# An Efficient Implementation of the Low-Complexity Multi-Coset Sub-Nyquist Wideband Radar Electronic Surveillance

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***Abstract*—The problem of efficient sampling of wideband radar signals for Electronic Surveillance (ES) using a parallel sampling structure will be investigated in this paper. Wideband radio frequency sampling, which is a necessary component of the modern digital radar surveillance systems, needs a sampling rate at least twice the maximum frequency of signals, *i.e.* Nyquist rate, which is generally very high. Designing an analog to digital converter which works with such a high sampling rate is difficult and expensive. The standard wideband ES receivers use the rapidly swept superheterodyne technique, which selects a subband of the spectrum at a time, while iterating through the whole spectrum sequentially. Such a technique does not explore the underlying structure of input RF signals. When the signal is sparsely structured, we can use the fact that signals do not occupy the whole spectrum. There indeed exists a parsimonious structure in the time-frequency domain in radar ES signals. We here use a recently introduced low-complexity sampling system, called LoCoMC [1], which is inspired by the compressive sampling (CS) of sparse signals and it uses the multi-coset sampling structure, while it does not involve a computationally expensive reconstruction step. A new implementation technique is here introduced, which further reduces the computational cost of the reconstruction algorithm by combining two filters, while improving the accuracy by implicitly implementing an infinite length filter.**

**We also describe the rapidly swept superheterodyne receiver and compare it with the LoCoMC algorithm. In a contrast to the former technique, LoCoMC continuously monitors the spectrum, which makes it much more robust in the short pulse detection.**

## I. INTRODUCTION

The radar ES signals are wideband and they normally exceed the sampling-rate/dynamic-range specifications of standard (single-unit) ADC's. The current civilian and military radars operate in a range of $100\,\text{MHz}$ to $18\,\text{GHz}$ [2]. As the future radars can easily send pulses up to $40\,\text{GHz}$, there is a need for the even wider-band receivers. While there exist some analog radar ES systems, the new ES systems are digital and we thus need a wideband ADC at the front end of the receiver. On the other hand, the unit cost and power consumption of the ADC's rapidly increase, when the maximum sampling rate exceeds 1GHz, which makes it impractical to directly use such ADC's in the ES systems.

One approach to sample such wideband signals, using some low rate ADC's, is to use a bank of bandpass filters, which partitions the whole spectrum, followed by some local oscillators (LO). This type of receivers are called the channelised or superheterodyne receivers. The issue here is that we need to have as many ADC's as the number of filterbank channels, which makes it impractical for radar ES systems. A modified version of this technique exists [2], which is based on time-sharing technique and the receivers based upon this technique are called Rapidly Swept Super-heterodyne Receivers (RSSR). This technique considers only a part of spectrum at each time interval, which makes it less sensitive to the short pulses.

Similarly, we can use a bank of ADC's, which iteratively samples delayed input signals. This type of ADC's are called time-interleaved ADC's [3]. With a similar argument, we need as many ADC's as the ratio of downsampling to the whole spectrum. We here use a structure which only uses a few ADC's, called a multi-coset sampling system [4]. For the recovery of the aliased signals, *i.e.* after undersampling, we have to incorporate some prior information about the input and use some non-linear reconstruction algorithms, which are generally computationally expensive. The computational cost does not allow us to use such techniques for the huge size problems like radar ES.

In [1], the authors propose a low-complexity technique for the signal recovery, which is more suitable for the structurally sparse signals. However, as the overall performance of the recovery technique heavily relies on the accuracy of implementation of the fractional delays, we need to implement a non-causal infinite length filter which is not possible. Any error in the implementation, causes some delay errors in the output. In this paper, a simple modification to the implementation of LoCoMC is introduced which improves the SNR of the received signal. We also present a side by side comparison with the RSSR to demonstrate the advantages of the LoCoMC, over a standard technique.

## II. SUB-NYQUIST SAMPLING SYSTEMS

We need to moderate the number of samples, or sampling rate, in many practical applications to reduce the sensing/imaging time or complexity. When the number of samples becomes less than the Nyquist rate, *i.e.* twice the maximum available frequency in the spectrum, we face an artefact, calling the aliasing or spectrum folding. The aim is to reduce the sampling rate without being badly affected by the aliasing. Most of the old techniques for sub-Nyquist sampling are based on the non-uniform or random sampling techniques, with linear reconstructions [5]. However, the application of

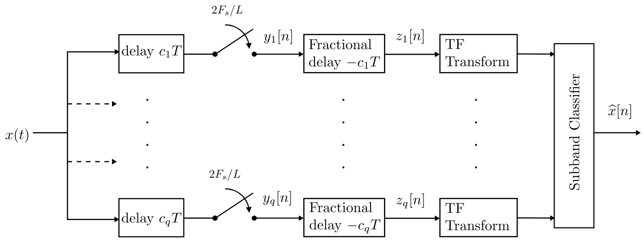


Fig. 1. The proposed low-complexity sub-Nyquist sampling system.

