Adaptive Bayesian Sensor Motion Planning for Hazardous Source Term Reconstruction

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Abstract: There has been a strong interest in emergency planning in response to an attack or accidental release of harmful chemical, biological, radiological or nuclear substances. Under such circumstances, it is of paramount importance to determine the location and release rate of the hazardous source to forecast the future harm it may cause and employ methods to minimize the disturbance. In this paper, a sensor data collection strategy is proposed whereby an autonomous mobile sensor is guided to address such a problem with a high degree of accuracy and in a short amount of time. First, the parameters of the release source are estimated using the Markov chain Monte Carlo sampling approach. The most informative manoeuvre from the set of possible choices is then selected using the concept of maximum entropy sampling. Numerical simulations demonstrate the superior performance of the proposed algorithm compared to traditional approaches in terms of estimation accuracy and the number of measurements required.

Keywords: Autonomous vehicles; Inverse problem; Information fusion; Parameter estimation; Optimal experiment design; Statistical inference; Motion planning

1. INTRODUCTION

In the event of an accidental or intentional release of a hazardous contaminant into the atmosphere, it is important to predetermine affected areas for a rapid emergency response. The spread of particles into the atmosphere can be estimated using atmospheric transport and dispersion (ATD) models (Panofsky and Dutton (1984)). Important input parameters to such models include meteorological variables and an estimate of the source term. The source term can feature a range of variables; however, in this work we focus on the location and release rate of the source.

Source seeking algorithms are the intuitive approach to determine the location of an emitting source by moving towards it. Biologically inspired strategies based on the male silkworm moth in search for a female and the foraging behaviour of Escherichia coli bacteria have been popular in the literature (Kowadlo and Russell (2008)). These algorithms find the source by climbing the chemical concentration gradient (chemotaxis). This proved to be a suitable approach under short distances from the source where concentration gradients are high. Recently, more complex probabilistic algorithms have been developed to handle difficult scenarios where sensing is noisy and sporadic. For instance, Vergassola et al. (2007) used Bayesian inference to update a probabilistic map of source location. The searcher was moved to maximise the expected reduction in entropy of the posterior distribution. Several papers have since been produced validating the approach and proposing extensions (Martin Moraud and Martinez (2010); Eggels et al. (2016)). A lacking feature within source seeking algorithms is an estimate of the release rate of a source which will be essential for future hazard prediction.

Source term estimation (STE) algorithms estimate at minimum the location and release rate of the source; nonetheless, in the literature, several additional uncertain parameters have been included such as meteorological variables (Allen et al. (2007)) and stochastic diffusion parameters (Senocak et al. (2008)). The most popular approach to STE features a static network of concentration sensors spread over a large region on the ground. Source estimation is then carried out using optimization (Long et al. (2010); Thomson et al. (2007)) or Bayesian inference algorithms (Keats et al. (2007); Senocak et al. (2008)) where inferred source parameters are run in a forward ATD model to generate predicted concentrations that are then compared with the data using a cost or likelihood function. A recent study by Platt and Deriggi (2010) based on data from the FFT07 experiment demonstrated some of the limitations of theses approaches when applied to real data. Large errors were witnessed in relatively simplified conditions.

An alternative to large static networks of sensors are sensor equipped unmanned vehicles. Mobile sensors can estimate the source term more efficiently as they can be rapidly deployed and collect observational data from more

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informative locations. Kuroki et al. (2010) used an expert system for sensor motion planning, while the genetic algorithm from Long et al. (2010) estimated the source term of the dispersing contaminant. Ristic et al. (2010) used mobile sensors to estimate the source term of a radiological release. Estimation was carried out using a particle filter and the next sampling location was selected based on maximizing the fisher information of the posterior. Madankan et al. (2014) used polynomial chaos quadrature to estimate source parameters of an atmospheric release, where dynamic programming was used to guide the sensor to the most informative measurement locations based on maximising the mutual information.

