Co-prime Arrays with Reduced Sensors (CARS) for Direction-of-Arrival Estimation

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***Abstract*—In traditional direction-of-arrival (DoA) estimation, the number of sources to be detected is widely assumed to be smaller than the number of physical sensors. In practice, however, the number of sources encountered may be greater than that of the sensors. To address this problem, nested array structure can be considered, however, the effects of mutual coupling between sensors may become significant when the sensors are located closely. Recently, the co-prime array structure has been developed to further enhance the uniform degrees-of-freedom (DoF) with less mutual coupling, by utilizing a co-prime pair of uniform linear subarrays. This paper proposes a pair of co-prime arrays with reduced sensors (CARS) structure, where the first array is shifted until symmetrical to the center (reference sensor) and the number of sensors in the second array can be reduced as compared to existing optimized co-prime array configurations, according to the odd and even of the co-prime number pair. Furthermore, with the CARS structure, much more uniform DoF can be detected. The feasibility of the proposed CARS structure is demonstrated by numerical results of DoA estimations for multiple stationary sources with noise.**

I. Introduction

Source localisation, such as direction-of-arrival (DoA) estimation, is an important issue in applications, including underwater acoustic detection, target tracking and environmental monitoring [1] [2]. This problem has been studied extensively, with many methods proposed, including the popular subspace-based methods, such as MUSIC and ESPRIT [3] [4]. However, with an N-element uniform linear array (ULA), the number of sources that can be detected by e.g. the MUSIC algorithm is $N-1$ [3]. As a result, such a method is limited when more sources than the number of sensors need to be detected. To address this problem, the nested array structure, which is obtained by combining two or more ULAs, was introduced in [5] to enhance the uniform degrees-of-freedom (DoF) through generating $O(N^2)$ co-array elements from $O(N)$ array elements. The uniform DoF here denotes the cardinality of the maximum contiguous ULA segments in the difference coarray set, which consists of differences between any pair of sensors positions in the array structure. The uniform DoF is required to implement MUSIC and ESPRIT algorithms. Although the nested array configuration is easy to construct, some of the sensors in a nested array may be located so closely that the effect of mutual coupling between sensors becomes significant [6] [7].

As an alternative, a co-prime pair of uniform linear subarrays structure based on [8] has been developed in [9], where one of the subarrays consists of M sensors with an inter-element spacing of N units and the other contains $2N-1$ sensors with an inter-element spacing of M units, with $M > N$ and both subarrays sharing the first sensor at the zero-th position. This co-prime array structure has been generalized in [10] via the compression of the inter-element spacing of one constituting subarray by a positive integer. The co-prime array structure contains a total number of $M+2N-1$ sensors to achieve $2MN+2N-1$ uniform DoF [10] by calculating the difference set $\pm(Mn-Nm)$, where $0 \le m \le M-1, 0 \le n \le 2N-1$ are two sequences to discribe the position of each sensor. Another center symmetric co-prime array is presented in [11], where the first subarray contains $2M$ sensors with $-(2M-1)/2 \le m \le (2M-1)/2$ and the second subarray contains $2N$ sensors with $-(2N-1)/2 \le n \le (2N-1)/2$. This pair of subarrays constitutes a full co-array spanning a contiguous region of $\pm(MN+(M+N)/2-1)$. Compared to the nested arrays, the co-prime array structure mitigates mutual coupling, however, it uses more sensors to attain the same uniform DoF [9] [10].

In this paper, we propose a new structure named co-prime arrays with reduced sensors (CARS), which can achieve more uniform DoF with fewer sensors than the one in [9]. In particular, when M is an even number, N is an odd number and $M > N$, the proposed coprime array can produce $2MN+3M-1$ uniform DoF with only $M+(3N+1)/2$ sensors (the result should be an integer as N is an odd number). That is, we can reduce the number of sensors by $(N-3)/2$, while increasing the uniform DOF by $3M-2N$, as a comparison to the method in [9]. When compared to the center symmetric structure in [11], our method increases $2M$ more uniform DoF and uses $M+(N-1)/2$ less sensors with only a half of the second subarray in [11]. Another different sensor arrangement has been presented in [12], using an unfolded coprime array structure, where the two subarrays are aligned along the positive axis and negative axis, respectively. With $M+N$ physical elements, the unfolded coprime array generates $2MN-1$ unique number of lags. However, the region is not continuous, i.e. there are holes in it. The MUSIC algorithm cannot be used directly for the coarray in [12].