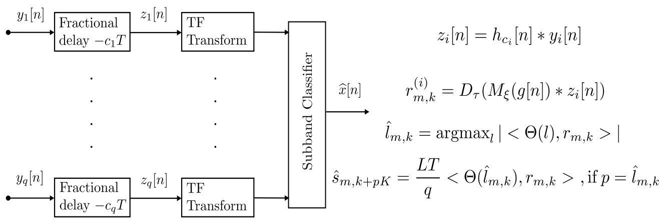


Fig. 2. The digital part of the LoCoMC reconstruction procedure.

such techniques are limited as the goal of such methods are not to remove the aliasing effect, but to compensate it by spreading the error effect over a wide frequency range. In this paper, we concentrate on the modern sub-Nyquist sampling techniques base on the signal sparsity, which are inspired from the theory of compressive sampling [6], [7]. In this framework, sampling structure is composed of two separate parts: the analog circuit, *i.e.* digitiser, and the digital processing unit. In the Random Demodulator (RD) technique [8], there exists a pre-multiplication with a random binary sequence to spread the input spectrum, in the analog part. The analog signal then low-pass filtered and sampled by a sub-Nyquist rate ADC. The reconstruction is based upon some non-linear reconstruction algorithms, *i.e.* $\ell_1$ convex optimisation or greedy algorithms. There exist some multichannel versions of this technique, called the Modulated Wideband Converter (MWC) [9] and Random-Modulation Pre-Integrator (RMPI) [10].

There is an alternative framework for sub-Nyquist sampling [4], which predates CS. Feng and Bresler in [4] suggest to use a digitiser which consists of a bank of parallel delayed signals, with distinguished delays. These channels are sampled with a fixed rate lower than the Nyquist. Signals of different channels are called cosets and the whole system is called a multi-coset (MC) sampling scheme [4]. This approach has a simple digitiser, but the reconstruction algorithm, based on a subspace method, is computationally expensive.

### *A. Low-Complexity Multi-Coset Sampling*

The digitiser of the LoCoMC framework has a similar structure to MC. However, unlike the work of [4], LoCoMC can be implemented using as few as two multi-coset channels, while increasing the number of channels, increases the robustness of the sampling technique to the noise. The entire sampling process can be pipelined, while each channel is non-iterative. It is therefore ideally suitable for a low Size, Weight And Power (SWAP) implementation.

The complete system diagram is shown in Figure 1 and consists of the following elements. The input signal $x(t)$ is sampled using a bank of sub-Nyquist track and hold (T/H) devices, each sampling at $L$ times lower than the Nyquist rate $1/T$. Although these T/H's are operating with sub-Nyquist rate, the tracking part should work with the Nyquist rate to support aliased sampling. Prior to the T/H, each channel is delayed by a unique time delay of $c_iT$ second. The delays can be selected to reduce the sensitivity to the input noise by using the parameters of a Harmonic Equiangular Tight Frame (HETF) $\mathbf{\Theta}$, see [11] for a sufficient condition for the feasibility of such HETF for a given number of channels. Following the T/H, the signal is digitized using an Analog to Digital Converter (ADC) for the subsequent digital processing and analysis.

The digital component of the receiver is composed of a digital fractional delay (DFD) filter and a Time Frequency (TF) transform per channel, followed by a joint detection and de-aliasing step. The class of TF transforms which has been considered for LoCoMC is the Gabor type TF's, which is a time-discretisation of the following functional:

$$g_{m,k}(t) = g(t - m\tau_0)e^{2\pi i k \xi_0 t} \tag{1}$$

where $\tau_0$ and $\xi_0$ are the Frame discretisation parameters, $g(t)$ defines the window function [12], which is assumed to be normalised, $\|g\|_2 = 1$, *essentially* band-limited to $\omega \in [0, 2\pi/LT)$ and have its temporal support in the interval $0 \leq t < LNT$, where $N$ is the window length.