In this work, an alternative approach to sensor motion planning for STE is proposed, inspired by work from the field of optimal experiment design known as Bayesian adaptive exploration (BAE) Loredo (2004). The BAE provides an iterative observation-inference-design framework for probabilistic and on-line experimental design. Keats et al. (2010) first applied the BAE method to the problem of STE for optimal placement of a single additional static sensor to an existing static sensor network. This BAE approach has been adapted in this work for motion planning of a mobile sensor to manoeuvre to the most informative measurement locations, which combines search for the contaminant source and STE under a single procedure. The proposed algorithm is compared with traditional techniques under various levels of noise while showing robustness to large amounts of noise as a result of Bayesian sampling techniques.

This paper is organised as follows. In Section 2, the problem is presented including information about the domain and the forward dispersion model used. In Section 3, the adaptive Bayesian sensor motion planning is described. In Section 4, the computational algorithms that were used to implement the conceptual solution are described. An illustrative run and Monte Carlo simulations with other strategies are given in Section 5. Finally, the paper is concluded and future work is proposed in Section 6.

2. PROBLEM DESCRIPTION

When signs of a possible harmful contaminant release are brought to the attention of emergency services, the responders must determine the location of the emitting source, and predict the spread of contamination in order to react efficiently. To avoid putting the emergency responders in danger, an unmanned vehicle equipped with an appropriate sensor can be sent into the search area to assess the severity of contamination. The vehicle is to navigate within the search area collecting concentration measurements which will be used in an estimation algorithm to determine the source term. Sensor measurements can require a long sampling time to gain an accurate concentration reading, so it is important to need as few as possible, whilst producing a high level of STE accuracy. Our aim is to rapidly gain a reliable estimate of the source term for its use in an ATD model.

In this work, the dispersion of contaminant is assumed to have reached a steady state. Due to its low computational burden and reasonable accuracy under short ranges, the Gaussian plume dispersion equation (Panofsky and Dutton (1984)) is used as the forward ATD model in the estimation of source parameters:

$$C\left(x_k, y_k, z_k, x_s, y_s, z_s, q_s\right) = \frac{q_s}{\bar{u}\sigma_y\sigma_z 2\pi}\left(\frac{-y_k^2}{2\sigma_y^2}\right) \times \left[\exp\left(\frac{-(z_k - z_s)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z_k + z_s)^2}{2\sigma_z^2}\right)\right], \quad (1)$$

where $C(x_k, y_k, z_k, x_s, y_s, z_s, q_s)$ is the concentration at a particular location (x_k, y_k, z_k) from a source positioned at (x_s, y_s, z_s) with release rate q_s. $\bar{u}$ is the mean wind speed. σ_y, σ_z are turbulent diffusion parameters that are estimated based on Pasquill's atmospheric stability class (Panofsky and Dutton (1984)).

Most meteorological variables can be known within a certain degree of accuracy from existing sensors across the globe. We assume that these variables have been provided and that the source is located on the ground ($z_s = 0$). The source term parameters remaining to be estimated are the location (x_s, y_s) and the release rate (q_s) of the source. We assume the source parameters are within a search space Ω. The source term vector θ is defined as:

$$\theta = [x_s, y_s, q_s]^\mathsf{T} \quad \text{where} \quad (x_s, y_s, q_s) \in \Omega. \quad (2)$$

We assume that the vehicle knows its location (x_k, y_k) at the current time step k and it is equipped with the appropriate concentration sensor. The available manoeuvres for the vehicle are $A_k = \{\uparrow, \downarrow, \leftarrow, \rightarrow\}$, referring to a move up, down, left or right, by a fixed jump size. The goal of the algorithm is to choose the manoeuvre $a_k \in A_k$ that provides the most information about the unknown source term θ.

3. ADAPTIVE BAYESIAN SENSOR MOTION PLANNING

Bayesian adaptive exploration, proposed by Loredo (2004), is adapted for mobile sensor motion planning. For the remainder of the paper, the approach shall be referred to as adaptive Bayesian motion planning (ABMP). The process iterates an observation, inference and design cycle illustrated in Fig. 1.

