

Joint Spatio-Temporal Bias Estimation and Tracking for GNSS-Denied Sensor Networks

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Abstract—Sensor parameter estimation is a key process that must be considered when performing data fusion in a multi-sensor object tracking scenario. For example, significant relative time delays in sensor data arriving at a fusion centre can result in a reduction of track accuracy, false tracks, or early termination of a true object track. The same issues may arise in the presence of some relative angular bias between sensors. This article presents a technique for simultaneous target tracking and estimation of relative time delays and angular biases in data for a multi-sensor system with no access to a global frame of reference. The proposed technique makes use of a hierarchical Bayesian model and couples a grid-based search method with an array of augmented state Kalman filters to accomplish this. Results are provided comparing the root-mean-squared error in a simulated single object tracking scenario. The performance of a single sensor, two sensors with correct registration, two sensors with incorrect registration, and two sensors with registration correction are compared. The results demonstrate a significant improvement in tracking performance when registration errors are corrected with the proposed method, as well as an increase in accuracy over object tracking with only a single sensor.

Index Terms—hierarchical Bayesian model, spatio-temporal alignment, single object tracking, GNSS-denied, bias estimation, Kalman filter, grid-based search

I. INTRODUCTION

A. Problem Overview

Sensor data fusion is a valuable technique in sensor networks. Use of multiple sensors to carry out a task – such as behaviour analysis – has advantages over the same task performed by only a single sensor [1]. Calibration of these sensors is key to their reliable performance. Consider the rapid deployment of radars on ships where there may be no time for calibration ahead of time - impromptu registration becomes a necessary capability. Sensors with unknown relative registration biases are likely to return data which can lead to, in the case of object tracking, false tracks or complete loss of tracks. In a crisis situation, this could be disastrous.

Sensors may not share a common spatial reference frame (spatial bias) and may not be synchronised in time (temporal bias). Therefore, to allow successful data fusion, it is important to remove these biases such that the sensors share a frame of reference (FoR). Additionally, the network may not have any reliable access to an external reference such as the Global Positioning System (GPS). GPS can provide much information about sensor position and pose, however, in a real scenario this will likely be with some error. Furthermore, regions

exist which are entirely GPS-denied. These include indoors, underwater, hostile regions (where signals may experience interference), and in space exploration.

Developed in 1981, the Network Time Protocol (NTP) is a method for the clock synchronisation of computer systems [2]. It requires access to the internet. The sensors considered in this work have no internet access and therefore NTP is not considered a solution to the problem this work addresses. Additionally, NTP deals with round trip delays and overall clock synchronisation, which is not considered here.

B. State-of-the-Art & Contributions

Registration methods for networks of sensors have been proposed previously in the literature. Some fall into the category of spatial alignment [3], [4], while others focus primarily on temporal alignment [5]. A new KF-based algorithm for spatial bias estimation among stationary time synchronised sensors tracking N targets is proposed in [6]. This is founded on the reconstruction of Kalman gains at the fusion centre. Fortunati et al. [7] consider the spatial alignment problem between local and remote sensors and derive a linear LS estimator to align the data. A modified exact maximum likelihood (MEML) registration algorithm, which was shown to outperform the standard exact maximum likelihood (EML) algorithm in a small radar network, has been presented [8]. This was achieved through the determination of an exact likelihood function. A neural EKF (NEKF) has been developed for the alignment of two-sensor systems [9], while in [10] a deep learning based 3D point cloud registration system is proposed. In 2013, [11] presented a Bayesian algorithm based on importance sampling to estimate sensor bias for asynchronous sensors. Recently, [12] formulate a nonlinear LS approach for the three dimensional asynchronous multi-sensor registration problem.