As $g_{m,k}(t)$ is essentially band limited, the discrete-time version of (1) can be derived as $g_{m,k}[n] = g[n - mM]e^{2\pi ikn/K} = g((n - mM)LT)e^{2\pi ikn/K}$, where $M$ and $K$ respectively adjust the overlap in time and frequency neighbouring TF-bins, see [1] for more detail.

The ADC of each MC channel is sampling with a rate less than the Nyquist and we thus have aliasing in the output of each channel. The assumption here is that, different active TF-bins do not overlap after downsampling. This condition is called the Approximate Disjoint Aliased Support (ADAS) [1]. This is not very restrictive condition for the TF sparse signals like radar ES. The task of subband classifier is to identify the correct location of each frequency bin in the aliased signals and move it back to the correct location in the full spectrum. This has been done using the fact that we have multiple instances of the signals, with known delays. The LoCoMC recovery algorithm is inspired from the DUET algorithm in the source separation field [13]. However, the performance of the recovery here is superior to the source separation algorithm, as the phase shift, which is caused by the time-delay, is constant over all bands of each channel. For the signal recovery, we then use the HETF corresponding to the one used for generating the time delays $c_i$'s, to map the vector $[r^{(i)}_{m,k}]_{1 \leq i \leq q} \in \mathbb{C}^q$, *i.e.* $(m, k)$-TF coefficient in channel $i$, to a vector with a peak at the correct frequency band number in the full spectrum, *i.e.* $1 \leq l \leq L$. We can then reconstruct the signal using the inverse TF transform or

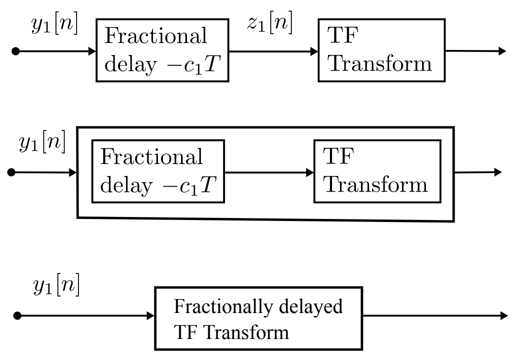


Fig. 3. The process of combining two filters of LoCoMC.

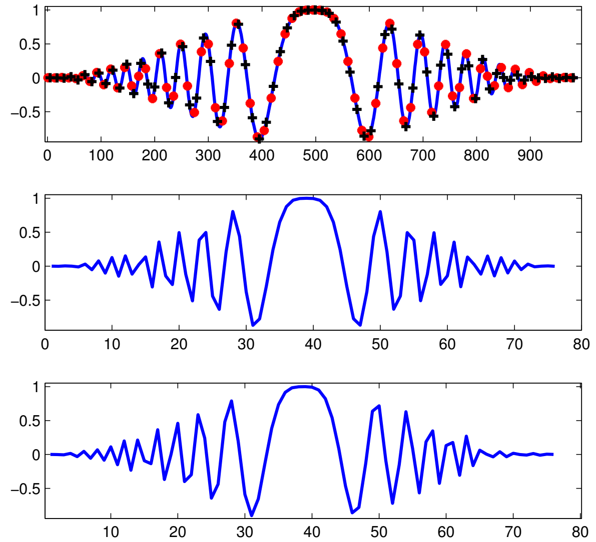

Fig. 4. Top plot: the chirped Hanning window, its discrete window without delay (red dots) and its discrete window with delay of 4/13 (black plus signs). Middle plot: discrete chirped Hanning window without delay (from red dots). Bottom plot: discrete chirped Hanning window with a delay of 4/13 (from black plus signs)

standard TF based detection/identification algorithms, applied to the full band TF representation. A brief representation of the LoCoMC is presented in Figure 2, where $h_{c_i}[n]$ is the impulse response of the DFD filter, $M_\xi$ is the frequency modulation operator, $D_\tau$ is a downsampling operator and $\mathbf{\Theta}(l)$ is the $l^{th}$ row of the HETF.

