Wireless Pers Commun
DOI 10.1007/s11277-012-0568-8

On Multiple Access Using H-ARQ with SIC Techniques for Wireless Ad Hoc Networks

Jinho Choi

Abstract In this paper, we consider non-orthogonal multiple access (NOMA) for wireless ad hoc networks over block fading channels where the performance is limited by interference and fading. In order to provide a reasonable performance, we can use re-transmission and interference cancellation techniques. Re-transmission techniques can provide a diversity gain over fading channels, while the successive interference cancellation (SIC) can improve the signal to interference ratio (SIR). Using the information outage probability, we show that the NOMA approach with re-transmissions can perform better than the orthogonal multiple access (OMA) approach with re-transmissions when the signal to noise ratio (SNR) is low. It is also shown that the outage probability of the NOMA with SIC is lower than that of OMA when the rate is sufficiently low where SIC can be facilitated.

Keywords Wireless ad hoc networks · Multiple access · H-ARQ · Successive interference cancellation

1 Introduction

In wireless ad hoc networks, medium access control (MAC) plays a key role in providing reliable communications between nodes. Various multiple access schemes are proposed based the request-to-send (RTS)/ clear-to-send (CTS) mechanism in IEEE 802.11 standards in [1]. For example, code division multiple access (CDMA) has been considered in [2,3]. CDMA is capable of dealing with uncoordinated multiple access interference that exists in wireless ad hoc networks and this advantage of CDMA over time division multiple access (TDMA) or frequency division multiple access (FDMA) is highlighted in [2].

J. Choi (✉)
College of Engineering, Swansea University, Singleton Park, Swansea, SA2 8PP, UK
e-mail: j.choi@swansea.ac.uk

From the channel capacity point of view, the superiority of non-orthogonal multiple access (NOMA) schemes (e.g., CDMA) to orthogonal multiple access (OMA) schemes (e.g., TDMA or FDMA) is well-known [4–6]. The use of stripping or cancellation plays a key role in achieving the channel capacity for NOMA [4,5]. In [6], the capacity of randomly spread CDMA systems is studied under various conditions. It is shown that the multiuser gain is crucial to offset the performance loss caused by multiuser interference.

In [7–9], it is shown that the linear minimum mean square error (MMSE) receiver with successive interference cancellation (SIC) can be used at the receiver for NOMA systems to achieve the channel capacity. This is an important observation as a relatively simple receiver can be used to achieve the channel capacity.

In [10], using the techniques proposed in [11], the comparison between TDMA and CDMA has been made. In general, it is shown that CDMA as one of the NOMA schemes is superior to TDMA as one of the OMA schemes although the difference vanishes as the power decreases for non-fading channels. However, as also pointed out in [6], since the multiuser gain can be exploited within NOMA schemes for fading channels, CDMA becomes preferable to TDMA.

Generally, the performance of NOMA schemes can be improved if joint rate and power control is used. In [12], rate and power control for coded CDMA with low-density parity check (LDPC) codes has been investigated to maximize the spectral efficiency. In [13], an optimization of interleave-division multiple access (IDMA) scheme, which is another NOMA scheme, is studied.

In this paper, we consider a NOMA scheme which is different from CDMA for wireless ad hoc networks. It is assumed that there are multiple nodes transmitting signals in a common channel (multiple access channel) and an automatic repeat request (ARQ) protocol is used. For reliable communications over fading channels, various ARQ protocols can be used [14,15] and they can also be used with scheduling [16]. In particular, we consider hybrid ARQ (H-ARQ) with incremental redundancy (IR) [17,18], which is called the H-ARQ with IR, in this paper. In [19–21], various coding approaches to implement H-ARQ with IR have been studied. We assume block fading [22] in this paper. Thus, the information outage probability [23] rather than the channel capacity is used as the performance index. In general, we consider the two key ingredients for NOMA schemes to improve the performance: H-ARQ with IR and SIC. The H-ARQ with IR is an effective means to adapt fading channel conditions without fixing a code rate in advance. The code rate can be effectively adjusted by feedback in the H-ARQ: the effective code rate decreases as more re-transmissions are carried out in the H-ARQ with IR. The SIC can effectively reduce the signal to interference-plus-noise ratio (SINR) in NOMA schemes. In fading channels, since SIC can cancel or strip strong interfering signals first, it becomes an effective means to exploit the multiuser gain and improve the performance. Note that the multiuser gain by using SIC is different from the multiuser diversity gain in [24]. The multiuser diversity gain can be achieved by selecting the user of the highest channel gain to access channel, while the multiuser gain from SIC could increase the SINR by cancelling the interfering signals (but not improve the diversity gain). Throughout the paper, we show that how the H-ARQ with IR and SIC can be adopted in NOMA and derive the outage probability to see the performance under an environment of wireless ad hoc networks. Furthermore, the performance comparisons with OMA are studied. Based on the results obtained by analysis, we can see that NOMA is a promising scheme for wireless sensor networks where sensors need to transmit information at a low rate, but frequently.

The rest of the paper is organized as follows. In Sect. 2, we present the system model for a NOMA system. Outage probability analysis is given in Sect. 3, where we drive upper and lower bounds on the information outage probability. Section 4 presents simulation results. We conclude this paper with some remarks in Sect. 5.

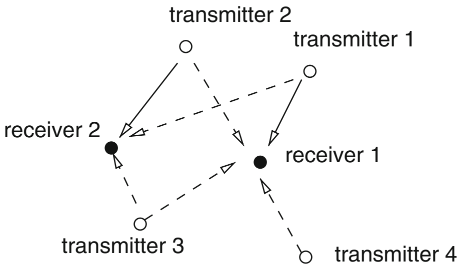


Fig. 1 An example of the system with $K = 4$ (*solid lines* represent the desired signals, while *dashed lines* represents undesired signals)

2 System Model

In this section, we consider a NOMA scheme based on the H-ARQ with IR for wireless ad hoc networks. If each node is considered a user, the resulting wireless ad hoc network can be considered as a multiuser system. In such a system, if multiple packets are transmitted, there is collision and re-transmission request is sent to the users in collision. Although the packets are collided, each packet can transmit a certain amount of its information. Thus, in order to improve the spectral efficiency, the H-ARQ with IR is utilized for combining all the retransmitted signals effectively. The resulting NOMA can be called the H-ARQ with IR based NOMA. However, for convenience, throughout the paper, the H-ARQ with IR based NOMA will be simply referred to as NOMA if there is no significant risk of confusion. In addition, SIC can be used to increase the SINR, which can also improve the spectral efficiency of NOMA.

Suppose that there are K senders or transmitters. We assume that all the signals share the same time and frequency channel. In addition, each transmitter has its dedicated receiver. At the receiver, the received signal is written as

$$r_m = \sum_{k=1}^{K} h_k s_{k,m} + n_m, \; m = 0, 1, \ldots, M-1, \tag{1}$$

where r_m denotes the mth received signal sample, h_k and $s_{k,m}$ denote the channel coefficient over the packet duration (block fading is assumed) and the mth data symbol from transmitter k, respectively, and n_m denotes the background zero-mean white Gaussian noise. We consider the following addition assumptions.

- A packet consists of M data symbols and can be decodable individually. That is, a packet corresponds to a codeword which is self-decodable and each re-transmitted packet can be considered the codeword which is generated by a different code from the the same message sequence for the IR. (This approach would be different from [19] where a single code is used.)
- All the packets are synchronized[1] (a slotted multiple access system is assumed).

In addition, it is assumed that the signal from transmitter 1 is the desired signal (i.e., it is assumed that receiver 1 is dedicated to transmitter 1). Figure 1 shows an example of the system where $K = 4$. The same multiple access is considered in [17]. Throughout the paper, we assume that no power control is used and the feedback information from the receiver to its corresponding transmitter is ACK (when the packet is successfully decoded) or NACK

[1] Synchronization is not necessary unless SIC is used. However, for convenience, we assume a synchronous NOMA system.

(when the packet is not successfully decoded). In this system, we can see that the rate control is adaptively carried out using any re-transmission protocols.

Note that in the paper, in order to focus on the performance of NOMA, we only consider one receiver node when $K \geq 1$ nodes transmit their signals simultaneously in wireless ad hoc networks. Although this simple case cannot reveal an overall performance of wireless ad hoc networks when NOMA is employed, the performance analysis becomes tractable with respect to outage probability. It is noteworthy that a multi-layer transmission based on H-ARQ with IR studied in [25] is similar to NOMA. In this multi-layer transmission, there are interfering signals and SIC can be employed to improve the performance. Since this multi-layer transmission is applied to point-to-point communications, optimal rate and power allocation can be formulated to maximize the throughput. Unfortunately, in a wireless ad hoc network in Fig. 1, such optimizations cannot be considered due to the lack of coordination between nodes. Thus, the resulting approaches in [25] are different from those in the paper.

2.1 H-ARQ with Incremental Redundancy

Assume that a codeword is transmitted over a packet duration. If the signal is not decodable due to interfering signals and/or background noise, the re-transmission request is sent to the transmitter. The re-transmitted packet is combined with the previous packet(s) for decoding. We assume that a capacity achieving channel code is employed. Note that in [26] the throughput of H-ARQ with IR is studied with LDPC codes and it is shown that a practical design using LDPC codes can closely approach the information-theoretical limit found in [17].

For the lth packet from transmitter k, the (normalized) instantaneous channel capacity can be given as follows [17]:

$$I_{k,l} = \log_2\left(1 + \frac{\gamma_k |h_{k,l}|^2}{1 + \sum_{q \neq k} \gamma_q |h_{q,l}|^2}\right), \; l = 1, 2, \ldots; k = 1, 2, \ldots, K, \tag{2}$$

where $h_{k,l}$ denotes the channel coefficient from transmitter k to the receiver over the lth data packet duration from transmitter k and γ_k is the normalized signal to noise ratio (SNR) of the signal from transmitter k. Suppose that the normalized rate of each data packet (from all the transmitters) is R_k. When the H-ARQ with IR is employed, the information outage probability for the desired signal with L re-transmissions (a transmission and $L-1$ re-transmissions) can be given as follows:

$$P_{\text{no,out}}(R_1, L) = \Pr(S_{1,L} < R_1),$$

where $S_{1,L} = \sum_{l=1}^{L} I_{1,l}$ [17].

