A Bayesian Game Theoretic Framework for Resource Allocation in Multistatic Radar Networks

Anastasios Deligiannis, Sangarapillai Lambotharan
Wolfson School of Mechanical and Electrical Engineering
Loughborough University, Leicestershire, UK, LE11 3TU
Emails: {A.Deligiannis, S.Lambotharan}@lboro.ac.uk

***Abstract*—We propose a Bayesian game-theoretic SINR maximization technique for a multistatic radar network. We consider a distributed network of radars, where the primary goal of each radar is to maximize their signal to interference plus noise ratio (SINR), within the constraint of its maximum transmission power. We assume no communication between the radars, hence we utilize a noncooperative game-theoretic approach. The channel gain between a radar and the target is assumed to be private information which characterizes the type of the player, whereas the distribution of the channel gain is common knowledge to every player in the game. Subsequently, we examine and prove the existence and the uniqueness of the Bayesian Nash equilibrium (BNE) for the aforementioned game. The simulation results also confirm the convergence of the algorithm to the unique solution.**

I. INTRODUCTION

Distributed radar networks benefit from spatial diversity in terms of radar cross section (RCS) variations, capture of the geometrical characteristics of the target, multiple targets detection, and slow moving targets tracking [1]. Nevertheless, multistatic radar networks suffer from multiple sources of interference induced at the receivers of each radar, namely the cross channel interference generated by other radars in the same network and the clutter interference. This interference seriously deteriorates the performance and the tracking capabilities of the radar system and thus an optimal power allocation strategy that minimizes the interference and maximizes the detection performance is necessary. Game theory is an appropriate and efficient tool to address this issue, as it constitutes a mathematical framework of confrontation and coordination among selfish, intelligent and rational players.

Game theoretic techniques have been utilized recently to address various radar problems. Especially optimal power allocation and distribution in radar networks motivated many authors to utilize different game theoretic techniques. The authors in [2] and [3] addressed the power allocation problem by formulating a non-cooperative game with predefined SINR constraints. Since it is difficult for a radar to obtain information regarding the transmission power of the remaining radars in the network, an SINR estimation technique was applied in [4], to extend the work in [2]. Various game theoretic techniques were applied in [5] to address a distributed beamforming and resource allocation problem for a radar system in the presence of multiple targets. In particular, strategic noncooperative, partially cooperative and Stackelberg game were used to obtain the optimal power allocation and satisfy a certain detection criterion for each of the targets. The authors in [6] exploited cooperative game theoretic techniques to solve the resource allocation problem through maximizing the Bayesian-Fisher information matrix (B-FIM) and utilizing the Shapley value solution. A combination of a water filling algorithm and a Stackelberg game was used in [7] for optimal power distribution. In addition, a noncooperative power allocation game between a multistatic radar network and multiple jammers was presented in [8], together with the proof of the existence and uniqueness of the Nash equilibrium.

In the aforementioned radar literature, the radars have been assumed to have exact knowledge of the channel gain in terms of the RCS parameters of the targets and clutter, which may not be feasible in a real system. In this paper we introduce uncertainty on the channel gains associated with the radars and the targets, which arises due to the RCS fluctuations of the targets. Bayesian game theory provides a framework to address this problem of incomplete information. Therefore, we consider a Bayesian game, where each player egotistically maximizes its SINR, under a predefined power constraint. Within this framework, we assume that each radar/player exactly knows the channel gain between itself and the target as private information, however we include uncertainty on the channel experienced by other radars in the network. Only the distribution of the channel gains is considered as common knowledge to every player and can be obtained by exploiting several target models, such as Swerling or extended-Swerling models, depending on the target's type. This problem is solved using a Bayesian noncooperative game (BNG) framework as considered for communication application in [9]. In order to prove the uniqueness of the BNE of the considered game, we exploit convex geometric programming techniques [10].

