Robust Cooperative Navigation for AUVs using the Student's t Distribution

Qian Li
School of Electrical Engineering and Automation
Harbin Institute of Technology
Heilongjiang Province, China
Email: liqianheu@163.com

Syed Mohsen Naqvi, Jeff Neasham, and Jonathon Chambers
School of Electrical and Electronic Engineering
Newcastle University
Newcastle Upon Tyne, UK
Email: {Mohsen.Naqvi, Jeff.Neasham, Jonathon.Chambers}@ncl.ac.uk

***Abstract*—In this paper, we propose a robust Student's t filtering algorithm designed for Cooperative Navigation (CN) for Autonomous Underwater Vehicles (AUVs). In a CN system, an acoustic communication technique is usually used to exchange information and measure range between AUVs, and many cheap but low-accuracy Micro-Electro-Mechanical Systems (MEMS)-based Inertial Measurement Units (IMUs) are used as Dead-Reckoning (DR) sensors on AUVs. The use of unreliable sensors and an acoustic communication technique can induce outliers leading to the probability densities of process and measurement noise having a heavier-tailed behavior than a Gaussian distribution. To cope with such non-Gaussian distributions, the process and measurement noises are modeled as Student's t distributions, and the Student's t filtering algorithm for CN is presented. Simulation results show the efficiency and superiority of the proposed robust CN algorithm as compared with the standard extended Kalman filtering-based CN algorithm.**

***Index Terms*—AUV, cooperative navigation, acoustic communication, Student's t distribution**

I. Introduction

Accurate navigation is a vital enabler for the operation of an Autonomous Underwater Vehicle (AUV) and it is also essential to improve the efficiency of AUV missions. Without a Global Navigation Satellite System (GNSS) signal as an external reference, an AUV has to rely on proprioceptive information, obtained through a compass, a Doppler Velocity Log (DVL) or an Inertial Navigation System (INS), and a pressure depth sensor to calculate a Dead-Reckoning (DR) navigation solution.

Recently, multiple AUV deployments are becoming more common as the technologies upon which the individual AUV relies become more stable and the acoustic communications technique that they use to share commands and information are standardized. Aiming at the deployment of multiple AUVs, Cooperative Navigation (CN) is a viable option for high-accuracy underwater navigation of AUVs. In a CN system, a fleet of AUVs exchange relative position measurements and their motion information to collectively estimate their states. The study indicates that the exchange of positioning information benefits all vehicles. Furthermore, if absolute geo reference information could be provided to one of the vehicles, the states of vehicles performing CN are observable in a connected Relative Position Measurement Graph (RPMG) [1]. Such increase in navigation accuracy is a major benefit to CN, and its advantages also include sensor coverage, robustness and flexibility, and thus it remains an active area of research.

Many CN algorithms which could make a consistent and accurate estimation of the positions of the full fleet of vehicles have been proposed [2]. Estimation algorithms and techniques such as the extended Kalman filter (EKF) [3], minimum mean-square estimator (MMSE) [4], and particle filter [5] have been used to enhance CN. Particularly, some CN algorithms have been proposed to cope with the problem of low data rates and high latency in underwater acoustic communication for AUV CN [6]. However, most approaches proposed assume that the process and measurement noises admit a Gaussian distribution. In fact, the low-accuracy and unreliable sensors present on slave AUVs, such as a Micro-Electro-Mechanical Systems (MEMS)-based Inertial Measurement Unit (IMU), together with the utilization of underwater acoustic communication usually induceS outliers [7], which will violate the Gaussian assumption of the process and measurement noises.

Fortunately, such noise including outliers can be modeled by a heavy-tailed Student's t distribution. Some Student's t filters have been studied in the signal processing community. A robust Student's t filter for a linear system is derived by approximating the posterior probability density function (pdf) as a Student's t distribution [8], and the process and measurement noise are assumed to have Student's t distributions. Aiming at nonlinear estimation, Huang et al. proposed a third-degree Student's t spherical-radial cubature rule, and developed a new robust Student's t based cubature filter to cope with nonlinear estimation [9]. In addition, nonlinear systems with additive noise, Gaussian state transitions, and Student's t observation noise are studied in [10] where a variational Bayes approach is employed.