Note that if K signals have their dedicated *orthogonal* channels (i.e., in OMA systems), each signal's instantaneous mutual information can be written as

$$I_{\text{orth},k,l} = \frac{1}{K} \log_2\left(1 + \gamma_k |h_{k,l}|^2\right), \; l = 1, 2, \ldots; k = 1, 2, \ldots, K. \tag{3}$$

Here, we assume that the channel is equally divided into K sub-channels. Then, for orthogonal channels, the outage probability becomes

$$P_{\text{orth,out}}(R_1, L) = \Pr(S_{\text{orth},1,L} < R_1),$$

where $S_{\text{orth},1,L} = \sum_{l=1}^{L} I_{\text{orth},1,l}$. For the sake of simplicity, we assume that the rate, R_k, is the same for all the signals, i.e., $R_k = R$ for all k (symmetric case in terms of rate). In practice, however, they can be different from one signal to another and depend on the actual data rate of each signal and coding scheme.

2.2 SIC Based Decoding

In (2), the other $(K-1)$ signals become the interference and lower the SINR. If some signals can be corrected decoded and cancelled or stripped, the SINR can be higher and the performance can be improved. For example, if the other $K-1$ packets are decodable, the cancellation of $K-1$ signals is possible and the resulting instantaneous channel capacity becomes

$$I_{1,l} = \log_2\left(1 + \gamma_1 |h_{1,l}|^2\right).$$

Thus, we can see that the probability of successful decoding can increase and the average number of re-transmissions can be smaller. This results in a higher throughput or spectral efficiency for the NOMA scheme.

Throughout the paper, we assume that all the re-transmitted packets from transmitter 1 are optimally combined, while no combining is used for the signals from the other $(K-1)$ transmitters. As the receiver is dedicated to the desired signal, the receiver does not know that whether another (interfering) transmitter re-transmits a packet or not. Thus, the decoding and cancellation of the other signals should be performed using the current packet without any combining with previously transmitted packets.

For SIC, the order of the detection/cancellation becomes important. For example, suppose that the packet from transmitter K is firstly decoded. If the decoding succeeds, that is,

$$\log_2\left(1 + \frac{\gamma_K |h_{K,l}|^2}{1 + \sum_{q=1}^{K-1} \gamma_q |h_{q,l}|^2}\right) \geq R,$$

the packet from transmitter K can be cancelled or stripped. Then, the packet from transmitter $K-1$ is to be decoded, and it can be decodable and cancelled if

$$\log_2\left(1 + \frac{\gamma_{K-1} |h_{K-1,l}|^2}{1 + \sum_{q=1}^{K-2} \gamma_q |h_{q,l}|^2}\right) \geq R.$$

To cancel more interfering signals through SIC, the signal (among the $(K-1)$ interfering signals) that has the highest SINR should be chosen first:

$$u_1^* = \arg \max_{k \in \{2,3,\ldots,K\}} \log_2\left(1 + \frac{\gamma_k |h_{k,l}|^2}{1 + \sum_{q \neq k} \gamma_q |h_{q,l}|^2}\right),$$

where u_m denotes the index for the mth cancelled signal. Let

$$\mathcal{W}_m = \{2, 3, \ldots, K\} \setminus \{u_1, u_2, \ldots, u_m\}.$$

Then, it follows

$$u_m^* = \arg \max_{k \in \mathcal{W}_{m-1}} \log_2\left(1 + \frac{\gamma_k |h_{k,l}|^2}{1 + (\gamma_1 |h_{1,l}|^2 + \sum_{q \in \mathcal{W}_{m-1} \setminus k} \gamma_q |h_{q,l}|^2)}\right),$$

where $\mathcal{W}_0 = \{2, 3, \ldots, K\}$. This SIC continues until the mutual information is less than R or decoding fails. If D interfering signals are cancelled, the resulting instantaneous mutual information for the desired signal is

$$\tilde{I}_{1,l} = \log_2\left(1 + \frac{\gamma_1 |h_{1,l}|^2}{1 + \sum_{q \in \mathcal{W}_D} \gamma_q |h_{q,l}|^2}\right). \tag{4}$$

Note that D depends on the channel realizations, $h_{k,l}$, SNRs, and R. When SIC is employed, the outage probability with L re-transmissions becomes

$$P_{\text{no-sic,out}}(R, L) = \Pr(\tilde{S}_{1,L} < R),$$

where $\tilde{S}_{1,L} = \sum_{l=1}^{L} \tilde{I}_{1,l}$.

In general, we can see that

$$P_{\text{no-sic,out}}(R, L) \leq P_{\text{no,out}}(R, L).$$

As expected, the NOMA with SIC can provide a better performance than that without SIC. Thus, in wireless ad hoc networks where multiple packets can be transmitted simultaneously, the role of SIC could be crucial.

Note that the rate, R, would be sufficiently small to facilitate SIC. For example, suppose that the amplitude of the channel coefficient is Rayleigh distributed and the probability density function (pdf) of the SNR is $f(x) = e^{-x}$, $x \geq 0$. Then, the probability that SIC can be performed, denoted by P_{sic}, becomes $P_{\text{sic}} = \Pr(\log_2(1+x) \geq R) = e^{-(2^R-1)}$. If $R = 1$ and $R = 0.25$, $P_{\text{sic}} = 0.37$ and 0.83, respectively. The lower R, the higher probability SIC can be performed. Consequently, a smaller R (i.e., $R < 1$) is preferred in the NOMA with SIC.

3 Outage Probability Analysis

In this section, we consider the information outage probability to understand the performance of the NOMA with/without SIC. We have some technical assumptions for the analysis as follows.

- A1) $\gamma_k = \gamma$ for all k (As previously assumed that $R_k = R$ for all k, we assume that the system is symmetric in terms of SNR and rate).
- A2) The $h_{k,l}$'s are independent and their amplitudes are Rayleigh distributed for all k and l.

As the derivation of a closed-form expression for the outage probability over block fading channels is quite involved, we mainly focus on upper and lower bounds.

As mentioned earlier, we focus on the outage analysis. In [17,27,28], the throughput or rate is analyzed for H-ARQ with IR.

3.1 Without SIC

In this subsection, we focus on the case where SIC is not applied. In this case, the diversity gain through re-transmission plays a key role in providing reliable transmission. We derive upper and lower bounds on the information outage probability.

3.1.1 Upper Bounds

The instantaneous mutual information is a function of multiple random variables and is not easy to analyze. Thus, to make the analysis tractable, we approximate the instantaneous mutual information $I_{1,l}$ in (2) as

$$X_l = \log_2(1 + |h_{1,l}|^2 \bar{\gamma}_K) \simeq I_{1,l},$$

where $\bar{\gamma}_K = \frac{\gamma}{1+(K-1)\gamma}$, which is obtained by taking the expectation of the power of the $(K-1)$ interfering signals. This approximation would be reasonable as K becomes larger. For convenience, let

$$Y_l = \log(1 + |h_{1,l}|^2 \bar{\gamma}_K) = X_l \log 2.$$

Then, the outage probability after L (re-) transmissions of IR becomes

$$\begin{aligned} P_{\text{no,out}}(R, L) &= \Pr(X_1 + X_2 + \cdots + X_L < R) \\ &= \Pr(Y_1 + Y_2 + \cdots + Y_L < \bar{R}) \\ &\leq \min_{\lambda \geq 0} E\left[e^{-\lambda \sum_{l=1}^{L} Y_l}\right] e^{\lambda \bar{R}} \\ &= \min_{\lambda \geq 0} \left(E\left[e^{-\lambda Y_l}\right]\right)^L e^{\lambda \bar{R}} \\ &= \min_{\lambda \geq 0} e^{\lambda \bar{R}} \left(E\left[\frac{1}{(1+|h|^2 \bar{\gamma}_K)^\lambda}\right]\right)^L, \end{aligned} \tag{5}$$

where $\bar{R} = R \log 2$. For the inequality in (5), we use the Chernoff inequality.

Noting that the pdf of $|h_{k,l}|^2$ (from A2)) is

$$f(x) = e^{-x}, \; x \geq 0,$$

we have

$$\begin{aligned} E\left[\frac{1}{(1+|h|^2 \bar{\gamma}_K)^\lambda}\right] &= \int_0^\infty \left(\frac{1}{1+x\bar{\gamma}_K}\right)^\lambda f(x) \mathrm{d}x \\ &= \int_0^\infty \frac{e^{-x}}{(1+x\bar{\gamma}_K)^\lambda} \mathrm{d}x \\ &= \frac{1}{\bar{\gamma}_K} A_\lambda \left(\frac{1}{\bar{\gamma}_K}\right), \end{aligned} \tag{6}$$

where

$$A_n(\mu) = \int_0^\infty \frac{e^{-\mu x}}{(1+x)^n} \mathrm{d}x.$$

Then, the upper bound on the outage probability becomes

$$\Pr\left(\sum_{l=1}^{L} X_l < R\right) \leq \min_{\lambda \geq 0} \xi(\lambda), \tag{7}$$

where

$$\xi(\lambda) = e^{\lambda \bar{R}} \left(\frac{1}{\bar{\gamma}_K} A_\lambda \left(\frac{1}{\bar{\gamma}_K}\right)\right)^L. \tag{8}$$

It is not easy to find the optimal solution that minimizes $\xi(\lambda)$ analytically. Thus, we need to use numerical approaches to find the solution. The following property becomes useful to develop numerical approaches.