This article is organised as follows. In Section II, the background about the co-prime sampling array structure in [9] and the MUSIC algorithm for DoA estimation are

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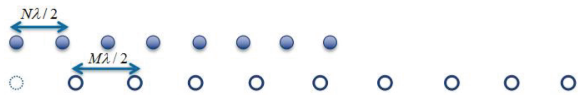

Fig. 1. The co-prime arrays structure in [9] when M is 8 and N is 5.

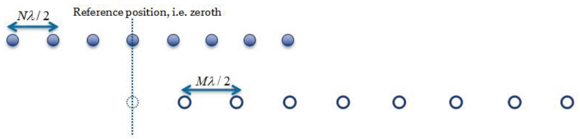

Fig. 2. The proposed CARS structure when M is 8 and N is 5.

presented. The proposed CARS is presented in Section III. In Section IV, numerical results on DoA estimation by using CARS are presented. Finally, the conclusion is drawn in Section V.

Notations: We use lower-case (upper-case) bold characters to denote vectors (matrices). In particular, $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and conjugate transpose of a matrix or vector, respectively. $\tilde{(\cdot)}$ is used to emphasize variables corresponding to the co-prime arrays with reduced sensors. In order to discuss the odd and even conveniently, we set a pair of operators $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$ as follows

$$\lfloor N/2 \rfloor = \begin{cases} \frac{N}{2} & \text{when } N \text{ is an even integer;} \\ \frac{N-1}{2} & \text{when } N \text{ is an odd integer.} \end{cases} \tag{1}$$

$$\lceil N/2 \rceil = \begin{cases} \frac{N}{2} & \text{when } N \text{ is an even integer;} \\ \frac{N+1}{2} & \text{when } N \text{ is an odd integer.} \end{cases} \tag{2}$$

II. Background

A. Original Co-prime Sampling Array

We set a co-prime number pair of M and N, where $M > N$, without loss of generality [9]. The unit inter-element spacing d equals $\lambda/2$, where λ denotes the wavelength. The array sensors are positioned at

$$\mathbb{P} = \{Mnd | 0 \leq n \leq 2N-1\} \cup \{Nmd | 0 \leq m \leq M-1\} \tag{3}$$

As an example, Figure 1 shows the array structure with $M = 8, N = 5$. Assume $\mathbf{p} = [p_1, \cdots, p_{M+2N-1}]^T$ as the positions of the array sensors where $p_q \in \mathbb{P}$, $q = 1, \cdots, M + 2N - 1$, and the first sensor is set as the reference which is shared by both subarrays, i.e. $p_1 = 0$. The maximum number of the difference lags is determined by the number of unique elements in the following set

$$\mathbb{L}_p = \{l_p | l_p d = u - v, u \in \mathbb{P}, v \in \mathbb{P}\} \tag{4}$$

The difference coarrays consist of either self-differences of the two subarrays or their cross-differences. The self-difference in the coarray has positions

$$\mathbb{L}_s = \{l_s | l_s = Mn\} \cup \{l_s | l_s = Nm\} \tag{5}$$

and the corresponding mirrored positions $\mathbb{L}_s^- = \{-l_s | l_s \in \mathbb{L}_s\}$, whereas the cross-difference has positions

$$\mathbb{L}_c = \{l_c | l_c = Mn - Nm\} \tag{6}$$

and the corresponding mirrored positions $\mathbb{L}_c^- = \{-l_c | l_c \in \mathbb{L}_c\}$, for $0 \leq m \leq M-1, 0 \leq n \leq 2N-1$. Thus the full set of lags in the virtual array is given by

$$\mathbb{L}_p = \mathbb{L}_s \cup \mathbb{L}_s^- \cup \mathbb{L}_c \cup \mathbb{L}_c^- \tag{7}$$

The number of elements in the difference co-array (given by the set $\mathbb{L}_p$) directly decides the distinct values of the cross correlation terms in the covariance matrix of the signal, however, there exist pairs of $u - v$ giving the same value of difference, which causes significant mutual coupling. Thus the concept of weight function $w(l_p)$ is considered.

$Definition$ $(weight\ function)$: The weight function $w(l_p)$, $l_p \in \mathbb{L}_p$ of an array is defined as the number of sensor pairs which have the same value of coarray index l_p.

