Performance Comparison of Localization Techniques For Sequential WSN Discovery

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***Abstract*—In this paper, the performance of different localization algorithms are compared in the context of the sequential Wireless Sensor Network (WSN) discovery problem. Here, all sensor nodes are at unknown locations except for a very small number of so called *anchor* nodes at known locations. The locations of nodes are sequentially estimated such that when the location of a given node is found, it may be used to localize others. The underlying performance of such an approach is largely dependent upon the localization technique employed. In this paper, several well-known localization techniques are presented using a unified notation. These methods are time of arrival (TOA), time difference of arrival (TDOA), received signal strength (RSS), direction of arrival (DOA) and large aperture array (LAA) localization. The performance of a sequential network discovery process is then compared when using each of these localization algorithms. These algorithms are implemented in the Java-DSP software package as part of a localization toolbox.**

I. INTRODUCTION

Wireless sensor networks (WSNs) are commonly employed for many applications including for environmental protection, structural monitoring and passive localization and tracking [1]. Here, a large number of inexpensive sensor nodes with low size, weight and power (SWAP) are randomly distributed across an area of interest. These nodes operate as transceivers, communicating with one another in an ad-hoc manner, and are at unknown locations. This is with the exception of a very small number of so called anchor nodes which are at known locations. In commercial applications such as water quality monitoring as well as in military applications such as gunshot detection, an accurate knowledge of the location where an event occurs is often critical to the users of the system. Therefore, the location of the sensors must be found.

The wireless sensor network discovery problem considered in this paper is concerned with estimating the location of all the nodes using only a small number of anchor nodes. The heart of this problem is in choosing which localization algorithm to use in the discovery process. In general, localization techniques can be classified into range-based and direction-based approaches [2], [3]. Common range-based approaches are time of arrival (TOA), time difference of arrival (TDOA), and received signal strength (RSS). Direction-based approaches include direction of arrival (DOA) estimation techniques implemented by employing antenna arrays at each sensor [3], [4]. In contrast, in [5], a novel large aperture array (LAA) localization algorithm is presented which jointly uses direction and range based information to localize a source by forming a single large aperture array of sensors.

WSN discovery may be performed in a centralized or distributed manner [3]. In the centralized approach, all nodes are localized using the same set of anchors at known locations. The main drawback of this approach is that anchors must be within the coverage area of all nodes which will lead to an undesirably large cost in power. Distributed discovery procedures such as the one used in this paper attempt to overcome this issue by allowing nodes which have previously been localized to be used to localize other nodes. The main drawback of this approach is that localization errors will propagate through the network during the iterative localization process. This is because it is assumed that the estimated locations of the nodes are the actual locations. However, due to the errors associated in localizing the nodes, this may not be the case. This makes the order in which nodes are localized markedly important as well as the localization algorithm used.

In this paper, the TOA, TDOA, RSS, DOA and LAA localization algorithms will be expressed as the solution to a set of linear equations of the same structure as presented in [5]. Following this, the performance of a sequential network discovery process when using these different localization algorithms will be compared.

The following notation is used in this paper: A scalar is represented as x or X, a column vector is represented by $\underline{x}$ or $\underline{X}$ and a matrix is represented by $\mathbf{X}$ or $\mathbb{X}$. Furthermore, $(\cdot)^T$ denotes a matrix transpose, $\otimes$ denotes the Kronecker product and $\|\underline{A}\|$ denotes the Euclidean norm of the vector $\underline{A}$. Finally, $\mathcal{R}$ denotes sets of real numbers.

The rest of this paper is organized as the follows. In Section 2, the WSN sequential discovery problem is formulated. Then, in Section 3, the TOA, TDOA, RSS, DOA and LAA localization algorithms are introduced. Section 4 contains simulation results. The Java-DSP toolbox for localization is presented in Section 5; and in Section 6, the paper is concluded.

