Reduced-Rank STAP for Airborne Radar Based on Switched Joint Interpolation, Decimation and Filtering Algorithm

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***Abstract*— We present an adaptive reduced-rank signal processing technique for airborne phased array radar applications. The proposed method performs dimensionality reduction by using a reduced-rank switched joint interpolation, decimation and filtering algorithm (RR-SJIDF). A multiple-processing-branch (MPB) framework, which contains a set of jointly optimized interpolation, decimation and filtering units, is employed to process the observations. The output is switched to the branch with the best performance among the available ones. In order to design the decimation unit, we present the optimal decimation scheme and also a low-complexity decimation algorithm. We then develop a low-complexity recursive least squares (RLS) algorithm for the proposed scheme. Simulations results show that the proposed RR-SJIDF STAP algorithm converges at a very fast speed and provides a considerable signal-to-interference-plus-noise-ratio (SINR) performance improvement over the state-of-the-art reduced-rank schemes.**

I. Introduction

Space-time adaptive processing (STAP) has been motivated as a key enabling technology for advanced airborne radar applications following the landmark publication by Brennan and Reed [1]. STAP techniques can improve slow-moving target detection through better mainlobe clutter suppression, provide better detection in combined clutter and jamming environment, and offer a significant increase in output signal-to-interference-plus-noise-ratio (SINR). However, due to their large computational complexity, the full-rank optimum and adaptive STAP techniques are prohibitive for practical use when the number of elements in the filter is large. It is well-known that $K \geq M$ independent and identically distributed (i.i.d) training samples are needed for the filter to achieve the steady performance, where M is the filter dimension [2]. Thus, in dynamic scenarios, full-rank STAP techniques with large M usually fails or provides poor performance in tracking target signals contaminated by interference and noise.

Reduced-rank adaptive signal processing, which has gained a great deal of attention in the last decades, is a key technique for dealing with large systems [3]- [17]. The basic idea of the reduced-rank algorithms is to reduce the number of adaptive coefficients by projecting the received vectors onto a lower dimensional subspace which consists of a set of basis vectors. The adaptation of the low-order filter within the lower dimensional subspace results in significant computational savings, faster convergence speed and better tracking performance. The first statistical reduced-rank method was based on a principal-components (PC) decomposition of the target-free covariance matrix [3]. Another class of eigen-decomposition methods was based on the cross-spectral metric (CSM) originally proposed in [4] and then also considered in [5]. Both the PC and CSM algorithms have a high computational load due to the eigen-decomposition. Another class of subspace methods has been investigated thoroughly in the recent years. Among them are the multistage Wiener filter (MSWF) [6], [7] which projects the observation data onto a lower-dimensional Krylov subspace and the auxiliary-vector filtering (AVF) [8], [9]. These methods are very complex to implement in practice and suffer from numerical problems despite their improved convergence and tracking performance. Recently, reduced-rank filtering algorithms based on joint iterative optimization of filters [11], [13], [14] and an adaptive diversity-combined decimation and interpolation scheme [15] have been proposed. In our prior work [14], a STAP scheme based on joint iterative optimization of filters has been applied to airborne radar. This technique provides a significant improvement both in convergence speed and SINR performance but with considerable complexity level, even compared with the existing reduced-rank STAP algorithms. However, the work in [15], [16] has not considered linearly constrained reduced-rank algorithms that are suitable for STAP in radar systems.

In this paper, we develop a reduced-rank approach to the STAP design utilizing a scheme based on a switched joint interpolation, decimation and filtering (SJIDF) algorithm for airborne radar systems. In this scheme, the number of elements for processing is substantially reduced, resulting in considerable computational savings and very fast convergence performance for radar applications. The proposed approach obtains the subspace of interest via a multiple processing branch (MPB) framework which consists of a set of simple interpolation, decimation and filtering operations. Unlike the previous work in [15], multiple interpolators and reduced-rank filters are employed in the MPB framework. We describe an optimal decimation scheme and a low-complexity decimation scheme for the proposed structure. We derive an RLS algorithm for the proposed scheme and evaluate its computational complexity. The results show that the proposed RR-SJIDF STAP converges at a very fast speed and obtains a considerable SINR improvement over the existing methods.

This paper is organized as follows. Section II states the signal model and the problem which concerns us. Section III presents the proposed reduced-rank adaptive filtering scheme, describes the proposed joint iterative optimization of the interpolation, decimation and filtering tasks, and details the proposed decimation schemes. In Section IV, we present the proposed adaptive RLS algorithm. The performance assessment examples of the proposed reduced-rank STAP are provided in Section V using simulated radar data. Finally, conclusions are given in Section VI.

This work is funded by the Ministry of Defence (MoD), UK. Project MoD, Contract No. RT/COM/S/021.