When the signals are ADAS with the selected downsampling factor, the SNR of the received signal depends on the input signal SNR and the accurate implementation of the DFD's. The effect of the input noise magnifies by downsampling [14]. We then need to be careful on the selection of downsampling factor, as a very large factor may case the signal be buried in the magnified noise, which makes it more challenging to do post-detection/identification task. However for a modest range of downsampling, *e.g.* 2 to 32, the output SNR is very related on how to implement the DFD. The ideal DFD filter is a shifted sinc function, which is non-causal and infinitely long. Any truncation of filter coefficients for practical purposes, generates artefacts in the output. While a frequency domain implementation of the filter may seem to be a good solution [15], *i.e.* a linear phase shift, it introduces delay distortion for the large fractional delays. The reason is that such a fractional delay has to be implemented using a discrete Fourier transform of a finite length. Such an implementation uses a circularly periodic sinc function, which introduces some time delay distortion. In the next section, we present an elegant implementation of the fractional delay with the less output distortion, which is also computationally more efficient for the class of TF transforms that we know their continuous kernel.

### B. Efficient Joint Implementation of the DFD and TF Transform

The digital part of the LoCoMC has two linear operators in each channel, *i.e.* before the subband classifier. These linear operators, *i.e.* fractional delay and TF transform, can be implemented using some linear filtering. The implementation of channel $i$ can thus be formulated as follows:

$$\mathbf{z}_i[n] = h_{c_i}[n] * \mathbf{y}_i[n], \tag{2}$$

and,

$$\begin{aligned} \mathbf{r}_\tau^{(i)} &= [\alpha_{k,\tau} D_\tau (M_k(g[n]) * \mathbf{z}_i[n])]_{1\le k\le K} \\ &= [\alpha_{k,\tau} D_\tau (g_k[n] * \mathbf{z}_i[n])]_{1\le k\le K} \\ &= [r_{m,k}^{(i)}]_{1\le k\le K}, \end{aligned} \tag{3}$$

where $\tau := mM$, $\alpha_{k,\tau} := e^{2\pi i k\tau/K}$ and

$$\begin{aligned} g_k[n] &:= M_k(g[n]) \\ &= g(nLT)e^{2\pi ikn/K}. \end{aligned}$$

By substituting $\mathbf{z}_i[n]$ from (2) in (3), we derive the following equation:

$$\begin{aligned} r_{m,k}^{(i)} &= \alpha_{k,\tau} D_\tau (g_k[n] * (h_{c_i}[n] * \mathbf{y}_i[n])) \\ &= \alpha_{k,\tau} D_\tau ((g_k[n] * h_{c_i}[n]) * \mathbf{y}_i[n]) \\ &= \alpha_{k,\tau} D_\tau ((h_{c_i}[n] * g_k[n]) * \mathbf{y}_i[n]), \end{aligned}$$

where the second and third equations are respectively derived using the associativity and the commutativity of the convolution operator. The operation $h_{c_i}[n] * g_k[n]$ is actually fractionally delaying $g_k[n]$. The implementation of this operation has a similar difficulty as before, if we only know $g_k(t)$ at discrete values. Fortunately, for the class of windows we consider here we know $g_k(t)$ over the period of $[0, N]$. The fractional delay in this case is very easy, as we only need to sample $g_k(t)$ at the delayed locations of interest. This process can be formulated as follows:

$$\begin{aligned} h_{c_i}[n] * g_k[n] &= h_{c_i}[n] * g(nLT)e^{2\pi ikn/K} \\ &= g((n + \frac{c_i}{N})LT)e^{2\pi ik(n+\frac{c_i}{N})/K} \\ &= g((n + \frac{c_i}{N})LT)e^{2\pi ikn/K}e^{2\pi ikc_i/NK}. \end{aligned} \tag{4}$$