II. SYSTEM MODEL

We consider a multistatic radar network, consisting of K widely separated radars. In radar field, a flying target and extended clutter is assumed. Hence, the primary objective of each radar is to maximize the SINR associated with the target, while satisfying the power constraint. In the noncooperative approach of the distributed radar network, each radar performs the optimization selfishly and autonomously, having complete knowledge only of its own channel gain realization as private information. On the other hand, only the distribution of the

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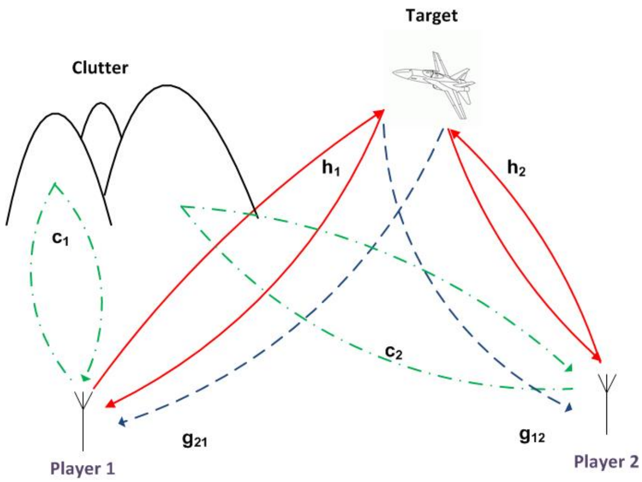

Fig. 1: A multistatic MIMO radar network with two radars, one target and clutter interference.

inter-radar channel gains is available to every radar. Since we assume that all radars belong to the same organization, there is no deliberate interference from any radars, yet we use a non cooperative game theoretic approach as this avoids the need for communication and coordination among radars.

In the presence of a target, the received signal for radar k is obtained by:

$$\mathbf{x}_k = \alpha_k \sqrt{p_k}\mathbf{s}_k + \sum_{j=1, j\neq k}^{K} \beta_{jk}\sqrt{p_j}\mathbf{s}_j + \nu_k \sqrt{p_k}\mathbf{s}_k + \hat{n} \quad (1)$$

where $\mathbf{s}_k = \psi_k \mathbf{a}_k$ describes the transmitted signal from radar k and $\mathbf{a}_k = [1, e^{j2\pi f_{D,k}}, \ldots, e^{j2\pi(N-1)f_{D,k}}]^T$ is the Doppler steering vector of radar k associated with the desired target, $f_{D,k}$ denotes the normalized Doppler shift as seen by radar k, N is the number of signal return samples that the radars receive at each time step and ψ_k corresponds to the predesigned waveform transmitted from radar k. The parameter α_k denotes the desired channel gain at the direction of the target, p_k stands for the transmission power of radar k, β_{jk} describes the cross-channel gain among radars k and j, ν_k and $\hat{n}$ denote the clutter channel gain and a zero-mean white Gaussian noise with variance σ_n^2. There is no direct path interference assumed among the radars of the system. Hence, the SINR for the k^{th} radar is written as:

$$\text{SINR}_{ki} = \frac{h_k p_k}{c_k p_k + \sum_{j=1, j\neq k}^{K} g_{jk} p_j + \sigma_n^2}. \quad (2)$$

where $\alpha_k \sim \mathcal{CN}(0, h_k)$, $\nu_k \sim \mathcal{CN}(0, c_k)$ and $\beta_{jk} \sim \mathcal{CN}(0, g_{jk})$, hence h_k, c_k and g_{jk} describe the variance of the desired channel gain, the clutter channel gain and the cross-channel gain, respectively.

III. Game Theoretic Formulation

In this section, we model the interactions between the K radars in the network as a Bayesian game, in which the main goal for each radar is to maximize its SINR for target detection under a power constraint and uncertainty on channel knowledge. More specifically, the incomplete information in the considered system model reflects the inability of radar k to obtain the exact value of the cross channel gains, i.e. $[g_{1k}, g_{2k}, \ldots, g_{Kk}]$. Nevertheless, since each radar knows the type of the target, then the distribution of the RCS of the target and subsequently the distribution of the cross channel gain is considered as common information. It is clear from the SINR equation (2) that although increased transmission power at a radar strengthens the desired signal, it induces higher cross interference to the remaining radars in the network. Thus, we model the aforementioned interaction as the noncooperative Bayesian game $\mathcal{G} =< \mathcal{R}, \mathcal{T}, \mathcal{P}, \Pi, \mathcal{U} >$, which can be fully characterized as:

- The set of radars is considered to be the player set: $\mathcal{R} = \{R_1, \ldots, R_K\}$.
- The type set is denoted as $\mathcal{T} = \mathcal{T}_1 \times \ldots \times \mathcal{T}_K$, and corresponds to each player's channel gain, i.e. for the case of two types, $\mathcal{T}_K = \{g_-, g_+\}$, where g_- and g_+ represent two possible channel states with $g_- < g_+$.
- The action set of the game is $\mathcal{P} = \mathcal{P}_1 \times \ldots \times \mathcal{P}_K$ with
$$\mathcal{P}_k = \{p_k \in \mathbb{R}^+ \mid p_k \in [0, P_k^{max}]\}, \quad \forall i \in \{1, \ldots, K\}$$
where P_k^{max} denotes the maximum available power for radar R_k.
- The common prior or probability set is defined as $\Pi = \Pi_1 \times \ldots \times \Pi_K$, where Π_k is the probability distribution of the channel gain for radar R_k and hence the distribution of the player's type and it is common knowledge to every player.
- The Bayesian game model is concluded by defining the utility function set as $\mathcal{U} = \{u_1, \ldots, u_K\}$, where u_k represents the k^{th} radar SINR from (2), as $u_k(p_1, \ldots, p_K) = \text{SINR}_{ki}$. It is evident that the utility function is a function of the power allocation of all K players.

In the considered BNG, player k egotistically maximizes its SINR, within the constraint of maximum transmission power, given the transmission power strategies of the remaining players. Therefore, the best response for player k is determined by solving the following optimization problem:

$$\max_{p_k \in \mathcal{P}_k} \mathbb{E}[u_k(p_1, \ldots, p_K)] \quad (3)$$

$$\text{s.t.} \quad \mathbb{E}[p_k] \leq P_k^{max}, p_k > 0$$

The study on the convergence of the game $\mathcal{G}$ to a stable solution is the most critical part of the game theoretic analysis, as it provides the ability to predict the performance and the stability of the distributed radar system under channel uncertainty. This specific solution defines the BNE, where no player could benefit by unilaterally changing its power allocation strategy. Hence, for the considered game $\mathcal{G}$ the BNE describes the action profile $(\mathbf{p}^*_{-k}, p^*_k)$, where $\mathbf{p}_{-k}$ denotes the transmission power adopted by all other players except player *k*, when:

$$\bar{u}_k(\mathbf{p}^*_{-k}, p^*_k) \geq \bar{u}_k(\mathbf{p}^*_{-k}, p_k), \quad \forall \mathbf{p}_k \in \mathcal{P}_k, \forall k \in \mathcal{R}.$$

where $\bar{u}_k$ defines the expected utility for player *k*. The next section presents a mathematical analysis on the existence and the uniqueness of the BNE.

IV. Existence and Uniqueness of the Bayesian Nash Equilibrium

Initially, it is important to underline that for a given set of opponent power strategies $\mathbf{p}_{-k}$, the optimization problem (3) is convex, since the objective is a quasiconcave function and the constraint is a convex set. Therefore, we can reformulate (3) as a standard form convex optimization problem, by changing the sign of the objective function, as follows:

$$\min_{p_k \in \mathcal{P}_k} -\mathbb{E}[u_k(p_1, \ldots, p_K)] \tag{4}$$

$$\text{s.t. } \mathbb{E}[p_k] - P_k^{max} \leq 0 \,, -p_k < 0$$

At this point, we may define the Lagrangian $\mathcal{L}$ corresponding to the convex optimization problem (4) as:

$$\mathcal{L}(p_k, \lambda_1, \lambda_2) = \mathbb{E}\left[-\frac{h_k p_k}{c_k p_k + \sum\limits_{\substack{j=1 \\ j\neq k}}^{K} g_{jk} p_j + \sigma_n^2}\right] + \lambda_1(p_k - P_k^{max}) - \lambda_2 p_k \tag{5}$$

where λ_1 and λ_2 are the Lagrange multipliers associated with the inequality constraints of (4). We assume that $(p_k^*, \lambda_1^*, \lambda_2^*)$ are the primal and dual optimal points of (4). Thus, the Karush-Kuhn-Tucker (KKT) conditions on convexity must be satisfied and we have:

$$\lambda_1^* = \mathbb{E}\left[\frac{h_k \sum\limits_{j=1, j\neq k}^{K} g_{jk} p_j + h_k \sigma_n^2}{\left(c_k p_k + \sum\limits_{j=1, j\neq k}^{K} g_{jk} p_j + \sigma_n^2\right)^2}\right] \tag{6}$$

$$\lambda_1^*(p_k^* - P_k^{max}) = 0 \tag{7}$$

From (6) it is straightforward that the optimal Lagrange multiplier λ_1 is strictly positive. Therefore, from (7) the optimal transmission power for radar *k* is equal to the maximum power constraint, i.e. $p_k^* = P_k^{max}$. However, it is evident from (6) that the optimal solution for radar *k* is a function of the transmission power of all K players, which is not common knowledge. Hence, in order for each player to obtain the optimal power allocation, each radar must optimize its transmission power based on the estimate of all the remaining radars' power allocation. The investigated BNG theoretic framework models exactly this kind of interaction.