Lemma 1 *$\xi(\lambda)$ is convex for $\lambda \geq 0$.*

Proof See Appendix A. □

As $\xi(\lambda)$ is convex, there exists a unique solution of the minimization in (7) and could be a number of computationally efficient techniques [29] to find the solution of the right hand side in (7).

We can also find another upper bound with a constraint.

Lemma 2

$$P_{\text{no,out}}(R, L) \leq \min_{\lambda \geq 0} \xi(\lambda) \leq e^{R \log 2} \left(\frac{cR}{\bar{\gamma}_K L} \right)^L, \tag{9}$$

where $c = e \log 2$ is a constant.

Proof From the following inequality:

$$A_\lambda(\mu) = \int_0^\infty \frac{e^{-\mu x}}{(1+x)^\lambda} \mathrm{d}x \leq \int_0^\infty \frac{1}{(1+x)^\lambda} \mathrm{d}x = \frac{1}{\lambda - 1}, \ \lambda \geq 1, \ \mu \geq 0, \tag{10}$$

we can show that

$$\xi(\lambda) \leq \bar{\xi}(\lambda) = \frac{1}{\bar{\gamma}_K^L} \frac{e^{\lambda \bar{R}}}{(\lambda - 1)^L}, \ \lambda > 1. \tag{11}$$

Consider the constraint that $\lambda > 1$ and the following optimization problem:

$$\lambda^* = \arg \min_{\lambda > 1} \bar{\xi}(\lambda).$$

The solution becomes

$$\lambda^* = \frac{L}{\bar{R}} + 1 \ (> 1).$$

Then, the last inequality in (9) is obtained as

$$\bar{\xi}(\lambda^*) = \left(\frac{\bar{R}}{\bar{\gamma}_K L} \right)^L e^{\bar{R} + L} = e^{R \log 2} \left(\frac{cR}{\bar{\gamma}_K L} \right)^L.$$

This completes the proof. □

Property 1 *The outage probability, $P_{\text{no,out}}(R, L)$, decreases with L faster than exponential decay. For convenience, define the asymptotic loglinear decay rate (ALDR) with respect to L as*

$$\mathsf{ALDR} = -\lim_{L \to \infty} \frac{\log P_{\text{no,out}}(R, L)}{L \log L}. \tag{12}$$

Then, we can see that

$$\mathsf{ALDR} \geq 1. \tag{13}$$

Proof This is a consequence of (9).

From (12), Eq. (13) implies that

$$P_{\text{no,out}}(R, L) \leq e^{-L \log L}, \quad L \gg 1.$$

Clearly, this asymptotic behavior is independent of R and $\bar{\gamma}_K$. □

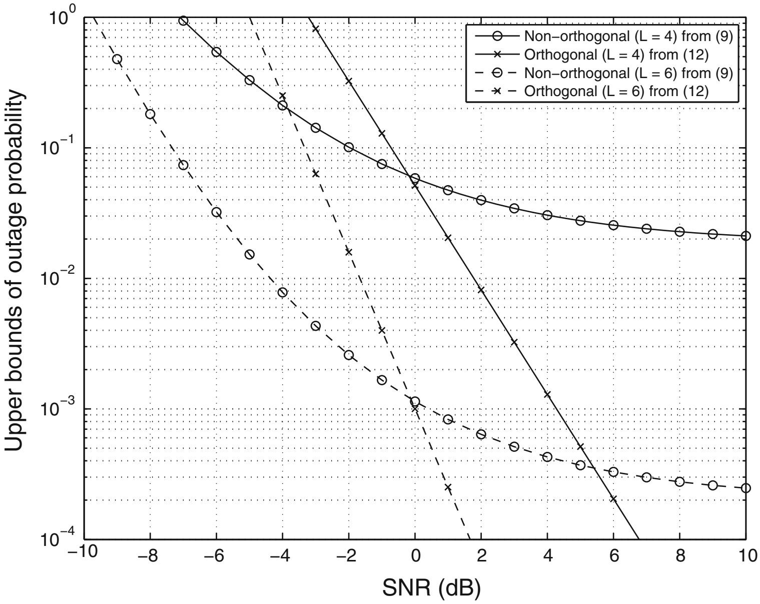


Fig. 2 Upper bounds on the outage probability of OMA and NOMA

Property 2 *For the case of K orthogonal subchannels, the upper bound on the outage probability in (9) becomes*

$$\bar{\xi}_{\text{orth}}(\lambda^*) = e^{(R \log 2)/K} \left(\frac{cRK}{\gamma L} \right)^L . \tag{14}$$

From Properties 1 and 2, we can observe that NOMA can asymptotically (i.e., for a large L) provide lower outage probability than OMA when $\gamma \leq 0$ dB. Figure 2 shows the upper bounds in (9) for OMA and NOMA schemes with different values of SNRs. We can see that the NOMA without SIC outperforms OMA when the SNR is less than 0 dB. Consequently, NOMA is preferable (even without joint decoding) when the SNR is sufficiently low and H-ARQ is actively used for reliable communications.

According to [10], it is shown that the capacity difference between TDMA (i.e., OMA) and CDMA (i.e., NOMA) decreases with the SNR. Thus, the results from Properties 1 and 2 would be different from that in [10]. This difference results from that no joint decoding is used in the NOMA without SIC where the interfering signal is considered as the noise; see (2).

3.1.2 Lower Bound

In this subsection, we derive a lower bound using the following relations:

- $$\log_2(1 + x) \leq x \log_2 e. \tag{15}$$
- In addition, if $X \leq Y$, then

$$\Pr(X < R) \geq \Pr(Y < R). \tag{16}$$

Using (15) and (16), we have

$$\begin{aligned}\Pr(X_1 + X_2 + \cdots + X_L < R) &\geq \Pr\left(\bar{\gamma}_K \sum_{l=1}^{L} |h_{1,l}|^2 < \bar{R}\right) \\ &= \Pr\left(\chi^2_{2L} < \bar{R}/\bar{\gamma}_K\right) \\ &= 1 - e^{-\bar{R}/\bar{\gamma}_K} \sum_{l=0}^{L-1} \frac{1}{l!} \left(\frac{\bar{R}}{\bar{\gamma}_K}\right)^l, \end{aligned} \tag{17}$$

where χ^2_n denotes a chi-square random variable with n degrees of freedom. In general, this low bound is tight when the SINR is sufficiently low according to (15).

Property 3 *For a large L, the outage probability is bounded as*

$$\frac{1}{\sqrt{2\pi L}} \left(\frac{cR}{\bar{\gamma}_K L}\right)^L \leq P_{\text{no,out}}(R, L) \leq e^{R \log 2} \left(\frac{cR}{\bar{\gamma}_K L}\right)^L. \tag{18}$$

Thus, the ALDR with respect to L is $\mathsf{ALDR} = 1$ *and*

$$P_{\text{no,out}}(R, L) = e^{-L \log L}, \quad L \gg 1.$$

Proof See Appendix B. □

If the same packet is re-transmitted (i.e., Type-1 H-ARQ), we can have the repetition time diversity (RTD) [17] and the outage probability becomes

$$\begin{aligned}P_{\text{rtd,out}}(R, L) &\simeq \Pr\left(\log_2\left(1 + \sum_{l=1}^{L} \bar{\gamma}_K |h_{1,l}|^2\right) < R\right) \\ &= \Pr\left(\chi^2_{2L} < \frac{2^R - 1}{\bar{\gamma}_K}\right) \\ &\simeq \frac{1}{\sqrt{2\pi L}} \left(\frac{e(2^R - 1)}{\bar{\gamma}_K L}\right)^L \quad \text{(for a large } L\text{)}. \end{aligned} \tag{19}$$

For the second approximation in (19), see Appendix B. Since

$$cR = e \log_2 R \leq e(2^R - 1), \ R \geq 0,$$

from (18) and (19), we can see that the IR can provide lower information outage probability than the RTD. However, the ALDR is the same for both IR and RTD. From this, we can see that the H-ARQ with RTD could be almost equally efficient as the H-ARQ with IR in terms of the error decay rate by re-transmission.

3.2 With SIC

As mentioned earlier, in wireless ad hoc networks where multiple users can transmit packets simultaneously, SIC can play a crucial role in improving an overall performance by canceling multiuser interference. In this subsection, we consider the outage probability when SIC is used. We derive an exact expression for the outage probability when there are two users (i.e., $K = 2$) and one transmission (i.e., $L = 1$). Although an exact expression for the outage probability is available for the case of $K = 2$ and $L = 1$, it is not easy to generalize the result for $K > 2$. Thus, we will consider approximations to derive bounds later.

3.2.1 Two Users

Suppose that $K = 2$ and $L = 1$. The instantaneous channel capacity can be given as follows:

$$I_{1,l} = \begin{cases} \log_2\left(1 + \gamma_1 |h_{1,l}|^2\right), & \text{if } \log_2\left(1 + \frac{\gamma_2 |h_{2,l}|^2}{1+\gamma_1 |h_{1,l}|^2}\right) \geq R; \\ \log_2\left(1 + \frac{\gamma_1 |h_{1,l}|^2}{1+\gamma_2 |h_{2,l}|^2}\right), & \text{if } \log_2\left(1 + \frac{\gamma_2 |h_{2,l}|^2}{1+\gamma_1 |h_{1,l}|^2}\right) < R. \end{cases} \tag{20}$$

For convenience, define a random variable as follows:

$$B_{1,l} = \begin{cases} \gamma_{1,l}, & \text{if } \frac{\gamma_{2,l}}{1+\gamma_{1,l}} \geq V; \\ \frac{\gamma_{1,l}}{1+\gamma_{2,l}}, & \text{if } \frac{\gamma_{2,l}}{1+\gamma_{1,l}} < V, \end{cases} \tag{21}$$

where $V = 2^R - 1$ and $\gamma_{k,l} = \gamma_k |h_{k,l}|^2$. It is easy to see that $I_{1,l} = \log_2(1 + B_{1,l})$.