II. WSN SEQUENTIAL DISCOVERY PROBLEM

Consider a homogeneous wireless sensor network with a large number of nodes at random unknown locations in $\mathcal{R}^2$

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space. Assume that a small number of anchor nodes at known locations are included in this network. Each node has a circular coverage area with radius D, and operates at the frequency F_c. The Cartesian coordinates of the i^{th} node is denoted by $\underline{r}_i \in \mathcal{R}^{2\times 1}$. The locations of all nodes at unknown locations are to be estimated by a sequential discovery algorithm using different localization algorithms. Each algorithm requires a different minimum number of receiving nodes, $N_{\min}$, in the coverage area of the transmitting node for localization to take place. In $\mathcal{R}^2$ space, for DOA, $N_{\min} = 2$, for TOA, TDOA and RSS, $N_{\min} = 3$ and for LAA, $N_{\min} = 4$.

The sequential discovery process used in the paper is described in [6] and may be summarized as follows: Initially, one node at an unknown location in the coverage area of the anchor nodes transmits. Other nodes operate as receivers. If at least $N_{\min}$ nodes (anchors or the previously localized nodes) are within the coverage area of the transmitting node then the node location is estimated. Here, if more than $N_{\min}$ nodes are available, all the available nodes are used. However, if fewer than $N_{\min}$ nodes are available, this node is skipped. Following this, another node transmits and the process continues. Once localization has been attempted at all nodes, the process is repeated from the beginning a number of times. This gives an opportunity for nodes which were previously skipped to be revisited in the hope that more nodes at known or estimated locations will now be available to meet the $N_{\min}$ node requirement for localization of the skipped node to take place. In addition, repeating the discovery process also allows node locations which were successfully estimated previously to be gradually refined using data fusion techniques (including simple schemes such as averaging) to diminish the effect of localization order and noise.

III. LOCALIZATION TECHNIQUES

In this section the TOA, TDOA, RSS, DOA and LAA localization approaches will be presented in turn as the solution to a set of linear equations as presented in [5]. In particular, the position of a source operating at a frequency F_c and located at $\underline{r}_0$ will be estimated in $\mathcal{R}^2$ space by solving,

$$\mathbb{H}\underline{r}_0 = \underline{b}. \tag{1}$$

This is achieved by collecting data from N receiving nodes at locations $\underline{r}_1, \underline{r}_2, \cdots, \underline{r}_N$ and extracting metrics based on the localization method to construct $\mathbb{H}$ and $\underline{b}$. If $N = N_{\min}$ receiving nodes are available then the solution to Equation 1 is,

$$\underline{r}_0 = \mathbb{H}^{-1}\underline{b}, \tag{2}$$

where $\mathbb{H}^{-1}$ denotes the inverse of the matrix $\mathbb{H}$. In contrast, if $N > N_{\min}$ receiving nodes are available then the solution to Equation 1 is,

$$\underline{r}_0 = \mathbb{H}^{\#}\underline{b}, \tag{3}$$

where $\mathbb{H}^{\#}$ denotes the pseudo inverse of the matrix $\mathbb{H}$. This will provide a least squares solution.

The range based localization approaches are illustrated in Figure 1. The TOA technique is one of the most popular techniques used for localization. Here, the propagation time of the line-of-sight (LOS) signal from the transmitting node to each of the N receiving nodes is measured to estimate the range between the source and all the sensors. This requires time synchronization between the transmitter and receiver [3],[7]. Denoting t_{i0} as the propagation time from the source to the i^{th} node and c as the signal propagation speed, Equation 1 may be used to estimate the location of the source where,

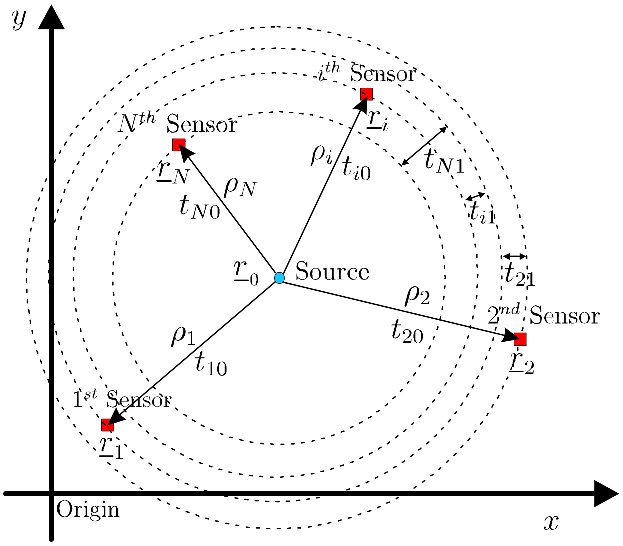


Fig. 1. TOA/TDOA/RSS based localization of a transmitting node at $\underline{r}_0$ using N sensors at locations $\underline{r}_1, \underline{r}_2, \ldots, \underline{r}_N$.