In this paper, the process and measurement noise contaminated by the outliers from unreliable MEMS-based IMU and the underwater acoustic range measurements are modeled by a Student's t distribution, and a new robust CN algorithm for multiple AUVs based on a Student's t distribution is proposed.

II. System Model and Noise Analysis

The navigation reference frame (x, y, z) is defined as a local-level frame with three axes pointing east, north and up, respectively. As the depth of the AUV can be accurately measured with an on-board depth sensor, the 3D navigation problem is thereby converted into the 2D navigation problem.

A. System Model

The discrete-time kinematic equations on the x-y horizontal plane for the i-th AUV of a fleet of AUVs are:

$$\begin{cases} x_{k+1}^i = x_k^i + t \cdot V_k^i \cdot \cos\left(\theta_k^i\right) \\ y_{k+1}^i = y_k^i + t \cdot V_k^i \cdot \sin\left(\theta_k^i\right) \\ \theta_{k+1}^i = \theta_k^i + t \cdot \omega_k^i \end{cases} \tag{1}$$

where the elements of $\mathbf{X}_k^i = \left[x_k^i, y_k^i, \theta_k^i\right]^T$ denote the east and north components of the i-th AUV location together with its heading angle at time step k. In addition, the fore-and-aft velocity V_k^i and rotational velocity ω_k^i constitute the control input vector $\mathbf{u}_k^i = \left[V_k^i, \omega_k^i\right]^T$, and t is the DR sampling period.

To facilitate the discussion, a typical framework of master-slave CN for three AUVs is considered in this paper. In this configuration, A and B are two slave AUVs, which are both equipped with low-cost and low-accuracy compass and speed sensor. To bound the navigation error of the slave AUVs, a master AUV (C AUV) with an expensive and accurate navigation suite is included in the CN system. With the information about ranges to the master AUV and the accurate position of master AUV, the unbounded navigation errors of the slave AUVs are corrected by a filter technique.

The state vector for the entire system is defined as the stacked vector comprised of the positions and heading angles of the three AUVs. Considering the discrete-time kinematic equations for the i-th AUV, the CN dynamic model is as below:

$$\mathbf{X}_{k+1} = \mathbf{f}\left(\mathbf{X}_k, \mathbf{u}_k, \mathbf{w}_k\right) = \mathbf{X}_k + t \cdot \mathbf{G}_k\left(\mathbf{u}_k + \mathbf{w}_k\right) \tag{2}$$

where $\mathbf{w}_k$ is the process noise, and $\mathbf{G}_k$ is the control input matrix including all control input matrices of the three AUVs. The i-th control input matrix is as below:

$$\mathbf{G}_k^i = \begin{bmatrix} \cos\theta_k^i & 0 \\ \sin\theta_k^i & 0 \\ 0 & 1 \end{bmatrix}, \quad i = (A, B, C) \tag{3}$$

It can be found that the process equation is nonlinear and it is necessary to provide the partial derivative matrices to be applied in Section III. The partial derivative matrices $\mathbf{F}_{\mathbf{X}_k^i} = \frac{\partial \mathbf{f}}{\partial \mathbf{X}_k^i}$, $\mathbf{F}_{\mathbf{u}_k^i} = \frac{\partial \mathbf{f}}{\partial \mathbf{u}_k^i}$ of the i-th AUV are as below:

$$\mathbf{F}_{\mathbf{X}_k^i} = \begin{bmatrix} 1 & 0 & -t \cdot \tilde{V}_k^i \cdot \sin\tilde{\theta}_k^i \\ 0 & 1 & t \cdot \tilde{V}_k^i \cdot \cos\tilde{\theta}_k^i \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{F}_{\mathbf{u}_k^i} = t \cdot \begin{bmatrix} \cos\tilde{\theta}_k^i & 0 \\ \sin\tilde{\theta}_k^i & 0 \\ 0 & 1 \end{bmatrix} \tag{4}$$

where the symbol $(\tilde{\cdot})$ denotes the measurement data from the DR sensors.

