Compressed Sensing Solutions for Airborne Low Frequency SAR

Mike E. Davies & Shaun I. Kelly
The University of Edinburgh

[dstl] UDRC EPSRC
Engineering and Physical Sciences Research Council

WF3 - Compressive sensing for radar applications

Overview

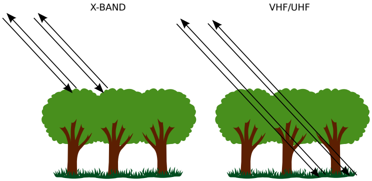
Motivation/Challenges

- Motivation: Why Airborne LF SAR?
- Challenges:
  - Notching on transmit
  - Radio Frequency Interference (RFI)
  - Phase errors
  - Near field imaging
- Solutions (CS based)

Motivation

Why use VHF/UHF Spectrum?

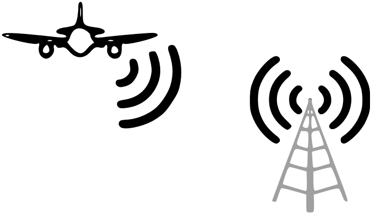
- Foliage Penetration (FoPEN) Radar
- Ground Penetration Radar (GPR)
- Scattering is dependent on wavelength.



Challenges

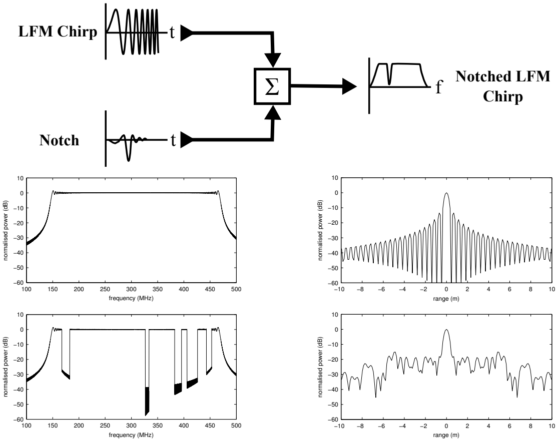
Issues which effect the VHF/UHF spectrum

- Interference between SAR systems and radio, television and communications systems.
- Radio frequency interference (RFI)
- Interference Types:
  1. SAR systems can interfere with other spectrum users.
  2. Other users in the spectrum can interfere with SAR system.



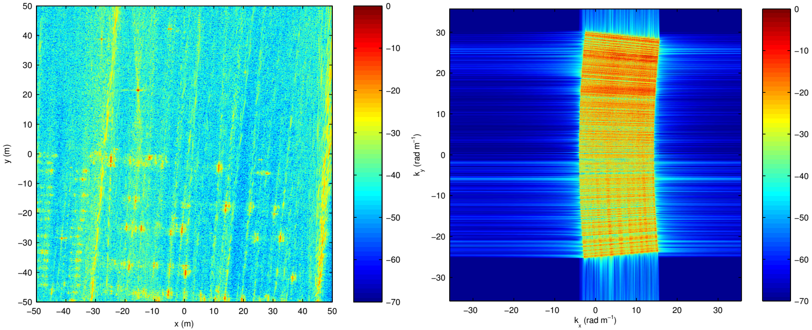
Challenges

Notched LFM on Transmit



Challenges

Standard Image Formation with Notching



Solution Outline

CS-based components

- Sparse Image Formation
- Fast forward/back projections
- Compressive Autofocus
- RFI suppression

System model

Notched LFM on Transmit

System model (after dechirping and deskewing):

$$\mathbf{Y} = \mathrm{diag}\,(\mathbf{w})h(\mathbf{X}) + \mathbf{N}$$

$$\mathbf{Y} \in \mathbb{C}^{M' \times N'},\ \mathbf{N} \in \mathbb{C}^{M \times N},\ \mathbf{w} \in \mathbb{R}^{M}$$

- $\mathbf{X}$ is the scene reflectivities
- $\mathbf{Y}$ is the phase history
- $h(\cdot)$ is the system model without notching
- $\mathbf{w}$ is a weighting that models the transmit notching
- $\mathbf{N}$ is the RFI and additive noise

Sparse Image Formation SAR

First Ingredient: Sparsity

- The signal/image must be sparse or well approximated by a sparse signal/image (compressible)

Will be considered later.

