Non-cooperative target localisation using rank based EDM approach

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***Abstract*—An analysis of the effect of the number of sensors on the non-cooperative target node localisation is presented. This work examines the target localisation using a centralised range based approach. In doing this, the work leverages an algorithm based on a class of matrix structure called Euclidean distance matrices (EDMs) for the specific purpose of improving localisation performance when the fusion centre (FC) cannot receive certain sensors' information due to fading or shadowing, etc. While this interesting approach to the problem of localisation has been found to be successful, it is also shown at high delay values the proposed alternating rank-based EDM algorithm outperforms the conventional linear least squares (LLS) based algorithm for the minimum number of sensors. The localisation error decreases when the conditioning of EDM is better, i.e., when the sensors are further apart from each other and closer to the target.**

I. Introduction

In a target localisation problem, it is often necessary to localise radio frequency (RF) emissions from radio communications systems to provide a greater level of situational awareness for units in an operational area. Techniques to perform localisation can be divided into various categories. Range based techniques rely on measurements of distances between nodes that are often based on received signal strength (RSS) [1], time of arrival (ToA) [2], and time difference of arrival (TDoA) [3], [4]. Range free techniques [5], on the other hand, rely on knowledge of connectivity possibilities, i.e., who is connected to whom, to ascertain locations. Furthermore, localisation can adopt a distributed approach, where the sensors themselves perform many calculations and analysis to locate the target or a centralised approach, where sensors perform limited analysis. In this case, there is a greater instance of sensors simply forwarding information to a central base station or fusion centre (FC), which would in turn locate the target. The wireless sensor network (WSN) configuration that is explored in this work is a Mobile Ad-Hoc Sensor Network (MASNET), which is a particular class of WSN [6], [7] that consists of a large number of inexpensive sensor nodes that are distributed over a large region. These sensor nodes pass their sensor data such as the sensor node identity and time information to FC to locate the target node. Each sensor node may be equipped with multiple sensors along with limited processing and wireless communications capabilities. Although the transmission range of individual sensor nodes is limited, they can communicate over long distances using multihop wireless transmissions within a network.

In this work, the range based information that is transmitted to the FC from the MASNET sensors are ToA estimates and sensor position information. The FC will then perform two initial analytical tasks, which will eventually achieve localisation of the RF emisiions. These are:

- Translate the ToA estimates and sensor position information into distances.
- Attempt to tabulate these distances into an Euclidian distance matrix (EDM) structure.

EDMs are matrices of the squared distances between points, (i.e., distances between sensor and target nodes, and distances between sensor nodes) and due to the fact that they have a certain structure, they have many useful properties and applications such as crystallography, wireless sensor networks, acoustics, etc [8]. The use of EDMs leads to two more key analytical steps:

- Given a matrix with noisy data, test if its structure is in fact EDM or not.
- Given an incomplete set of distances, determine whether a configuration of points exists that generates a matrix that is EDM.

These last two steps are in fact two fundamental problems associated with EDM completion and denoising problems. Assuming that an attempt at creating an EDM has been made in the initial first two steps, it is necessary to examine it carefully to verify if it is indeed EDM and also try to exploit EDM structure to compensate for sensors that may be hidden from the FC due to fading or shadowing, etc.

Recent approaches to localisation involving EDM structure in [8]–[11] have only focused on 2 dimensional position information and to the best of the authors' knowledge, EDM localisation has never been undertaken in 3 dimensional space. In [8], the fundamental properties of EDM were reviewed and algorithms for denoising and completing distance data were shown. While a cooperative joint synchronization and localisation algorithm in ad-hoc networks was proposed in

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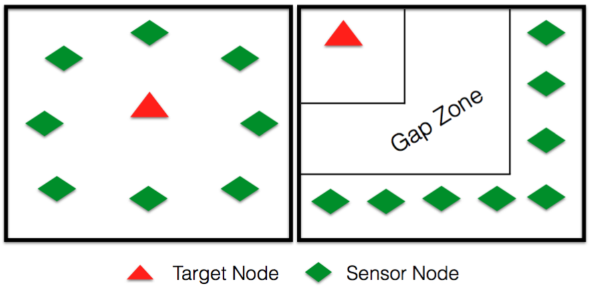

Fig. 1: System model for two different scenarios mixed (LHS) and separated (RHS).

[9], the complexity of the approach was prohibitive. The main focus of this paper is the performance of sensor links in the context of non-cooperative enemy targets and the ability to localise such targets based on noisy sensor information. The entire analysis will be supported by WINNER II channel models [12], [13].