$$\mathbb{H} = \left[\underline{r}_2 - \underline{r}_1, \underline{r}_3 - \underline{r}_1, \cdots, \underline{r}_N - \underline{r}_1\right]^T, \tag{4}$$

$$\underline{b} = \frac{1}{2}\begin{bmatrix} \|\underline{r}_2\|^2 - \|\underline{r}_1\|^2 - c^2\left(t_{20}^2 - t_{10}^2\right) \\ \|\underline{r}_3\|^2 - \|\underline{r}_1\|^2 - c^2\left(t_{30}^2 - t_{10}^2\right) \\ \vdots \\ \|\underline{r}_N\|^2 - \|\underline{r}_1\|^2 - c^2\left(t_{N0}^2 - t_{10}^2\right) \end{bmatrix}. \tag{5}$$

In contrast to TOA, TDOA measures the difference in propagation time at which the LOS signal arrives at the N receiving nodes. This removes the need for synchronization between the transmitter and receiver [8], [9]. Denoting t_{i1} as the difference in propagation time from the i^{th} node to the first node and t_{10} as the propagation time from the source to the first node,

$$\mathbb{H} = \left[\underline{r}_2 - \underline{r}_1, \underline{r}_3 - \underline{r}_1, \cdots, \underline{r}_N - \underline{r}_1\right]^T, \tag{6}$$

$$\underline{b} = \frac{1}{2}\begin{bmatrix} \|\underline{r}_2\|^2 - \|\underline{r}_1\|^2 - c^2\left(t_{21}^2 + 2t_{10}\cdot t_{21}\right) \\ \|\underline{r}_3\|^2 - \|\underline{r}_1\|^2 - c^2\left(t_{31}^2 + 2t_{10}\cdot t_{31}\right) \\ \vdots \\ \|\underline{r}_N\|^2 - \|\underline{r}_1\|^2 - c^2\left(t_{N1}^2 + 2t_{10}\cdot t_{N1}\right) \end{bmatrix}. \tag{7}$$

Note that both TOA and TDOA algorithms are known to suffer if there is insufficient bandwidth. In RSS, a path loss model is used to infer the location of a source based on power loss measurements [10] associated with the LOS signal arriving at each of the N receiving nodes. This is a simple and inexpensive technique to implement but suffers from problems in the presence of channel impairments such as multipath and frequency flat fading. Defining P_T as the transmit power and P_{R_i} as the received power at the i^{th} node, assuming a free

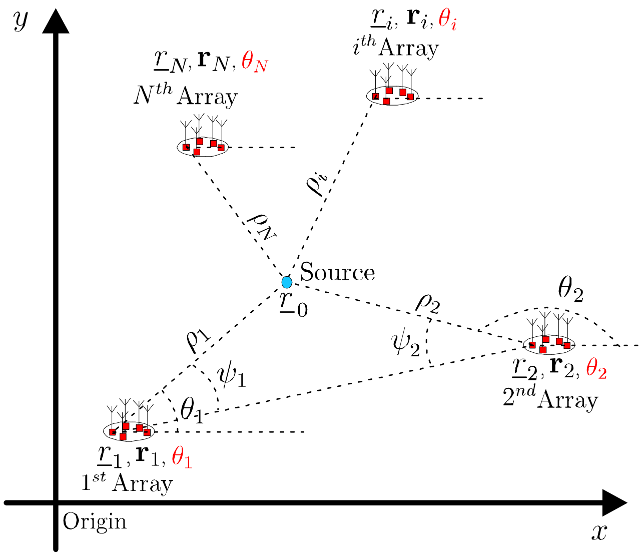


Fig. 2. DOA based localization of a transmitting node at $\underline{r}_0$ using N arrays with reference points at $\underline{r}_1$, $\underline{r}_2$, …, $\underline{r}_N$, local geometries $\mathbf{r}_1$, $\mathbf{r}_2$, …, $\mathbf{r}_N$ and DOAs $\theta_1, \theta_2, \ldots, \theta_N$.

