Synthetic Signals for Signal Processing

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***Abstract*—Modern signal processing algorithms exploit spatial and temporal features of signals and such algorithms can be highly sensitive to structured noise. Measured data is routinely used to evaluate algorithms but this approach is often limited by the expense and impracticality of collecting data and the presence of unknown events in the data. It is often necessary to use simulated signals as an interim stage. The paper introduces a method to generate spatially correlated noise as a first step towards developing a fast and efficient scheme to generate *non-stationary, spatially and temporally correlated noise*. This method is tested on standard (Bartlett) and high resolution (minimum variance) beamforming algorithms to demonstrate how the performance is changed and the advantage to using correlated noise to evaluate signal processing algorithms which require more realistic test signals.**

I. Introduction

Modern signal processing algorithms exploit spatial and temporal features of signals for detection, classification and localisation. However, such algorithms can be highly sensitive to structured noise, which is prevalent in many applications such as sonar and radar.

It is common to test signal processing algorithms in the laboratory with simple sets of data, such as spatially uncorrelated, non-fluctuating white noise and perfectly coherent and non-fluctuating signals. While these types of data have value in determining some performance aspects of the algorithm, they are unlikely to be able to provide information on the performance that can be expected practically. On the other hand, while measured data is routinely used to evaluate algorithms, and can be very useful to assess practical situations, it is rare to have complete knowledge of the contents of the data, it is unlikely that controlled data can be collected and real data is very often expensive or impractical to collect, if not altogether impossible.

There becomes a need to provide a halfway point between simple data sets and measured data. The intention is not to simulate a specific environment but replicate (in a controlled manner) key components that advanced signal processing algorithms are attempting to exploit. The advantage of a method for generating synthetic noise is that signal processing algorithms assigned for front-line operations can be more rigorously tested in the laboratory prior to being implemented in operational systems. This reduces front-line algorithm tuning problems. In addition, with the transition towards open systems where hardware and software can be replaced efficiently and more frequently it is necessary that prior assessment of algorithms is more robust.

Time series analysis and forecasting has been an extensive area of research in a variety of different disciplines. Much of this research involves reducing clutter and modelling the underlying pattern (amplitude fluctuations) of the data rather than producing signals with realistic noise fluctuations. The following presents the development of a method for generating correlated noise. This paper presents the initial foundation work for this project with some results. It is the authors' intention to build upon this work to include progressively complex signal and noise features to assess their complexity and utility for advanced signal processing algorithms. Some example results are provided to demonstrate how the performance of typical beam-formers is changed using correlated noise. Finally, some provisional comments are offered on the next stages of the scheme.

II. Spatially correlated noise

Methods to generate certain types of spatially correlated noise have been available for some time and can be incorporated in large environmental simulation models. However, these models have the disadvantage that they generally take a long time to run and need to be re-run for any physical changes in the environment or, for example, vehicle or sensor combinations. Generating spatially correlated noise has a key value in assessing time series algorithms. The method developed here (similar to [1]) uses cross-spectral matrices. The formulation of this method is presented below.

A. Formulation

For a number of sensors placed in an environment, one can write their vector of noise time series from each sensor (with the corresponding Fourier transform) as

$$\underline{s_v} \Longleftrightarrow \underline{S_v}, \tag{1}$$

where $\underline{s_v}$ is the time series vector, $\underline{S_v}$ is the frequency vector, and v is the position of the sensor[1].

The cross spectral matrices can be formed for these sensors as a function of frequency: $\underline{\mathbf{C}}_s$. It can be shown that, over a timescale where the noise is stationary, one can generate a set of synthetic time series:

$$\underline{w_v} \Longleftrightarrow \underline{W_v}, \tag{2}$$

[1]Note that lower case indicates time domain and upper case indicates frequency domain