II. Problem Formulation

Two proposed scenarios are shown in Fig. 1. One is a mixed scenario where the target and sensor nodes are in the same region and the other is a separated scenario, where the target node and sensor nodes are separated by a gap zone. It is assumed that the location of the sensor nodes is known as well as the distances between them. Once the target starts to transmit RF signals, the sensor nodes acquire the time information of the received signal. The time information is then transmitted to a FC, which converts this information into corresponding distances to localise the target node. Since 3-dimensional space is considered, at least 4 sensor nodes are needed to localise the target node. There are two sets of nodes, where the first set is a single target node ($m = 1$) and the second set comprises the s sensor nodes. This problem is similar to that in [8], thus the structure of EDM may be defined here as:

$$\mathbf{D} = \begin{bmatrix} \mathrm{edm}(\mathbf{T}) & \mathrm{edm}(\mathbf{T},\mathbf{S}) \\ \mathrm{edm}(\mathbf{S},\mathbf{T}) & \mathrm{edm}(\mathbf{S}) \end{bmatrix}, \tag{1}$$

where the definitions are given in Table I. Since there is only one target, so there is no squared distances between target nodes, i.e. $\mathrm{edm}(\mathbf{T}) = 0$. The squared distances between the sensor nodes $\mathrm{edm}(\mathbf{S})$ are known since it is assumed that the sensor nodes' locations are known. This is another advantage of rank-based EDM approach, where the distances between sensors and their locations act as reference and additional information. According to the symmetric property of an EDM: $\mathrm{edm}(\mathbf{T},\mathbf{S}) = \mathrm{edm}(\mathbf{S},\mathbf{T})$, which are the squared distances between the target and sensor nodes measured using the time information of received signal at each sensor, the measured distances are subject to noise and delay due to multipath components. Following the approach in [8], a mask matrix $\mathbf{W}$ is defined as follows:

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_{\mathbf{T}} & \mathbf{W}_{\mathbf{T},\mathbf{S}} \\ \mathbf{W}_{\mathbf{S},\mathbf{T}} & \mathbf{W}_{\mathbf{S}}, \end{bmatrix} \tag{2}$$

TABLE I: Notations and Definitions

Notation	Meaning
d	Dimensional space
$\mathrm{edm}(\mathbf{D})$	EDM created from target nodes
$\mathrm{edm}(\mathbf{S})$	EDM created from sensor nodes
$\mathrm{edm}(\mathbf{S},\mathbf{T})$	EDM created from sensor nodes and target node
ϕ	Initial value for unobserved distances
$\mathbf{W}$	Mask matrix
$\mathbf{D}_{\mathbf{W}}$	Restriction of $\mathbf{D}$ to non-zero entries in $\mathbf{W}$
$\mathbf{J}$	Geometric Centering Matrix
$\mathbf{G}$	Gram Matrix, $\mathbf{G} = \mathbf{T}^T\mathbf{S}$
$\mathbf{1}$	Column vector of all ones
$\mathbf{U}$	Eigenvectors Matrix
$\hat{\mathbf{X}}$	Estimated nodes position matrix
$\mathrm{diag}(\mathbf{G})$	Column vector of the diagonal entries of $\mathbf{G}$
$\lVert . \rVert_F$	Frobenius Norm

where the elements in $\mathbf{W}_{\mathbf{T},\mathbf{S}}$ and $\mathbf{W}_{\mathbf{S},\mathbf{T}}$ are defined as:

$$w_{i,j} = \begin{cases} 1, & \text{if} \quad (m,j) \in K \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Here m is the index of target node, j is the index of sensor nodes (i.e. $j = 1, 2, ...s$), and K is the set of observed distances between target and sensor nodes. Furthermore, the indexing $\mathbf{D}_{\mathbf{W}}$ is defined as the restriction of $\mathbf{D}$ to the entries where $\mathbf{W}$ is non-zero as stated in Table I. Thus the goal of the mask matrix, $\mathbf{W}$, is simply to differentiate between the observed and unobserved entries in $\mathbf{D}$.

III. Alternating Rank Based EDM Algorithm

According to the rank property established in Theorem 1 in [8], the rank of an EDM corresponding to points in $\mathcal{R}^d$ is at most $d+2$, where d is defined in Table I. In other words, the rank of an EDM does not depend on the number of points generating it. The number of sensor nodes s can be in hundreds or thousands, while d is three or less. This EDM rank property can be exploited to develop tractable algorithms for EDM completion and denoising problems. Using the two pieces of information available, which are a subset of potentially noisy distances and the desired embedding dimension, the relative position of the target node can be calculated. This is done by alternating between these two properties in the hope that the algorithm converges to an EDM from the produced sequence of matrices. The approach is shown in detail in Algorithm 1 and appropriate definitions may be seen in Table I. $\mathbf{E}_{\mathbf{W}} \leftarrow \mathbf{D}_{\mathbf{W}}$ means assigning the observed part of $\mathbf{D}$ to the observed part of $\mathbf{E}$.