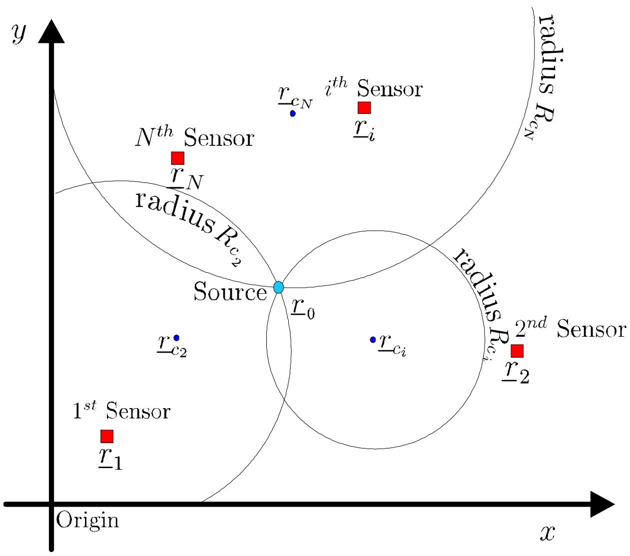


Fig. 3. LAA based localization of the transmitting node $\underline{r}_0$ using N sensors at known locations $\underline{r}_1, \underline{r}_2, \ldots, \underline{r}_N$. R_{c_j} and $\underline{r}_{c_j}$ denote the radius and center of the j^{th} circular locus which may be used to estimate the source location.

space path loss model, the range ρ_i between the source and the i^{th} node is estimated by,

$$\rho_i = \frac{c}{4\pi F_c}\sqrt{\frac{P_T}{P_{R_i}}}. \tag{8}$$

Then,

$$\mathbb{H} = \left[\underline{r}_2 - \underline{r}_1, \underline{r}_3 - \underline{r}_1, \cdots, \underline{r}_N - \underline{r}_1\right]^T, \tag{9}$$

$$\underline{b} = \frac{1}{2}\begin{bmatrix} \|\underline{r}_2\|^2 - \|\underline{r}_1\|^2 - \left(\rho_2^2 - \rho_1^2\right) \\ \|\underline{r}_3\|^2 - \|\underline{r}_1\|^2 - \left(\rho_3^2 - \rho_1^2\right) \\ \vdots \\ \|\underline{r}_N\|^2 - \|\underline{r}_1\|^2 - \left(\rho_N^2 - \rho_1^2\right) \end{bmatrix}. \tag{10}$$

The DOA based localization approach is illustrated in Figure 2. Here, small aperture arrays are employed at each sensor node to estimate the direction of the transmitting source [1]. This does not require nodes to be synchronized but has increased hardware and processing overheads as a result of using more antennas. Defining θ_i as the DOA of the LOS source signal from the i^{th} array/node measured with respect to its array reference point at $\underline{r}_i$ using a DOA algorithm (e.g. MUSIC [11]), the range ρ_i between the source and the i^{th} array reference point may be estimated using the sine rule. For example, with reference to Figure 2, for the triangle $\underline{r}_1, \underline{r}_2$ and $\underline{r}_0$,

$$\rho_1 = \frac{\|\underline{r}_2 - \underline{r}_1\| \sin\left(\psi_2\right)}{\sin\left(\psi_1 + \psi_2\right)}, \tag{11}$$

$$\rho_2 = \frac{\|\underline{r}_2 - \underline{r}_1\| \sin\left(\psi_1\right)}{\sin\left(\psi_1 + \psi_2\right)}. \tag{12}$$

Then,

$$\mathbb{H} = \underline{1}_N \otimes \mathbb{I}_2, \tag{13}$$

$$\underline{b} = \begin{bmatrix} \underline{r}_1 + \rho_1 \cdot \left[\cos\theta_1, \sin\theta_1\right]^T \\ \underline{r}_2 + \rho_2 \cdot \left[\cos\theta_2, \sin\theta_2\right]^T \\ \vdots \\ \underline{r}_N + \rho_N \cdot \left[\cos\theta_N, \sin\theta_N\right]^T \end{bmatrix}. \tag{14}$$