Lemma 3 *Suppose that the pdf of $\gamma_{k,l}$ is given as follows:*

$$f(\gamma_{k,l}) = \frac{1}{\bar{\gamma}_k} e^{-\gamma_{k,l}/\bar{\gamma}_k}, \quad k = 1, 2. \tag{22}$$

Then, the cumulative distribution function (cdf) of $B_{1,l}$ is

$$F_1(x) = \Pr(B_{1,l} \leq x) = P_1(x) + P_2(x), \tag{23}$$

where

$$P_1(x) = \frac{e^{-V/\bar{\gamma}_2}}{\bar{\gamma}_1 C_1}\left(1 - e^{-C_1 x}\right). \tag{24}$$

and $C_1 = \frac{1}{\bar{\gamma}_1} + \frac{V}{\bar{\gamma}_2}$. The second term, $P_2(x)$, is given as follows:

$$P_2(x) = \begin{cases} 1 - e^{-V/\bar{\gamma}_2} - \frac{e^{-x/\bar{\gamma}_1}}{\bar{\gamma}_2 D(x)}\left(1 - e^{-D(x)V}\right) + \frac{e^{1/\bar{\gamma}_1}}{\bar{\gamma}_2 C_2} e^{-C_2 V} - \frac{e^{-x/\bar{\gamma}_1}}{\bar{\gamma}_2 D(x)} e^{-D(x)V}, & \text{if } x > 1/V; \\ \frac{e^{1/\bar{\gamma}_1}}{\bar{\gamma}_2 C_2}\left(e^{-C_2 V} - e^{-C_2 B(x)}\right) - \frac{e^{-x/\bar{\gamma}_1}}{\bar{\gamma}_2 D(x)}\left(e^{-D(x)V} - e^{-D(x)B(x)}\right), & \text{if } x < 1/V. \end{cases} \tag{25}$$

where $D(x) = \frac{x}{\bar{\gamma}_1} + \frac{1}{\bar{\gamma}_2}$, $B(x) = (x + 1)/(V^{-1} - x)$, and $C_2 = \frac{1}{\bar{\gamma}_2} + \frac{1}{V\bar{\gamma}_1}$.

Proof See Appendix C. □

From the cdf of $B_{1,l}$ in (23), the information outage probability can be found as

$$P_{\text{no-sic,out}}(R, 1) \Pr(I_{1,l} < R) = \Pr(B_{1,l} < 2^R - 1) = F_1(2^R - 1). \tag{26}$$

For comparison, we can consider the outage probability of OMA when $K = 2$ and $L = 1$. The outage probability becomes

$$\begin{aligned} P_{\text{orth,out}}(R, 1) &= \Pr\left(\frac{1}{2}\log_2(1 + \gamma |h_{k,l}|^2) < R\right) \\ &= 1 - e^{-(2^{2R}-1)/\gamma} \end{aligned} \tag{27}$$

if the channel is equally divided.

In Fig. 3, we show the outage probability of the NOMA with SIC and OMA when $K = 2$ and $L = 1$. In general, it is shown that the NOMA with SIC can have a lower outage probability than OMA. We can show a precise result for the superiority of the NOMA with SIC to OMA as follows.

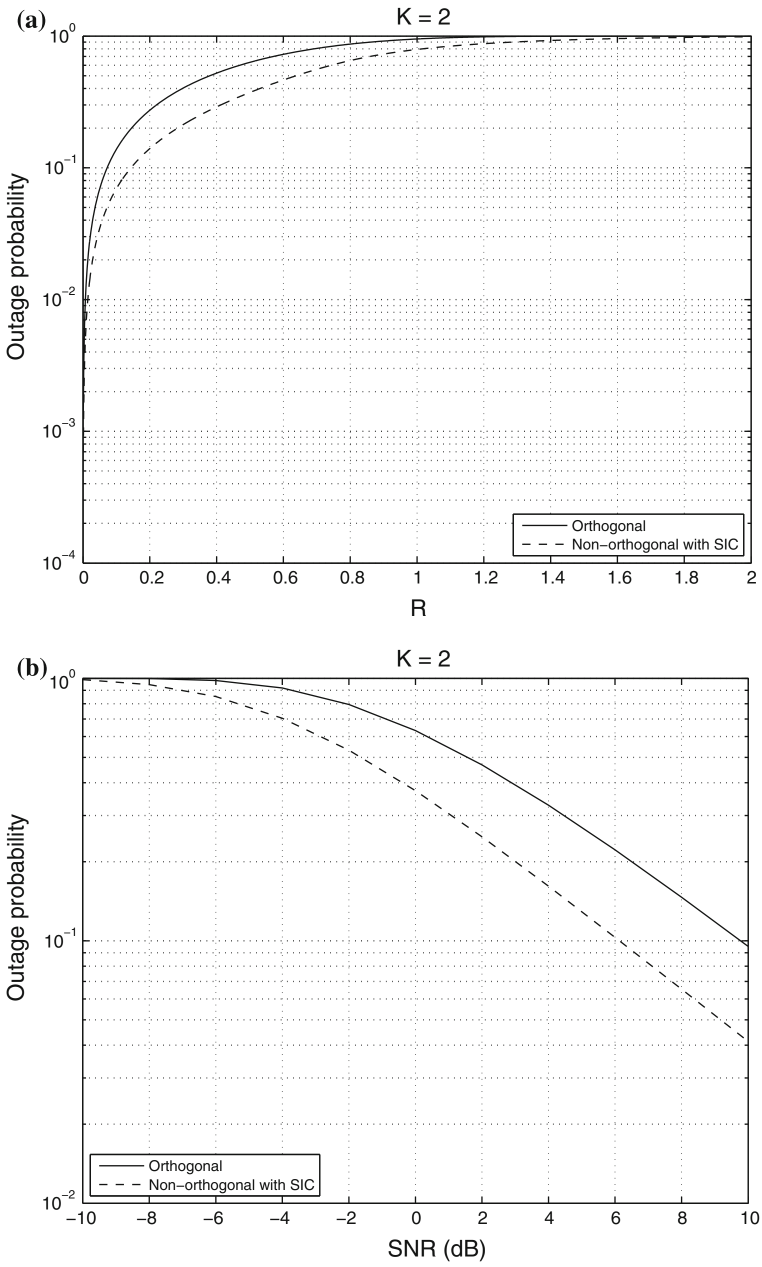


Fig. 3 Outage probability for OMA and NOMA with SIC: **a** Outage probability versus R when $\gamma_1 = \gamma_2 = 0$ dB; **b** Outage probability versus SNR ($\gamma_1 = \gamma_2$) when $R = 0.5$

Property 4 *Suppose that $K = 2$ and $L = 1$. The outage probability of the NOMA with SIC is lower than or equal to that of OMA if $0 \leq \gamma < \frac{V(V+1)}{\log V}$ when $V > 1$ (or $R > 1$) and for all $\gamma \geq 0$ when $V \leq 1$ (or $R \leq 1$).*

Proof See Appendix D. □

Again, the result of Property 4 shows the superiority of NOMA to OMA in terms of the information outage probability when $V < 1$. Note that OMA can perform better when $V > 1$ and $\gamma > \frac{V(V+1)}{\log V}$. However, in general, for the case of $V > 1$ or $R > 1$, the outage probability is high and this case would not be practically important.

For $L > 1$, according to A2), we can have the upper bounds as follows:

$$\begin{aligned} P_{\text{no-sic,out}}(R, L) &\leq P^L_{\text{no-sic,out}}(R, 1) \\ P_{\text{orth,out}}(R, L) &\leq P^L_{\text{orth,out}}(R, 1). \end{aligned} \tag{28}$$

3.2.2 Approximation: Effective Interference After SIC

As shown in Appendix C, the derivation of the cdf (or pdf) of $X_{1,l}$ is not straightforward even if $K = 2$. For a large K, the derivation of the cdf of $S_{k,L}$ becomes difficult. Thus, we need to consider approximations to derive bounds.

Among $K - 1$ interfering signals, some signals can be cancelled if their instantaneous channel capacity is greater than R. According to (4), D depends on the channel realizations, $h_{k,l}$, R, and γ. For approximation, we assume that D is an independent random variable. In Appendix E, we derive an approximate of the probability mass function of D.

Assuming that D is given, the mutual information can be approximated becomes

$$\tilde{I}_{1,l} = \log_2\left(1 + \frac{\gamma|h_{1,l}|^2}{1 + \gamma\sum_{q=D+1}^{K-1}\alpha_q}\right) \simeq \hat{I}_{1,l}(D) = \log_2\left(1 + \frac{\gamma|h_{1,l}|^2}{1 + \gamma\sum_{q=D+1}^{K-1}\bar{\alpha}_q}\right), \tag{29}$$

where α_q denotes the qth largest $|h_{k,l}|^2$, $k = 2, 3, \ldots, K$, and $\bar{\alpha}_q = E[\alpha_q]$. Taking D is an independent random variable, the average mutual information can be an approximate of $\tilde{I}_{1,l}$ in (4) as

$$\tilde{I}_{1,l} \simeq \hat{I}_{1,l} = E_D[\hat{I}_{1,l}(D)]. \tag{30}$$

Property 5 *$\hat{I}_{1,l}$ is bounded as follows:*

$$\log_2\left(1 + \frac{\gamma|h_{1,l}|^2}{E_D\left[1 + \gamma\sum_{q=D+1}^{K-1}\bar{\alpha}_q\right]}\right) \leq \hat{I}_{1,l} \leq \log_2\left(1 + E_D\left[\frac{\gamma|h_{1,l}|^2}{1 + \gamma\sum_{q=D+1}^{K-1}\bar{\alpha}_q}\right]\right). \tag{31}$$

For convenience, let

$$\begin{aligned} \gamma_{\text{L}} &= \frac{\gamma}{E_D\left[1 + \gamma\sum_{q=D+1}^{K-1}\bar{\alpha}_q\right]} \\ \gamma_{\text{U}} &= E_D\left[\frac{\gamma}{1 + \gamma\sum_{q=D+1}^{K-1}\bar{\alpha}_q}\right]. \end{aligned} \tag{32}$$

If γ_{sic} denotes the effective SNR after SIC, γ_{L} and γ_{U} become lower and upper bounds on γ_{sic}, respectively. Using γ_{L} and γ_{U}, we can find upper and lower bounds on the outage probability.