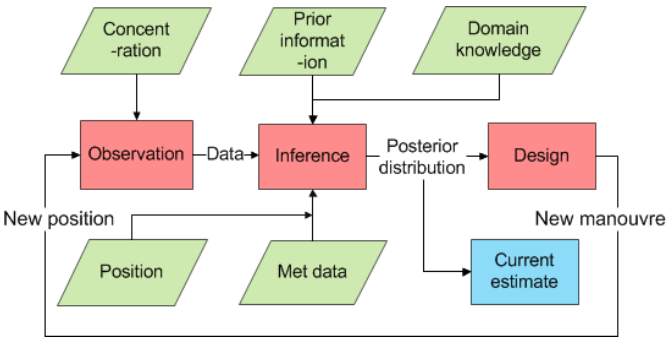

Fig. 1. Adaptive Bayesian motion planning algorithm flowchart

The observation phase is rather simple and essentially involves taking a measurement of the phenomena. Which is the contaminant concentration in this paper. In the inference phase, Bayesian inference is used to gain an estimate of the source term to reveal the current state of knowledge about the release. During the design phase, the optimal manoeuvre is selected, which is expected to

yield the most information for the next inference cycle. The optimal manoeuvre is determined using the idea of maximum entropy sampling, where it is believed the most is learnt by sampling from where the least is known, (Sebastiani and Wynn (2000)). In the following section, the steps used are described in more detail.

3.1 Observation

Concentration observations D_k and predictions C_k are assumed to be composed of the true signal $\bar{D}_{true,k}$ and noise ν_k.

$$D_k = C_k(\theta_{true}) + \nu_k = \bar{D}_{true,k} + \nu_k. \tag{3}$$

Noise can arise from errors in meteorological data, sensing, atmospheric turbulence or modelling discrepancies (Rao (2005)). In this work, meteorological data and errors due to atmospheric turbulence are neglected. The sensing and modelling errors are assumed to be normally distributed with zero mean.

3.2 Inference

In such a scenario where input variables and underlying models are uncertain, a probabilistic approach is preferred over optimisation so that uncertainty in the source term can be captured within a posterior probability density function (pdf). Bayes' theorem is used to define the posterior pdf of the source term θ given the observations D. In this work, Bayes' theorem is expressed as (4), where I represents prior information about the release and the ATD model used. Essentially, this means that the posterior distribution is proportional to the product of the prior and the likelihood.

$$P(\theta|D, I) \propto P(\theta|I)P(D|\theta, I). \tag{4}$$

The prior on all parameters is assumed to be uniformly distributed within the search domain Ω. A Gaussian form of the likelihood is used similarly to Keats et al. (2007):

$$P(D_{1:k}|\theta_k, I_{1:k}) = \sum_{i=1}^{k} \frac{1}{\sigma_i\sqrt{2\pi}} \exp\left[-\frac{(D_i - C(x_i, y_i, \theta_i))^2}{2\sigma_i^2}\right], \tag{5}$$

where D_i are the observed concentration data, and $C(x_i, y_i, \theta_i)$ are predicted concentrations obtained by running inferred parameters θ in an ATD model (1). σ_i refers to the noise variance, which has a strong effect on the acceptance rate of the algorithm. The Gaussian likelihood allows measurements of zero to be easily incorporated into the likelihood function without the additional treatment that is needed in a log-normal form (Senocak et al. (2008)).

3.3 Design

The goal of the design phase is to choose the manoeuvre a_k that is expected to be the most informative, by following a similar approach described in (Loredo (2004)):

$$a_k^* = \underset{a_k \in A_k}{\arg\max}\{E\mathscr{I}(a_k)\}, \tag{6}$$

where $E\mathscr{I}(a_k)$ is the expected information provided by taking manoeuvre a_k. This is defined as the information gained about the posterior distribution given the new data $d_k^{a_k}$, multiplied by the probability of the new data (7):

$$E\mathscr{I}(a_k) = \int \mathscr{I}(\theta_k|d_k^{a_k}, D_{1:k}, I_{1:k}) \times P(d_k^{a_k}|D_{1:k}, I_{1:k})\,\mathrm{d}d_k^{a_k}. \tag{7}$$