$Definition$ $(uniform\ DoF)$: Let the set $\mathbb{U}$ denote the maximum contiguous ULA segment in $\mathbb{L}_p$. The number of elements in $\mathbb{U}$ is called the number of uniform degrees-of-freedom (uniform DoF). In this paper, in order to implement the MUSIC algorithm, we only consider the number of uniform DoF.

B. MUSIC Algorithm

With the original co-prime array structure described above, the DoA can be detected using the MUSIC algorithm [9]. We assume that D narrowband far-field sources with the i-th signal ($i = 1, 2, \cdots, D$) arriving in one half of the plane and the sensor array is expected to have a perfect baffle, which means the arrival directions are from -90 degrees to +90 degrees along the plane of the array elements, and 0 degree is the axis of symmetry. The number of samples in a limited time is defined as snapshot K. The directions of source signals remain the same during snapshotting. For the k-th snapshot ($k = 1, 2, \cdots, K$) on D sources, its complex amplitude is expressed as $\mathbf{x}(k) \in \mathbb{C}^D$ and the DoA is denoted by $\theta_i \in [-90°, 90°]$. The received sensor signal $\mathbf{y}(k) \in \mathbb{C}^{(M+2N-1)}$ at the coprime array is expressed as

$$[\mathbf{y}(1), \mathbf{y}(2), \cdots, \mathbf{y}(K)] = \mathbf{AX} + \mathscr{N} \tag{8}$$

where $\mathbf{A} = [\mathbf{a}(\tilde{\theta}_1), \mathbf{a}(\tilde{\theta}_2), \cdots, \mathbf{a}(\tilde{\theta}_D)] \in \mathbb{C}^{(M+2N-1)\times D}$, $\mathbf{a}(\tilde{\theta}_i) = [1, e^{j2\pi\tilde{\theta}_i p_2}, \cdots, e^{j2\pi\tilde{\theta}_i p_{M+2N-1}}]^T_{j\equiv\sqrt{-1}} \in \mathbb{C}^{(M+2N-1)}$ are steering vectors, $p_q \in \mathbf{p}$, $\mathbf{X} = [\mathbf{x}(1), \mathbf{x}(2), \cdots, \mathbf{x}(K)] \in \mathbb{C}^{D\times K}$ and $\mathscr{N} = [\mathbf{n}(1), \mathbf{n}(2), \cdots, \mathbf{n}(K)] \in \mathbb{C}^{(M+2N-1)\times K}$, $\mathbf{n}(k) \in \mathbb{C}^{(M+2N-1)}$ is assumed to be independent and identically distributed (i.i.d) random noise vector. $\tilde{\theta}_i = (d/\lambda)\sin\theta_i$ is the normalized DoA. We can obtain $-1/2 \leq \tilde{\theta}_i \leq 1/2$. Both $\mathbf{x}$ and $\mathbf{n}$ are assumed to be vectors of zero-mean, uncorrelated random variables with covariance matrices of $\mathbf{R_{xx}} = E[\mathbf{xx}^H]$ and $\mathbf{R_{nn}} = E[\mathbf{nn}^H]$, and $\tilde{\theta}_i$ is fixed but unknown.

The covariance matrix of data vector $\mathbf{y}$ is obtained as

$$\mathbf{R_{yy}} = \sum_{i=1}^{D} \sigma_i^2 \mathbf{a}(\tilde{\theta}_i)\mathbf{a}^H(\tilde{\theta}_i) + \sigma^2\mathbf{I} \tag{9}$$

where σ_i^2 is the power of the i-th source, σ^2 is the noise power, $\mathbf{a}(\tilde{\theta}_i)\mathbf{a}^H(\tilde{\theta}_i)$ consists of entries in the form of

$e^{j2\pi\tilde{\theta}_i(p_1-p_2)}$, where $p_1, p_2 \in \mathbf{p}, p_1 - p_2 \in \mathbb{L}_p$, and the convariance matrix in (9) can be reshaped into an autocorrelation vector $\mathbf{y}_D$ as

$$\mathbf{y}_D = \sum_{i=1}^{D} \sigma_i^2 \mathbf{a}_{\mathbb{L}_p}(\tilde{\theta}_i) + \sigma^2 \mathbf{e}_0 \tag{10}$$

where the noise $\sigma^2\mathbf{e}_0$ at the sensor location pd, $p \in \mathbb{L}_p$ follows normal distribution with the average of 0 and the noise power of σ^2.