The works mentioned above do not consider joint tracking and spatio-temporal alignment. This paper builds on the work in [13]. An algorithm is proposed for the spatial and temporal alignment of co-located radars with relative registration errors between them. The method allows simultaneous joint bias estimation and object tracking and does not rely on access to a global FoR. Instead, the radars are calibrated relative to each other. A grid-based search method is implemented. A two dimensional grid represents the bias hypothesis state space, where one dimension represents temporal bias hypotheses and the other represents spatial bias hypotheses. In this work

temporal bias is a fixed, integer value of radar sampling interval, whilst spatial bias is a relative angular offset between the radars. The method is implemented within a hierarchical Bayesian model (HBM) [14] – which is a powerful tool for state prediction – and an array of augmented state Kalman filters (ASKFs) utilised for the object tracking. A likelihood function suited for KFs is derived and evaluated during the data filtering stage. Values assigned to bias pairs are then used to update the corresponding weights of points on the grid. This work considers centralised networks – in other words, all measurements collected by all sensors are transmitted to a data fusion centre. A plot fusion architecture is employed. The novel contributions of this work include a joint sensor calibration and object tracking method implemented within an HBM, and the derivation of a likelihood function for the parameter estimation. An analysis of a range of simulations where sensor configuration is varied is provided.

C. Paper Organisation

Section II presents the joint spatio-temporal estimation problem and an overview of the HBM processes; Section III provides model definitions, scenarios, and implementation; results are displayed in Section IV; and Section V provides a conclusion and brief discussion of future work.

II. PARAMETER ESTIMATION AND DATA FUSION

The main problem addressed in this work is that of estimating a relative temporal delay, τ , in sensor data and a relative angular offset, ϕ , between multiple sensors. These are estimated jointly and in a recursive manner alongside the object tracking procedure. Consider the scenario of two co-located sensors tracking the same object. Both sensors collect measurements with some time-varying measurement noise. The FoR of one sensor is rotated by ϕ relative to that of the other sensor and there is relative delay τ in data transmission from this sensor. These biases must be corrected so that accurate sensor fusion can be accomplished. The framework of choice is the HBM. This hierarchical approach has been successfully applied to the solution of problems in a wide range of fields. These include spatio-temporal forecasting in urban traffic modelling [15], unsupervised learning and estimation of crowd emotional states in crowd monitoring [16], and the modelling of the brain cortex for pattern recognition [17], amongst others. In the field of target tracking, HBMs have been used for joint multiple-target tracking (MTT) and registration parameter estimation [18] and for simultaneous localisation and mapping (SLAM) [19]. The HBM utilised in this work has two levels: the high-level process (known as the parent process) estimates the unknown (or ‘hidden’) parameters, i.e. the sensor calibration parameters; and the low-level process (known as the offspring process) estimates the object states, i.e. the tracking function. The two processes are linked by a likelihood function calculated in the offspring process and employed for parameter estimation in the parent process. This function is problem dependent.

A. Offspring Process

The tracking problem considered is linear when the calibration is known and so the offspring process utilises a bank of ASKFs for the tracking procedure. It is important to describe it here as this is where the bias hypotheses are incorporated. An ASKF is a Kalman filter with augmented state vector and extended transition and observation matrices. Augmentation of the state vector at time k involves the concatenation of previous state estimates with the current state estimate in the following manner:

$$\hat{\mathbf{X}}_k = [\hat{\mathbf{x}}_k \quad \cdots \quad \hat{\mathbf{x}}_{k-\hat{\tau}_{max}}]^T \quad (1)$$

Here, $\hat{\tau}_{max}$ denotes the largest temporal bias hypothesis made in units of the sampling interval, δt , and:

$$\hat{\mathbf{x}}_k = [\hat{p}_{x,k} \quad \hat{v}_{x,k} \quad \hat{p}_{y,k} \quad \hat{v}_{y,k}]^T \quad (2)$$

Here, $\hat{p}_{x/y,k}$ is the filter object x and y position estimates, and $\hat{v}_{x/y,k}$ is the filter object x and y velocity estimates. The transition matrix of the system model, \mathbf{F}_k , is extended to become:

$$\mathbf{F}_k = \begin{bmatrix} \mathbf{F}_0 & 0 & 0 & \cdots & 0 & 0 \\ \mathbb{I} & 0 & 0 & \cdots & 0 & 0 \\ 0 & \mathbb{I} & 0 & \cdots & 0 & 0 \\ & & & \vdots & & \\ 0 & 0 & 0 & \cdots & \mathbb{I} & 0 \end{bmatrix} \quad (3)$$

where \mathbb{I} is a 4×4 unit matrix, and:

$$\mathbf{F}_0 = \begin{bmatrix} 1 & \delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta t \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