From (31) and using (16), we have the following upper bounds:

$$\Pr\left(\sum_{l=1}^{L} \hat{I}_{1,l} < R\right) \leq \Pr\left(\sum_{l=1}^{L} \log_2(1+\gamma_{\mathrm{L}}|h_{1,l}|^2) < R\right) \leq \min_{\lambda \geq 0} \xi_{\mathrm{sic}}(\lambda), \tag{33}$$

where the last inequality results from (5) and

$$\xi_{\mathrm{sic}}(\lambda) = e^{\lambda \bar{R}} \left(\frac{1}{\gamma_{\mathrm{L}}} A_\lambda \left(\frac{1}{\gamma_{\mathrm{L}}}\right)\right)^L. \tag{34}$$

For a lower bound, from (31), we can use (16) and (17):

$$\Pr\left(\sum_{l=1}^{L} \hat{I}_{1,l} < R\right) \geq \Pr\left(\sum_{l=1}^{L} \log_2(1+\gamma_{\mathrm{U}}|h_{1,l}|^2) < R\right) = 1 - e^{-\bar{R}/\gamma_{\mathrm{U}}} \sum_{l=0}^{L-1} \frac{1}{l!}\left(\frac{\bar{R}}{\gamma_{\mathrm{U}}}\right)^l. \tag{35}$$

We have some remarks as follows.

- As shown in (32), the multiuser gain is shown through the SINR gain. Thus, the diversity order shown in (33) and (35) is L which is the same as that without SIC.
- The upper and lower bounds in (33) and (35), respectively, would be loose as the two inequalities (one for the SINR and the other for the outage probability) are subsequently used.
- As the upper and lower bounds in (33) and (35), respectively, are based on approximations, they are not strictly upper and lower bounds on the information outage probability.

4 Simulation Results

In this section, we present simulation results for OMA and NOMA with/without SIC. Figure 4 shows the outage probability with $K = 4$. In general, we can see that NOMA has lower outage probability than OMA and SIC can improve the performance of NOMA. In addition, we can also confirm that the re-transmission is a very effective means to reduce the outage probability.

Figure 5 shows the outage probability for different number of transmitters. In NOMA, it is expected that the performance becomes worse as the number of signals increases. Furthermore, in OMA, since the channel resource decreases with K, the outage probability increases with K for a fixed rate, R. As shown in Fig. 5, the outage probability of OMA and NOMA without SIC increases with K. However, the NOMA with SIC has less performance degradation with respect to the increase of K when the rate is sufficiently low: see Fig. 5a, b. As each signal experiences independent fading, there would be some dominant interfering signals which can be successfully decoded and cancelled as their SINRs would be sufficiently high. Therefore, in the NOMA with SIC, the impact of strong interfering signals can be small as they can be stripped. On the other hand, if R is high, SIC may not be applicable to strip interfering signals. From this, for a higher R, the outage probability of the NOMA with SIC can increase rapidly with K as shown Fig. 5c.

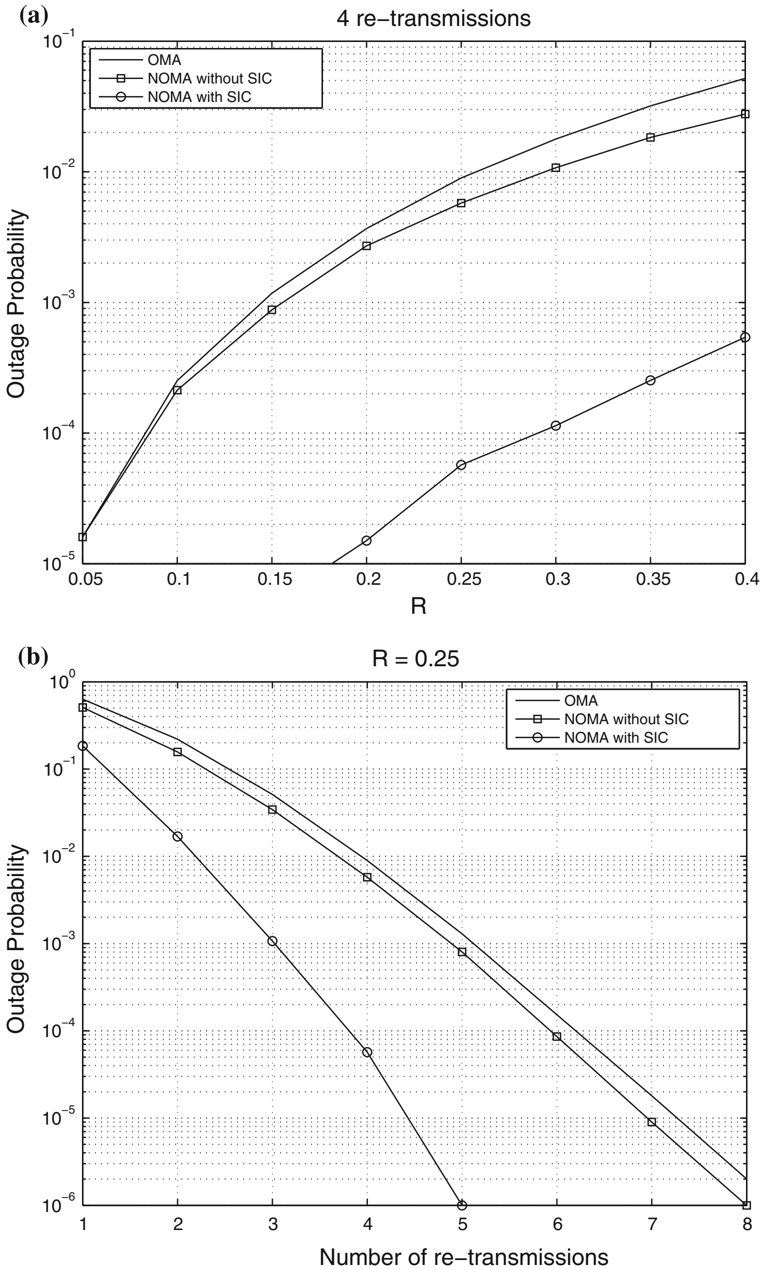


Fig. 4 Outage probability for OMA and NOMA with/without SIC when $\gamma = 0$ dB: (a) Outage probability versus R; (b) Outage probability versus L

The throughput of the NOMA with SIC can be improved if SIC is successfully performed. As mentioned earlier, a lower R would be preferable to facilitate SIC. From this and the results in Fig. 5, we can see that the throughput can be improved by increasing K for a sufficiently

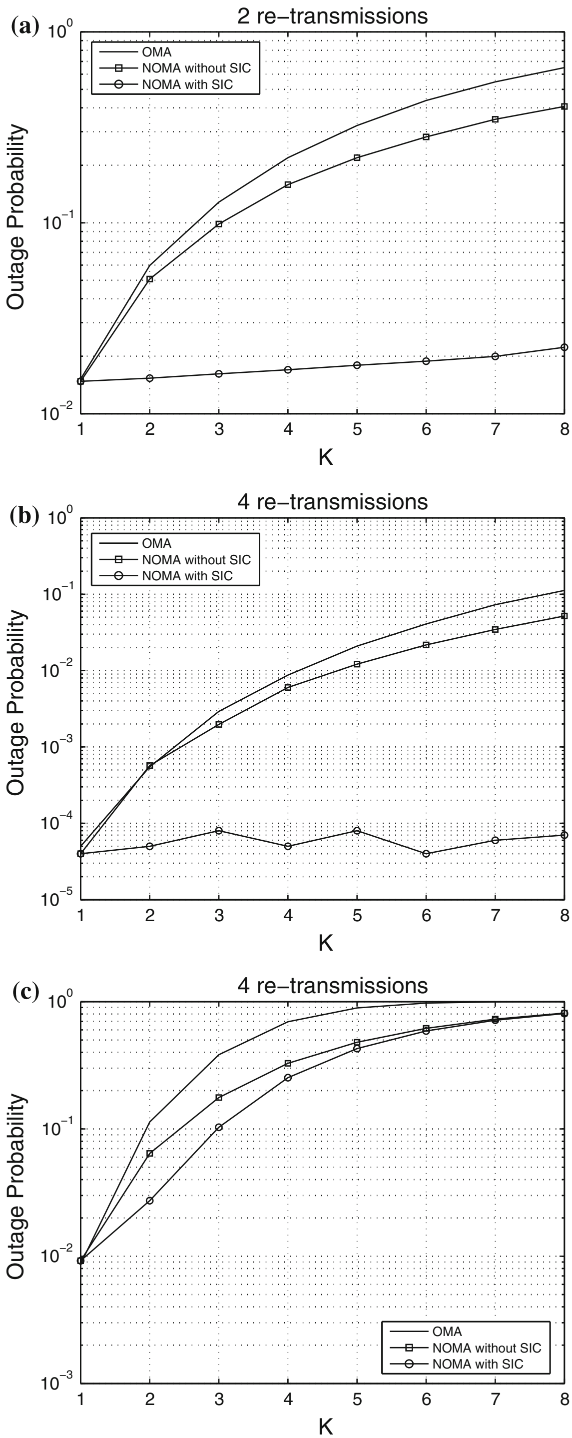


Fig. 5 Outage probability versus K for OMA and NOMA with/without SIC when $\gamma = 0$ dB: **a** $R = 0.25$ and $L = 2$; **b** $R = 0.25$ and $L = 4$; **c** $R = 1$ and $L = 4$