Finally, the LAA approach presented in [5] is illustrated in Figure 3. Here, single element nodes are aggregated to form an array system of large aperture. This allows data received from each node to be used in a more statistically efficient manner compared to other techniques. By constructing the second order statistics of the array signal when the array reference point is rotated to be at each of its elements, corresponding signal eigenvalues of these matrices may be used to construct a metric $\underline{\mathcal{K}}$. This is related to the ratio of the range from the array elements to the source and the range of the primary reference point (taken as the first node) to the source. With reference to Figure 3, $\underline{\mathcal{K}}$ may be used to construct $N-1$ circular loci to estimate the source location with the i^{th} locus of radius R_{c_i} and center $\underline{r}_{c_i}$. Alternatively, $\underline{\mathcal{K}}$ may be used in the form of Equation 1 such that,

$$\mathbb{H} = \begin{bmatrix} 2\left(\underline{1}_{N-1}\underline{r}_1^T - \left[\underline{r}_2, \underline{r}_3, \cdots, \underline{r}_N\right]^T\right) \\ \left(\underline{1}_{N-1} - \underline{\mathcal{K}}^2\right) \end{bmatrix}, \tag{15}$$

$$\underline{b} = \left[\|\underline{r}_1\|^2 \underline{1}_{N-1} - \left[\|\underline{r}_2\|^2, \|\underline{r}_3\|^2, \cdots, \|\underline{r}_N\|^2\right]^T\right], \tag{16}$$

with Equation 1 modified to be

$$\mathbb{H}\begin{bmatrix} \underline{r}_0 \\ \rho_1^2 \end{bmatrix} = \underline{b}. \tag{17}$$

This approach is robust to frequency flat fading [5].