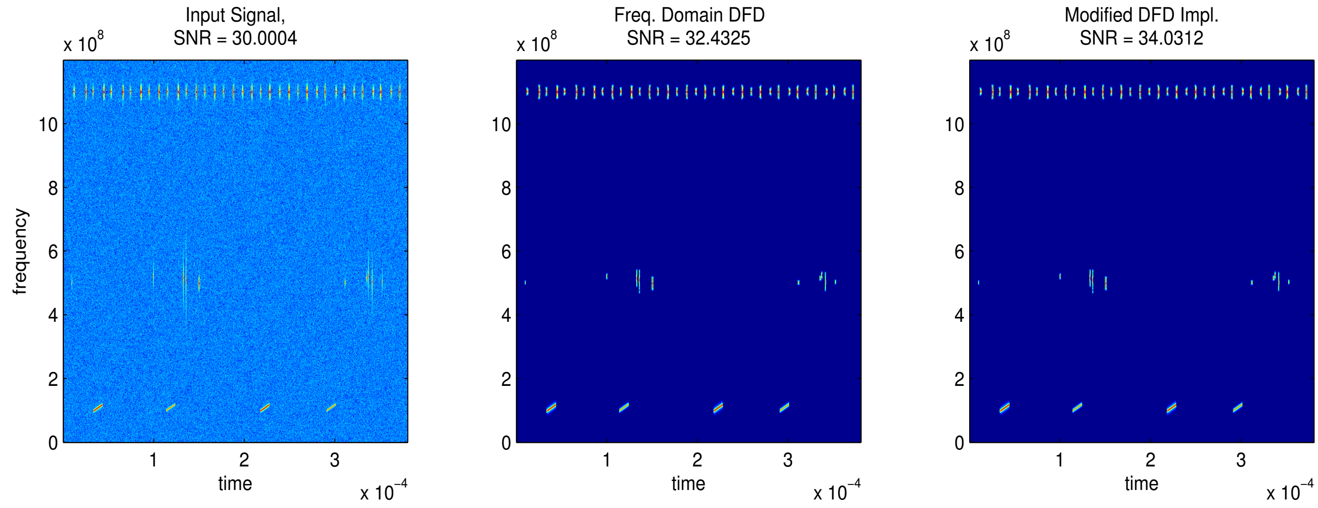


Fig. 5. Spectrogram of the noisy input signal (left panel), LoCoMC reconstruction with the Fourier domain phase shift (middle panel) and LoCoMC reconstruction with the modified TF transform (right panel)

An interested reader may notice the relation between formulation of (4) and the phase shifting technique. In the phase shifting technique, we only use the r.h.s. term, *i.e.* $e^{2\pi ikc_i/NK}$, while ignoring the window shift. The difference will be more important when the window has high-frequency components, *e.g.* Chirplet window [16]. A diagram which demonstrates the process of combining DFD filter and TF filter is shown in Figure 3. As an example, we have plotted a chirped Hanning window and its discrete versions with and without delay in Figure 4. It is clear that the delayed window is not only shifted with the delay factor, it has also some deformations.

Another potential application of the new implementation is that we can precisely do the fractional delay, which is necessary for the digital calibration of the analog delays, caused by the fabrication tolerance. Such a calibration is necessary as we normally have slight clock synchronisation error in the fabrication process.

## III. Rapidly Swept Super-heterodyne Receiver

In this section, we briefly describe the RSSR technique for channelised receivers, which is a popular technique for sub-Nyquist radar ES. This technique is based on the time-sharing and resource allocation principles. There is a bank of bandpass filters (BPF) at the front-end of the receiver, such that the output of each filter falls in the frequency bandwidth which we are able to sample with Nyquist rate. We then down-convert the signals to the baseband or IF band, depending on the structure of digitiser, using LO's. This channelised receiver can be time-division multiplexed [17] and digitised with a single ADC. There are some dual channel ADC's which sample the signal and its $90^\circ$ phase shifted version, to double the instantaneous bandwidth [2].

To minimise the switch-over period artefact, we can also use a second ADC with half a period of two consecutive switch. In this case, we constantly monitor a part of spectrum. The period of monitoring each frequency band is related to the shortest pulse-width. For the simulations of this paper, we used this technique for the comparison, as the comparison in SNR with the LoCoMC is more fair, *i.e.* no windowing edge artefact caused by band switch-over in the RSSR.

## IV. Simulations

In this part, we first demonstrate the advantages of using the proposed fractional delay implementation. For this reason, we used some simulated Radar ES signals with an active band between 10 and 11.2 GHz. The signal was preprocessed by downconverting to the baseband. We used a 4-channel multicoset sampling structure with 13 times undersampling in each track and hold, *i.e.* $1200/13 \approx 92\,\mathrm{MHz}$, and the delay factors were $\mathbf{c} = [6, 7, 10, 12]$ and the STFT was used as the TF transform. We then reconstructed the signal using the LoCoMC technique using the Fourier based and proposed DFD techniques. The simulation results shown in Figure 5 displays the output of LoCoMC, using an approximately 30 $\mathrm{dB}$ SNR noisy input signal. Although both techniques are successful in the recovery of the input pulses and chirps, the SNR of the output using the proposed DFD technique is more than 1.5 $\mathrm{dB}$ higher than the other method in this experiment. We repeated the simulation 100 times, with different additive noise, while the noisy signals had roughly the same SNR level, for an average case analysis. The average output SNR was 34.07 $\mathrm{dB}$ for the proposed technique, while it was 32.44 $\mathrm{dB}$ for the Fourier based DFD.

