MULTI-DIMENSIONAL FAST MARCHING APPROACH TO WAVE PROPAGATION IN HETEROGENOUS MULTIPATH ENVIRONMENT

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Abstract: *Sound propagation is described by the wave equation. If in an homogenous free field its resolution is straightforward, any variation from this hypothesis makes the wave equation solution not analytically tractable especially in a shallow water environment. The direct resolution of the wave equation requires in most cases numerical methods such as FDTD (Finite Difference Time Domain) or PSTD (Pseudo Spectral Time Domain). A direct approach however is often extremely computationally expensive (the spatial and temporal discretisation has to be small, of the order of $\lambda/10$, for stability criterion) and approximations are necessary for practical reasons. For low frequencies applications, mathematical models for shallow water propagation include Normal Mode Model or Parabolic Equation Model. For higher frequencies (above 1kHz), the most popular method for wave propagation in shallow water is based on Ray theory and geometrical acoustics. Thanks to the infinite frequency assumption, the wave equation simplifies to the eikonal equation which propagates the wavefront of the acoustic pulse. The ray trajectories are computed as perpendicular to the wavefront. In the ideal case of a constant sound velocity profile and perfectly flat interfaces for the surface and the seafloor, an elegant solution is derived from the Mirror theorem: source images are easily geometrically computed by successive symmetries of the source itself. In a second step folding the straight paths linking all the source images to a target computes the multipath. Unfortunately this method fails for non flat seabeds, non constant depth or non constant velocity profile. In this paper we propose an extension to the Mirror theorem to take into account any interface geometry or sound velocity variation (horizontally or vertically) by solving the eikonal equation using the Fast Marching algorithm. We will show that multipath can then be solved by wrapping the wavefront propagation at each interface.*

Keywords: *Fast Marching, Multipath environment, Propagation simulation.*

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1. SHALLOW WATER PROPAGATION MODELLING

Different models have been developed for acoustic wave propagation during the past half-century [1, 2]. Propagation models include Ray tracing, Normal Mode or Parabolic Equation. The starting point of any model is the wave equation given by:

$$\Delta p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \tag{1}$$

Solving directly Eq. (1) can be archived using FDTD (Finite Difference Time Domain) or PSTD (Pseudo Spectral Time Domain) [3] for example. Looking for harmonic solutions of the wave equation, *i.e.* where p can be written as $p(\vec{r},t) = A(\vec{r})\mathrm{e}^{-i\omega t}$, leads to the Helmholtz equation:

$$\Delta A + k^2 A = 0 \tag{2}$$

In this paper we are interested on the Ray theory formalism, where the solutions of the Helmholtz equation can be written as:

$$A(\vec{r}) = F(\vec{r})\mathrm{e}^{iG(\vec{r})} \tag{3}$$

$F(\vec{r})$ and $G(\vec{r})$ are called respectively the amplitude and phase functions. By substituting Eq. 3 into Eq. 2, and considering the real part only, we arrive to:

$$\frac{\Delta F}{F} - [\nabla G]^2 + k^2 = 0 \tag{4}$$

The Ray theory assumes that $k^2 \gg \frac{\Delta F}{F}$. Physically this assumption means that the sound speed is approximatively constant over one wavelength [2]. Eq. (3) then becomes the Eikonal equation:

$$[\nabla G]^2 = k^2 \tag{5}$$

This equation is the foundation of Ray theory and geometrical acoustics. In Eq. (5) the surfaces of constant phase are then the wavefronts. Ray trajectoryies are computed as perpendicular to the wavefront [4]. Considering high frequencies scenarios (above 1 kHz), Ray tracing offers a rapid solution to the wave propagation problem even in complex environment such as shallow water environments.