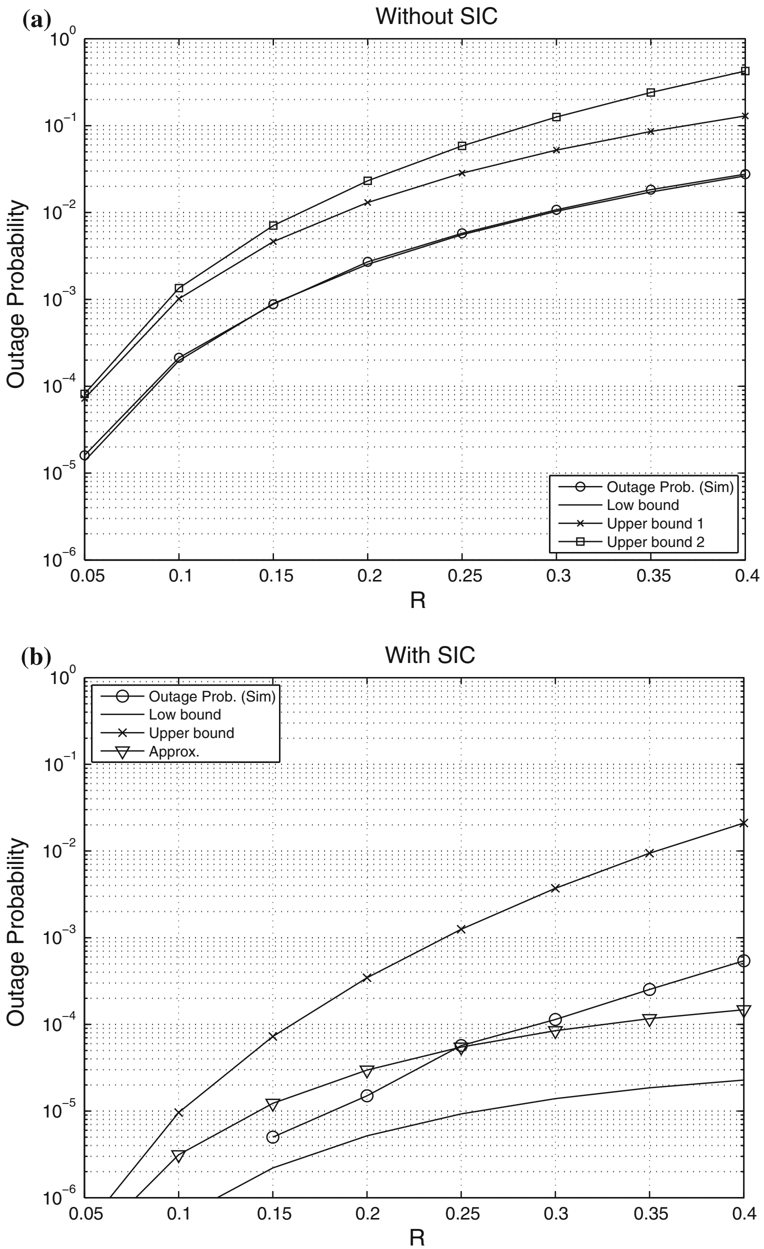


Fig. 6 Outage probability versus rate when $\gamma = 0$ dB: **a** without SIC; **b** with SIC

low R. For example, consider the case, say Case A, that $K = 8$, $L = 4$, and $R = 0.25$ in Fig. 5b. The corresponding outage probability is less than 10^{-4}. For another case, say Case B, that $K = 2$, $L = 4$, and $R = 1$ in Fig. 5c, the outage probability is about 0.03. If we define

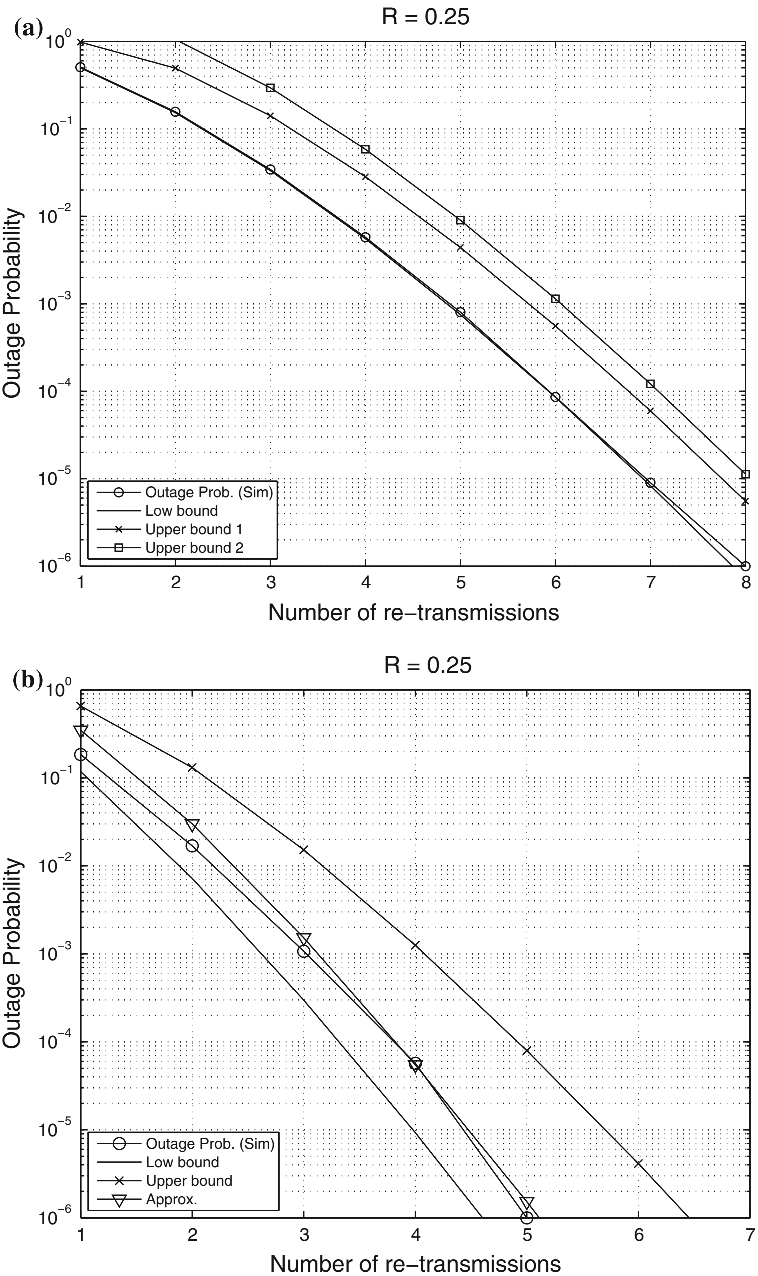


Fig. 7 Outage probability versus the number of re-transmissions when $\gamma = 0$ dB: **a** without SIC; **b** with SIC

the total throughput[2] as $\frac{KR}{L}(1 - P_{\text{out}}(R, L))$, Case A (a lower R and a larger K) has higher

[2] As we assumed that the system is symmetric in terms of rate in A1), this throughput can be a total throughput as the outage probability would be the same for all users.

throughput as its outage probability is lower (while the product of K and R is the same for the two cases).

The results in Fig. 5 show that NOMA with SIC is effective for low R and performs much better than OMA for a large K. This implies that NOMA with SIC can be an effective multiple access scheme for wireless ad hoc networks when a number of nodes can transmit their low data rate packets simultaneously.

In Figs. 6 and 7, we present the outage probability from simulation results and its upper and lower bounds from the results of Sect. 3 with $K = 4$. In general, it is shown that the lower bound is tighter than the upper bound for the NOMA without SIC: see Figs. 6a and 7a. As shown previously, when the SINR is low, (15) is tight and it results in a tight lower bound. However, when SIC is used, the SINR becomes higher and the lower bound is no longer tight. This is shown in Figs. 6b and 7b. Then, the upper bound becomes useful together with the lower bound to predict the performance of the NOMA with SIC.

As the gap between the lower and upper bounds of the information outage probability for the NOMA with SIC is large, approximations can be considered using the pair of the upper and lower bounds on the SINR, (γ_U, γ_L), and the pair of the upper and lower bounds on the outage probability. It is shown that the following approximation using a combination of the upper bound on the outage probability and γ_U is more accurate:

$$\Pr\left(\sum_{l=1}^{L} \hat{I}_{1,l} < R\right) \simeq \min_{\lambda \geq 0} e^{\lambda \bar{R}} \left(\frac{1}{\gamma_U} A_\lambda \left(\frac{1}{\gamma_U}\right)\right)^L .$$

This approximate outage probability is presented in Figs. 6b and 7b.

5 Concluding Remarks

In this paper, we studied NOMA when the H-ARQ protocol with IR is used and derived upper and lower bounds on the information outage probability to see the performance. It was shown that the outage probability decreases with L as $e^{-L \log L}$, which is faster than the exponential decay. In addition, the ALDR with respect to the number of re-transmissions is identical for both H-ARQ protocols with IR and RTD. From the outage probability, it was observed that the NOMA without SIC outperforms OMA when the SNR is low (less than 0 dB). In addition, when $R < 1$, the NOMA with SIC outperforms OMA for all SNR. To facilitate SIC, a low rate is preferable in the NOMA with SIC. It was shown that the outage probability increases slowly with K for a sufficiently low R. Thus, the throughput can be improved by increasing K with a sufficiently low R in the NOMA with SIC.