In the finite-snapshot setting, where the measurenment vectors $\mathbf{y}(k), k = 1, 2, \cdots, K$ are given, the covariance matrix can be estimated by

$$\tilde{\mathbf{R}}_{\mathbf{yy}} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{y}(k)\mathbf{y}^H(k) \tag{11}$$

The finite-snapshot version of the autocorrelation function can be averaged from the convariance matrix by

$$\langle \mathbf{y}_D \rangle_{l_p} = \frac{1}{w(l_p)} \sum_{(p_1,p_2), p_1-p_2 \in \mathbb{U}} \langle \tilde{\mathbf{R}}_{\mathbf{yy}} \rangle_{p_1,p_2} \tag{12}$$

where the angle bracket $\langle \mathbf{y} \rangle_p$ represents the value of the signal at the sensor location pd, $\langle \tilde{\mathbf{R}}_{\mathbf{yy}} \rangle_{p_1,p_2} = E[\langle \mathbf{y} \rangle_{p_1} \langle \mathbf{y} \rangle_{p_2}^H]$, only $p_1 - p_2 \in \mathbb{U}$ is considered. The weight function $w(l_p), l_p \in \mathbb{U}$ is defined earlier.

In order to estimate the DoA $\tilde{\theta}_i$ from $\mathbf{y}_D$, a variation of the rank-enhanced spatial smoothing MUSIC algorithm will be used in this paper [5] [9] [13]. The spatial smoothing step can be modified so that the finite-snapshot autocorrelation vector equals $\langle \mathbf{y}_D \rangle$ according to [13]. A Hermitian Toeplitz matrix $\tilde{\mathbf{R}}$ can be constructed as

$$\langle \tilde{\mathbf{R}} \rangle_{p_1,p_2} = \langle \mathbf{y}_D \rangle_{p_1-p_2} \tag{13}$$

where $p_1, p_2 \in \mathbb{U}^+$, the set $\mathbb{U}^+$ denotes the non-negative part of the maximum contiguous ULA segment. The proof provided in [13] shows that the MUSIC spectrum over $\tilde{\mathbf{R}}$ gives the same performance as that over the spatially smoothed matrix $\tilde{\mathbf{R}}_{ss}$, if the noise subspace is classified by the magnitudes of the eigenvalues of $\tilde{\mathbf{R}}$.

III. Co-prime Arrays with Reduced Sensors (CARS) Structure

We present a new co-prime arrays structure named CARS. The sensors in CARS are located at

$$\begin{aligned}\tilde{\mathbb{P}} = \{Mnd | 1 \leq n \leq \lceil 3N/2 \rceil\} \cup \\ \{Nmd | -M/2+1 \leq m \leq M/2 \text{ when } M \text{ is even,} \\ -\lfloor M/2 \rfloor \leq m \leq \lfloor M/2 \rfloor \text{ when } M \text{ is odd.}\}\end{aligned} \tag{14}$$

where M and N are co-prime, both subarrays share the reference sensor at the zero-th position. Figure 2 gives an example with $M = 8, N = 5$, from which we can see that the first array is shifted until symmetrical to the center (reference sensor) and the number of sensors in the second array is $\lceil 3N/2 \rceil = 8$. Thus the number of physical sensors in CARS is smaller than that in [9], but it is able to handle more uniform DoF (see $Property$ 2).

In this array configuration, both subarrays share the reference sensor at the zero-th position, the self-lags of the two subarrays are given by the following sets

$$\tilde{\mathbb{L}}_s = \{\tilde{l}_s | \tilde{l}_s = Mn\} \cup \{\tilde{l}_s | \tilde{l}_s = Nm\} \tag{15}$$

and the correspoding mirrored positions $\tilde{\mathbb{L}}_s^-$, whereas the cross-lags between the two subarrays are given by

$$\tilde{\mathbb{L}}_c = \left\{\tilde{l}_c | \tilde{l}_c = Mn - Nm\right\} \tag{16}$$

and the corresponding mirrored positions $\tilde{\mathbb{L}}_c^-$, where n and m are give in (14).