Sound reflection on the boundaries of the propagating medium (the sea surface and the sea bottom) creates multi paths which can get extremely severe as the ratio range / depth increases. An interesting case scenario is the ideal model where the sound velocity profile is constant along the water column and the sea surface and seafloor are considered as perfectly flat. In that case an elegant geometrical solution called the Mirror theorem can be applied. Because the sound speed is constant, acoustic rays propagate in straight lines, and because the interfaces are perfectly flat, the different multiple paths can be geometrically computed by considering the images of the source (or the target) obtained by successive symmetry through bottom and surface reflections and folding back the multipath image trajectories. Figure 1 draws a sketch of multiple paths geometrical computed using the Mirror theorem.

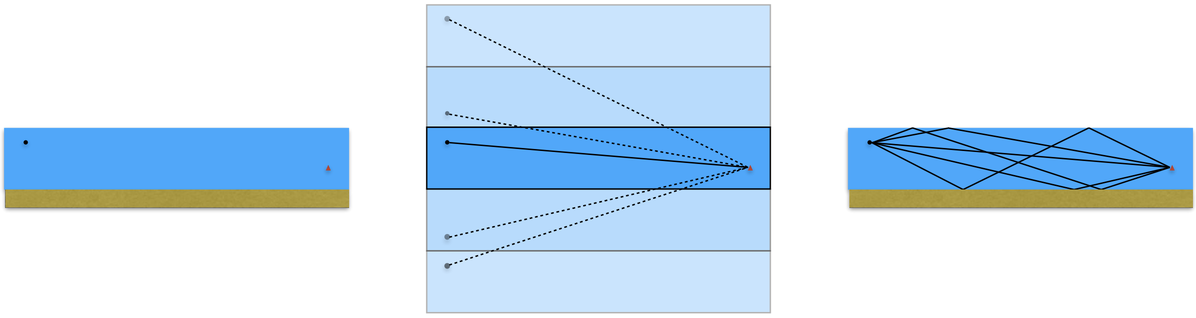

Fig. 1: Geometrically computing multipath using the Mirror theorem. The black circle represents the source and the red star represents the receiver.

Unfortunately the previous hypotheses are too strong to hold against real environment scenarios. And Ray tracing techniques, where 1000's of rays are drawn *blindly*, are applied to find the propagation pattern of a particular environment. In this paper we aim to extend the Mirror theorem to take into account any interface geometry or sound velocity variation (horizontally or vertically).

2. FAST MARCHING ALGORITHM

Ray tracing is not the only method to solve the Eikonal equation. Solutions to this equation are particularly useful in computer vision or path planning, for example, and have been studied in diverse scientific fields. An interesting solution method to Eq. (5) called Fast Marching Method (FMM) is given by Adalsteinsson and Sethian in [5]. Using the notation $T = G/\omega$, Eq. (5) then writes:

$$|\nabla T(\vec{r})|\, c(\vec{r}) = 1 \tag{6}$$

$T(\vec{r})$ is then the arrival time of the wavefront. In this section, we briefly present the FMM algorithm. We only consider here the isotropic case, where the speed is independent of the direction of propagation. The discrete approximation of Eq. (6) in 2D is the viscosity equation given by [6]:

$$\max(\nabla_x^+ T_{n,m}, -\nabla_x^- T_{n,m}, 0)^2 + \max(\nabla_y^+ T_{n,m}, -\nabla_y^- T_{n,m}, 0)^2 = \frac{1}{c_{n,m}^2} \tag{7}$$

where ∇^+ and ∇^- are the discrete forward and backward discrete differentials. (n,m) is the pixel coordinates. Let h be the grid spacing. Using first order differentials in Eq. (7) leads to:

$$\sum_{k=1}^{2} \max\left(T_{n,m} - T_k, 0\right)^2 = \frac{h^2}{c_{n,m}^2} \tag{8}$$

where

$$T_1 = \min(T_{n-1,m}, T_{n+1,m}) \tag{9}$$

$$T_2 = \min(T_{n,m-1}, T_{n,m+1}) \tag{10}$$

The FMM algorithm proposed by [5] is a one-pass algorithm and works as follows:

Three categories of pixels are considered: *known:* the arrival time has already be computed and validated, *narrow band:* an estimated arrival time has been computed and can be updated and *far:* no arrival time computed. The Fast Marching algorithm pseudocode is shown in Algorithm 1:

Algorithm 1 Fast Marching Method pseudo code

while end condition **do**
  Find the *narrow band* pixel P with minimal arrival time
  Update P status from *narrow band* to *known*
  Update arrival time for P's *narrow band* and *far* neighbours using Eq. (8)
end while

The *end condition* in Algorithm 1 can be when a particular point P_{finish} is reached or when all the map has been covered. Figure 2 sketches the wavefront propagation in a 2D grid using Algorithm 1. Once the wavefront arrival time has been computed for the map, a steepest gradient descent algorithm such as the fourth-order Runge-Kutta method computes the shortest path from any point on the map to the starting point. FMM have been applied for path planning for autonomous underwater vehicles [7]. For a known map, FMM provides the optimum trajectory for an AUV to reach a given destination.

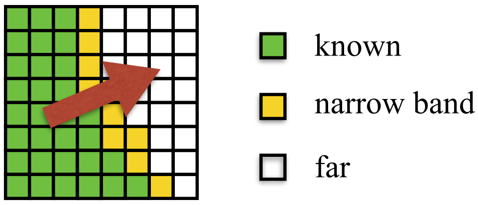

Fig. 2: Drawing of the wavefront propagation (the red arrow) using FMM algorithm. The known, narrow band and far pixel are colour coded to green, yellow and white respectively.

3. MULTI-DIMENSION FAST MARCHING APPROACH

Several Fast Marching Methods have been developed during the last decade to improve the initial FMM algorithm from [5] in term of accuracy or computation time. In this paper we compute the wavefront propagation using the FMM multistencils approach from Hassouna [8] which takes into account two stencils to benefit from the 8-neighbours topology of a pixel in a 2D grid. We are also using the second order approximation of Eq. (7) proposed by Sethian in [9] and also implemented in [8]. With these FMM variants, we seek for more precision and accuracy in the time of arrival computation.

A naive implementation of FM to acoustic wave propagation, however, only propagates the acoustic wavefront (*i.e.* the shortest path according to Fermat principle). It does not take into account boundary reflections nor the multipath effect however. For that reason we introduce here the Multi-Dimension Fast Marching (MDFM) approach. We suppose that there is only one path per bounce. Let TS_n^m be the acoustic wavefront propagated from the sea surface after m bounces on the sea surface and n bounces on the sea bottom. Let TB_m^n be the acoustic wavefront propagated from the sea bottom after n bounces on the sea surface and m bounces on the sea bottom. Note that for a constant number of bounces $k = n+m$, there are two contributing multi-paths originating from TS_n^m and TB_m^n. It is also interesting to note that $m = n$ or $m = n+1$. The MDFM algorithm is described in Algorithm 2.

Algorithm 2 Multi-Dimension Fast Marching Method pseudo code

$\mathrm{T}_0 \leftarrow$ solution of Eq. (5) using Algorithm 1
Initiate TS_0^1: all pixels to *far* except the surface interface *known* with T_0 values
Initiate TB_1^0: all pixels to *far* except the bottom interface *known* with T_0 values
while end condition **do**
  $\mathrm{TS}_n^m \leftarrow$ solution of Eq. (5) with appropriate init values
  $\mathrm{TB}_m^n \leftarrow$ solution of Eq. (5) with appropriate init values
  Initiate TS_m^{n+1}: all pixels to *far* except the surface interface to *known* with TB_m^n values
  Initiate TB_{n+1}^m: all pixels to *far* except the bottom interface to *known* with TS_n^m values
  $n \leftarrow n+1$
  $n \leftrightarrow m$
end while

The main idea behind the MDFM algorithm is that the acoustical wave is propagating from interface to interface (from sea surface to sea bottom and vice-versa), the algorithm then proceeds recursively: once TS_n^m and TB_m^n have been computed using FM methods, the time of arrival at the sea bottom and at the sea surface respectively computed from TS_n^m and TB_m^n act as radiating surfaces for TB_{n+1}^m and TS_m^{n+1}.