The measurement information is the accurate position of the C AUV and the ranges between A/B AUV and the C AUV. The range between A AUV and C AUV is calculated as below:

$$z_k^{C \leftrightarrow A} = \sqrt{\left(x_k^C - x_k^A\right)^2 + \left(y_k^C - y_k^A\right)^2} + v_k \tag{5}$$

where v_k is the measurement noise.

Similarly, the measurement equation is nonlinear and the linearized measurement matrix is represented by:

$$\mathbf{H}_k^{C \leftrightarrow A} = \begin{bmatrix} \frac{\hat{x}_k^A - \hat{x}_k^C}{r_{AC}} & \frac{\hat{y}_k^A - \hat{y}_k^C}{r_{AC}} & \mathbf{0}_{1\times 4} & \frac{\hat{x}_k^C - \hat{x}_k^A}{r_{AC}} & \frac{\hat{y}_k^C - \hat{y}_k^A}{r_{AC}} & 0 \end{bmatrix} \tag{6}$$

where $r_{AC} = \sqrt{\left(\hat{x}_k^C - \hat{x}_k^A\right)^2 + \left(\hat{y}_k^C - \hat{y}_k^A\right)^2}$, and the symbol $(\hat{\cdot})$ denotes the estimated value.

The derivation of the measurement matrix describing the range between the B and C AUV is similar and it is omitted.

B. Distribution of Process and Measurement Noise

Most algorithms proposed for CN of multiple AUVs assume that the states and measurements are distributed normally around the true mean, namely the Gaussian distribution. However, for the underwater acoustic communication system, which is the main communication method underwater, this is most certainly not the case. Signal reflections from the surface of the water as well as from temperature or salinity discontinuities within the water column itself lead to outliers existing in the acoustic measurement data. In addition to the outliers existing in the measurement data, due to the unreliability of the low-accuracy MEMS-based IMU, the output of the MEMS-based IMU often includes high peaks, namely the outliers. These outliers also violate the Gaussian assumption of the process noise.

To cope with the outliers in the process and measurement noise, we introduce the multivariate Student's t distribution to describe the process and measurement noises as below:

$$\begin{aligned} St\left(\mathbf{X}; \hat{\mathbf{X}}, \mathbf{P}, \nu\right) = & \frac{\Gamma\left(\frac{\nu+n}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{(\nu\pi)^{\frac{n}{2}}} \frac{1}{\sqrt{det\left(\mathbf{P}\right)}} \\ & \left(1 + \frac{1}{\nu}\left(\mathbf{X} - \hat{\mathbf{X}}\right)^T \mathbf{P}^{-1}\left(\mathbf{X} - \hat{\mathbf{X}}\right)\right)^{-\frac{n+\nu}{2}} \end{aligned} \tag{7}$$

As shown in (7), an n-vector Student's t random variable $\mathbf{X}$ is fully characterized by its degrees of freedom (dof) ν, which determines the tail behavior of the density, a mean $\hat{\mathbf{X}}$ and a symmetric matrix $\mathbf{P}$. The symmetric matrix $\mathbf{P}$ is related to the covariance of the random variable $\mathbf{X}$ as $E[(\mathbf{X}-\hat{\mathbf{X}})(\mathbf{X}-\hat{\mathbf{X}})^T]=\frac{\nu}{\nu-2}\mathbf{P}$ [9].