To exploit the uniform DoF of the CARS structure completely, we summarize the properties of $\tilde{\mathbb{L}}_s$ and $\tilde{\mathbb{L}}_c$. We also give the proof of each property.

$Property$ 1: There are $(\lceil 3N/2 \rceil + 1)M$ distinct integers in set $\tilde{\mathbb{L}}_c$.

$Proof$: Suppose that M is even, N is odd, both subarrays share the zero-th position as the reference sensor, $\tilde{l}_{c1} = Mn_1 - Nm_1$ and $\tilde{l}_{c2} = Mn_2 - Nm_2$ as two arbitrary differences in set $\tilde{\mathbb{L}}_c$, where $-M/2+1 \leq m_1 \leq M/2, 0 \leq n_1 \leq \lceil 3N/2 \rceil$ and $-M/2+1 \leq m_2 \leq M/2, 0 \leq n_2 \leq \lceil 3N/2 \rceil$. If $\tilde{l}_{c1} = \tilde{l}_{c2}$, we would have

$$M/N = (m_1 - m_2)/(n_1 - n_2) \tag{17}$$

As $-M/2+1 \leq m_1 \leq M/2, -M/2+1 \leq m_2 \leq M/2$, we obtain $m_1 - m_2 \leq M - 1$. However M and N are co-prime, the equation (17) cannot be held, which means $\tilde{l}_{c1} \neq \tilde{l}_{c2}$. Thus, there are $(3NM + 3M)/2$ distinct integers in set $\tilde{\mathbb{L}}_c$, which is similar to the case when M is odd, N is even, and when M is odd, N is odd. ■

$Property$ 2: The uniform DoF of the proposed array structure is as follows:

(1). When M is even, N is odd, $\tilde{\mathbb{L}}_c \cup \tilde{\mathbb{L}}_c^-$ contains the contiguous (hole-free) region spanning within $\pm(MN + 3M/2 - 1)$ and the uniform DoF is $2MN + 3M - 1$.

(2). When M is odd, N is even, $\tilde{\mathbb{L}}_c \cup \tilde{\mathbb{L}}_c^-$ contains the contiguous (hole-free) region spanning within $\pm(MN + N/2 + M - 1)$ and the uniform DoF is $2MN + N + 2M - 1$.

(3). When M is odd, N is odd, $\tilde{\mathbb{L}}_c \cup \tilde{\mathbb{L}}_c^-$ contains the contiguous (hole-free) region spanning within $\pm(MN + N/2 + 3M/2 - 1)$ and the uniform DoF is $2MN + N + 3M - 1$.

$Proof$: We consider the situation when M is even, N is odd. Given

$$0 \leq \tilde{l}_c \leq MN + 3M/2 - 1 \tag{18}$$

Since

$$-M/2+1 \leq m \leq M/2 \Rightarrow -MN/2 + N \leq Nm \leq MN/2 \tag{19}$$

and

$$\tilde{l}_c = Mn - Nm \Rightarrow Mn = \tilde{l}_c + Nm \tag{20}$$

We have

$$-MN/2 + N \leq Mn \leq 3MN/2 + 3M/2 - 1 \tag{21}$$

Since M and N are integers, we get

$$-MN/2 + N \leq Mn < 3M(N+1)/2 \tag{22}$$

$$\Rightarrow -N/2 + N/M \leq n < 3(N+1)/2 \tag{23}$$

When $n < 0, Mn < 0$. If $\tilde{l}_c = Mn - Nm > 0$, $m < 0$. As $N(-m) - M(-n) = N\check{m} - M\check{n}$, which can be regarded as the flipped positive values in $Mn - Nm$. So we only need to consider $n \geq 0$ and obtain

$$1 \leq n \leq (3N+1)/2 \tag{24}$$

Thus $\tilde{\mathbb{L}}_c \cup \tilde{\mathbb{L}}_c^-$ contains all the contiguous integers in the range $-MN - 3M/2 + 1 \leq \tilde{\mathbb{L}}_c \cup \tilde{\mathbb{L}}_c^- \leq MN + 3M/2 - 1$, where $-M/2 + 1 \leq m \leq M/2, 1 \leq n \leq \lceil 3N/2 \rceil$.