From this, we can conclude that NOMA with SIC can be a good candidate for multiple access in wireless ad hoc networks. In particular, if the data rate between nodes is low but there could be frequent transmissions as in wireless sensor networks, NOMA with SIC seems promising.

Appendix A

Proof of Lemma 1

For convenience, let

$$g(\lambda) = A_\lambda \left(\frac{1}{\gamma_K}\right).$$

Then, $\phi(\lambda) = e^{\lambda \bar{R}} g^L(\lambda)$ and $\xi(\lambda) = \phi(\lambda)\gamma_K^{-L}$. We have

$$\phi'(\lambda) = \frac{\mathrm{d}}{\mathrm{d}\lambda}\xi(\lambda) = e^{\lambda \bar{R}} g^{L-1}(\lambda)\left(\bar{R}g(\lambda) + Lg'(\lambda)\right) \tag{36}$$

and

$$\begin{aligned}\phi''(\lambda) &= \frac{\mathrm{d}^2}{\mathrm{d}^2\lambda}\phi(\lambda)\\ &= e^{\lambda \bar{R}} g^{L-2}(\lambda)\left((\bar{R}g(\lambda) + Lg'(\lambda))^2 + L(g''(\lambda)g(\lambda) - (g'(\lambda))^2)\right).\end{aligned} \tag{37}$$

A sufficient and necessary condition that $\phi(\lambda)$ is convex is

$$\phi''(\lambda) \geq 0. \tag{38}$$

According to (37), the following two conditions are equivalent:

$$\phi''(\lambda) \geq 0 \Leftrightarrow g''(\lambda)g(\lambda) \geq (g'(\lambda))^2.$$

Since

$$g(\lambda) = A_\lambda(\mu) = \int_0^\infty \frac{e^{-\mu x}}{(1+x)^\lambda}\mathrm{d}x, \tag{39}$$

where $\mu = 1/\gamma_K$, we have

$$\begin{aligned}g'(\lambda) &= -\int_0^\infty \frac{e^{-\mu x}}{(1+x)^\lambda}\log(1+x)\mathrm{d}x \leq 0;\\ g''(\lambda) &= \int_0^\infty \frac{e^{-\mu x}}{(1+x)^\lambda}\log^2(1+x)\mathrm{d}x \geq 0.\end{aligned} \tag{40}$$

For convenience, let

$$\begin{aligned}a(x) &= \frac{e^{-\frac{\mu x}{2}}}{(1+x)^{\frac{\lambda}{2}}}\log(1+x)\\ b(x) &= \frac{e^{-\frac{\mu x}{2}}}{(1+x)^{\frac{\lambda}{2}}}.\end{aligned}$$

Then, using the Cauchy–Schwarz inequality, it follows

$$(g'(\lambda))^2 = \left(\int_0^\infty a(x)b(x)\mathrm{d}x\right)^2 \leq \left(\int_0^\infty a^2(x)\mathrm{d}x\right)\left(\int_0^\infty b^2(x)\mathrm{d}x\right) = g''(\lambda)g(\lambda). \tag{41}$$

This shows that (38) is true and it completes the proof. □

Appendix B

Proof of Property 3

From (17), using a Taylor series expansion, we can show that

$$\Pr\left(\chi^2_{2L} < \frac{\bar{R}}{\bar{\gamma}_K}\right) = 1 - e^{-\bar{R}/\bar{\gamma}_K} \sum_{l=0}^{L-1} \frac{1}{l!}\left(\frac{\bar{R}}{\bar{\gamma}_K}\right) \quad = \frac{1}{L!}\left(\frac{\bar{R}}{\bar{\gamma}_K}\right)^L + O\left(\left(\frac{\bar{R}}{\bar{\gamma}_K}\right)^{L+1}\right). \quad (42)$$

For a large L, using Stirling's formula, $n! \simeq \sqrt{2\pi n} n^n e^{-n}$, we have

$$\frac{1}{L!}\left(\frac{\bar{R}}{\bar{\gamma}_K}\right)^L \simeq \frac{1}{\sqrt{2\pi L}}\left(\frac{cR}{\bar{\gamma}_K L}\right)^L.$$

This completes the proof. □

Eq. (19) can be obtained by the same approach.

Appendix C

Proof of Lemma 3 : Derivation of the cdf

For convenience, let

$$a_1 = \gamma_1 |h_{1,l}|^2 \quad \text{and} \quad a_2 = \gamma_2 |h_{2,l}|^2.$$

From (21), the cdf of $B_{1,l}$ becomes

$$\begin{aligned} F(x) &= \Pr(B_{1,l} \le x) \\ &= \Pr\left(\frac{a_1}{1+a_2} \le x, \frac{a_2}{1+a_1} < V\right) + \Pr\left(a_1 \le x, \frac{a_2}{1+a_1} \ge V\right). \end{aligned} \quad (43)$$

We can show that

$$\begin{aligned} P_1(x) &= \Pr\left(\frac{a_1}{1+a_2} \le x, \frac{a_2}{1+a_1} < V\right) \\ &= \Pr(a_1 < x, a_2 > V(a_1+1)) \\ &= \int_0^x \int_{V(a_1+1)}^{\infty} f(a_1, a_2) \mathrm{d}a_2 \mathrm{d}a_1, \end{aligned}$$

where $f(a_1, a_2)$ is the joint pdf of a_1 and a_2 which is given as

$$f(a_1, a_2) = \frac{1}{\gamma_1 \gamma_2} e^{-\frac{a_1}{\gamma_1}} e^{-\frac{a_2}{\gamma_2}}.$$

After some manipulations, we can obtain (24).

We can also show that

$$\begin{aligned} P_2(x) &= \Pr\left(a_1 \le x, \frac{a_2}{1+a_1} \ge V\right) \\ &= \Pr\left(\frac{a_2}{V} - 1 \le a_1 < x(a_2+1)\right) \\ &= \int_0^{\infty} \int_{\frac{a_2}{V}-1}^{x(a_2+1)} f(a_1, a_2) \mathrm{d}a_1 \mathrm{d}a_2. \end{aligned}$$

After some manipulations, we can obtain (25). This completes the proof. □

Appendix D

Proof of Property 4

If we have $\gamma_1 = \gamma_2$ and $x = V$, we have

$$C_1 = D(V) = \frac{V+1}{\gamma}, \; C_2 = \frac{V+1}{\gamma V}, \quad \text{and} \quad B(V) = \frac{V}{1-V}.$$

In addition, we can show that

$$P_{\text{orth,out}}(R) = 1 - e^{2^{2R}-1/\gamma} = 1 - e^{-\frac{V(V+2)}{\gamma}}. \tag{44}$$

Firstly, we consider the case of $V \ge 1$. When $V \ge 1$, we have

$$\Pr(I_{1,l} < R) = P_1(V) + P_2(V) = 1 - \frac{1}{V+1}\left(e^{-V/\gamma} + e^{-\frac{V(V+2)}{\gamma}}\right).$$

After some manipulations, we have

$$P_{\text{orth,out}}(R) - \Pr(I_{1,l} < R) = \frac{e^{-V/\gamma}}{V+1}\left(1 - Ve^{-\frac{V(V+1)}{\gamma}}\right) \tag{45}$$

Thus, if

$$\gamma < \frac{V(V+1)}{\log V},$$

we have

$$P_{\text{orth,out}}(R) \ge \Pr(I_{1,l} < R).$$

Secondly, we consider the case of $V < 1$. From (25), we can show that

$$P_1(V) + P_2(V) = \frac{e^{-\frac{V}{\gamma}}}{V+1}\left((V+1) - 2e^{-\frac{V(V+1)}{\gamma}} + e^{-\frac{V(V+1)}{\gamma(1-V)}}\right) - \frac{V}{V+1}e^{-\frac{2V}{\gamma(1-V)}}. \tag{46}$$

After some manipulations, we have

$$\Pr(I_{1,l} < R) - P_1(V) - P_2(V) = \left(1 - e^{-\frac{V}{\gamma}}\right) + \left(\frac{1-V}{1+V}\right)e^{-\frac{V(V+2)}{\gamma(1-V)}}\left(1 - e^{-\frac{V^2(V+1)}{\gamma(1-V)}}\right). \tag{47}$$

When $V \leq 1$, we can see that $1 - e^{-\frac{V}{\gamma}} \geq 0$, $1 - e^{-\frac{V^2(V+1)}{\gamma(1-V)}} \geq 0$, and $\frac{1-V}{1+V} \geq 0$. Thus,

$$P_{\text{orth,out}}(R) \geq \Pr(I_{1,l} < R).$$

This completes the proof. □

Appendix E

Derivation of An Approximate of the Probability Mass Function of D

In order to find the probability mass function of D, we consider the following $K-1$ channel gains of the interfering signals:

$$G_k = \{|h_{2,\bullet}|^2, |h_{3,\bullet}|^2, \ldots, |h_{K,\bullet}|^2\} \tag{48}$$

and denote by α_k the kth largest channel gain:

$$\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_{K-1}. \tag{49}$$

Suppose that the signal which has the largest channel gain is cancelled first as the corresponding SINR is the highest (thus, the probability that this signal can be cancelled is the highest). Let

$$Z_m = \log_2\left(1 + \frac{\gamma\alpha_m}{1 + \gamma(|h_{1,\bullet}|^2 + \sum_{q=m+1}^{K-1} \alpha_q)}\right) \simeq \log_2\left(1 + \frac{\gamma\alpha_k}{1 + \gamma(1 + \sum_{q=m+1}^{K-1} \bar{\alpha}_q)}\right), \tag{50}$$

where $\bar{\alpha}_q = E[\alpha_q]$.