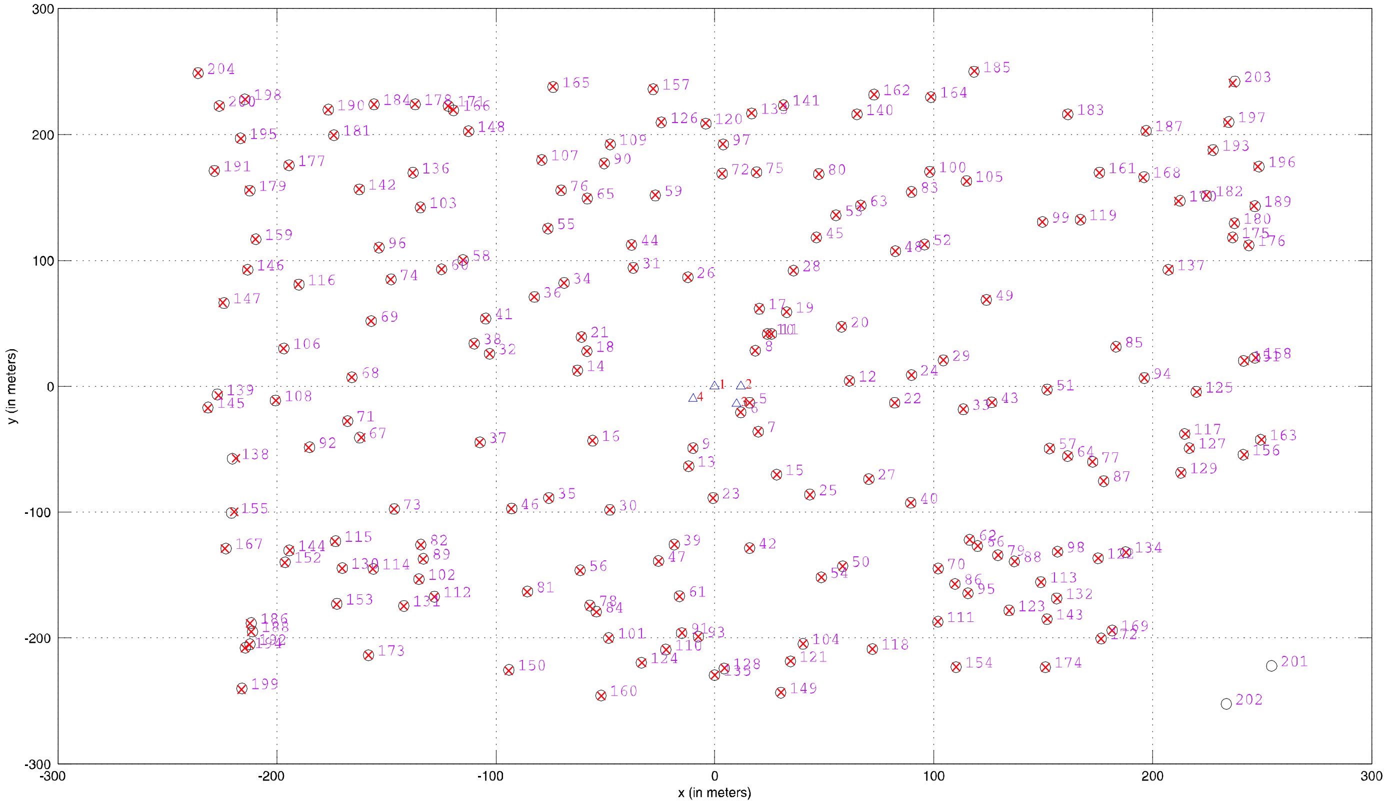


Fig. 4. Sequential discovery using LAA technique with each node having a coverage of $D = 100$m; $L = 1000$; SNR=30dB. Black circles represent actual node locations, blue triangles anchor nodes at known locations, and red crosses location estimates. All but nodes 201 and 202 are localized. Location uncertainties can be seen more predominantly at the edges of the network.

IV. SIMULATION RESULTS

With reference to Figure 4, consider a homogeneous wireless sensor network of 204 nodes located in $\mathcal{R}^2$ space with 4 nodes as the anchors at known locations represented by blue triangles. The other 200 nodes are at random unknown locations represented by black circles. All nodes operate (as transceivers) at frequency $F_c = 2.45GHz$. Each node transmits a sinusoid as its positioning signal over a transmission range of $D = 100$m. Assume a free space propagation constant in the simulation environment and that there is no frequency flat fading or multipath. Localization of each node is attempted using $L = 1000$ snapshots under an SNR of 30dB. Each node is revisited several times to maximize the total number of discovered nodes and to refine location estimates. In Figure 4, the results of the sequential discovery process using LAA localization is shown. Estimated node locations are represented using red crosses. Two of the nodes (node 201 and node 202) remain undiscovered because there are not enough nodes in their coverage range for localization.

The RMSE performance of each of the localization algorithms in solving the wireless sensor network discovery problem is now investigated. In particular, the average RMSE of all node locations is plotted (see Figure 5) when using the different localization algorithms under different numbers of snapshots from 10^1 to 10^9 under an SNR of 30dB. The same node locations, transmission range, and propagation model is used as in the previous simulation shown in Figure 4. For the DOA approach, an $N = 5$ element Uniform Linear Array is employed at each node. One can see that for all techniques, average RMSE decreases as SNR $\times$ L increases. This is expected as noise reduces and more data is available to produce more statistically efficient metrics. When SNR $\times$ L is below 10^7, DOA has the best performance. However, when

