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Adaptive Waveform Design for Cognitive Radar

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Cognitive Radar Characteristics

Cognitive Radar is the next generation intelligent radar which utilizes new technologies to enhance the performance of the system. Cognitive Radar is a highly adaptive and autonomous multifunctional system which can adapt the operating algorithm on the dynamically changing environment. It therefore requires diverse waveform design in order to achieve its mission such as detecting moving targets, imaging terrain or measuring accurately range, angle and Doppler.



The measurement noise covariance matrix *R* is given by:

$$R = E[(Y - \bar{Y})(Y - \bar{Y})^T] = BJ^{-1}B^T \qquad B = diag({^C/_2}, {^C/_{2f_c}})$$
(5)

T denotes the conjugate and B the transformation matrix between the received parameter vector of time delay τ and Doppler shift ω [τ, ω] and the tracking system measurement vector of range r and velocity v[r, v].

The MSE is given by the trace of the $P_{k|k}$ matrix which using the Kalman filters is iteratively calculated by:

$$P_{k|k} = P_{k|k-1} - P_{k|k-1} H_k^T \left[H_k P_{k|k-1} H_k^T + R(\theta_k) \right]^{-1} H_k P_{k|k-1}$$
(6)

where $P_{k|k-1}$ is the predicted MSE matrix at time k and H is the observation matrix and θ_k the vector containing the adaptive parameters of the waveform. The aim is to appropriately choose θ_k in order to minimize the trace of $P_{k|k}$.

• Feedback – information loop from the receiver to the

• Memory – keep past data of

Adaptive Waveform Design

The adaptive waveform design is based on the computation of Cramer Rao Lower Bound (CRLB) aiming to decrease the mean square error (MSE) of the response estimation based on the target impulse response and its dynamic model. It is obtained by adjusting the pulse width and the chirp rate of an LFM waveform.



The complex envelop of the received signal:

$$\tilde{r}(t) = a\sqrt{E_T}\tilde{f}(t-\tau)e^{-j\omega t} + w(t)$$
(1)

where a is a random Gaussian variable with zero mean, E_T is the transmitted energy, \tilde{f} is the complex envelop of the transmitted waveform and w is noise.

 $+\infty$

The Ambiguity Function:
$$x(\tau, \omega) = \int_{-\infty} \tilde{r}(t) \tilde{f}^*(t-\tau) e^{-j\omega t} dt$$
 (2)

Simulation

The first scenario simulated includes a constant velocity target moving away from the radar. The adaptive waveform is a rectangular LFM pulse with duration λ adjusted from 0.01s to 0.3s and the bandwidth B from 1 KHz to 10 KHz. The fixed waveform was set to $\lambda = 0.01s$ and B = 1KHz.



The ability to choose between three different classes of waveforms was simulated in the second scenario. The available waveforms are: 1) Triangular, 2) Gaussian and 3) Gaussian LFM pulse. The target in this case approaches the radar moving with variable acceleration. Pulse Duration of Adaptive Waveforn



with τ and ω representing the time delay and Doppler frequency respectively.

The Likelihood Function: $\ln \Lambda(\tau, \omega) = n\{|x(\tau, \omega)|^2\}$ (3)

with *n* representing the signal to noise ratio. Then the Fisher matrix of the CRLB has the following form:

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \qquad J_{12} = J_{21} = -E \begin{bmatrix} \frac{\partial^2 \ln \Lambda(\tau, \omega)}{\partial \tau \partial \omega} \end{bmatrix}$$
(4)
$$L = E \begin{bmatrix} \partial^2 \ln \Lambda(\tau, \omega) \end{bmatrix} \qquad L = E \begin{bmatrix} \partial^2 \ln \Lambda(\tau, \omega) \end{bmatrix}$$

$$J_{11} = -E\left[\frac{\partial^2 \ln \Lambda(\tau,\omega)}{\partial \tau^2}\right] \qquad \qquad J_{22} = -E\left[\frac{\partial^2 \ln \Lambda(\tau,\omega)}{\partial \omega^2}\right]$$

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Future Work

- Develop a method for target tracking with unknown target motion model •
- Incorporation of data provided by other sensors or databases (i.e. high resolution cameras, environmental and clutter statistics) to the waveform design process
- Communication and information sharing between radar platforms •
- Integration of the above techniques to MIMO radars

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