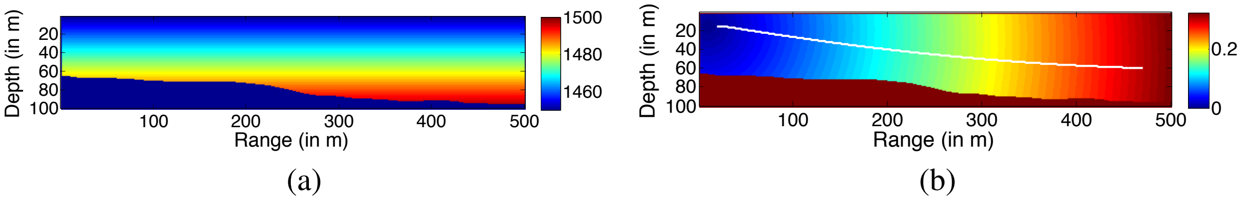

Fig. 3: (a) *Sound Velocity Profile (in m.s^{-1}),* (b) *Shortest path (white curve) computed with gradient descent algorithm on the T_0 field.*

Figure 3(a) displays a non-constant sound speed profile example in a shallow environment. In figure 3(b) we compute the wavefront T_0. The direct path (in white in the figure) from a target in the scene to the source is computed thanks to the Runge-Kutta gradient descent.

In figure 4 we show how to compute the multiple paths with one bounce on the seafloor or one bounce on the surface using respectively the TB_1^0 and TS_0^1 spaces. It is important to note that for each multipath, we compute only one ray. Finally figure 5 draws the analogy between our method and the Mirror theorem. The *white* curve corresponds to the direct path, the *yellow* curve corresponds to the shortest path with one bounce on the surface and the *magenta* curve corresponds to the shortest path with one bounce on the sea bottom. In this paper we are using isotropic FM methods. So when TB_n^m or TS_m^n are computed using the wavefront radiating from the sea bottom and the sea surface respectively, every point on the interface diffuses the acoustic wave in all directions. Interface interactions are then naturally taken into account using the multiple dimensions approach.

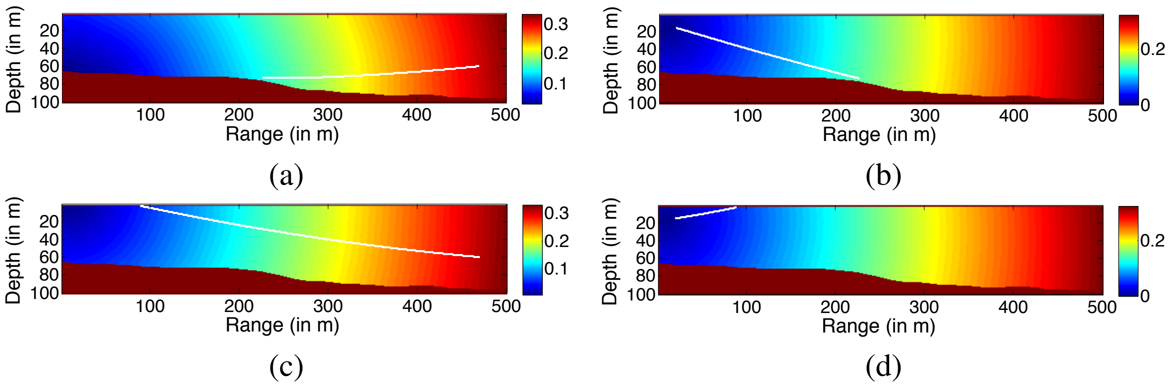

Fig. 4: (a) *Steepest gradient descent in the TB_1^0 space,* (b) *Steepest gradient descent from the bottom reflection from* (a) *to the source in the T_0 space ,*(c) *Steepest gradient descent in the space TS_0^1,* (d) *Steepest gradient descent from the surface reflection from* (c) *to the source in the T_0 space.*