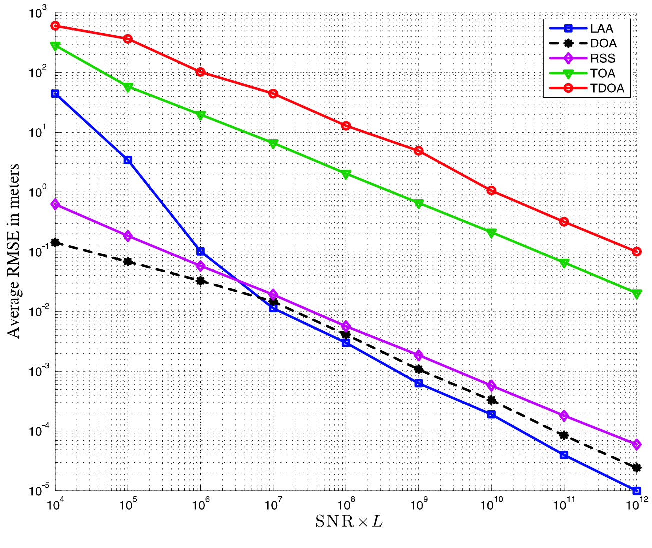
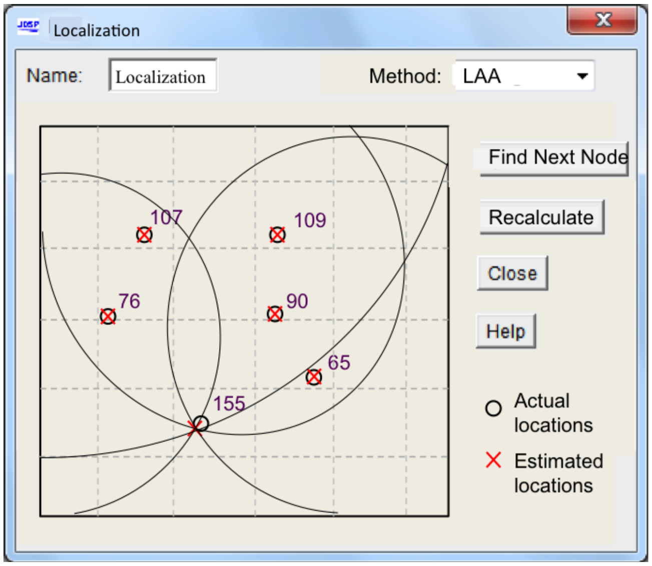


Fig. 5. Performance comparison of different localization techniques for the network discovery problem in Figure 4. System parameters are $D = 100$m; SNR= 30dB. RMSE (averaged across all nodes) is plotted vs. SNR$\times L$.



Name: Localization

Method: LAA

Fig. 6. Java-DSP interface for network discovery. In this example the LAA algorithm is used to estimate the location of node 155 using nodes 65, 76, 90, 107 and 109 to form an array.

the SNR $\times L$ is greater than 10^7, the LAA algorithm exceeds its performance. Here, however, it should be noted that since for the DOA approach, each node consists of $N = 5$ array elements, the hardware overhead will be significantly larger. Note that the TOA and TDOA schemes are markedly poor due to bandwidth limitations. While the performance of the RSS regime is good in this simulation environment, under scenarios with more complex channel effects, performance will be degraded.

V. Development of a Java Toolbox

In this section, we briefly discuss the Java modules that have been developed for the implementation of the discovery process presented in this paper. These modules have been integrated as a separate toolbox in the Java-DSP package. Java-DSP is an NSF sponsored online programming environment for signal processing and communications education and research [12]. Java-DSP was initially designed to enable students and distance learners to perform laboratories over the Internet. However, toolboxes that support multidisciplinary research are also being developed. As part of this effort, a toolbox has been developed that includes Java modules for: (a) aggregating data collected from listening nodes at known locations; (b) Estimating node locations; (c) Visualizing the results; and (d) Estimating RMSE. Figure 6 shows the Java-DSP interface used to visualize the discovery process for a small example. Here, the same setup is used as in Figure 4. In this example, the LAA algorithm is employed to estimate the location of node 155 by forming an array with nodes 65, 76, 90, 107 and 109. The program allows users to select different localization algorithms, to re-estimate node locations, and to move from node to node sequentially. The toolbox is currently under testing, and is planned to be released as a public beta shortly.

VI. Conclusions

In this paper, the use of different localization algorithms when performing sequential wireless sensor network discovery has been investigated. This discovery process is employed to estimate the location of a large number of low powered nodes in a distributed fashion such that when a node location has been estimated, it may be used to localize other nodes. The localization algorithm employed is central to the performance of this process. Range based (i.e. TOA, TDOA and RSS), direction based (i.e. DOA) and hybrid (i.e. LAA) localization algorithms were considered and it was shown that DOA is the preferred scheme at low SNR and the LAA localization algorithm provides better performance for network discovery at higher SNR.

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