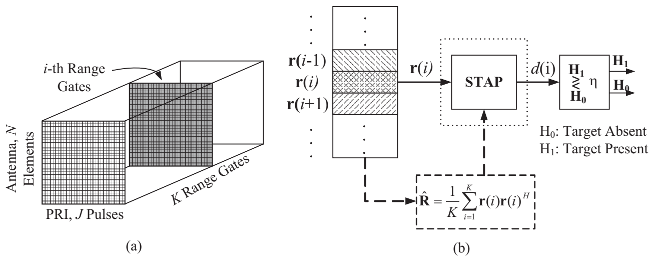

Fig. 1. (a) The Radar CPI datacube. (b) The STAP schematic.

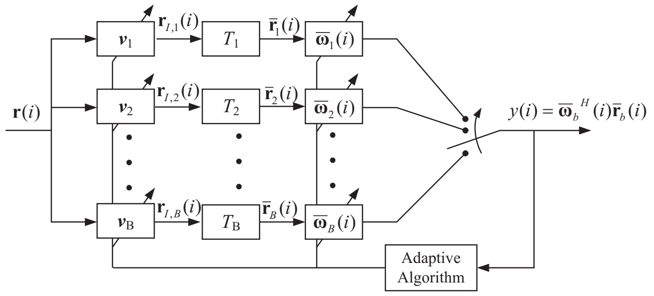

Fig. 2. Proposed Adaptive Reduced-Rank Filtering Scheme (RR-SJIDF).

II. Problem Statement

The system under consideration is a pulsed Doppler radar residing on an airborne platform. The radar antenna is a uniformly spaced linear array antenna consisting of N elements. Radar returns are collected in a coherent processing interval (CPI), which is referred to as the 3-D radar datacube shown in Fig. 1(a), where K denotes the number of samples collected to cover the range interval. The data is then processed at one range of interest, which corresponds to a slice of the CPI datacube. This slice is a $J \times N$ matrix which consists of $N \times 1$ spatial snapshots for J pulses at the range of interest. It is convenient to stack the matrix column-wise to form the $M \times 1, M = JN$ vector $\mathbf{r}(i)$, termed the i-th range gate space-time snapshot, $1 \leq i \leq K$ [1].

A. Signal Model

The function of a radar is to ascertain whether targets are present in the data. Thus, given a space-time snapshot, radar detection is a binary hypothesis problem, where hypothesis $\mathbf{H}_0$ corresponds to target absence and hypothesis $\mathbf{H}_1$ corresponds to target presence. The radar space-time snapshot is then expressed for each of the two hypotheses in the following form,

$$\begin{aligned} \mathbf{H}_0 &: \mathbf{r}(i) = \mathbf{v}(i) \\ \mathbf{H}_1 &: \mathbf{r}(i) = a\mathbf{s} + \mathbf{v}(i), \end{aligned} \tag{1}$$

where a is a zero-mean complex Gaussian random variable with variance σ_s^2, $\mathbf{v}(i)$ denotes the input interference-plus-noise vector which consists of clutter $\mathbf{r}_c(i)$, jamming $\mathbf{r}_j(i)$ and the white noise $\mathbf{r}_n(i)$. These three components are assumed to be mutually uncorrelated. Thus, the $M \times M$ covariance matrix $\mathbf{R}$ of the undesired clutter-plus-jammer-plus-noise component can be modelled as

$$\mathbf{R} = \mathbb{E}\{\mathbf{v}(i)\mathbf{v}^H(i)\} = \mathbf{R}_c + \mathbf{R}_j + \mathbf{R}_n \tag{2}$$

where H represents Hermitian transpose, $\mathbf{R}_c = \mathbb{E}\{\mathbf{r}_c(i)\mathbf{r}_c^H(i)\}$, $\mathbf{R}_j = \mathbb{E}\{\mathbf{r}_j(i)\mathbf{r}_j^H(i)\}$ and $\mathbf{R}_n = \mathbb{E}\{\mathbf{r}_n(i)\mathbf{r}_n^H(i)\}$ denote the clutter, jamming and noise covariance matrices respectively, and $\mathbb{E}$ denotes expectation. The vector $\mathbf{s}$, which is the $M \times 1$ normalized space-time steering vector in the space-time look-direction, can be defined as:

$$\mathbf{s} = \mathbf{b}(\varpi_t) \otimes \mathbf{a}(\vartheta_t), \tag{3}$$

where $\mathbf{b}(\varpi_t)$ is the $K \times 1$ normalized temporal steering vector at the target Doppler frequency ϖ_t and $\mathbf{a}(\vartheta_t)$ is the $N \times 1$ normalized spatial steering vector in the direction provided by the target spatial frequency ϑ_t. The notation $\otimes$ denotes Kronecker product.