To facilitate the discussion later, let $m \sim Gam\left(\alpha,\beta\right)$ and $\mathbf{y}\sim\mathcal{N}\left(\mathbf{0},\mathbf{P}\right)$, then $\mathbf{X}=\hat{\mathbf{X}}+m^{-\frac{1}{2}}\mathbf{y}$ admits a Student's t distribution with density

$$St\left(\mathbf{X};\hat{\mathbf{X}},\frac{\beta}{\alpha}\mathbf{P},2\alpha\right)=\frac{\Gamma\left(\alpha+\frac{n}{2}\right)}{\Gamma\left(\alpha\right)}\frac{1}{\left(2\beta\pi\right)^{\frac{n}{2}}}\frac{1}{\sqrt{det\left(\mathbf{P}\right)}}\left(1+\frac{\Delta^2}{2\beta}\right)^{-\frac{n}{2}-\alpha} \tag{8}$$

where $\Delta^2=\left(\mathbf{X}-\hat{\mathbf{X}}\right)^T\mathbf{P}^{-1}\left(\mathbf{X}-\hat{\mathbf{X}}\right)$, and it is straightforward to show that (8) turns into (7) for $\alpha=\beta=\frac{\nu}{2}$.

III. Cooperative Navigation Algorithm Based on Student's t Distribution

In this section, we propose a robust CN algorithm based on a Student's t filter. Roth et al. proposed a Student's t filter for linear systems by approximating the posterior pdf as a Student's t distribution [8]. However, for a CN system, the process and measurement equations are nonlinear as presented in (2), (5). Thus, directly porting the linear Student's t filter algorithm to the cooperative navigation case is not straightforward. Fortunately, both the Gaussian distribution and the Student's t distribution are closed under linear transformation, thus the framework proposed by Roth et al. can be extended to nonlinear systems in a manner similar to the development of the EKF for Gaussian systems.

We consider a nonlinear process equation and a measurement equation as follows:

$$\begin{aligned}\mathbf{X}_k&=\mathbf{f}_k\left(\mathbf{X}_{k-1},\mathbf{u}_{k-1},\mathbf{w}_{k-1}\right)\\ \mathbf{Z}_k&=\mathbf{h}_k\left(\mathbf{X}_k,\mathbf{v}_k\right)\end{aligned} \tag{9}$$

The initial state and the process and measurement noise are mutually uncorrelated and Student's t distributed with marginal densities $p(\mathbf{X}_0)=St(\mathbf{X}_0;\hat{\mathbf{X}}_0,\mathbf{P}_0,\eta_0)$, $p(\mathbf{w}_k)=St(\mathbf{w}_k;\mathbf{0},\mathbf{Q}_k,\gamma)$, and $p(\mathbf{v}_k)=St(\mathbf{v}_k;\mathbf{0},\mathbf{R}_k,\delta)$. The dof parameters η_0,γ,δ determine the tail behavior of the related densities. Similar to the EKF, the nonlinear Student's t filtering recursion includes a time update and a measurement update. In the time update, we first assume the dof for $p(\mathbf{X}_{k-1}|\,\mathbf{Z}_{1:k-1})$ is η_{k-1}, and $\eta_{k-1}=\gamma$ for all k, then we can formulate a joint Student's t density because of the common dof as below:

$$\begin{aligned}&p\left(\mathbf{X}_{k-1},\mathbf{w}_{k-1}\middle|\,\mathbf{Z}_{1:k-1}\right)=\\&St\left(\begin{bmatrix}\mathbf{X}_{k-1}\\ \mathbf{w}_{k-1}\end{bmatrix};\begin{bmatrix}\hat{\mathbf{X}}_{k-1}^{+}\\ \mathbf{0}\end{bmatrix},\begin{bmatrix}\mathbf{P}_{k-1}^{+}&\mathbf{0}\\ \mathbf{0}&\mathbf{Q}_{k-1}\end{bmatrix},\eta_{k-1}=\gamma\right)\end{aligned} \tag{10}$$