Second Ingredient: "Good" Measurements

- Measurement equation $\mathbf{Y} = h(\mathbf{X})$

An approximate sub-sampling of the k-space!

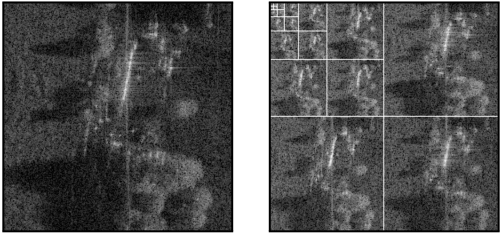
Third Ingredient: Reconstruction Algorithm

- Sparse reconstruction algorithm, e.g. constrained ℓ_1 min., greedy algorithms - OMP, IHT, etc.

Many fast algorithms available if there are fast operators available!

Sparsity in Wavelets?

Image Domain | Wavelet Domain



SAR images are not significantly compressible in any basis!

Sparse Image Formation SAR

Interaction of Reflectors in a Range Cell

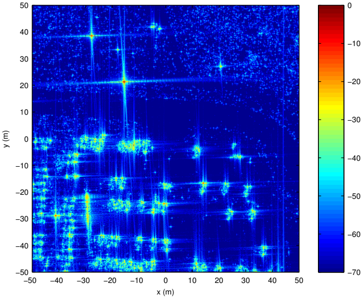
- *Random interference:* Speckle dominates images due to many random reflectors in a range cell inducing multiplicative noise in the reconstructed image - not compressible.
- *Coherent interference:* Coherent reflectors (often targets of interest) whose intensity tend to be much larger than incoherent reflections - compressible in spatial domain.

Sparse Image Formation SAR

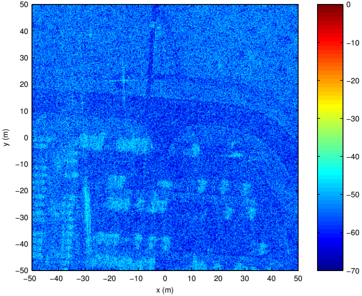
Compressed Sensing Image Formation

$$\underset{\mathbf{X}_s}{\text{argmin}} \|\mathbf{X}_s\|_1 \text{ s.t. } \|\mathbf{Y} - h(\mathbf{X}_s)\|_F \leq \epsilon$$

$$\underset{\mathbf{X}_{bg}}{\text{argmin}} \left\|\mathbf{Y} - h\left(\mathbf{X}_{bg} + \mathbf{X}_s\right)\right\|_F$$



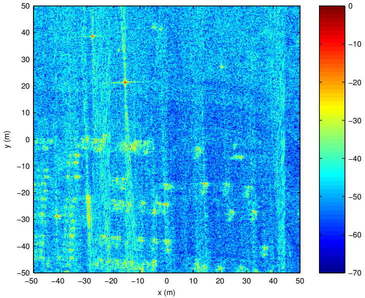
Bright Targets



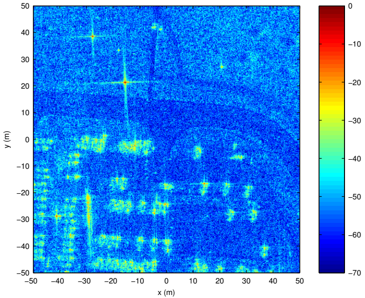
Background Speckle

Sparse Image Formation SAR

Compressed Sensing Image Formation



Standard Image Formation

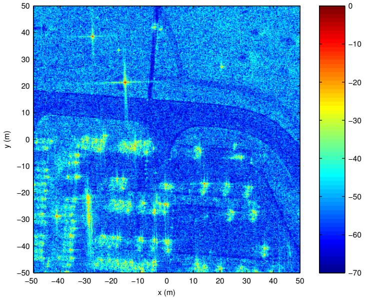


Compressed Sensing Image Formation

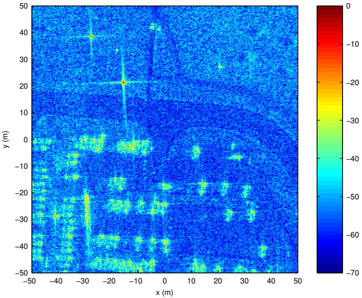
Significant improvement in imaging of bright targets!