$Remark$: The configuration proposed in [9] and [10] can achieve a maximum of $2MN + 2N - 1$ uniform DoF with $M + 2N - 1$ sensors, whereas our CARS structure can detect $2MN + 3M - 1$ uniform DoF with $M + (3N+1)/2$ sensors. Thus, our proposed system can use fewer sensors to significantly improve the number of uniform DoF. The center symmetric co-prime array in [11] can achieve $2MN+M+N-1$ uniform DoF with $2M+2N$ sensors when both subarrays have an odd number of sensors, due to the center symmetric structure of both subarrays. However, in our method, the first subarray is center symmetric with only M sensors, and the second subarray is not center symmetric with a sensor reduction of $\lfloor N/2 \rfloor$, to achieve $2M$ more uniform DoF.

Similarly, we can prove for the other two situations, of which the one is when M is odd, N is even and the other is when M and N are both odd. ■

$Property$ 3: The self differences set is contained in the cross differences set, i.e. $(\tilde{\mathbb{L}}_s^- \cup \tilde{\mathbb{L}}_s) \subseteq (\tilde{\mathbb{L}}_c^- \cup \tilde{\mathbb{L}}_c)$.

$Proof$: Because the two subarrays Mn, Nm are set to share the reference sensor at the zero-th position, the self differences can be regarded as the cross differences between every sensor of one subarray and the reference sensor of the other subarray. Thus, we obtain $(\tilde{\mathbb{L}}_s^- \cup \tilde{\mathbb{L}}_s) \subseteq (\tilde{\mathbb{L}}_c^- \cup \tilde{\mathbb{L}}_c)$. ■

According to the above three properties, we can draw a conclusion that, for a pair of co-prime subarrays with M and N sensors, compared to reconstructing a subarray with $2N - 1$ sensors in [9], we are able to get a reduction of sensors with increased degrees of design freedom using the CARS structure.

IV. Numerical Results

In this section, the performance of the proposed CARS structure is investigated for narrowband DoA estimation through a pair of co-prime linear arrays. All the experiments are based on simulated data, including ten stationary sources with the DoA profiles of $\tilde{\theta}_1 = -0.1, \tilde{\theta}_2 = -0.08, \tilde{\theta}_3 = -0.06, \tilde{\theta}_4 = -0.04, \tilde{\theta}_5 = -0.02, \tilde{\theta}_6 = 0, \tilde{\theta}_7 = 0.02, \tilde{\theta}_8 = 0.04, \tilde{\theta}_9 = 0.06, \tilde{\theta}_{10} = 0.08$. The level of noise in terms of Signal to Noise Ratio (SNR) is 5 dB and the number of snapshots K is 800. M is chosen to be 12 and N is 11, which means, for the co-prime array configuration in [9], the total number of sensors is $M + 2N - 1 = 33$. For the proposed CARS structure, the total number of sensors is $M + \lceil 3N/2 \rceil = 29$, where the first subarray has $M = 12$ senosrs and the second subarray has $\lceil 3N/2 \rceil = 17$ sensors. Both subarrays share the zero-th position as the reference point. The DoAs are estimated from the measurement vectors and the MUSIC algorithm introduced in Section II is used.

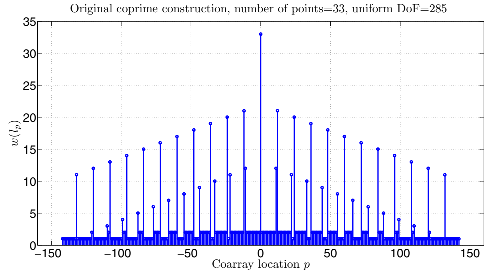

Fig. 3. The values of weight function $w(l_p)$ given in co-prime structure in [9], as well as the maximum contiguous segments $\mathbb{U}$.

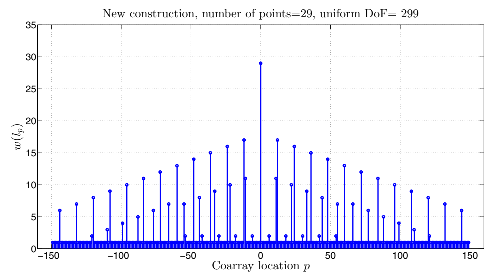

Fig. 4. The values of weight function $w(l_p)$ given in the proposed CARS structure, as well as the maximum contiguous segments $\mathbb{U}$.