To quantify the measure of information $\mathscr{I}$, several derivations have been proposed from the literature on information theory. In this work, the negative Shannon entropy has been used given by the following Eq. (8). This quantity of information provides a measure of the spread of a distribution:

$$\mathscr{I}(\theta|d, D, I) = \int P(\theta|d, D, I) \log P(\theta|d, D, I)\,\mathrm{d}\theta, \tag{8}$$

where $P(\theta|d, D, I)$ is the posterior for source term parameters considering future data d. In order to represent (7) in another form, let us look at the joint distribution for θ and d and using the product rule to factor it as given:

$$\begin{aligned}
\mathscr{I}(d_k^{a_k}, \theta_k) &= \int\int P(d_k^{a_k}, \theta_k|D_{1:k}, I_{1:k}) \\
&\qquad \times \log P(d_k^{a_k}, \theta_k|D_{1:k}, I_{1:k})\,\mathrm{d}\theta_k\mathrm{d}d_k^{a_k} \\
&= \int\int P(d_k^{a_k}, \theta_k|D_{1:k}, I_{1:k}) \log P(\theta_k|D_{1:k}, I_{1:k})\,\mathrm{d}\theta_k\mathrm{d}d_k^{a_k} \\
&+ \int\int P(d_k^{a_k}, \theta_k|D_{1:k}, I_{1:k}) \log P(d_k^{a_k}|\theta_k, D_{1:k}, I_{1:k})\,\mathrm{d}\theta_k\mathrm{d}d_k^{a_k} \\
&= \mathscr{I}(\theta_k|D_{1:k}, I_{1:k}) + \int P(\theta_k|D_{1:k}, I_{1:k})\mathscr{I}(d_k^{a_k}|\theta_k, D_{1:k}, I_{1:k})\,\mathrm{d}\theta_k.
\end{aligned} \tag{9}$$

Repeating the above calculation but switching the order of factorising d and θ gives:

$$\begin{aligned}
\mathscr{I}(d_k^{a_k}, \theta_k) &= \mathscr{I}(d_k^{a_k}|D_{1:k}, I_{1:k}) \\
&\quad + \int P(d_k^{a_k}|D_{1:k}, I_{1:k})\mathscr{I}(\theta_k|d_k^{a_k}, D_{1:k}, I_{1:k})\,\mathrm{d}d_k^{a_k}.
\end{aligned} \tag{10}$$

Equating Eqs. (9) and (10) and noting that the integral in (10) is simply the expected information from (7) yields:

$$\begin{aligned}
E\mathscr{I}(a_k) &= \mathscr{I}(\theta_k|D_{1:k}, I_{1:k}) \\
&\quad + \int P(\theta_k|D_{1:k}, I_{1:k})\mathscr{I}(d_k^{a_k}|\theta_k, D_{1:k}, I_{1:k})\,\mathrm{d}\theta_k \\
&\quad - \mathscr{I}(d_k^{a_k}|D_{1:k}, I_{1:k}).
\end{aligned} \tag{11}$$

The first term in (11) refers to the information in the posterior distribution from the previous time step which is independent from the future measurement, so it shall remain constant. The second term refers to the average information contained in the sampling distribution. In cases where the noise variance varies with the signal, this is an important quantity; however, if the noise is constant regardless of the signal, then this term is also constant. The final term is the entropy (considering a minus sign) in the predictive distribution which needs to be calculated. Using (8), the expected information can be represented as:

$$\begin{aligned}
E\mathscr{I}(a_k) &= c_k - \mathscr{I}(d_k^{a_k}|D_{1:k}, I_{1:k}) \\
&= c_k - \int P(d_k^{a_k}|D_{1:k}, I_{1:k}) \log P(d_k^{a_k}|D_{1:k}, I_{1:k})\,\mathrm{d}d_k^{a_k},
\end{aligned} \tag{12}$$

where c_k represents a constant formed from the first two terms of (11). In order to choose the most informative manoeuvre using (6), we need to maximise (12). This means that the best move is to go toward the location whose predictive distribution has maximum entropy (equivalently, the least information); it is called as the principle of maximum entropy sampling (Sebastiani and Wynn (2000)). In other words, the most informative manoeuvre is where the predictive distribution has the most spread and uninformative.

4. COMPUTATIONAL APPROACH

The inference and the design stages of the algorithm involve solving multidimensional integrals that cannot be done analytically. In this section, the computational approach used to implement the conceptual solution for ABMP is described.