A. Existence

The existence of a BNE follows from the result of [11] on abstract economies. According to this result, for our problem a BNE exists as the following hold: for all players $k = 1, \ldots, K$ the set P_k is compact, nonempty and convex, the utility function $u_k(p_{-k}, p_k)$ is continuous on $\mathcal{P}$ and quasi-convex in p_k. For every p_{-k} the set-valued function $\mathcal{P}_k$ is continuous with closed graph and for every p_{-k} the set $\mathcal{P}(p_{-k})$ is non-empty and convex.

B. Uniqueness

One method to prove the uniqueness of the BNE is to verify that the second derivative of the utility function of radar *k* is strictly concave with respect to its action set. Therefore, we utilize geometric programming techniques [10] to prove the uniqueness of the solution, as the following Lemma suggests:

Lemma 1: The Bayesian game $\mathcal{G}$ has a unique solution.

Proof. Following [10], we can maximize a nonzero monomial utility function, by minimizing its inverse. Thus, we restate the best response optimization problem (3) for player *k* following geometric programming techniques as:

$$\min_{p_k \in \mathcal{P}_k} \mathbb{E}\left[(h_k p_k)^{-1}\left(c_k p_k + \sum_{j=1, j\neq k}^{K} g_{jk} p_j + \sigma_n^2\right)\right] \tag{10}$$

$$\text{s.t. } \mathbb{E}[p_k] - P_k^{max} \leq 0, -p_k < 0$$

At this point, by redefining the utility function as $u_k^{'}(\mathbf{p}_{-k}, p_k) = (h_k p_k)^{-1}(c_k p_k + \sum\limits_{j=1, j\neq k}^{K} g_{jk} p_j + \sigma_n^2)$, game $\mathcal{G}$ becomes $\mathcal{G}^{'} = < \mathcal{R}, \mathcal{T}, \mathcal{P}, \Pi, \mathcal{U}^{'} >$, where $\mathcal{U}^{'} = \{u_1^{'}, \ldots, u_K^{'}\}$. Since we have shown from the KKT conditions (6) and (7) that the optimal transmission power is obtained when the power constraint is satisfied with equality, we can define the transmission power when the channel gain is g_+ as $\pi_+ p_k(g_+) = P_k^{max} - \pi_- p_k(g_-)$ in the case when two possible channel states are considered (g_- and g_+, $g_+ > g_-$), where $p_k(g_+)$ and $p_k(g_-)$ denote the power transmitted from radar *k* regarding the higher and lower channel gains, respectively. Hence we define the average utility function $\bar{u}_k^{'}$ as a weighted sum function as:

$$\bar{u}_k^{'}(\mathbf{p}_{-k}, p_k) = \sum_i \phi_i (h_k^i p_k)^{-1}(c_k p_k + \sum_{j=1, j\neq k}^{K} g_{jk}^i p_j + \sigma_n^2) \tag{11}$$

where i stands for the different probability realizations of the channel gains [1], ϕ_i represents the respective probability for

[1] i represents a certain combination of the channel gains associated to each radar.

$$\frac{\partial \bar{u}_k^{'}(\mathbf{p}_{-k}, p_k)}{\partial p_k} = \sum_i \phi_i \left(-(h_k^i)^{-1} p_k^{-2} \sum_{j=1, j\neq k}^{K} g_{jk}^i p_j - (h_k^i)^{-1} p_k^{-2} \sigma_n^2 \right) \tag{8}$$

$$\frac{\partial^2 \bar{u}_k^{'}(\mathbf{p}_{-k}, p_k)}{\partial^2 p_k} = \sum_i \phi_i \left(2(h_k^i)^{-1} p_k^{-3} \sum_{j=1, j\neq k}^{K} g_{jk}^i p_j + 2(h_k^i)^{-1} p_k^{-3} \sigma_n^2 \right) \tag{9}$$

event i, h_k^i and g_{jk}^i denote the desired and cross-channel gains for event i. At this point, we can derive the first and the second derivatives of the utility function of player k with respect to its strategy p_k, as shown in (8) and (9).