The observation matrix, \mathbf{H}_i , is also extended and arranged to reflect the bias hypothesis of any grid point i , $[\hat{\phi}_s^i \quad \hat{\tau}_s^i]$ for $s \in S = \{1, \dots, n_s\}$, the set of sensor IDs, where n_s is the total number of sensors. $\hat{\phi}_s^i$ represents the spatial bias (angular offset) hypothesis at grid point i for sensor s and $\hat{\tau}_s^i$ represents the temporal bias hypothesis at grid point i for sensor s . A rotation matrix, Θ^i , is then applied to incorporate the angular offset estimate:

$$\mathbf{H}_i = \Theta^i \mathbf{H}^i \quad (5)$$

where:

$$\Theta^i = \begin{bmatrix} \theta_1^i & 0 & 0 & \cdots & 0 \\ 0 & \theta_2^i & 0 & \cdots & 0 \\ 0 & 0 & \theta_3^i & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & \theta_{n_s}^i \end{bmatrix} \quad (6)$$

and:

$$\theta_s^i = \begin{bmatrix} \cos(\hat{\phi}_s^i) & -\sin(\hat{\phi}_s^i) \\ \sin(\hat{\phi}_s^i) & \cos(\hat{\phi}_s^i) \end{bmatrix} \quad (7)$$

\mathbf{H}^i takes the following general form:

$$\mathbf{H}^i = [\mathbf{H}_1^i \quad \mathbf{H}_2^i \quad \cdots \quad \mathbf{H}_{n_s}^i]^T \quad (8)$$

where:

$$\mathbf{H}_s^i = [\mathcal{O}_{2 \times 4\hat{\tau}_s^i} \quad \mathbf{H}_0 \quad \mathcal{O}_{2 \times (4\hat{\tau}_{max} + 4)}] \quad (9)$$

and

$$\mathbf{H}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (10)$$

In the above, $\mathcal{O}_{m \times n}$ is an $m \times n$ zero matrix. The offspring process is responsible for estimating an object state as it evolves through time. Here, this obeys the nearly constant velocity (NCV) motion model. Measurements from both sensors are fused and supplied to the filter, along with a spatio-temporal bias hypothesis. With these inputs, the filter performs the state estimate update and evaluates the likelihood function. This is then passed to the parent process for the weights update.

B. Parent Process

The parent process performs the estimation of the registration parameters, ϕ and τ . With a grid-based approach the hypotheses of the registration parameters at a time-step k for sensor s are represented by a set of points and their corresponding weights $\Psi_{k,s} = \{\psi_s^i, w_{k,s}^i\}_{i=1}^N = \{[\hat{\phi}_s^i \quad \hat{\tau}_s^i]_{s=1}^{n_s}, w_{k,s}^i\}_{i=1}^N$. Each i references a different bias pair (grid point) and N is the total number of pairs. Every pair has some corresponding weight, $w_{k,s}^i$, which reflects the belief that registration parameters ψ_s^i are closest to the true values, ϕ and τ . Because the grid is not time-varying, a grid-based search method ([20], pg. 9) can be used to update the weights at each time-step. Weights are predicted and updated recursively, following [13] equations (2a) and (2b). Equation (2a) can be understood as a convolution of prior weights with a kernel function. The kernel function is selected to be the binomial distribution $r \sim \mathcal{B}(n, p)$ and, following investigation, the parameters $n = N$ and $p = 0.5$ were found to be suitable in this work. These were selected as a compromise between an uninformative distribution and one which searches the state space efficiently. The parent process is initialised with the assumption that, before any data processing, all sensor bias pairs are equally likely. Therefore, the supplied prior distribution is flat. Equation (2b) requires a likelihood function to update weights and here the Kalman filter likelihood function, $\ell_k(\psi_k^i | \mathbf{Z}_k)$, is derived from the integral form of the KF likelihood conditioned on ψ_k^i (\mathbf{Z}_k is the set of all sensor measurements up to time-step k , and \mathbf{z}_k and \mathbf{x}_k are the set of sensor measurements and the object state at time k , respectively):