4.1 Inference

Since the posterior distribution of source parameters (4) cannot be obtained analytically, it shall be approximated using a numerical techniques such as Monte Carlo methods. However, as it is computationally expensive, an efficient sampling technique is required to approximate the posterior distribution. Within the literature on STE, several techniques have been used: i) Markov chain Monte Carlo (MCMC) (Keats et al. (2007); Senocak et al. (2008)); ii) sequential Monte Carlo (SMC) (Johannesson et al. (2005)); and iii) differential evolution Monte Carlo (DEMC) (Robins et al. (2009)). In this work, we use the Metropolis-Hastings MCMC algorithm (Hastings (1970)). As this is a popular approach used in the majority of MCMC based STE algorithms, it will not be described any further in this paper. For more information on MCMC for STE, the reader is directed to Keats et al. (2007).

The output of the MCMC algorithm is a posterior distribution for the source parameters (4), represented by a Markov chain. In subsequent iterations of the ABMP algorithm, a new Markov chain is initiated each time new data has been collected. The starting point of the new Markov chain is at the mean value of each source parameter from the previous iteration.

4.2 Design

Once a posterior distribution of source parameters has been obtained using MCMC; The pdf in (12) can be approximated using a set of N samples $\{\theta_k^n\}_{n=1}^N$ for which the information can be estimated:

$$P(d_k^{a_k}|D_{1:k}, I_{1:k}) = \int P(d_k^{a_k}|\theta_k, I_{1:k}) P(\theta_k|D_{1:k}, I_{1:k})\, \mathrm{d}\theta_k \approx \frac{1}{N}\sum_{n=1}^{N} P(d_k^{a_k}|\theta_k^n, I_{1:k}) = \bar{P}(d_k^{a_k}). \quad (13)$$

The average information from a set of samples for a specific manoeuvre is used as a measure of the expected information (Loredo (2004)):

$$E\mathscr{I}(a_k) \approx -\frac{1}{M}\sum_{m=1}^{M} \log \bar{P}(d_k^{a_k,m}). \quad (14)$$

The overall ABMP algorithm is described in Algorithm 1.