From [30], we can find that

$$\bar{\alpha}_q = \sum_{i=q}^{K-1} \frac{1}{i}.$$

Define the following event:

$$A_m = \{\alpha_m \geq \psi_m\},$$

where

$$\psi_m = \frac{2^R - 1}{\gamma}\left(1 + \gamma\left(1 + \sum_{q=m+1}^{K-1} \bar{\alpha}_q\right)\right).$$

Let

$$U_m = \bigcap_{q=1}^{m} A_q.$$

Then, U_m represents the random event that the first m signals can be cancelled. Then, from U_m, we can find the probability that the m signals are cancelled as follows:

$$\begin{aligned} \Pr(D = K-1) &= \Pr(U_{K-1}) \\ \Pr(D = m) &= \Pr(U_m) - \Pr(U_{m+1}),\ m = 1, 2, \ldots, K-2. \end{aligned} \tag{51}$$

To find the $\Pr(U_m)$'s, the joint pdf of α_m is required:

$$f(\alpha_1, \alpha_2, \ldots, \alpha_{K-1}) = (K-1)! \exp\left(-\sum_{q=1}^{K-1} \alpha_q\right), \ \alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_{K-1}.$$

It follows that

$$\Pr(U_m) = \int_{\psi_1}^{\infty} \int_{\psi_2}^{x_1} \cdots \int_{\psi_m}^{x_{m-1}} f_m(x_1, x_2, \ldots, x_m) \mathrm{d}x_m \ldots \mathrm{d}x_2 \mathrm{d}x_1, \tag{52}$$

where $f_m(x_1, x_2, \ldots, x_m)$ denotes the marginalized pdf which is given as

$$\begin{aligned} f_m(x_1, x_2, \ldots, x_m) &= (K-1)! e^{-(x_1+x_2+\cdots+x_m)} \times \\ &= \int_0^{x_m} \int_0^{x_{m+1}} \cdots \int_0^{x_{K-2}} e^{-(x_{m+1}+x_{m+2}+\cdots+x_{K-1})} \mathrm{d}x_{K-1} \ldots \mathrm{d}x_{m+2} \mathrm{d}x_{m+1}. \end{aligned}$$

Once $\Pr(U_{K-1})$ is obtained as a function of the ψ_m's, we can also easily find $\Pr(U_m)$ using the marginalization. That is, letting

$$\Pr(U_{K-1}) = F(\psi_1, \psi_2, \ldots, \psi_{K-1}),$$

we have

$$\Pr(U_m) = F(\psi_1, \psi_2, \ldots, \psi_m, 0, \ldots, 0), \ m = 1, 2, \ldots, K-2.$$

Although a closed-form expression for $\Pr(U_{K-1})$ is available, it becomes lengthy as K increases. We show some examples when K is small as follows:

- $K = 2$:

$$P(U_1) = e^{-\psi_1}.$$

- $K = 3$:

$$P(U_2) = e^{-\psi_1}(2e^{-\psi_2} - e^{-\psi_1}).$$

- $K = 4$:

$$P(U_3) = e^{-3\psi_1-2\psi_2-\psi_3}(6e^{2\psi_1+\psi_2} - 3e^{\psi_1+2\psi_2} - 3e^{2\psi_1+\psi_3} + e^{2\psi_2+\psi_3}).$$

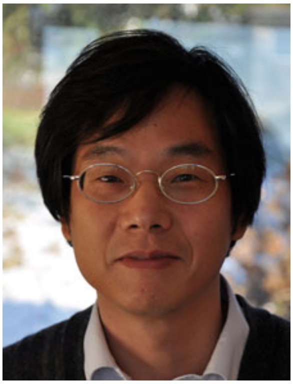
Acknowledgment This work has been supported by EPSRC-DSTL, Grant No. EP/H011919/1.

References

1. IEEE 802.11 Working Group. (1999). *Wireless LAN medium access control (MAC) and physical layer (PHY) specification*.
2. Muqattash, A., & Krunz, M. (2003). CDMA-based MAC protocol for wireless ad hoc networks. In *Proceedings of MobiHoc'03* (pp. 153–164).
3. Al-Meshhadany, T., & Ajib, W. (2007). New CDMA-based MAC protocol for ad hoc networks. In: *Proceedings of IEEE VTC Fall 2007* (pp. 91–95).
4. Cover, T. M., & Thomas, J. A. (1991). *Elements of information theory*. New York: Wiley.
5. Gallager, R. G. (1994). An inequality on the capacity region of multiple access multipath channels. In R. E. Blahut, D. J. Costello, U. Maurer, & T. Mittelholzer (Eds.), *Communications and cryptography: Two sides and one tapestry* (pp. 129–139). Boston: Kluwer.
6. Shamai, S., & Verdu, S. (2001). The impact of frequency-flat fading on the spectral efficiency of CDMA. *IEEE Transactions on Information Theory, 47*, 1302–1327.

7. Guess, T., & Varanasi, M. K. (2005). An information-theoretic framework for deriving canonical decision-feedback receivers in Gaussian channels. *IEEE Trans. Inform. Theory, 51*, 173–187.
8. Forney, G. D., Jr. (2004). Shannon meets Wiener II: On MMSE estimation in successive decoding schemes. In: *Proceedings of the 42nd annual allerton conference communication, control, and computing*. Monticello.
9. Tse, D., & Viswanath, P. (2005). *Fundamentals of wireless communication*. Cambridge: Cambridge University Press.
10. Caire, G., Tunietti, D., & Verdu, S. (2004). Suboptimality of TDMA in the low-power regime. *IEEE Transactions on Information Theory, 50*, 608–620.
11. Verdu, S. (2002). Spectral efficiency in the wide-band regime. *IEEE Transactions on Information Theory, 48*, 1319–1343.
12. Caire, G., Guemghar, S., Roumy, A., & Verdu, S. (2004). Maximizing the spectral efficiency of coded CDMA under successive decoding. *IEEE Transactions on Information Theory, 50*, 152–164.
13. Li, K., Wang, X., & Ping, L. (2007). Analysis and optimization of interleave-division multiple-access communication systems. *IEEE Transactions on Wireless Communications, 6*, 1973–1983.
14. Lin, S., & Costello, D. (2004). *Error control coding, 2nd ed*. Englewood Cliffs: Prentice Hall.
15. Wicker, S. B. (1995). *Error control systems for digital communication and storage*. Englewood Cliffs: Prentice Hall.
16. Hou, F., She, J., Ho, P.-H., & Shen, X. (2010). Performance analysis of the cumulative ARQ in IEEE 802.16 networks. *Wireless Networks, 16*, 559–572.
17. Caire, G., & Tunietti, D. (2001). The throughput of hybrid-ARQ protocols for the Gaussian collision channel. *IEEE Transactions on Information Theory, 47*, 1971–1988.
18. Tunietti, D., & Caire, G. (2002). The throughput of some wireless multiaccess systems. *IEEE Transactions on Information Theory, 48*, 2773–2785.
19. Hagenauer, J. (1988). Rate-compatible punctured convolutional codes (RCPC codes) and their applications. *IEEE Transactions on Communications, 36*, 389–400.
20. Ha, J., Kim, J., & McLaughlin, S. W. (2004). Rate-compatible puncturing of low-density parity-check codes. *IEEE Transactions on Information Theory, 50*, 2824–2836.
21. Cheng, J.-F. (2010). Coding performance of HARQ with BICM Part I: Unified performance analysis. In: *Proceedings of IEEE PIMRC* (pp. 976–981)
22. Biglieri, E., Proakis, J., & Shamai, S. (1998). Fading channels: Information-theoretic and communication aspects. *IEEE Transactions on Information Theory, 44*, 2619–2692.
23. Ozarow, L., Shamai, S., & Wyner, A. D. (1994). Information theoretic considerations for cellular mobile radio. *IEEE Transactions on Vehicular Technology, 43*, 359–378.
24. Viswanath, P., Tse, D. N. C., & Laroia, R. (2002). Opportunistic beamforming using dumb antennas. *IEEE Transactions on Information Theory, 48*, 1277–1294.
25. Shen, C., Liu, T., & Fitz, M. P. (2009). On the average rate performance of hybrid-ARQ in quasi-static fading channels. *IEEE Transactions on Communications, 57*, 3339–3352.
26. Sesia, S., Caire, G., & Vivier, G. (2004). Incremental redundancy hybrid ARQ schemes based on low-density parity-check codes. *IEEE Transactions on Communications, 52*, 1311–1321.
27. Wu, P., & Jindal, N. (2010). Performance of hybrid-ARQ in block-fading channels: A fixed outage probability. *IEEE Transactions on Communications, 58*, 1129–1141.
28. Makki, B., & Eriksson, T. (2010). On the average rate of quasi-static fading channels with ARQ and CSI feedback. *IEEE Communications Letters, 14*, 806–808.
29. Boyd, S., & Vandenberghe, L. (2004). *Convex optimization*. Cambridge: Cambridge University Press.
30. David, H. A. (1981). *Order statistics, 2nd ed*. New York: Wiley-Interscience.

Author Biography



Jinho Choi (Senior Member of IEEE) was born in Seoul, Korea. He received B.E. (magna cum laude) degree in electronics engineering in 1989 from Sogang University, Seoul, and the M.S.E. and Ph.D. degrees in electrical engineering from Korea Advanced Institute of Science and Technology (KAIST), Daejeon, in 1991 and 1994, respectively. He is now with School of Engineering, Swansea University, United Kingdom, as a Professor/Chair of Wireless. His research interests include wireless communications and array/statistical signal processing. He authored two books published by Cambridge University Press in 2006 and 2010. Prof. Choi received the 1999 Best Paper Award for Signal Processing from EURASIP, 2009 Best Paper Award from WPMC (Conference), and is a Senior Member of IEEE. Currently, he is an Editor of Journal of Communications and Networks (JCN) since 2005 and served an Associate Editor of IEEE Transactions on Vehicular Technology from 2005 to 2007 and ETRI journal. Since 2009, he joined the Editorial Board of International Journal of Vehicular Technology.