B. Optimum Radar Signal Processing

To detect the presence of targets, each range bin is processed by an adaptive 2D beamformer (to achieve maximum output SINR) followed by a hypothesis test to determine the target presence or absence. The optimum full-rank STAP (or Neyman-Pearson optimal under Gaussian disturbance) [1] obtained by an unconstrained optimization of the SINR is given as follows:

$$\boldsymbol{\omega}_{opt} = k\mathbf{R}^{-1}\mathbf{s} \tag{4}$$

where k is an arbitrary nonzero complex number. The optimal constrained weight vector for maximizing the output SINR, while maintaining a normalized response in the target spatial-Doppler look-direction was originally given in [18] by

$$\boldsymbol{\omega}_{opt} = \frac{\mathbf{R}^{-1}\mathbf{s}}{\mathbf{s}^H\mathbf{R}^{-1}\mathbf{s}}. \tag{5}$$

It is obvious that the solution in (5) can also be obtained by solving the linearly constrained minimum variance (LCMV) problem as

$$\boldsymbol{\omega}_{opt} = \arg\min_{\boldsymbol{\omega}(i)} \boldsymbol{\omega}^H(i)\mathbf{R}\boldsymbol{\omega}(i) \quad \text{s. t.} \quad \mathbf{s}^H\boldsymbol{\omega}(i) = 1. \tag{6}$$

III. Proposed RR-SJIDF Scheme

In this section, we detail the proposed adaptive reduced-rank filtering scheme based on the switched joint interpolation, decimation and filtering (RR-SJIDF). The reduced-rank adaptive filtering scheme based on combined decimation and interpolation filtering was presented in [15]. In this work, we develop a reduced-rank STAP algorithm based on the SJIDF scheme for airborne radar applications, whose schematic is shown in Fig. 2. The motivation for designing a projection matrix based on interpolation and decimation comes from two observations. The first is that rank reduction can be performed by eliminating (decimating) samples that are not useful in the filtering process and then attempting to recreate the eliminated samples with an interpolator. The second comes from the structure of the projection matrix, whose columns are a set of bases formed by the interpolators and the decimators.

A. Overview of the RR-SJIDF Scheme

In this part, we briefly introduce the principle of the proposed RR-SJIDF algorithm. In this scheme, the number of elements for filtering is substantially reduced, resulting in considerable computational savings and very fast convergence performance for the radar applications. The proposed approach straightforwardly obtains the subspace of interest via a multiple processing branch (MPB) framework. The $M \times 1$ received vector $\mathbf{r}(i) = [r_0(i), r_1(i), \cdots, r_{M-1}(i)]^T$ is processed by a MPB framework with B branches, where each processing branch contains an interpolator filter, a decimation unit and a reduced-rank filter. In the b-th branch $b \in \{1, 2, ..., B\}$, the received vector $\mathbf{r}(i)$ is filtered by the interpolator filter $\bar{\boldsymbol{v}}_b(i) = [v_{0,b}(i), v_{1,b}(i), \cdots, v_{I-1,b}(i)]^T$ with filter length I, yielding the interpolated received vector $\mathbf{r}'_b(i)$ with M samples, which is expressed by

$$\mathbf{r}'_b(i) = \mathbf{V}_b(i)\mathbf{r}(i) \tag{7}$$

where the $M \times M$ Toeplitz matrix $\mathbf{V}_b(i)$ is given by

$$\mathbf{V}_b(i) = \begin{bmatrix} v_{0,b}(i) & 0 & \ldots & 0 \\ \vdots & v_{0,b}(i) & \ldots & 0 \\ v_{I-1,b}(i) & \vdots & \ldots & 0 \\ 0 & v_{I-1,b}(i) & \ldots & 0 \\ 0 & 0 & \ddots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & v_{0,b}(i) \end{bmatrix}. \tag{8}$$

In order to facilitate the description of the scheme, let us introduce an alternative way of expressing the vector $\mathbf{r}'_b(i)$, which will be useful in the following through the equivalence:

$$\mathbf{r}'_b(i) = \mathbf{V}_b(i)\mathbf{r}(i) = \boldsymbol{\mathcal{R}}_0(i)\bar{\boldsymbol{v}}_b(i), \tag{9}$$

where the $M \times I$ matrix $\boldsymbol{\mathcal{R}}_0(i)$ with the samples of $\mathbf{r}(i)$ has a Hankel structure [19] and is described by

$$\boldsymbol{\mathcal{R}}_0(i) = \begin{bmatrix} r_0(i) & r_1(i) & \ldots & r_{I-1}(i) \\ r_1(i) & r_2(i) & \ldots & r_I(i) \\ \vdots & \vdots & \ldots & \vdots \\ r_{M-I}(i) & r_{M-I+1}(i) & \ldots & r_{M-1}(i) \\ r_{M-I+1}(i) & r_{M-I+2}(i) & \ddots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ r_{M-2}(i) & r_{M-1}(i) & 0 & 0 \\ r_{M-1}(i) & 0 & 0 & 0 \end{bmatrix}. \tag{10}$$