Algorithm 1 Adaptive Bayesian motion planning

```
for k = 1,2,..., max time steps do
    D_k ← take new measurment
    P(θ_k|D_{1:k}, I_{1:k}) ← run MCMC algorithm
    {θ_k^n} ← draw N samples from above distribution
    choose integer M ≤ N
    for all a_k ∈ A_k do
        consider potential position (x_k^{a_k}, y_k^{a_k})
        for m = 1:M do
            θ_k^{a_k,m} ← draw uniformly from {θ_k^n}
            d_k^{a_k,m} ← sample from P(d|θ_k^{a_k,m}, I_k^{a_k,m})
            determine P̄(d_k^{a_k,m}) ← Eq. (13)
        determine EI(a_k) ← Eq. (14)
    a_k^* = arg max{EI(a_k)} ← new manoeuvre
    (x_{k+1}, y_{k+1}) = (x_k, y_k) + a_k^* ← new position
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5. NUMERICAL SIMULATIONS

5.1 Illustrative run

An example run of the algorithm at various time steps is presented in Fig. 2. Synthetic data were created using the Gaussian plume dispersion model (1) infected with normally distributed noise with mean zero and standard deviation equal to 50% of the signal. In the early stages of the simulation, the sensor moved crosswind before moving towards the location of the source. In simulations under higher noise, as illustrated in Fig. 3, the algorithm was naturally more explorative without any tuning of parameters. This showed a strong balance between explorative and exploitative behaviour which is crucial for efficient yet robust autonomous search behaviour.

By using ABMP, the unmanned vehicle is capable of estimating the source term regardless of its starting location or the location of the plume, provided it existed within the search domain. Towards the end of the search, the acceptance rate of the MCMC inference decreases, this is not a problem as the results produced are still accurate. However, addressing this in the future could yield a better approximation of the posterior distribution.

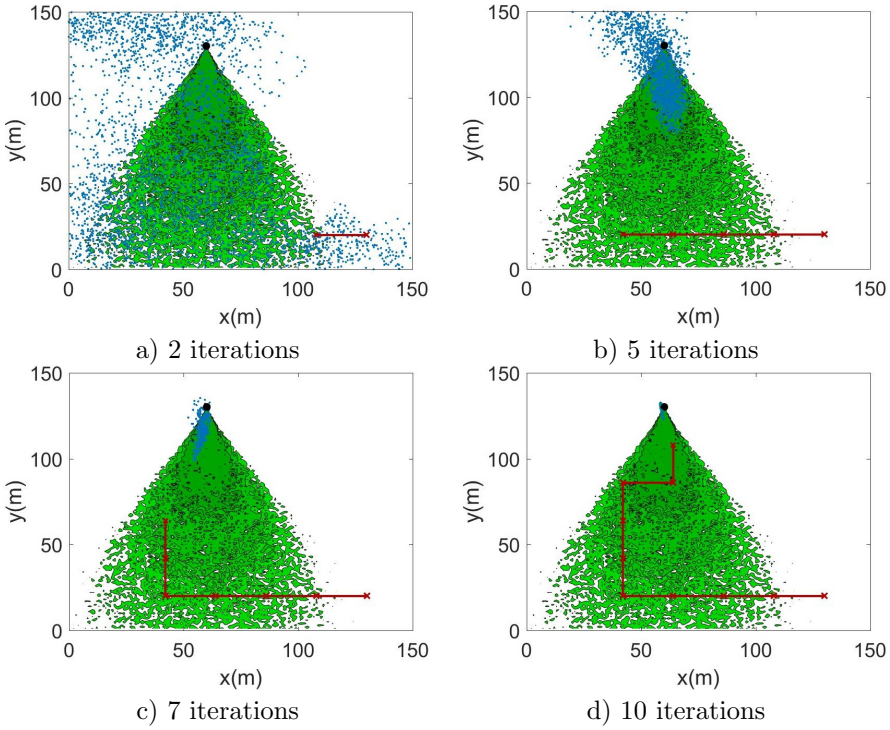
5.2 Monte Carlo comparison

Monte Carlo simulations were used to assess the performance of the algorithm in comparison to a uniform sweep, random movement and a source seeking algorithm. The source seeking algorithm followed the ABMP procedure partially; however, it moves towards the current estimate of the source position based on the mean values from the inference algorithm. Examples of the paths of each are shown in Fig. 4 under identical conditions as Fig. 2. For the Monte Carlo comparisons, the Gaussian plume equation (1) was used to generate synthetic data infected with normally distributed noise with mean zero and standard deviations equal to 10% and 50% of the signal. The contaminant plume was generated at random locations, with randomly varying wind direction. The results of the average root mean squared error (RMSE) for the mean parameter estimates after 100 Monte Carlo simulations are presented in Table 1.

Plots of the number of measurements versus the average RMSE for the Monte Carlo simulations subject to 10% normally distributed noise have been plotted in Fig. 5. The graphs clearly demonstrate the overall benefit with regards to the accuracy and convergence time of the algorithm in estimation of the source term parameters.