To ensure convergence to a matrix that is EDM, rank thresholding, i.e., truncating the eigenvalues $[\lambda]_{i=1}^{k} = \lambda_1, ...\lambda_d, 0, ...0$, where k is the number of target and sensor nodes, is performed on the Gram matrix, $\mathbf{G}$ which is defined in Table I, for centered locations. A linear map from a $\mathbf{G}$ to an EDM is given in step 8 in Algorithm 1 [14]. After computing EDM $\mathbf{E}$, the next stage is to apply a multidimensional scaling (MDS) algorithm in order to determine the best point set representation of a given set of distances. Note that the positions resulting from MDS are relative positions, which may be rotated, or translated.

Next a procrustes analysis [15] is applied, which is the problem of finding the optimum alteration that happened to the nodes. Since the initial locations of sensor nodes are known, these locations are compared to the new locations that were recomputed after the convergence of the algorithm. The details of this analysis can be found in [8] [Section II (C)]. After computing the appropriate rotation matrix and translation vectors, the initial insight into the position target node can be altered based on this change to provide a more appropriate estimate for the target node location.

Algorithm 1 Alternating Rank Based EDM Algorithm

Inputs: D, W, d, max.tolerance
Output: E

Initialization and Definitions:
$\mathbf{D}_{\mathbf{1}\mathbf{1}^T-\mathbf{W}} \leftarrow \phi$ Initialize unknown entries
$\mathbf{J} \leftarrow \mathbf{I} - \frac{1}{k}\mathbf{1}\mathbf{1}^T$ Define Geometric centering matrix
repeat
  $\mathbf{G} \leftarrow \frac{-1}{2}\mathbf{JDJ}$ Compute Gram Matrix
  $\mathbf{U}, [\lambda]_{i=1}^{k} \leftarrow \mathrm{EVD}(\mathbf{G})$ Eigenvalue Decomposition
  $\sum \leftarrow diag(\lambda_1, ...\lambda_d, 0, ...0)$
  Compute $\mathbf{G} \leftarrow \mathbf{U}\sum\mathbf{U}^T$
  Compute $\mathbf{E} = \mathrm{diag}(\mathbf{G})\mathbf{1}^T - 2\mathbf{G} + \mathbf{1}diag(\mathbf{G})$
  Compute $e_1 = ||\mathbf{E} - \mathbf{D}||_F$
  $\mathbf{E}_\mathbf{W} \leftarrow \mathbf{D}_\mathbf{W}$ Enforce known entries
  $\mathbf{E}_\mathbf{I} \leftarrow 0$ Set Diagonal to zero
  $(\mathbf{E})_- \leftarrow 0$ Assign zeros to the negative entries
  Compute $e_2 = ||\mathbf{E} - \mathbf{D}||_F$
  if $(e_1 <$ max. tolerance$) \vee (e_2 <$ max.tolerance$)$ **then**
    return E
  else
    $\mathbf{D} \leftarrow \mathbf{E}$
  end if
until Convergence or MaxIter
$\widehat{\mathbf{X}} = \sum^{1/2}\mathbf{U}^T$

IV. Results

In this section, we present a comparison of localisation error for using all ten sensors with that obtained using four randomly picked sensors out of the available ten. We assume that all the sensors detected the transmitted signal from the target, i.e., there is no missing information. The height of the target node is set to 5 m and the heights of the sensor nodes are varying between 0 and 4 m. The sensors and target nodes are randomly distributed in an area of 200×200 m^2 for the mixed scenario. For the separated scenario, the target node is randomly located in an area of 20×20 m^2, whereas the sensors are divided into two equal perpendicular rectangular areas of 200×20 m^2 and 20×180 m^2 with a gap zone in between as shown in Fig. 1. The position of target and the ten sensors nodes are randomly chosen and fixed throughout the simulation. The delay parameter, μ is the mean of the exponential random variable in [12], [13]. The amount of added delay can be