For the second experiment, we compared the LoCoMC, using the proposed DFD technique, with the RSSR technique. We used two ADC's which were operating with the $1200/6 = 200\,\mathrm{MHz}$ sampling rate. We chose such a setting as we have roughly the same average sampling rate, *i.e.* undersampling factor over the number of ADC's $13/4 \approx 6/2$. The simulation results are presented in Figure 6 We have plotted the output of

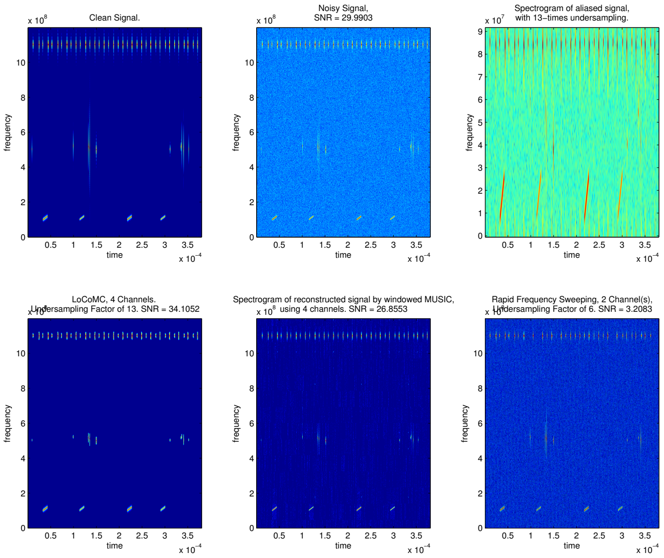


Fig. 6. The radar ES signals and their reconstructions. From left to right and top to bottom, clean signal, noisy input signal, a downsampled signal with the factor of 13, LoCoMC reconstruction, MC reconstruction with the MUSIC type algorithm and the RSSR reconstructed signals.

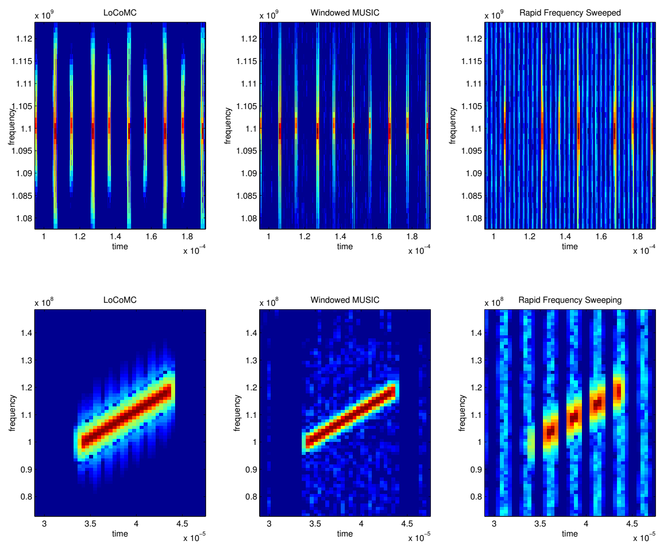


Fig. 7. Zoomed versions of the second row of Figure 6. Top row: stream of pulses. Bottom row: the first chirp.

LoCoMC, using a windowed MUSIC algorithm. We observe that SNR of the RSSR is very low in a comparison with MC methods. If we look at the zoomed plots of these results in Figure 7, we can see that some of the pulses are missing in the RSSR, because of the multiplexing, and we also have some processing gain loss in the chirps, as some parts of each chirp is missing, for the very same reason. The MC methods are behaving roughly the same, but the LoCoMC is computationally much simpler than a subspace method like MUSIC.

## V. Conclusion

We here investigated a particular type of sub-Nyquist sampling technique for the radar ES receivers, where the reconstruction algorithm is computationally suitable for implementation on embedded systems. A challenge of accurate implementation of the algorithm was investigated in this paper, where we presented a new approach which can reduce the system induced errors. These errors are mainly coming from the implementation of the DFD's and the time-delay tolerances in hardware fabrication. The proposed digital fractional delay implementation can be very accurate and compensate the fabrication tolerance by manual calibration. We also compared the performance of the LoCoMC with the subspace and rapidly swept superheterodyne based receivers in some simulated radar ES scenarios, which showed the advantages of LoCoMC.

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