It is evident from (9) that the second derivative of the payoff function regarding the k^{th} player is strictly positive $\forall p_k > 0$ and hence the Bayesian game $\mathcal{G}^{'}$ has a unique solution. Consequently, the initial game $\mathcal{G}$ admits a unique BNE. □

V. Simulation Results

In this section, simulation results are presented to validate the theoretical background. We consider a bistatic radar network consisting of two radars and two possible channel states $g_- = 1$ and $g_+ = 4$. There is also a target assumed at the far-field of the radars and substantial clutter, whose gain is set to $c_1 = 0.5$ and $c_2 = 0.3$. We presume that both radars have initial information regarding the target's location and thus each radar exactly knows the channel gain between itself and the target. For the simplicity of the algorithm, we assume that the desired channel gain at the direction of the target has also two possible states, identical to the cross-channel gain states, $h_- = g_- = 1$ and $h_+ = g_+ = 4$. The noise power is set to $\sigma_n^2 = 0.1$. Initially, in Fig. 2 and Fig. 3 we display the convergence of the power allocated to the higher channel gain ($p_k(g_+)$) to the unique solution for two different starting strategies when $\pi_- = \pi_+ = 0.5$. The starting power allocation is set to $p_1(g_+) = 0.5$, $p_2(g_+) = 0.00001$ for the first simulation and $p_1(g_+) = 0.8$, $p_2(g_+) = 0.2$ for the second simulation. It is clear that the proposed Bayesian geometric programming game converges swiftly to the unique solution, regardless the initial strategy of the radars.

In Fig.4 and Fig. 5 we confirm the convergence of the algorithm for different channel gain probabilities, hence we set $\pi_- = 0.25$ and $\pi_+ = 0.75$. Similar to the first example, the convergence is secured, whatever the starting power allocation of the radars. In addition, one can observe that when the belief regarding the higher channel gain is stronger, both players allocate more power to the higher channel gain. This fact is further analyzed in Table 1, where the Bayesian equilibrium for different values of the probability π_+ is displayed along with the SINRs of the two radars. As expected, the higher the belief for g_+, the players transmit with increased power corresponding to the stronger channel and also the SINR for both players is increasing with respect to the confidence of the high channel gain π_+.

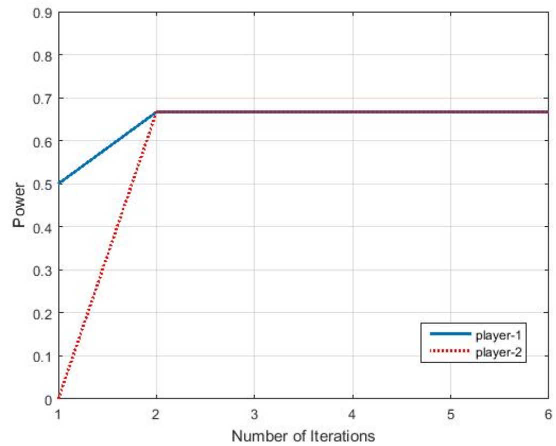

Fig. 2: Convergence of the power allocation corresponding to g_+ for $\pi_- = \pi_+ = 0.5$ when $p_1(g_+) = 0.5$ and $p_2(g_+) = 0.00001$.

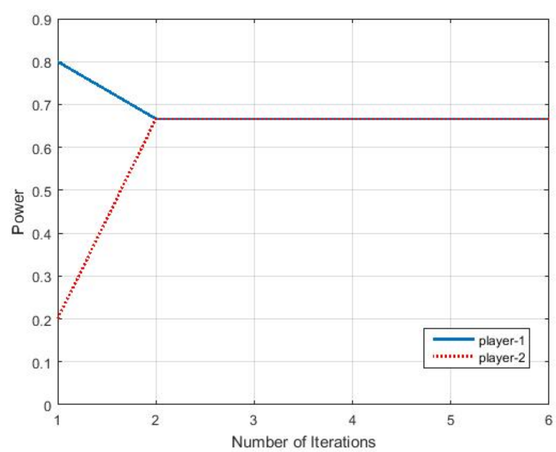

Fig. 3: Convergence of the power allocation corresponding to g_+ for $\pi_- = \pi_+ = 0.5$ when $p_1(g_+) = 0.8$ and $p_2(g_+) = 0.2$.

