Compressed Sensing Solutions for Airborne Low Frequency SAR

Mike E. Davies & Shaun I. Kelly
The University of Edinburgh

WF3 - Compressive sensing for radar applications
Overview

Motivation/Challenges

- Motivation: Why Airborne LF SAR?
- Challenges:
  - Notching on transmit
  - Radio Frequency Interference (RFI)
  - Phase errors
  - Near field imaging
- Solutions (CS based)
Motivation

Why use VHF/UHF Spectrum?

- Foliage Penetration (FoPEN) Radar
- Ground Penetration Radar (GPR)
- Scattering is dependent on wavelength.
Challenges

Issues which effect the VHF/UHF spectrum

- Interference between SAR systems and radio, television and communications systems.
- Radio frequency interference (RFI)
- Interference Types:
  1. SAR systems can interfere with other spectrum users.
  2. Other users in the spectrum can interfere with SAR system.
Challenges

Notched LFM on Transmit

![Diagram showing Notched LFM Chirp]

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Challenges

Standard Image Formation with Notching
Solution Outline

CS-based components

- Sparse Image Formation
- Fast forward/back projections
- Compressive Autofocus
- RFI suppression
System model

Notched LFM on Transmit

System model (after dechirping and deskewing):

\[ Y = \text{diag} (w) h(X) + N \]

\[ Y \in \mathbb{C}^{M' \times N'}, \quad N \in \mathbb{C}^{M \times N}, \quad w \in \mathbb{R}^M \]

- \( X \) is the scene reflectivities
- \( Y \) is the phase history
- \( h(\cdot) \) is the system model without notching
- \( w \) is a weighting that models the transmit notching
- \( N \) is the RFI and additive noise
Sparse Image Formation SAR

First Ingredient: Sparsity

▶ The signal/image must be sparse or well approximated by a sparse signal/image (compressible)

*Will be considered later.*

Second Ingredient: “Good” Measurements

▶ Measurement equation $Y = h(X)$

*An approximate sub-sampling of the k-space!*

Third Ingredient: Reconstruction Algorithm

▶ Sparse reconstruction algorithm, e.g. constrained $\ell_1$ min., greedy algorithms - OMP, IHT, etc.

*Many fast algorithms available if there are fast operators available!*
Sparse Image Formation SAR

Sparsity in Wavelets?

Image Domain  Wavelet Domain

SAR images are not significantly compressible in any basis!
Interaction of Reflectors in a Range Cell

- **Random interference:** Speckle dominates images due to many random reflectors in a range cell inducing multiplicative noise in the reconstructed image - not compressible.

- **Coherent interference:** Coherent reflectors (often targets of interest) whose intensity tend to be much larger than incoherent reflections - compressible in spatial domain.
Sparse Image Formation SAR

Compressed Sensing Image Formation

\[
\text{arg min}_{X_s} \|X_s\|_1 \text{ s.t. } \|Y - h(X_s)\|_F \leq \epsilon
\]

\[
\text{arg min}_{X_{bg}} \|Y - h(X_{bg} + X_s)\|_F
\]

Bright Targets

Background Speckle
Sparse Image Formation SAR

Compressed Sensing Image Formation

Significant improvement in imaging of bright targets!
Sparse Image Formation SAR

Compressed Sensing Image Formation

Fully Sampled Reference Image

Compressed Sensing Image Formation

Degradation in background speckle!

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Fast SAR Operators

Compressed Sensing Image Formation

- LF SAR typically has long apertures and large beam width making the aperture non-linear and the imaging near field.
- Efficient iterative reconstruction requires fast forward/backward operators:
  - Direct Forward/Backward Projection - too slow: $O(N^3)$
  - Polar Format Algorithm - far field imaging only
  - Range Migration Algorithm - flat terrain model and linear aperture
- Fast decimation-based Forward/Backward Projection Algorithms, e.g. [McCorkle et al. ’96],...
Fast SAR Operators

Decimation-in-phase-history

- Recursive splitting of image and decimating of phase history.
- $O(N^2 \log N)$ operations.
- e.g. [McCorkle et al. 1996], [Wahl et al. 2008].