$$\ell_k(\psi_k^i | \mathbf{Z}_k) = \int p(\mathbf{z}_k | \mathbf{x}_k, \psi_k^i) \times p(\mathbf{x}_k, \psi_k^i | \mathbf{Z}_{k-1}) d\mathbf{x}_k \quad (11)$$

where:

$$p(\mathbf{x}_k, \psi_k^i | \mathbf{Z}_{k-1}) = \int p(\mathbf{x}_k | \mathbf{x}_{k-1}, \psi_k^i) p(\mathbf{x}_{k-1}, \psi_k^i | \mathbf{Z}_{k-1}) d\mathbf{x}_{k-1} \quad (12)$$

The Gaussian propagation identity [21] is used to evaluate both of these integrals and the resulting function simplifies to:

$$\ell_k(\psi_k^i | \mathbf{Z}_k) = \mathcal{N}(\mathbf{z}_k | \mathbf{H}_i \mathbf{F}_k \hat{\mathbf{x}}_{k-1}, \mathbf{\Lambda}_k) \quad (13)$$

where $\mathbf{\Lambda}_k$ is the covariance of $\ell_k(\psi_k^i | \mathbf{Z}_k)$ with form:

$$\mathbf{\Lambda}_k = \mathbf{R}_k + \mathbf{H}_i (\mathbf{Q}_k + \mathbf{F}_k \mathbf{P}_{k-1} \mathbf{F}_k^T) \mathbf{H}_i^T \quad (14)$$

where \mathbf{R}_k is the measurement noise covariance and defined in the measurement model in Section III-B.2) in Equation (20), and \mathbf{P}_{k-1} is the KF estimate covariance at time-step $k-1$. \mathbf{Q}_k is the filter process noise covariance, defined in Equation (17). The pseudocode for the parent and offspring prediction and update steps of this work are based loosely on that found in [13], where a HBM is used for joint registration and fusion of heterogeneous sensors.

III. MODELING, DATA, AND SCENARIOS

A. Implementation

The parent process of this hierarchical model is represented by an evenly distributed, two-dimensional grid of distinct points. A single grid point represents a joint spatio-temporal bias hypothesis with a calculated weight. The weights are continuously updated based on the value of the likelihood function, $\ell_k(\psi_k^i | \mathbf{Z}_k)$, that is the output of the offspring process.

B. Model Definitions

1) *Object Motion Model*: Multiple sensors track a single object which evolves through time according to the NCV model. This is defined as:

$$\mathbf{x}_k = \mathbf{F}_0 \mathbf{x}_{k-1} + \mathbf{w}_k \quad (15)$$

where \mathbf{x}_k is a four dimensional Cartesian state vector, with the following elements:

$$\mathbf{x}_k = [p_{x,k} \quad v_{x,k} \quad p_{y,k} \quad v_{y,k}]^T \quad (16)$$

and \mathbf{F}_0 is the state transition matrix – previously defined in Equation (4). \mathbf{w}_k represents zero-mean white Gaussian process noise with covariance matrix given by:

$$\mathbf{Q}_k = \begin{bmatrix} \frac{1}{3} \delta t^3 & \frac{1}{2} \delta t^2 & 0 & 0 \\ \frac{1}{2} \delta t^2 & \delta t & 0 & 0 \\ 0 & 0 & \frac{1}{3} \delta t^3 & \frac{1}{2} \delta t^2 \\ 0 & 0 & \frac{1}{2} \delta t^2 & \delta t \end{bmatrix} \tilde{q} \quad (17)$$

Here, δt is the sampling interval and \tilde{q} is the process noise intensity level ([20], pg. 181 and [22], pg. 269). It dictates how closely the object adheres to the CV model: a value $\tilde{q} = \mathbf{Q}_{22}/\delta t = 1$ ([22], pg. 270) produces CV motion.