6. CONCLUSIONS AND FUTURE WORK

A motion planning strategy based on adaptive Bayesian exploration is used to guide a mobile sensor to the most



a) 2 iterations b) 5 iterations

c) 7 iterations d) 10 iterations

Fig. 2. Example run of the ABMP algorithm. The shaded green region represents the contaminant with source position indicated by the black circle. Blue dots represent the Markov chain posterior for source location. Red crosses represent the measuring locations of the sensor following the red lined vehicle path.

Table 1. Performance comparison over a hundred Monte Carlo simulations

Strategy	Random search		Uniform search		Source seeking		ABMP	
Noise	10%	50%	10%	50%	10%	50%	10%	50%
RMSE in x (m)	11.96	15.54	7.50	11.16	9.35	13.42	1.74	5.95
RMSE in y (m)	12.10	14.63	7.34	8.79	9.42	13.09	1.95	6.67
RMSE in q (g/s)	0.29	0.33	0.21	0.30	0.26	0.32	0.15	0.28
Number of moves	21.55	22.89	19.08	20.65	12.71	12.87	10.68	11.71

informative sensing locations for STE of a harmful atmospheric release. MCMC was used for inference of the source parameters whilst motion planning was achieved through posterior sampling to find the location of maximum entropy. The approach is capable of handling a high amount of noise and efficiently finds the source even under poor starting positions outside of the contaminated area. Simulation results demonstrated the performance improvements of the proposed approach compared to traditional methods. In future work, it will be important to consider the more probable scenario where the dispersion of contaminant is still evolving. This will result in an expansion of the parameter space in the inference phase of the algorithm, greatly increasing the complexity of the problem. In such a scenario, it will be useful to extend the method to intelligently guide multiple cooperating vehicles. Other useful extensions include: i) adding vehicle dynamics considerations to the algorithm; ii) a multiple step ahead strategy (however, the performance benefit may be outweighed by computational expense); and iii) to overcome the low acceptance rate towards the end of the search, an adaptive inference algorithm shall be implemented to accompany the changing input data.

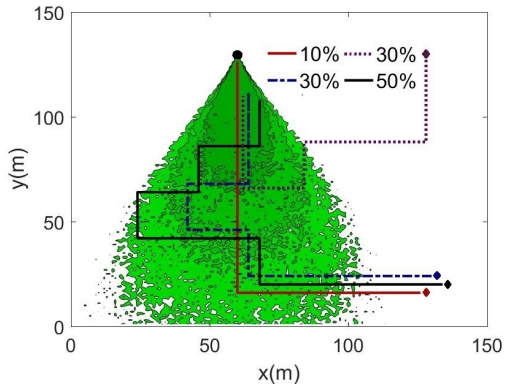


Fig. 3. Example paths of ABMP under various levels of noise (two 30% paths are generated from different initial positions, indicated by coloured diamonds).

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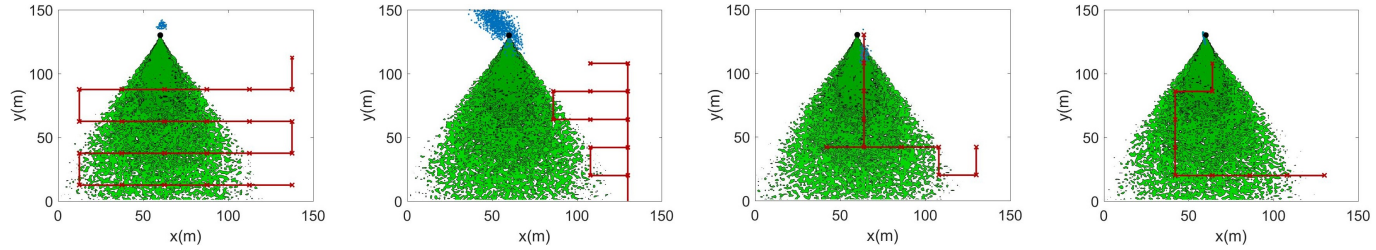
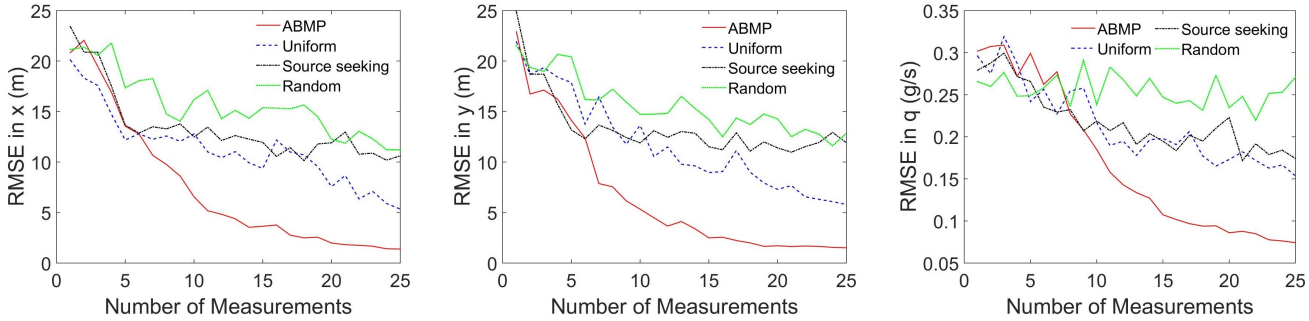


a) Uniform search b) Random movement c) Source seeking d) ABMP

Fig. 4. Example path and search results for a) uniform sweep; b) random movement; c) source seeking; and d) ABMP.



a) RMSE for x c) RMSE for y b) RMSE for q

Fig. 5. Monte Carlo results of RMSE vs the number of measurements.

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