The dimensionality reduction is performed by a decimation unit with $D \times M$ decimation matrices $\mathbf{T}_b(i)$ that projects $\mathbf{r}'_b(i)$ onto $D \times 1$ vectors $\bar{\mathbf{r}}_b(i)$ with $b = 1, \ldots, B$, where $D = M/L$ is the rank and L is the decimation factor. The $D \times 1$ vector $\bar{\mathbf{r}}_b(i)$ for branch b is expressed by

$$\bar{\mathbf{r}}_b(i) = \mathbf{T}_b(i)\mathbf{r}'_b(i) = \mathbf{T}_b(i)\boldsymbol{\mathcal{R}}_0(i)\bar{\boldsymbol{v}}_b(i), \tag{11}$$

where the vector $\bar{\mathbf{r}}_b(i)$ for branch b is used in the minimization of the output power for branch b

$$|y_b(i)|^2 = |\bar{\boldsymbol{\omega}}_b^H(i)\bar{\mathbf{r}}_b(i)|^2.$$

The output at the end of the MPB framework $y(i)$ is selected according to:

$$y(i) = y_{b_s}(i) \quad \text{when} \quad b_s = \arg\min_{1 \le b \le B} |y_b(i)|^2, \tag{12}$$

where B is a parameter to be set by the designer. Essential to the derivation of the joint iterative optimization that follows is to express the output of the RR-SJIDF STAP $y_b(i) = \bar{\boldsymbol{\omega}}_b^H(i)\bar{\mathbf{r}}_b(i)$ as a function of $\bar{\boldsymbol{v}}_b(i)$, the decimation matrix $\mathbf{T}_b(i)$ and $\bar{\boldsymbol{\omega}}_b^H(i)$ as follows:

$$\begin{aligned} y_b(i) &= \bar{\boldsymbol{\omega}}_b^H(i)\mathbf{T}_b(i)\boldsymbol{\mathcal{R}}_0(i)\bar{\boldsymbol{v}}_b(i) = \bar{\boldsymbol{\omega}}_b^H(i)\bar{\mathbf{r}}_{\bar{\omega},b}(i) \\ &= [\bar{\boldsymbol{v}}_b^H(i)\boldsymbol{\mathcal{R}}_0^H(i)\mathbf{T}_b^H(i)\bar{\boldsymbol{\omega}}_b(i)]^* = [\bar{\boldsymbol{v}}_b^H(i)\bar{\mathbf{r}}_{\bar{v},b}(i)]^*. \end{aligned} \tag{13}$$

where $\bar{\mathbf{r}}_{\bar{\omega},b}(i) = \mathbf{T}_b(i)\boldsymbol{\mathcal{R}}_0(i)\bar{\boldsymbol{v}}_b(i)$ denotes the reduced-rank signal with respect to $\bar{\boldsymbol{\omega}}_b(i)$ and $\bar{\mathbf{r}}_{\bar{v},b}(i) = \boldsymbol{\mathcal{R}}_0^H(i)\mathbf{T}_b^H(i)\bar{\boldsymbol{\omega}}_b(i)$ denotes the reduced-rank signal with respect to $\bar{\boldsymbol{v}}_b(i)$, $(\cdot)^*$ denotes the conjugate operation. The expression (13) indicates that the dimensionality reduction carried out by the proposed scheme depends on finding appropriate $\bar{\boldsymbol{v}}_b(i)$, $\mathbf{T}_b(i)$ and $\bar{\boldsymbol{\omega}}_b(i)$, as shown next.

B. Optimization of the Filters

In this part, we describe the proposed joint and iterative optimization algorithm that adjusts the parameters of the interpolator filter $\bar{\boldsymbol{v}}_b(i)$ and the reduced-rank filter $\bar{\boldsymbol{\omega}}_b(i)$ with the decimation pattern $\mathbf{T}_b(i)$. The objective of the LCMV criterion is the minimization of the cost function defined as

$$\begin{aligned} \mathcal{L}(\bar{\boldsymbol{\omega}}_b(i), \bar{\boldsymbol{v}}_b(i)) = \mathbb{E}\left[\left|\bar{\boldsymbol{\omega}}_b^H(i)\mathbf{T}_b(i)\boldsymbol{\mathcal{R}}_0(i)\bar{\boldsymbol{v}}_b(i)\right|^2\right] \\ + 2\Re\left\{\lambda\left[\bar{\boldsymbol{\omega}}_b^H(i)\mathbf{T}_b(i)\boldsymbol{\mathcal{S}}_0\bar{\boldsymbol{v}}_b(i) - 1\right]\right\} \end{aligned} \tag{14}$$