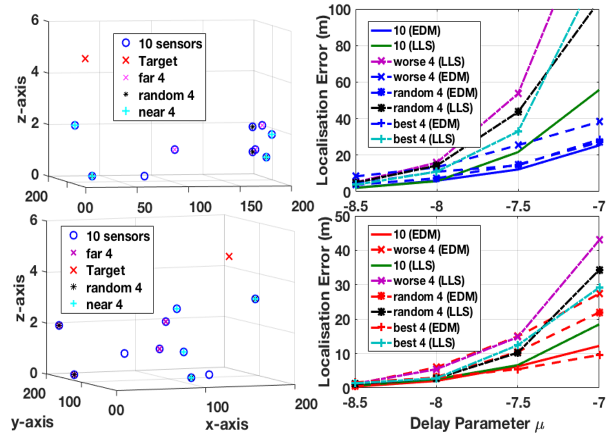

Fig. 2: Localisation error vs μ for the two scenarios: separated (top) and mixed (bottom), with different number of sensors ($N_s = 4, 10$). The solid lines are for the ten sensors and the dotted lines are for two different picks of four sensors.

treated as a positive bias to the theoretical time of flight that a signal would take to travel between the transmitter and the receiver if they were in free space. Hence, it represents noise included in the time information measurements. For example, if $\mu = -8.5$, it is equivalent to $10^{-8.5} \approx 3ns$ and if $\mu = -7$, it is equivalent to $10^{-7} \approx 100ns$. Consequently, low values of μ translates to low mean values of additional random time delays and therefore less noise and vice versa for high values of μ. In a three-dimensional space, the minimum number of sensors required to localise the target is four. The total number of available sensors is ten. A comparison between the proposed alternating rank-based EDM algorithm and the conventional method based on linear least squares (LLS) [16] is indicated in Fig. 2 and Fig. 3.

The first set in Fig. 2 compares the localisation error over four randomly picked sensors and when all the ten sensors are used, in terms of average location error, i.e., error between the estimated and actual target node position. First, we observe that the localisation errors for the separated scenarios are larger than mixed scenarios. This is because the sensors in the separated scenarios experience similar receiving time information, hence calculate similar distances. This leads to an EDM which is not well-separated in terms of distances and hence not well-conditioned [17]. However, in the mixed scenarios, the sensors experience varying receiving times and corresponding distances, improving the conditioning of the EDM, leading to better performance [17].

Moreover, for smaller values of μ (low noise), both the proposed alternating rank-based EDM algorithm and the conventional LLS algorithm with the minimum number of sensors (four), achieve localisation errors closer to the ten sensor results. However, the performance gap between using all ten sensors and only four sensors, is prominent for higher values for μ. This is because more information (obtained using all ten sensors) is beneficial at higher noise levels. Also, there are

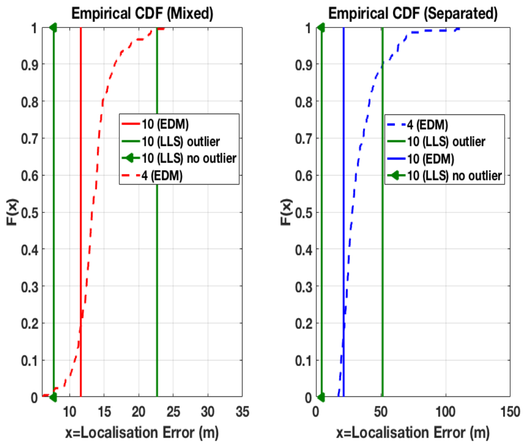

Fig. 3: CDF plot for the random choice of four sensors from the ten sensors in the mixed (left) and separated (right) scenarios with high delays $\mu = -7$.

some rare cases, such as the case using the best four sensors in high noise, where four sensors perform better than ten using the proposed algorithm. This occurs when these sensors are very close to the target and experience smaller overall delays. As the amount of noise increases, the proposed alternating rank-based EDM algorithm outperforms the corresponding LLS algorithm. The high noise levels result in large deviation of the measured distances from the expected value, thereby causing outliers in the system. These outliers in turn degrade the performance of the conventional LLS algorithm, resulting in large errors in the estimated target node position. This behaviour is analysed in detail in Fig. 3.

In Fig. 3, the cumulative distributive function (CDF) of the localisation error is plotted for the proposed alternating rank-based EDM algorithm evaluated using all possible combinations of four out of ten sensors. The figure also shows the LLS performance using all ten sensors with and without outliers. These results confirm the observations from Fig. 2, that for approximately 90% of the combinations, the proposed algorithm outperforms the LLS algorithm with outliers at high delay $\mu = -7$. However, the LLS algorithm without outliers performs better than the proposed algorithm. This indicates that the proposed alternating rank-based EDM algorithm is more robust towards handling outliers than the conventional LLS algorithm.