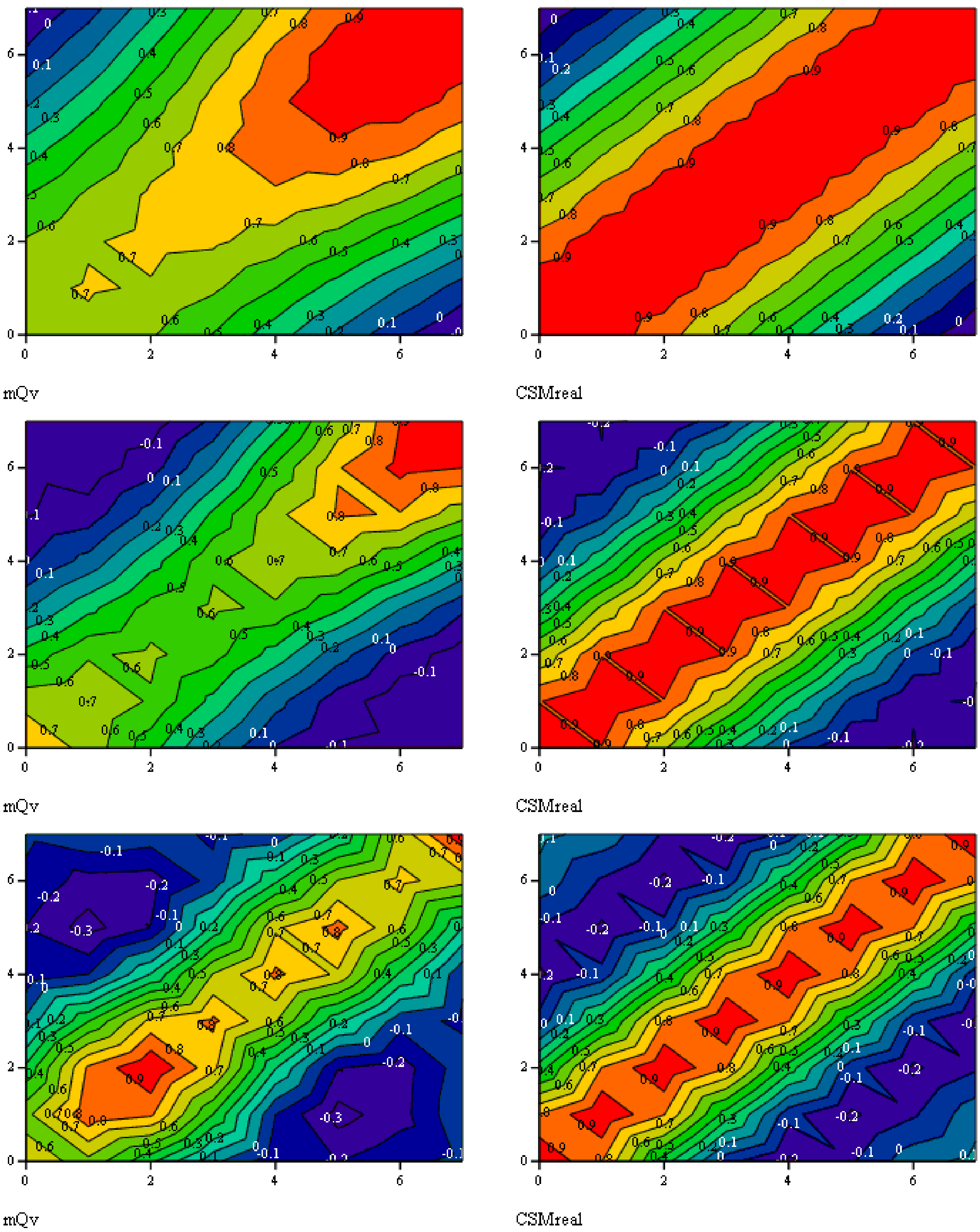

Fig. 1. Comparisons between simulated (left hand side) and required (right hand side) cross spectral matrices: at 5.875 Hz (top panel), at 9.625 Hz (middle panel), and at 13.375 Hz (bottom panel). Axis are element numbers

with cross spectral matrices given by $\underline{\mathbf{C}}_w$, such that the expected values of the cross spectral matrices are equal i.e.

$$\mathrm{E}|\underline{\mathbf{C}}_w| = \mathrm{E}|\underline{\mathbf{C}}_s|. \tag{3}$$

This can be achieved by generating a vector of random coefficients $\underline{U}_v$, (in this paper the random numbers are drawn from a Gaussian distribution) that are uncorrelated across the sensors and simple to generate, filtered by the operation

$$\underline{W}_v = \underline{\mathbf{Q}}.\underline{U}_v, \tag{4}$$

such that the expectation equality holds and $\underline{\mathbf{Q}} = \sqrt{\underline{\mathbf{C}}_s}$; this operation is known as taking the "square root" of a matrix and, for particular types of matrices, can be efficiently performed by using, for example, eigenvector decomposition. Note that

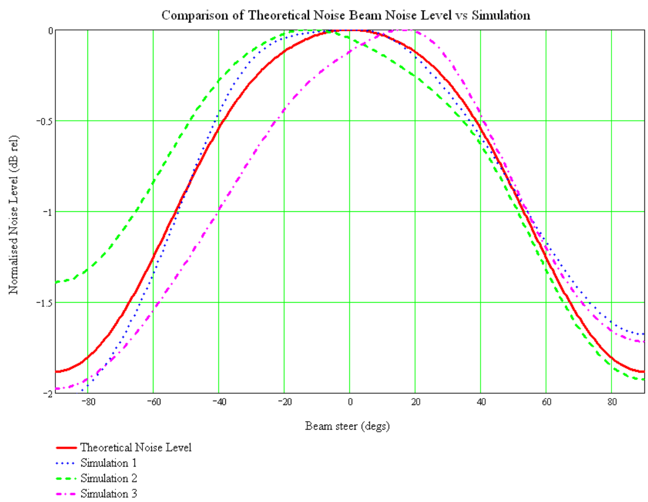

Fig. 2. Comparison between simulated and true beam noise level for a 16 element line array with 10 m spacing between each element.