Then, we assume that the prediction density is also a Student's t distribution and the dof η_{k-1} is not changed by nonlinear transformation $\mathbf{f}_k\left(\cdot\right)$. The prediction density can be represented as below:

$$p\left(\mathbf{X}_k\middle|\,\mathbf{Z}_{1:k-1}\right)=St\left(\mathbf{X}_k;\hat{\mathbf{X}}_k^{-},\mathbf{P}_k^{-},\eta_{k-1}=\gamma\right) \tag{11}$$

To apply the framework of a linear Student's t filter [8], we perform a first order linearization of the process equation. Thus, we can perform the time update of the state estimate and symmetric matrix estimate as follows:

$$\begin{aligned}\hat{\mathbf{X}}_k^{-}&=\mathbf{f}_k\left(\hat{\mathbf{X}}_{k-1}^{+},\mathbf{u}_{k-1},\mathbf{0}\right)\\ \mathbf{P}_k^{-}&=\mathbf{F}_{\mathbf{X}_{k-1}}\mathbf{P}_{k-1}^{+}\mathbf{F}_{\mathbf{X}_{k-1}}^T+\mathbf{F}_{\mathbf{u}_{k-1}}\mathbf{Q}_{k-1}\mathbf{F}_{\mathbf{u}_{k-1}}^T\end{aligned} \tag{12}$$

where $\mathbf{F}_{\mathbf{X}_{k-1}},\mathbf{F}_{\mathbf{u}_{k-1}}$ are obtained as shown in (4).

In the measurement update, we make assumptions on the dof in the same fashion. For $\eta_{k-1}=\delta$, then we can formulate a joint Student's t density for the predicted state and the measurement noise as below:

$$\begin{aligned}&p\left(\mathbf{X}_k,\mathbf{v}_k\middle|\,\mathbf{Z}_{1:k-1}\right)=\\&St\left(\begin{bmatrix}\mathbf{X}_k\\ \mathbf{v}_k\end{bmatrix};\begin{bmatrix}\hat{\mathbf{X}}_k^{-}\\ \mathbf{0}\end{bmatrix},\begin{bmatrix}\mathbf{P}_k^{-}&\mathbf{0}\\ \mathbf{0}&\mathbf{R}_k\end{bmatrix},\eta_{k-1}=\delta\right)\end{aligned} \tag{13}$$

Similarly, we linearize the measurement equation by performing a Taylor series expansion to obtain $\mathbf{H}_k$, and the joint Student's t density of the state and the measurement is:

$$\begin{aligned}&p\left(\mathbf{X}_k,Z_k\middle|\,\mathbf{Z}_{1:k-1}\right)=\\&St\left(\begin{bmatrix}\mathbf{X}_k\\ \mathbf{Z}_k\end{bmatrix};\begin{bmatrix}\hat{\mathbf{X}}_k^{-}\\ \mathbf{h}_k\left(\hat{\mathbf{X}}_k^{-},\mathbf{0}\right)\end{bmatrix},\begin{bmatrix}\mathbf{P}_k^{-}&\mathbf{P}_k^{-}\mathbf{H}_k^T\\ \mathbf{H}_k\mathbf{P}_k^{-}&\mathbf{H}_k\mathbf{P}_k^{-}\mathbf{H}_k^T+\mathbf{R}_k\end{bmatrix},\eta_{k-1}\right)\end{aligned} \tag{14}$$

Then, the conditional density of the estimated state given all measurements $\mathbf{Z}_{1:k}$ can be calculated using (8) as below:

$$\begin{aligned}&p\left(\mathbf{X}_k|\mathbf{Z}_k\right)=\frac{p\left(\mathbf{X}_k,\mathbf{Z}_k\right)}{p\left(\mathbf{Z}_k\right)}=\frac{\Gamma\left(\alpha+\frac{n}{2}\right)}{\Gamma\left(\alpha\right)}\frac{1}{\left(2\beta\pi\right)^{\frac{n}{2}}}\\&\frac{1}{\sqrt{det\left(\mathbf{P}\right)}}\left(1+\frac{\Delta^2}{2\beta}\right)^{-\frac{n}{2}-\alpha}\\&\left(\frac{\Gamma\left(\alpha+\frac{n_{dz}}{2}\right)}{\Gamma\left(\alpha\right)}\frac{1}{\left(2\beta\pi\right)^{\frac{n_{dz}}{2}}}\frac{1}{\sqrt{det\left(\mathbf{P}_{22}\right)}}\left(1+\frac{\Delta_z^2}{2\beta}\right)^{-\frac{n_{dz}}{2}-\alpha}\right)^{-1}\\&=\frac{\Gamma\left(\tilde{\alpha}+\frac{n_{dx}}{2}\right)}{\Gamma\left(\tilde{\alpha}\right)}\frac{1}{\left(2\tilde{\beta}\pi\right)^{\frac{n_{dx}}{2}}}\frac{1}{\sqrt{det\left(\tilde{\mathbf{P}}\right)}}\left(1+\frac{\Delta_x^2}{2\tilde{\beta}}\right)^{-\frac{n_{dx}}{2}-\tilde{\alpha}}\end{aligned} \tag{15}$$

where n_{dx} and n_{dz} are dimensions of the state and measurement vectors, and $n=n_{dx}+n_{dz}$. $\mathbf{P}$ is the symmetric matrix of the joint density as shown in (14), and $\mathbf{P}_{ij}\left(i,j=1,2\right)$ is the i-th row and j-th column element of the block matrix $\mathbf{P}$. In addition, $\tilde{\alpha}=\alpha+\frac{n_{dz}}{2}$, $\tilde{\beta}=\frac{1}{2}\left(2\beta+\Delta_z^2\right)$ and $\tilde{\boldsymbol{P}}=\boldsymbol{P}_{11}-\boldsymbol{P}_{12}\boldsymbol{P}_{22}^{-1}\boldsymbol{P}_{12}^T$.

The derived result (15) reveals that $p\left(\mathbf{X}_k|\mathbf{Z}_k\right)$ is also a Student's t distribution, and parameterized in terms of $\tilde{\alpha}$ and $\tilde{\beta}$. Then, considering (12), (14), and converting back to the ν

parameterization as (7), we obtain

$$
\begin{aligned}
&\eta_k = 2\tilde{\alpha} = \eta_{k-1} + n_{dz} \\
&\hat{\mathbf{X}}_k^+ = \hat{\mathbf{X}}_k^- + \mathbf{P}_k^- \mathbf{H}_k^T \left(\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k\right)^{-1} \left(\mathbf{Z}_k - \mathbf{h}_k\left(\hat{\mathbf{X}}_k^-, \mathbf{0}\right)\right) \\
&\mathbf{P}_k^+ = \frac{\tilde{\beta}}{\tilde{\alpha}} \tilde{\mathbf{P}} = \\
&\frac{\eta_{k-1} + \Delta_z^2}{\eta_{k-1} + n_{dz}} \left(\mathbf{P}_k^- - \mathbf{P}_k^- \mathbf{H}_k^T \left(\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k\right)^{-1} \mathbf{H}_k \mathbf{P}_k^-\right)
\end{aligned}
\tag{16}
$$

where $\Delta_z^2 = \left(\mathbf{Z}_k - \mathbf{h}_k\left(\hat{\mathbf{X}}_k^-, \mathbf{0}\right)\right)^T \left(\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k\right)^{-1}$ $\left(\mathbf{Z}_k - \mathbf{h}_k\left(\hat{\mathbf{X}}_k^-, \mathbf{0}\right)\right)$.