where λ is the Lagrangian multiplier and $\boldsymbol{\mathcal{S}}_0$ is $M \times I$ steering matrix with a Hankel structure with the same form as $\boldsymbol{\mathcal{R}}_0(i)$

$$\boldsymbol{\mathcal{S}}_0 = \begin{bmatrix} s_0 & s_1 & \ldots & s_{I-1} \\ s_1 & s_2 & \ldots & s_I \\ \vdots & \vdots & \ldots & \vdots \\ s_{M-I} & s_{M-I+1} & \ldots & s_{M-1} \\ s_{M-I+1} & s_{M-I+2} & \ddots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ s_{M-2} & s_{M-1} & 0 & 0 \\ s_{M-1} & 0 & 0 & 0 \end{bmatrix}. \tag{15}$$

By fixing $\bar{\boldsymbol{\omega}}(i)$ and $\bar{\boldsymbol{v}}(i)$, respectively, (14) can be rewritten into two equations as

$$\mathcal{L}(\bar{\boldsymbol{v}}_b(i)) = \mathbb{E}\left[\left|\bar{\boldsymbol{v}}_b^H(i)\bar{\mathbf{r}}_{\bar{v},b}(i)\right|^2\right] + 2\Re\left\{\lambda\left[\bar{\boldsymbol{v}}_b^H(i)\bar{\mathbf{s}}_{\bar{v},b}(i) - 1\right]\right\}$$

$$\mathcal{L}(\bar{\boldsymbol{\omega}}_b(i)) = \mathbb{E}\left[\left|\bar{\boldsymbol{\omega}}_b^H(i)\bar{\mathbf{r}}_{\bar{\omega},b}(i)\right|^2\right] + 2\Re\left\{\lambda\left[\bar{\boldsymbol{\omega}}_b^H(i)\bar{\mathbf{s}}_{\bar{\omega},b}(i) - 1\right]\right\}$$

where we define $\bar{\mathbf{s}}_{\bar{v},b}(i) = \mathbf{T}_b^H(i)\boldsymbol{\mathcal{S}}_0^H\bar{\boldsymbol{\omega}}_b(i)$ and $\bar{\mathbf{s}}_{\bar{\omega},b}(i) = \mathbf{T}_b(i)\boldsymbol{\mathcal{S}}_0\bar{\boldsymbol{v}}_b(i)$ to denote the reduced-rank steering vectors with respect to $\bar{\boldsymbol{v}}(i)$ and $\bar{\boldsymbol{\omega}}(i)$, respectively. By minimizing $\mathcal{L}(\bar{\boldsymbol{v}}_b(i))$ and solving for λ, we get

$$\bar{\boldsymbol{v}}_b(i) = \frac{\bar{\mathbf{R}}_{\bar{v},b}^{-1}\bar{\mathbf{s}}_{\bar{v},b}(i)}{\bar{\mathbf{s}}_{\bar{v},b}(i)^H\bar{\mathbf{R}}_{\bar{v},b}^{-1}\bar{\mathbf{s}}_{\bar{v},b}(i)}. \tag{16}$$

where $\bar{\mathbf{R}}_{\bar{v},b} = \mathbb{E}\left[\bar{\mathbf{r}}_{\bar{v},b}(i)\bar{\mathbf{r}}_{\bar{v},b}^H(i)\right]$. By minimizing $\mathcal{L}(\bar{\boldsymbol{\omega}}_b(i))$ and solving for λ, we get

$$\bar{\boldsymbol{\omega}}_b(i) = \frac{\bar{\mathbf{R}}_{\bar{\omega},b}^{-1}\bar{\mathbf{s}}_{\bar{\omega},b}(i)}{\bar{\mathbf{s}}_{\bar{\omega},b}(i)^H\bar{\mathbf{R}}_{\bar{\omega},b}^{-1}\bar{\mathbf{s}}_{\bar{\omega},b}(i)}. \tag{17}$$

where $\bar{\mathbf{R}}_{\bar{\omega},b} = \mathbb{E}\left[\bar{\mathbf{r}}_{\bar{\omega},b}(i)\bar{\mathbf{r}}_{\bar{\omega},b}^H(i)\right]$. Note that the joint iterative optimization of the interpolation filters $\{\bar{\boldsymbol{v}}_b(i)|b = 1, ..., B\}$ and the reduced-rank filters $\{\bar{\boldsymbol{\omega}}_b(i)|b = 1, ..., B\}$ are performed seperately in the all processing branches.