2) *Measurement Model*: Sensors collect x and y data and the measurement model is defined as:

$$\mathbf{z}_k = \mathbf{H}_k^{obs} \mathbf{x}_k + \mathbf{v}_k \quad (18)$$

where:

$$\mathbf{z}_k = [\mathbf{z}_{k,1} \quad \mathbf{z}_{k,2} \quad \cdots \quad \mathbf{z}_{k,n_s}]^T \quad (19)$$

$$\mathbf{H}_k^{obs} = [\mathbf{H}_1 \quad \mathbf{H}_2 \quad \cdots \quad \mathbf{H}_{n_s}]^T$$

\mathbf{z}_k is the measurement vector at time k made up of measurements collected by each sensor, $\mathbf{z}_{k,s}$. \mathbf{H}_s is the observation matrix associated with sensor s and identical to the matrix \mathbf{H}_0 (see Equation (10)). In this work, only x - y components

of the object trajectory are observed. \mathbf{v}_k is the measurement noise and drawn from a zero-mean white Gaussian distribution with parameters: $\mathcal{N}(\mathbf{v}_k|\mathbf{0}, \mathbf{R}_k)$. \mathbf{R}_k is given by:

$$\mathbf{R}_k = \mathbb{I}_{n_s \times |\mathbf{z}_k|} \sigma_R^2 \quad (20)$$

where $\mathbb{I}_{n \times m}$ is an $n \times m$ identity matrix. This definition of \mathbf{R}_k assumes that each sensor returns the same type of observation vector (see (19)), although note that it is possible for the sensors to collect different types of measurements. σ_R is the standard deviation of the multivariate Gaussian distribution that describes the measurement noise, \mathbf{v}_k .

C. Scenarios

The proposed algorithm is tested in an artificial scenario with the goal of demonstrating that the method is able to calibrate sensors effectively to the baseline. This is a preliminary test and therefore a basic setup is considered: two co-located radars and a fusion centre receiving updates from them every $\delta t = 0.5s$. Data is simulated of a single NCV object travelling in two dimensional space – i.e. only x and y coordinates and their associated rates of change, \dot{x} and \dot{y} , are measured by the radars. The simulation is run for a total of $t = 2000s$. Clutter and false alarms are not considered and, for all cases, probability of detection, p_d , is assigned the value 1. In this work, $n_s = 2$ and it is assumed that sensor $s = 2$ possesses the spatio-temporal biases, whilst sensor $s = 1$ is treated as the reference sensor. Standard deviation of measurement noise variance, σ_R , is selected to be $10m$ following those assigned by comparable works [23], [24], [25], whilst the process noise intensity, \tilde{q} , is set to 1 .¹ The initial object state is $[10 \ 10 \ 10 \ 10]^T$ and the initial object state tracker estimate is $[10 \ 5 \ 10 \ 5]^T$. Both radars are placed and fixed at co-ordinates $(0,0)$. Four distinct cases for the single object tracking are analysed. These are as follows:

- 1) single sensor;
- 2) two centralised sensors, correctly registered;
- 3) two centralised sensors, with relative spatio-temporal bias, and the proposed method for bias correction;
- 4) two centralised sensors, with bias, and no correction.

In cases 3) and 4) the temporal bias is assigned to $1 \times \delta t$ and angular bias to 10° . Sampling rate for all cases is $\delta t = 0.5s$.

IV. RESULTS

The results that follow have been averaged over 100 Monte Carlo (MC) runs and the temporal and angular biases jointly estimated using the maximum a posteriori (MAP) estimate of the likelihood function. The RMSE is the chosen metric for evaluation of tracking performance. RMSE is calculated for Euclidean distance over a range of σ_R values and \tilde{q} levels over the final quarter of the full simulation time, t (to allow for filter stabilisation). Figure (1) provides log plots of the RMSE of Euclidean distance as σ_R and \tilde{q} vary, respectively. Each data point represents the average over all 100 MC trials. A new

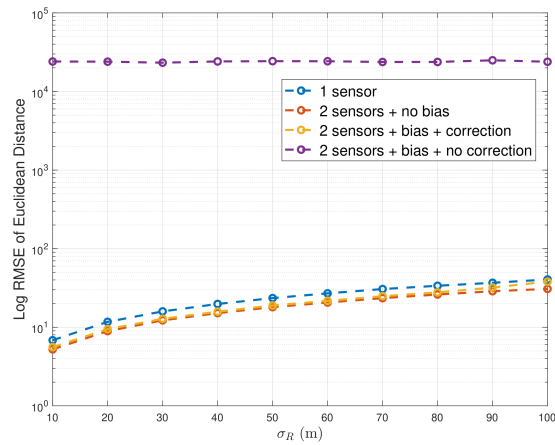
¹Note that this \tilde{q} value is the one used to generate tracking performance results against measurement noise standard deviation, σ_R .