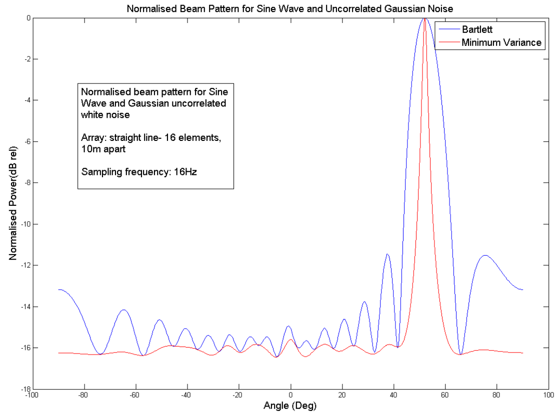

Fig. 3. Comparison between response from Bartlett and Minimum Variance beamformers for high SNR.

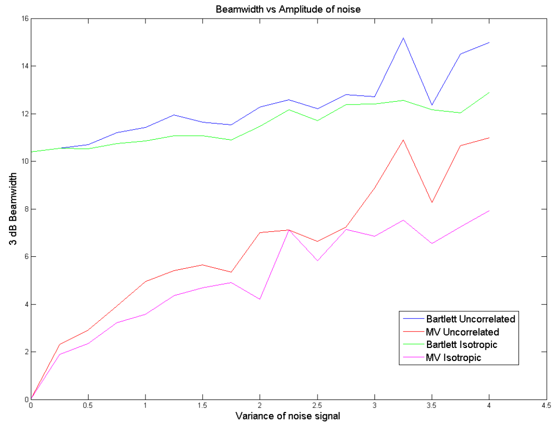

Fig. 4. Comparison between 3 dB beamwidth for isotropic and uncorrelated white noise for different values of noise.

other, more efficient, methods may be available by considering that the cross spectral matrices may be sparse Hermitian.

Once the frequency vectors $\underline{W}_v$ have been determined, the time series $\underline{w}_v$ are generated by the inverse Fourier transform. Signals generated in this manner implicitly have the required spatial and short term temporal properties.

B. Comparison

This scheme has been implemented in Matlab and, to compare with real time series, it has been used to generate the time series for a simple line array containing 8 elements with 20 m spacing between each element. Fig. 1 shows a comparison between the simulated and required cross spectral matrices at three different frequencies. The general agreement is good but it should be noted that each simulation is a statistical instance of the requirement; that is, for each instance we are not expecting an exact match since, by definition, only the expected values of the simulation equal the requirement.

III. Results

A. Noise

To demonstrate the utility of "pseudo-real" noise over uncorrelated noise Fig. 2 shows the normalised beam output for steering angles between $\pm$ 90° for broadband isotropic noise (16 Hz bandwidth) received on a line array with 16 elements with 10 m spacing between each element. It can be seen that the maximum noise level at broadside (0°) rolls off to end-fire (90°) by nearly 2 dB. The simulations show a similar pattern to the true result but, as expected, there is a variation between simulations.

B. Signals and noise

The use of synthetic data has been further investigated by adding the noise to a plane wave signal and assessing the output of two beamformers: 1. a plane wave beamformer, typical of many front line systems, often referred to as a Bartlett beamformer; and 2. a Minimum Variance (MV) beamformer, designed to optimise resolution (see [2]). Fig. 3 is an example of the beamformer outputs for a sine wave in uncorrelated noise (16 Hz bandwidth) for a 16 element line array with 10 m spacing between each element. As expected, the Minimum Variance beamformer can provide better resolution than the Bartlett beamformer.

To investigate the effects of Signal-to-Noise Ratio (SNR) and correlation on the beamformers, isotropic (spatially correlated) and uncorrelated white noise were beamformed and the 3 dB beamwidth was calculated. The results are shown in Fig. 4: 1. both types of noise result in a degradation in resolution as the noise level increases; 2. the isotropic noise results in higher resolution for both types of beamformers; 3.the resolution of the MV beamformer is better than the Bartlett; 4. the Bartlett beamformer is less sensitive to increases in noise(this can be seen by the shallower gradient).

IV. Comments

The results so far have shown that correlated noise is necessary to test signal processing algorithms in the absence of measured data. The next stage in the scheme is to develop modelling and forecasting methods. Forecasting has been an extensive area of research across multiple disciplines and many algorithms exist ranging from simple linear models like Auto Regressive Moving Average (ARMA) to more complex non-linear neural networks. The applicability of these methods is dependent on the properties of the signal and, if the appropriate methods are selected, the forecasted signal should have similar properties to the original. While it is possible to optimise a forecasting algorithm for a specific type of time series (e.g. a financial time series) there does not exist a single algorithm that can efficiently and effectively forecast all types of signal (see [3], [4]). One option is to build a database of forecasting algorithms, where algorithms can be selected based on the properties of the signal; currently, the algorithms and methods that should be included in such a database (and implemented in the noise algorithm) and the criteria for selecting the most appropriate is unknown.

V. Concluding remarks

This paper has outlined the initial stage of a scheme to generate non-stationary, spatially and temporally correlated noise, and has shown that synthetic noise containing spatial correlation properties can be generated using efficient processes. The results that have been presented have illustrated some of the differences in the outputs from signal processing algorithms between using more realistic spatially correlated noise and uncorrelated noise. More work is needed to extend this to the next stage and include a more practical forecasting capability.

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