C. Design of the Decimation Unit

In this part, we consider two strategies for the design of the decimation unit $\mathbf{T}_b(i)$. We constrain the design of $\mathbf{T}_b(i)$ so that the elements of the matrix only take the value 0 or 1. This corresponds to the decimation unit simply keeping or discarding the samples. The first strategy exhaustively explores all possible decimation patterns which select D samples out of M samples, this is therefore the optimal approach. In this case, the scheme can be viewed as a combinatorial problem and the total number of patterns B, equal to

$$B = M \cdot (M-1) \cdots (M-D+1) = \begin{pmatrix} M \\ D \end{pmatrix}. \quad (18)$$

However, the optimal decimation scheme described above is too complex for practical use since it needs D permutations of M samples for each snapshot and carries out an exhaustive search over all possible patterns. Therefore, an alternative decimation scheme with low-complexity that renders itself to practical use is of great interest. To this end, we consider the second decimation scheme that we call pre-stored decimation unit (PSDU). The PSDU scheme employs a structure formed in the following way

$$\mathbf{T}_b = [\phi_{b,1} \quad \phi_{b,2} \quad \dots \quad \phi_{b,D}] \quad (19)$$

where the $M \times 1$ vector $\phi_{b,d}$ denotes the dth basis vector of the bth decimation unit, $d = 1, ..., D$, $b = 1, ..., B$, and is composed of a single 1 and $(M-1)$ 0s, according to the following

$$\phi_{b,d} = [\underbrace{0, \ \dots \ , \ 0}_{z_{b,d}}, \ 1, \ \underbrace{0, \ \dots \ , \ 0}_{M - z_{b,d} - 1}] \quad (20)$$

where $z_{b,d}$ is the number of zeros before the only element equal to one. We set the value of $z_{b,d}$ in a deterministic way which can be expressed as

$$z_{b,d} = \frac{M}{D} \times (d-1) + (b-1). \quad (21)$$

It should be remarked that other designs have been investigated and this structure has been adopted due to an excellent trade-off between performance and complexity.

IV. Adaptive Algorithm

Here, we describe an RLS algorithm that adaptively adjusts the coefficients of the interpolation filters $\{\bar{\boldsymbol{v}}_b(i) | b = 1, ..., B\}$ and the reduced-rank filters $\{\bar{\boldsymbol{\omega}}_b(i) | b = 1, ..., B\}$ based on the least squares (LS) cost functions, which are shown as below:

$$\begin{aligned} \mathcal{L}_{LS}(\bar{\boldsymbol{v}}_b(i)) &= \sum_{n=1}^{i} \alpha^{i-n} \left| \bar{\boldsymbol{v}}_b^H(n) \bar{\mathbf{r}}_{\bar{v},b}(n) \right|^2 \\ &\quad + 2\Re \left\{ \lambda \left[\bar{\boldsymbol{v}}_b^H(i) \bar{\mathbf{s}}_{\bar{v},b}(i) - 1 \right] \right\}, \\ \mathcal{L}_{LS}(\bar{\boldsymbol{\omega}}_b(i)) &= \sum_{n=1}^{i} \alpha^{i-n} \left| \bar{\boldsymbol{\omega}}_b^H(n) \bar{\mathbf{r}}_{\bar{\omega},b}(n) \right|^2 \\ &\quad + 2\Re \left\{ \lambda \left[\bar{\boldsymbol{\omega}}_b^H(i) \bar{\mathbf{s}}_{\bar{\omega},b}(i) - 1 \right] \right\}, \end{aligned} \quad (22)$$

where α is the forgetting factor. By computing the gradients of $\mathcal{L}_{LS}(\bar{\boldsymbol{v}}_b(i))$ and $\mathcal{L}_{LS}(\bar{\boldsymbol{\omega}}_b(i))$ and equating them to zero, respectively, we obtain

$$\begin{aligned} \bar{\boldsymbol{v}}_b(i) &= \frac{\hat{\bar{\mathbf{R}}}_{\bar{v},b}^{-1}(i) \bar{\mathbf{s}}_{\bar{v},b}(i)}{\bar{\mathbf{s}}_{\bar{v},b}(i)^H \hat{\bar{\mathbf{R}}}_{\bar{v},b}^{-1}(i) \bar{\mathbf{s}}_{\bar{v},b}(i)}, \\ \bar{\boldsymbol{\omega}}_b(i) &= \frac{\hat{\bar{\mathbf{R}}}_{\bar{\omega},b}^{-1}(i) \bar{\mathbf{s}}_{\bar{\omega},b}(i)}{\bar{\mathbf{s}}_{\bar{\omega},b}(i)^H \hat{\bar{\mathbf{R}}}_{\bar{\omega},b}^{-1}(i) \bar{\mathbf{s}}_{\bar{\omega},b}(i)}, \end{aligned} \quad (23)$$