It is not hard to find that the dof η_k increases with the filter recursion according to (16). However, the most important assumption, which is the premise of the nonlinear Student's t filter recursion, is the state and process/measurement noise are jointly Student's t distributed with a common dof. To validate the assumption, it is clearly required that the noise dof increases with the recursion process, which will lead to a conventional EKF after a few time steps. It also can be observed when letting the dof approach infinity, we can recover the Gaussian conditional covariance as below:

$$
\lim_{\eta_k \to \infty} \mathbf{P}_k^+ = \mathbf{P}_k^- - \mathbf{P}_k^- \mathbf{H}_k^T \left(\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k\right)^{-1} \mathbf{H}_k \mathbf{P}_k^- \tag{17}
$$

Thus, the growth of the dof should be prevented so that a heavy-tailed posterior density can be retained throughout time. To bound the dof, we first select a common dof for the jointly Student's t distribution of state $\hat{\mathbf{X}}_{k-1}^+$ and process noise $\mathbf{w}_k$. The common dof can be chosen as $\eta'_{k-1} = min\left(\eta_{k-1}, \gamma\right)$. Generally, it follows that $\eta_{k-1} > \gamma$ and therefore $\eta'_{k-1} = \gamma$. However, because the dof of the marginal density $p\left(\mathbf{X}_{k-1}\right)$ is reduced to a new one η'_{k-1}, then we should adjust the marginal density $p\left(\mathbf{X}_{k-1}\right)$ given the new dof. This problem is formulated as finding a scalar c such that the densities $p(\mathbf{X}_{k-1}) = St(\mathbf{X}_{k-1}; \hat{\mathbf{X}}_{k-1}^+, \mathbf{P}_{k-1}^+, \eta_{k-1})$ and $\tilde{p}(\mathbf{X}_{k-1}) = St(\mathbf{X}_{k-1}; \hat{\mathbf{X}}_{k-1}^+, c\mathbf{P}_{k-1}^+, \eta'_{k-1})$ are similar in some sense to the specified one. To find the scalar c, the moment matching method can be used, which is simple and suitable for a real-time application.

Then, the adjusted estimation-error covariance $\tilde{\mathbf{P}}_{k-1}^+ = c\mathbf{P}_{k-1}^+$ is obtained to replace the corresponding one in (12). Note that if $\gamma \neq \delta$, the moment matching method should be performed again before the measurement update step, which is omitted here.

IV. Simulation and Studies

In this section, we present the simulation results that demonstrate the validity of the robust cooperative navigation algorithm for AUVs based on the Student's t distribution proposed in Section III. In the simulation test, the two slave AUVs (A AUV and B AUV) received the range information and the accurate position information of the master AUV (C AUV) periodically, the standard extended Kalman filter (EKF) and the nonlinear Student's t filter algorithm (NSTF) are used to estimate all the states and their performances are compared, as the EKF has been commonly used in CN.

The AUVs were commanded to move at a constant velocity of $V_A = V_B = 2m/s$, $V_C = 2m/s$. The orientations of three AUVs are set to be $\theta_A = 25deg$, $\theta_B = 145deg$, $\theta_C = 75deg$, and the rotational velocity is set to be zero for all AUVs.

The process noise and measurement noise are set to follow Student's t distributions, and the dof for the Student's t distributions were both chosen as 3 as in [9] in order to preserve as heavy tails as possible. The standard deviations of the range measurement and position measurement are $\sigma_r = 4m$, $\sigma_p = 2m$, respectively. The standard deviations of the velocity and rotational velocity of A and B AUV are set to be $\sigma_V = 1m/s$, $\sigma_\omega = 0.2deg/s$.

The measurement noise and its pdf (range between A AUV and C AUV) are drawn as in Fig. 1. It can be seen that some outliers, which are marked by red squares, exist in the measurement noise which lead to the heavy-tailed distribution.

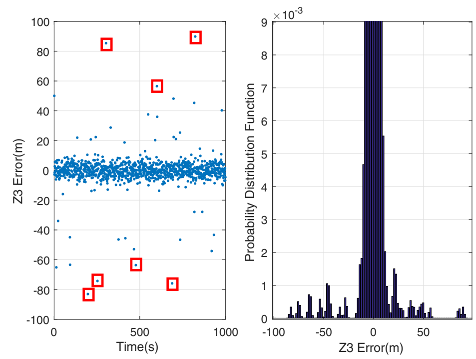

Fig. 1. Measurement noise of range between A AUV and C AUV and its pdf, and the dots marked by red squares are some outliers.