V. Conclusion

In this work, an analysis of the effect of the number of sensors on the target node localisation was presented. It was shown that at high delay values the proposed alternating rank-based EDM algorithm outperformed the conventional LLS algorithm for the minimum number of sensors. The results indicate that the localisation error depends on the amount of delay and the operating scenario for the sensors (separated or mixed). The sensor nodes in the separated scenario experience similar receiving time information which results in larger localisation error than in the mixed scenario where the receiving times are much different. Also, the error decreases when the conditioning of EDM is better, i.e., when the sensors are further apart from each other and closer to the target. Furthermore, unlike the LLS algorithm, the proposed algorithm is less sensitive to outliers resulting from larger errors in the measured distances, thereby making it suitable for the required application.

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References

[1] Y. I. Wu, H. Wang, and X. Zheng, "WSN localisation using RSS in three-dimensional space; A geometric method with closed-form solution," *IEEE Sensors J.*, vol. 16, no. 11, pp. 4397–4404, Jun. 2016.

[2] Z. Ma and K. C. Ho, "ToA localisation in the presence of random sensor position errors," in *IEEE Int. Conf. Acoust., Speech and Signal Process. (ICASSP)*, 2011, pp. 2468–2471.

[3] P. Singh and S. Agrawal, "TDoA based node localisation in wsn using neural networks," in *Int. Conf. Commun. Syst. and Network Technologies*, 2013, pp. 400–404.

[4] A. Beck, P. Stoica, and J. Li, "Exact and approximate solutions of source localisation problems," *IEEE Transactions on Signal Processing*, vol. 56, no. 5, pp. 1770–1778, May 2008.

[5] L. Gui, A. Wei, and T. Val, "A range-free localization protocol for wireless sensor networks," in *Int. Symp. Wireless Commun. Syst. (ISWCS), Paris*, 2012, pp. 496–500.

[6] Y. Xu and H. Qi, "Performance evaluation of distributed computing paradigms in mobile ad hoc sensor networks," in *9th Int. Conf. Proc. Parallel and Distributed Syst.*, 2002, pp. 451–456.

[7] J. Teng, R. Zhou, and Y. Zhang, "Cooperative simultaneous localisation and tracking for mobile ad-hoc sensor networks," in *23rd European Sig. Process. Conf. (EUSIPCO)*, 2015, pp. 2152–2156.

[8] I. Dokmanic, R. Parhizkar, J. Ranieri, and M. Vetterli, "Euclidean distance matrices: Essential theory, algorithms, and applications," *IEEE Signal Processing Mag.*, vol. 32, no. 6, pp. 12–30, Nov. 2015.

[9] H. Naseri and V. Koivunen, "Cooperative joint synchronization and localization using time delay measurements," in *IEEE Int. Conf. Acoust., Speech and Signal Process. (ICASSP)*, 2016, pp. 3146–3150.

[10] P. Drineas, A. Javed, M. Magdon-Ismail, G. Pandurangant, R. Virrankoski, and A. Savvides, "Distance matrix reconstruction from incomplete distance information for sensor network localization," in *3rd Annu. IEEE Commun. Soc. on Sensor and Ad Hoc Commun. and Networks*, 2006, pp. 536–544.

[11] L. Nguyen, S. Kim, and B. Shim, "Localization in internet of things network: Matrix completion approach," in *Inform. Theory and Applicat. Workshop (ITA)*, 2016, pp. 1–4.

[12] *WINNER* **II** *Channel Models Part* **I**. Deliverable D1.1.2 Radio Channel Models, ver 1.2, 30.9.2007.

[13] *WINNER* **II** *Channel Models Part* **II**. Deliverable D1.1.2 Radio Channel Measurement and Analysis Results, ver 1.1, 30.9.2007.

[14] J. Dattorro, *Convex Optimization and Euclidean Distance Geometry*. Meboo Publishing, 2005.

[15] P. H. Schnenmann, *A Solution of the Orthogonal Procrustes Problem with Applications to Orthogonal and Oblique Rotation*. Ph.D. dissertation, Univ. Illinois, Urbana-Champaign, 1964.

[16] G. S. Kuruoglu, M. Erol, and S. Oktug, "Localization in wireless sensor networks with range measurement errors," in *2009 Fifth Advanced International Conference on Telecommunications*, May 2009, pp. 261–266.

[17] K. Ball, "Eigenvalues of euclidean distance matrices," *Journal of Approximation Theory*, vol. 68, no. 1, pp. 74–82, 1992.