where $\hat{\bar{\mathbf{R}}}_{\bar{v},b}(i) = \sum_{n=1}^{i} \alpha^{i-n} \bar{\mathbf{r}}_{\bar{v},b}(n) \bar{\mathbf{r}}_{\bar{v},b}^H(n)$ and $\hat{\bar{\mathbf{R}}}_{\bar{\omega},b}(i) = \sum_{n=1}^{i} \alpha^{i-n} \bar{\mathbf{r}}_{\bar{\omega},b}(n) \bar{\mathbf{r}}_{\bar{\omega},b}^H(n)$ denote the time averaged correlation matrices with respect to $\bar{\boldsymbol{\omega}}_b(i)$ and $\bar{\boldsymbol{v}}_b(i)$, respectively. By employing the matrix inversion lemma, $\hat{\bar{\mathbf{R}}}_{\bar{\omega},b}^{-1}(i)$ and $\hat{\bar{\mathbf{R}}}_{\bar{v},b}^{-1}(i)$ can be recursively obtained as follows

$$\begin{aligned} \hat{\bar{\mathbf{R}}}_{\bar{v},b}^{-1}(i) &= \alpha^{-1} \hat{\bar{\mathbf{R}}}_{\bar{v},b}^{-1}(i-1) - \alpha^{-1} \bar{\mathbf{K}}_{\bar{v},b}(i) \bar{\mathbf{r}}_{\bar{v},b}^H(i) \hat{\bar{\mathbf{R}}}_{\bar{v},b}^{-1}(i-1), \\ \hat{\bar{\mathbf{R}}}_{\bar{\omega},b}^{-1}(i) &= \alpha^{-1} \hat{\bar{\mathbf{R}}}_{\bar{\omega},b}^{-1}(i-1) - \alpha^{-1} \bar{\mathbf{K}}_{\bar{\omega},b}(i) \bar{\mathbf{r}}_{\bar{\omega},b}^H(i) \hat{\bar{\mathbf{R}}}_{\bar{\omega},b}^{-1}(i-1), \end{aligned} \quad (24)$$

where

$$\begin{aligned} \bar{\mathbf{K}}_{\bar{v},b}(i) &= \frac{\hat{\bar{\mathbf{R}}}_{\bar{v},b}^{-1}(i-1) \bar{\mathbf{r}}_{\bar{v},b}(i)}{\alpha + \bar{\mathbf{r}}_{\bar{v},b}^H(i) \hat{\bar{\mathbf{R}}}_{\bar{v},b}^{-1}(i-1) \bar{\mathbf{r}}_{\bar{v},b}(i)}, \\ \bar{\mathbf{K}}_{\bar{\omega},b}(i) &= \frac{\hat{\bar{\mathbf{R}}}_{\bar{\omega},b}^{-1}(i-1) \bar{\mathbf{r}}_{\bar{\omega},b}(i)}{\alpha + \bar{\mathbf{r}}_{\bar{\omega},b}^H(i) \hat{\bar{\mathbf{R}}}_{\bar{\omega},b}^{-1}(i-1) \bar{\mathbf{r}}_{\bar{\omega},b}(i)}, \end{aligned} \quad (25)$$

where $\hat{\bar{\mathbf{R}}}_{\bar{v},b}^{-1}(0)$ and $\hat{\bar{\mathbf{R}}}_{\bar{\omega},b}^{-1}(0)$ are initialized to $\delta^{-1}\mathbf{I}$, where δ is a small constant and $\mathbf{I}$ is the identity matrix. It is worth remarking that $\bar{\mathbf{r}}_{\bar{\omega},b}^H(n)(i)$, $\bar{\mathbf{r}}_{\bar{v},b}^H(n)(i)$,$\bar{\mathbf{s}}_{\bar{\omega},b}^H(n)(i)$ and $\bar{\mathbf{s}}_{\bar{v},b}^H(n)(i)$ have to be updated as soon as $\bar{\boldsymbol{v}}_b(i)$ and $\bar{\boldsymbol{\omega}}_b(i)$ are updated since they are dependent on $\bar{\boldsymbol{\omega}}_b(i)$ and $\bar{\boldsymbol{v}}_b(i)$, respectively. The output of the MPB scheme $y(i)$ is selected according to:

$$y(i) = y_{b_s}(i) \quad \text{when} \quad b_s = \arg \min_{1 \le b \le B} |y_b(i)|^2, \quad (26)$$

where

$$y_b(i) = \bar{\boldsymbol{\omega}}_b^H(i) \mathbf{T}_b(i) \boldsymbol{\mathcal{R}}_0(i) \bar{\boldsymbol{v}}_b(i). \quad (27)$$

V. Performance Assessment

In this section we assess the proposed RR-SJIDF STAP algorithm in an airborne radar application. The parameters of the radar platform are shown in Table I. For all simulations, we assume the presence of a mixture of two broadband jammers at -45° and 60° with jammer-to-noise-ratio (JNR) equal to 40 dB. The clutter-to-noise-ratio (CNR) is fixed at 40 dB. We compare both the SINR performance against the number of snapshots and the P_D performance against the signal-to-noise-ratio (SNR) for the different designs of linear receivers using the full-rank filter with the RLS algorithm, the MSWF with the RLS algorithm, the AVF and our proposed technique. The radar receiver provides an estimate to determine whether the target is present or not. All presented results are averages over 1000 independent Monte-Carlo runs.