The positioning errors of A AUV and B AUV are drawn as in Fig. 2 and Fig. 3. As it can be seen, the average positioning errors of A AUV and B AUV are $10.13m$ and $12.42m$ using EKF, respectively. Estimated by the NSTF, the average positioning errors are reduced to be $6.14m$ and $3.14m$, and the positioning accuracy is improved by 39.4% and 74.7%, respectively. It can be found that the cooperative navigation algorithm based on NSTF outperformed the EKF-based cooperative navigation algorithm in terms of positioning error when outliers exist in the process and measurement noise. Note that when the outliers of measurement noise are enlarged, namely the heavy tailed behavior is worse, the EKF fails and the positioning error of B AUV is unbounded as shown in Fig. 4. Conversely, the NSTF still works well and the output is stable and bounded.

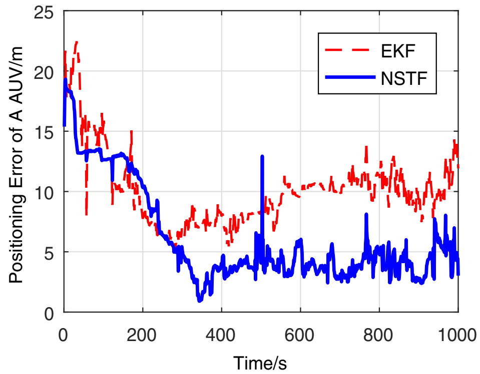

Fig. 2. Positioning error of A AUV in the CN system.

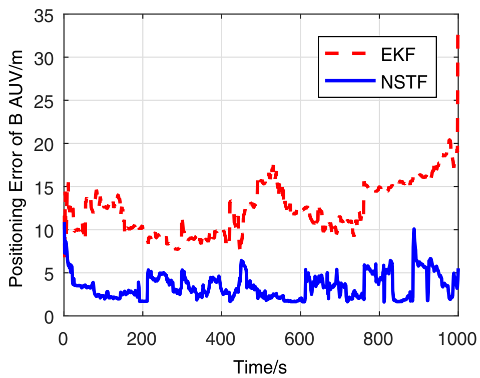

Fig. 3. Positioning error of B AUV in the CN system.

V. Conclusion

In this paper, a robust master-slave CN algorithm for AUVs based on nonlinear Student's t filter was proposed. The heavy-tailed process and measurement noise induced by outliers from MEMS-based IMU and underwater acoustic range measurements were considered, and the Student's t filter algorithm for the nonlinear system was derived to estimate the states of the slave AUVs, which will bound the positioning errors of the CN system. The resulting algorithm was compared to a standard EKF in a simulation evaluation. It was found that the CN algorithm based on the Student's t distribution outperformed the standard EKF in terms of positioning error when the process and measurement noise had heavy-tailed behavior.

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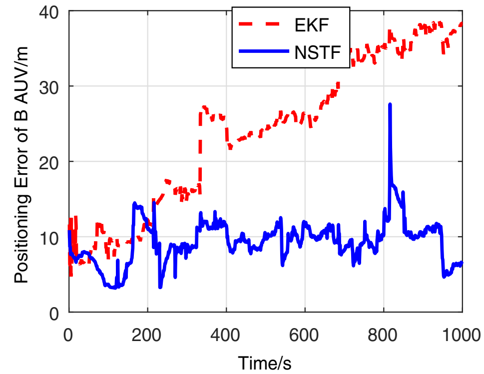

Fig. 4. Positioning error of B AUV when the outliers of measurement noise are enlarged, namely the heavy-tailed behavior is worse.

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