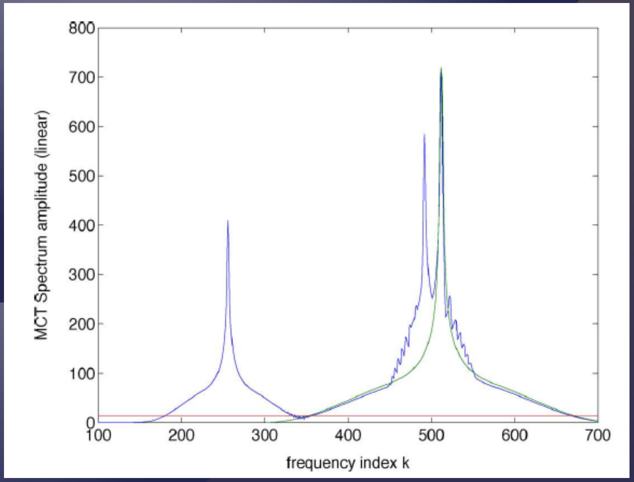
Iterative masking

Adapt masking to redundant transforms… but still only use a single chirplet transform

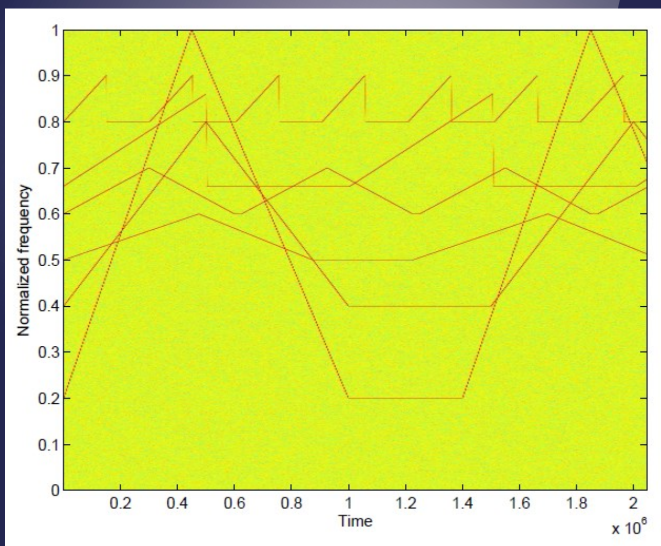
1. Calculate Maximum Chirplet Transform
2. Define noise threshold (Neyman-Pearson detection)
3. **While** coefficients above threshold:
    1. Select maximum coefficient
    2. Subtract the upper-bound spectral window

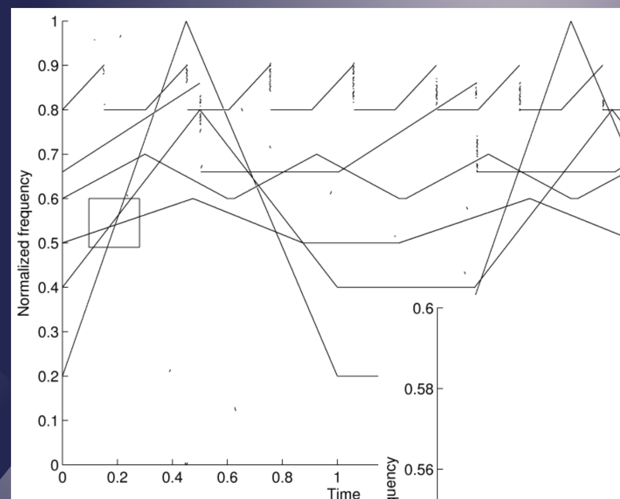
   end
4. Group coefficients into chirps



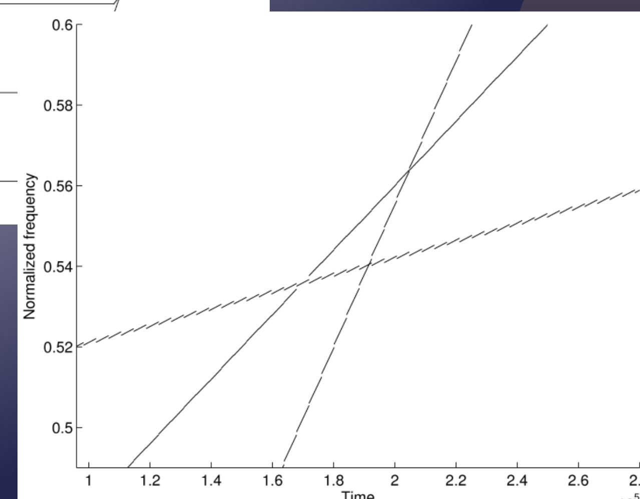
Recovered TF Representation

Recovered components: better coherent gain than STFT





zoomed section



Future perspectives

Further applications in SAR & ES

- RFI supression in SAR (SKs talk)
- 3D SAR imaging from through single or few nonlinear flight paths
- Combined SAR + GMTI
- Target classification exploiting non-isotropic scattering & spectral dependencies
- Wideband sensing through subNyquist sampling (MYs talk)

Questions?

Other Potential Defence applications

Sparsity is being investigated in a wide range of applications of interest to defence, including:

- Multispectral/Lidar imaging
- Blind Sensor Calibration
- Machine Learning (robust classification/estimation)
- Novel Computation (randomized linear algebra…fast matrix multiplication/SVD/etc.)

Compressive target classification

fully sampled reference with tank

ATR PERFORMANCE OF DIFFERENT SCENARIOS (Pr_{cc})

missing data pattern	data amount	CS-framework	back-projection
	full data	96.5%	95.5%
random	25% data	93%	85.5%
	10% data	83%	69%
gap	25% data	82%	64%
	10% data	57%	40.5%