Firstly, as shown in Fig. 3, we evaluate the SINR against the number of snapshots K performance of our proposed algorithm with different setting parameters and compare with the other schemes. The schemes are simulated over $K = 500$ snapshots and the SNR is set at 0 dB. The curves show an excellent performance by the proposed algorithm, which also converges much faster than other schemes. With the number of

TABLE I

RADAR SYSTEM PARAMETERS

Parameter	Value
Antenna array	Sideway-looking array (SLA)
Carrier frequency (f_c)	450 MHz
Transmit pattern	Uniform
PRF (f_r)	300 Hz
Platform velocity (v)	50 m/s
Platform height (h)	9000 m
Clutter-to-Noise ratio (CNR)	40 dB
Jammer-to-Noise ratio (JNR)	40 dB
Elements of sensors (N)	10
Number of Pulses (J)	8

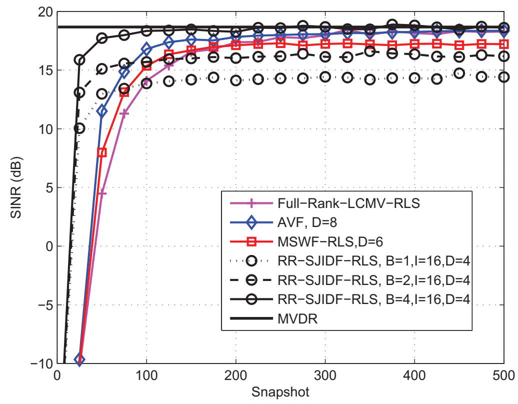

Fig. 3. SINR performance against snapshot with $M = 80$, SNR = 0 dB, $\alpha = 0.9998$. All algorithms are initialized with $\delta^{-1}\mathbf{I}$, where δ is a small constant.

branches $B = 4$, the proposed scheme approaches the optimal MVDR performance after 50 snapshots. As one may expect, with an increasing the number of branches, the steady SINR performance improves. An improvement of performance is also possible with the use of model order selection algorithms [16], [17] or an adjustment in the number of auxiliary vectors for the AVF [9]. In the second experiment, in Fig. 4, we present P_D versus SNR performance for all schemes using 50 snapshots as the training data. The false alarm rate P_{FA} is set to 10^{-6}. The figure illustrates that the proposed algorithm provides sub-optimal detection performance using very short support data, but remarkably, obtains a 90 percent detection rate, beating 50 percent for the AVF, 40 percent for the MSWF with the RLS and 30 percent for the full rank filter with the RLS at an SNR level of 15 dB.

VI. CONCLUSIONS

In this paper, we proposed an RR-SJIDF STAP algorithm for airborne radar systems. The proposed method performs dimensionality reduction by employing a MPB framework, which jointly optimizes interpolation, decimation and filtering units. The output is switched to the branch with the best performance according to the minimum variance criterion. In order to design the decimation unit, we have considered the optimal decimation scheme and also a low-complexity pre-stored decimation units scheme. Furthermore, we have developed an adaptive RLS algorithm for efficient implementation of the proposed scheme. Simulations results have shown that the proposed RR-SJIDF STAP scheme converges at a very fast speed and provides a considerable SINR improvement, outperforming existing state-of-the-art reduced-rank schemes.

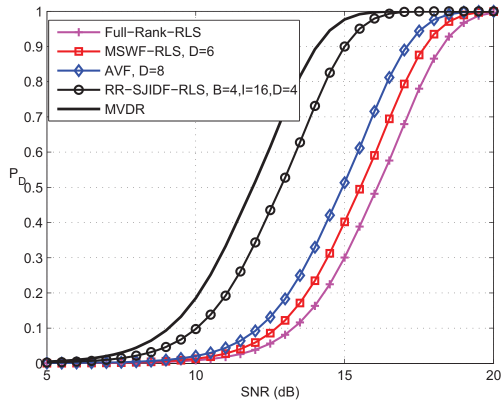

Fig. 4. Probability of detection performance vs SNR with $M = 80$, $\alpha = 0.9998$, $K = 50$ snapshots, $P_{FA} = 10^{-6}$.

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