In Fig. 6, we highlight the importance of the prior belief of a player regarding the channel gains on the resulting power allocation, by studying the convergence of the power allocated to g_+ from player 2 for different high channel gain probabilities π_+. As the belief for a better channel gain gets more robust, the player is more confident of deciding a mixed strategy, where the transmission power is increased. On the

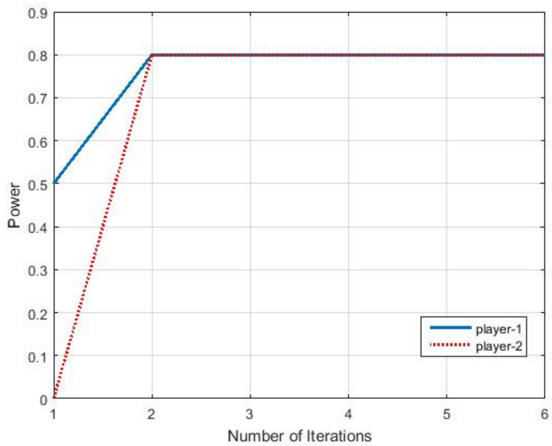

Fig. 4: Convergence of the power allocation corresponding to g_+ for $\pi_- = 0.25$, $\pi_+ = 0.75$ when $p_1(g_+) = 0.5$ and $p_2(g_+) = 0.00001$.

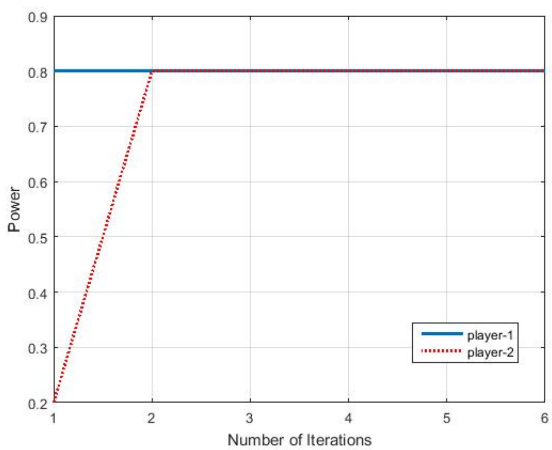

Fig. 5: Convergence of the power allocation corresponding to g_+ for $\pi_- = 0.25$, $\pi_+ = 0.75$ when $p_1(g_+) = 0.8$ and $p_2(g_+) = 0.2$.

other hand, when the aforementioned probability gets slimmer, the transmission power is restrained, as a worse channel gain is more probable.

VI. Conclusion

We investigated a Bayesian game theoretic SINR maximization and resource allocation technique within a distributed radar network. The radars are considered to have private information only about their own channel gains and only the distribution of the other radars' channel gains is assumed to be public knowledge. Initially, we modeled the interactions within the aforementioned multistatic network as a Bayesian game and then we presented a proof of the existence and uniqueness of the Bayesian Nash equilibrium. The simulation results also demonstrated the convergence to the unique solution, regardless the initial resource allocation strategy of the players. Furthermore, it was shown that the higher the confidence of a player regarding a better channel gain associated with the remaining players the higher the SINR and the transmission power of this player.

TABLE I: Bayesian Nash equilibrium and SINRs for the two players for different values of π_+.

Probability π_+	**0.1**	**0.5**	**0.75**
BNE	(0.5263,0.5263)	(0.6667,06667)	(0.8000,0.8000)
SINR 1	0.7830	0.8886	0.9009
SINR 2	0.8442	0.9493	0.9540

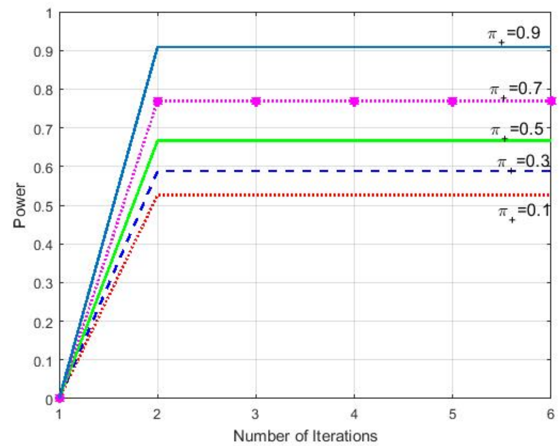

Fig. 6: Transmission power ($p_2(g_+)$) convergence for player 2 for different channel gain probabilities π_+.

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