The associated MUSIC spectra $P(\tilde{\theta})$ and root-mean-squared error (RMSE), are performed, where the RMSE (Error) is defined as

$$Error = \sqrt{\frac{1}{D}\sum_{i=1}^{D}(\hat{\tilde{\theta}}_i - \tilde{\theta}_i)^2} \tag{25}$$

where $\hat{\tilde{\theta}}_i$ denotes the estimated normalized DoA of the i-th source signal, according to the root MUSIC algorithm, and $\tilde{\theta}_i$ is the designed normalized DoA.

In Figure 3 and Figure 4, we show the values of the weight functions and the maximum contiguous segments in $\mathbb{L}_p$, which consists of the pair of co-prime arrays. For the co-prime array configuration in [9], there are 33 sensors in total and the number of uniform DoF is 285 (the sensor in coarray locates continuously from -142 to 142). While in the proposed CARS structure, there are 29 sensors which is less than 33, and a total of 299 uniform DoF are achieved (coarray locations are continuously from -149 to 149). We also found that for the proposed CARS structure, the values of weight function decrease and are more evenly distributed. Decreasing these weights reduces the number of sensor pairs that have significant mutual coupling. Less mutual coupling implies the mutual coupling matrix is closer to the identity matrix, which makes the RMSE likely to decrease.

In Figure 5, a comparison among CARS, the original co-prime construction in [9] and the center symmetric structure

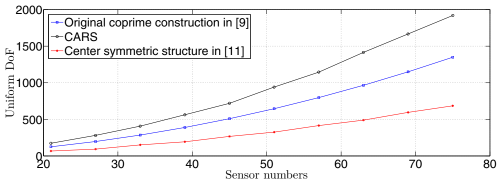

Fig. 5. Number of sensors vs uniform DoF for CARS, Original coprime construction [9] and center symmetric structure [11].

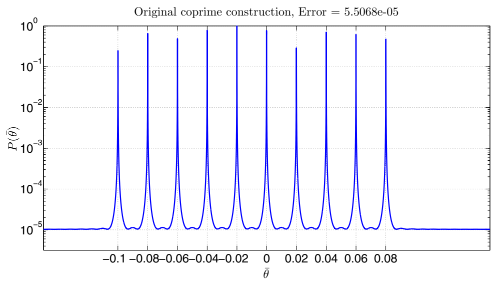

Fig. 6. The associated MUSIC spectra $P(\tilde{\theta})$ in co-prime structure in [9] for the DoAs estimation.

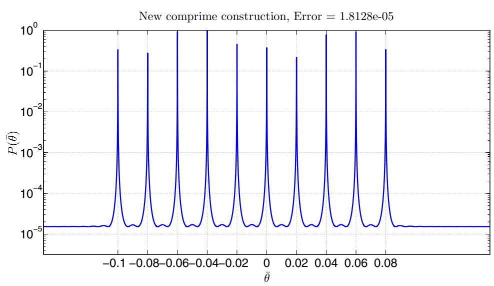

Fig. 7. The associated MUSIC spectra $P(\tilde{\theta})$ in the proposed CARS structure for the DoAs estimation.

in [11] is showed, considering achieved uniform DoF when using 21, 27, 33, 39, 45, 51, 57, 63, 69, 75 sensors. It is obviously when using same number of sensors, the proposed CARS can achieve most number of uniform DoF.

In Figure 6 and Figure 7, the associated MUSIC spectra $P(\tilde{\theta})$ are illustrated and the RMSEs are calculated. We found that the proposed CARS structure gives good DoA estimate and offers lower RMSE than the co-prime configuration in [9]. It can be concluded that the proposed CARS co-prime structure can achieve higher uniform DoF and smaller weight functions, which help decrease the RMSE.

V. Conclusion

A new structure of co-prime arrays with reduced sensors (CARS) has been presented to exploit the coarray distribution for source localisation. The structure contains a pair of co-prime subarrays, where the first array is shifted until symmetrical to the center (reference sensor) and the number of sensors in the second array is set according to the odd and even of the co-prime number pair. Both subarrays share the reference sensor at the zero-th position. This new co-prime array structure achieves more uniform DoF than previous work with a reduction in the number of physical sensors. The DoA estimation results by the MUSIC algorithm evaluated for multiple sources with noise show good performance of the proposed structure. For the future work, we will present other properties of the generalized CARS structure.

Acknowledgment

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