object trajectory and set of sensor measurements is generated in each trial. Both plots provide a legend to link the data with the different test cases (for reference, see Section III-C). From these graphs, we can see that the worst performance, by far, is demonstrated by the configuration of two sensors with relative registration errors that are not corrected. This clearly shows how important it is to address the registration problem. Using the proposed method for correction, performance of the sensors is very close to that of the benchmark scenario, and outperforms the single sensor for all tested values of σ_R and \tilde{q} . For example, at the lowest \tilde{q} value of 10 the corrected case differs from the benchmark case by $\approx 0.4m$; whilst the single sensor case differs by $\approx 1.6m$ ($\approx 4 \times$ the difference of the corrected case); and the no correction case is out by $\approx 5,500m$. Whereas, when \tilde{q} is at its greatest tested value of 80, the correction case differs by $\approx 1.5m$ from the benchmark; the single sensor by $\approx 2.6m$; and the uncorrected case by $\approx 42,000m$. A similar trend can be seen in the measurement noise graph. However, all scenarios display an increase in RMSE with increasing σ_R and \tilde{q} .

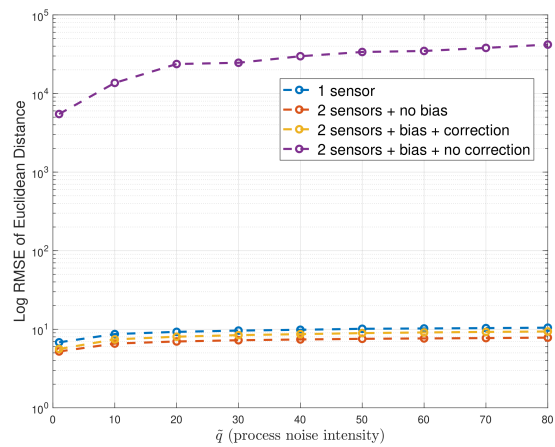
Although the key result has been demonstrated with this method and sensors are calibrated to the baseline, it is important to note that the issue of scalability must be addressed in the future. Due to the use of grid-based search, the number of grid points N (i.e. number of ASKFs required in the offspring process) relates to the number of registration errors, $|\psi_k^i|$, present as $N = \prod_{j=1}^{|\psi_k^i|} \chi_j$. χ_j represents the number of hypotheses made per error j . This shows that with every additional error, an additional dimension appears on the grid. A particle filter method may improve scalability.

V. CONCLUSION & FUTURE WORK

This article demonstrates the successful application of a method for simultaneous joint spatio-temporal alignment of sensors and object tracking. The registration parameters are continuously estimated as the object tracker runs, based on the performance of the tracker itself. The metric for the tracker performance is a KF likelihood function. It is used to update grid weights in the parent process of the HBM. Simulation results demonstrate how vital it is to have correctly calibrated sensors, and also show that tracking with multiple sensors is more accurate than tracking with only a single sensor. The work described in this article acts as a foundation for further investigation of the sensor alignment problem. The method can be extended to non-linear systems where, rather than collecting and processing data in the Cartesian frame, data is collected and processed in the polar frame. This is more realistic for radar systems. A new likelihood function would be required for the non-linear tracker. Varying probability of detection and introducing false alarms will also make the model more realistic. Increasing the number of sensors and targets is another useful avenue of investigation. Additionally, resampling and propagation can be introduced to the grid-based search method. A suitable resampling technique may allow for faster selection of a bias state - although it may add computational complexity. The usefulness of this method for



(a)



(b)

Fig. 1: 1(a) shows performance comparison for cases 1)-4) for increasing σ_R and 1(b) for increasing \tilde{q} .

non-co-located sensors can be explored, as well as for time-varying spatio-temporal biases and fractional temporal biases.

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