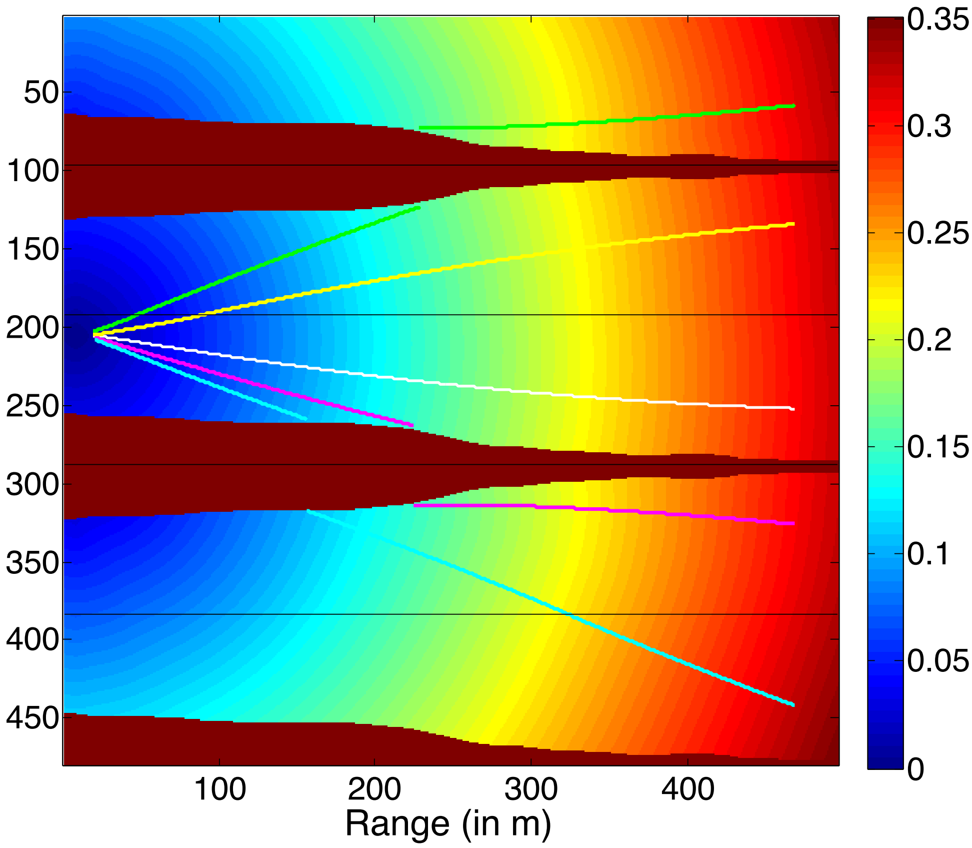

Fig. 5: Multi-dimension Fast Marching analogy with the Mirror theorem. white *curve: direct path,* yellow *curve: one bounce (surface),* green *curve: two bounces (surface, bottom),* magenta *curve: one bounce (bottom) and* cyan *curve: two bounces (bottom, surface).*

4. CONCLUSIONS

In this paper we extend the Mirror theorem used for acoustic wave propagation in shallow water environment to a multi-dimension Fast Marching approach which can take into account any sound velocity profile and any bathymetry profile. Thanks to the MDFM approach, only one ray per bounce is computed reducing drastically the computation time. MDFM then offers an elegant solution to acoustic wave propagation in shallow water.

5. ACKNOWLEDGEMENT

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