Decimation-in-image

- Recursive splitting of phase history and decimating of image.
- $O(N^2 \log N)$ operations.
- e.g. [Kelly and D. 2014]
### Image Formation Times (seconds)

<table>
<thead>
<tr>
<th>N</th>
<th>BP (dec.-in-image)</th>
<th>fast BP (dec.-in-image)</th>
<th>fast BP (dec.-in-phase-history)</th>
<th>PFA</th>
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</thead>
<tbody>
<tr>
<td>256</td>
<td>18.26</td>
<td>4.75</td>
<td>4.64</td>
<td>0.90</td>
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<tr>
<td>512</td>
<td>140.22</td>
<td>18.77</td>
<td>18.74</td>
<td>3.24</td>
</tr>
<tr>
<td>1024</td>
<td>1120.96</td>
<td>77.18</td>
<td>76.98</td>
<td>13.15</td>
</tr>
<tr>
<td>2048</td>
<td>9052.47</td>
<td>318.43</td>
<td>317.36</td>
<td>69.15</td>
</tr>
</tbody>
</table>

- NFFT algorithm with an interpolation kernel length of 24 samples.
- Time on a single core of 2.5 GHz Intel Xeon processor with $N^2$ element images and $N^2$ element phase histories.
- $\log_2 N - \log_2 64$ decomposition stages.
Images formed using fast decimation-in-image and decimation-in-phase-history BP algorithms with three decomposition stages.

Pixel-wise/k-space wise relative errors in the fast BP algorithms with respect to the BP algorithm.
Phase Errors

- Inaccuracies in the propagation delay estimates introduce unknown phase errors, $\phi_{\tau_{ek}}$:

$$\phi_{\tau_{ek}} \approx \omega_0 \tau_{ek} - \alpha \tau_{ek}^2$$

with, $\tau_{ek}$ - delay error at aperture position $k$
$\omega_0$ - carrier freq. and $\alpha$ - chirp rate.

- Modified SAR observation model with phase errors

$$Y = h(X) \text{diag} \{ e^{j\phi} \}$$

- If not corrected, phase errors can defocus targets and degrade reconstructed image.
Classical Autofocus

Classical (image based) autofocus assumes far field small aperture model

- System model $\sim$ fully determined and separable:
  \[ Y = h(X) \text{ diag } \{e^{i\phi}\} \approx AX\Psi B \]

- $A$ and $B \sim$ Fourier
- Autofocus $\sim$ deconvolution
- $X$ is recovered from $X\Psi$ using classical autofocus methods, e.g. Map Drift (MD) or Phase Gradient Autofocus (PGA)
Phase Calibration (Autofocus)

Undetermined System Model

\[ Y = A'X\Psi \]

- \( A' \in \mathbb{C}^{N \times S} \) is undetermined, e.g. due to notching

Post-Reconstruction Autofocus

- Can \( X\Psi \) be recovered from \( Y \) followed by a post-reconstruction autofocus?
  - CS Stable Sparse Recovery [Rudelson, Vershynin '08]:
    \[
    S \geq CK_{\Psi}K_X \log^4(N)
    \]
    with original sparsity \( K_X \) and blurring factor \( K_{\Psi} \)

Reconstruction quality deteriorates as phase errors increases!
Compressive Autofocus

- Better Solution: perform joint reconstruction

\[
\begin{align*}
\text{minimise} & \quad \|X\|_1 \\
\text{subject to} & \quad \|Y \text{ diag } \{d\} - h(X)\|_F \leq \sigma \\
& \quad d_n^* d_n = 1, \ n = 1, \ldots, N.
\end{align*}
\]

- Fast Block-relaxation algorithms via majorisation-minimisation exist [Kelly et al 2012/14]

- No far field/small aperture assumptions

- Theoretical guarantees: open problem
Phase Calibration (Autofocus)

Reconstruction performance versus under-sampling ratio

--- increasing phase errors ---

‘○’ oracle reconstruction, ‘□’ compressive auto-focus, ‘×’ sparse image formation with post-processing autofocus.
Phase Calibration (Auto-focus)

Figure: LF SAR image formations: (a) was formed using the BP algorithm; (b) was formed using sparse reconstruction (no autofocus); and (c) was formed using Compressive Auto-focus.
Radio Frequency Interference

RFI suppression

- Strong interference from AM/FM transmitters.
- RFI pre-processing suppression methods:
  1. Estimate-and-subtract: estimate the frequencies and phases of the RFI and then abstract. *Computationally expensive and approximation dependent.*
  2. Linear filter: minimise RFI using linear filter, e.g. LMS filter and Wiener filter. *Can produce large side lobes.*
Radio Frequency Interference

Dechirping

After dechirping and deskewing:

- narrowband interferes become concentrated in time and
- spectral notches become notches in time.

Before dechirping

After dechirping and deskewing
Filter-based RFI suppression

Linear RFI Filtered Reconstruction:

\[ \hat{X} = g(H \text{vec}(Y)) \]
\[ H = \text{diag}([H_1, \cdots, H_{N'}]) \]

- \( g(\cdot) \) is the filtered back-projection algorithm.
- \( H_{n'} \) are the Wiener filters for each slow-time position, i.e.
  \[ H_{n'} = I - Q_{n'} (Q_{\tilde{y}_{n'}} + Q_{n'})^{-1} \text{ for } Q_x = E[xx^H] \]
- \( Q_{\tilde{y}_{n'}} \) are the covariance matrices of the received signal at each slow-time position
- \( Q_{n'} \) are the covariance matrices of the RFI at each slow-time position
Radio Frequency Interference

RFI-aware Sparse Image Formation

Incorporate RFI into the Basis Pursuit Denoising:

\[
\hat{X} = \min_X \|X\|_1
\]

subject to \(\|Y - h(X)\|_{Q_N^{-1}} \leq \epsilon\),

where, \(\|A\|_Q = \text{vec}(A)^H Q \text{vec}(A)\)

- \(Q_N\) is full covariance matrix of the RFI and additive noise.
- \(Q_N\) is well approximated using a diagonal matrix so the data fidelity term becomes a weighted Frobenius norm.
RFI-aware Sparse Image Formation Implementation

*Estimate Noise Covariance:*

Estimate $Q_N$ using ten “dead-time” measurements.

*Assume elements of $N$ are independent.*

*Unconstrained Optimisation:*

$$\hat{X} = \underset{X}{\text{minimise}} \|X\|_1 + \lambda(\|Y - \text{diag}(w)h(X)\|_{Q_N^{-1}} - \epsilon)$$

*Approximately solved using thirty iterations of a fast iterative shrinkage thresholding algorithm.*

*Project onto Domain of $h(\cdot)$:*

$$\hat{X} \leftarrow g(h(\hat{X}))$$
# Radio Frequency Interference

## VHF/UHF SAR simulation Parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>carrier frequency ($\omega_0/2\pi$)</td>
<td>308 MHz</td>
</tr>
<tr>
<td>altitude</td>
<td>7000 m</td>
</tr>
<tr>
<td>number of targets</td>
<td>20</td>
</tr>
<tr>
<td>chirp bandwidth ($\alpha T/\pi$)</td>
<td>324 MHz</td>
</tr>
<tr>
<td>stand-off distance</td>
<td>7000 m</td>
</tr>
<tr>
<td>number of interferes</td>
<td>80</td>
</tr>
<tr>
<td>IF bandwidth</td>
<td>60 MHz</td>
</tr>
<tr>
<td>aperture length</td>
<td>7000 m</td>
</tr>
<tr>
<td>signal to noise ratio (SNR)</td>
<td>60 dB</td>
</tr>
<tr>
<td>scene radius ($L$)</td>
<td>75 m</td>
</tr>
<tr>
<td>number of aperture samples</td>
<td>300</td>
</tr>
<tr>
<td>signal to interference ratio (SIR)</td>
<td>-30 dB</td>
</tr>
<tr>
<td>transmit notch centre frequencies</td>
<td>175, 330, 389, 416 and 448 MHz</td>
</tr>
<tr>
<td>transmit notch bandwidths</td>
<td>15, 7, 13, 20 and 10 MHz</td>
</tr>
</tbody>
</table>
Radio Frequency Interference

Reconstructed Images

Filtered back-projection

Wiener filtered followed by filtered back-projection

RFI-aware sparse image formation

Range compression RFI-aware sparse image formation
Conclusions

- Iterative CS-based algorithms provide a good solution to LF SAR image formation with notch on transmit.
- Compressive Autofocus can be performed simultaneously.
- Receiver RFI suppression easily incorporated using a weighted Frobenius norm.
- The proposed technique is superior to previous approaches as it does not suffer from poor range side lobes and it can accommodate a wide range of RFI.
References


▶ S. I. Kelly and M. E. Davies, 2013, "RFI suppression and sparse image formation for UWB SAR," Radar Symposium (IRS), 14th International 2, pp. 655-660.