Sparse Image Formation SAR

Compressed Sensing Image Formation



Fully Sampled Reference Image



Compressed Sensing Image Formation

Degradation in background speckle!

Fast SAR Operators

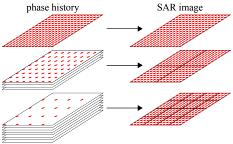
Compressed Sensing Image Formation

- LF SAR typically has long apertures and large beam width making the aperture non-linear and the imaging near field.
- Efficient iterative reconstruction requires fast forward/backward operators:
  - × Direct Forward/Backward Projection - too slow: $\mathcal{O}\left(N^3\right)$
  - × Polar Format Algorithm - far field imaging only
  - × Range Migration Algorithm - flat terrain model and linear aperture
  - ✓ Fast decimation-based Forward/Backward Projection Algorithms, e.g. [McCorkle et al. '96],...

Fast SAR Operators

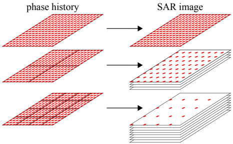
Decimation-in-phase-history

- Recursive splitting of image and decimating of phase history.
- $\mathcal{O}\left(N^2 \log N\right)$ operations.
- e.g. [McCorkle et al. 1996], [Wahl et al. 2008].



Decimation-in-image

- Recursive splitting of phase history and decimating of image.
- $\mathcal{O}\left(N^2 \log N\right)$ operations.
- e.g. [Kelly and D. 2014]



Fast SAR Operators

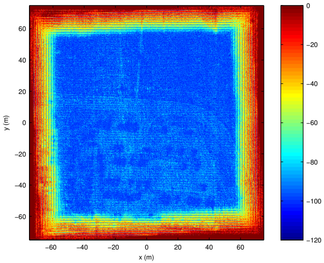
Image Formation Times (seconds)

N	BP	fast BP (dec.-in-image)	fast BP (dec.-in-phase-history)	PFA
256	18.26	4.75	4.64	0.90
512	140.22	18.77	18.74	3.24
1024	1120.96	77.18	76.98	13.15
2048	9052.47	318.43	317.36	69.15

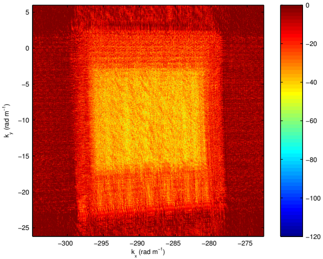
- NFFT algorithm with an interpolation kernel length of 24 samples.
- Time on a single core of 2.5 GHz Intel Xeon processor with N^2 element images and N^2 element phase histories.
- $\log_2 N - \log_2 64$ decomposition stages.

Fast SAR Operators

Pixel-wise Relative Errors



decimation-in-image



decimation-in-phase-history

- Images formed using fast decimation-in-image and decimation-in-phase-history BP algorithms with three decomposition stages.
- Pixel-wise/k-space wise relative errors in the fast BP algorithms with respect to the BP algorithm.

Phase Calibration (Autofocus)

Phase Errors

- Inaccuracies in the propagation delay estimates introduce unknown phase errors, $\phi_{\tau_{e_k}}$:

$$\phi_{\tau_{e_k}} \approx \omega_0 \tau_{e_k} - \alpha \tau_{e_k}^2$$

with, τ_{e_k} - delay error at aperture position k
ω_0 - carrier freq. and α - chirp rate.

- Modified SAR observation model with phase errors

$$\mathbf{Y} = h(\mathbf{X}) \,\mathrm{diag}\left\{ e^{j\phi} \right\}$$

- If not corrected, phase errors can defocus targets and degrade reconstructed image.

Phase Calibration (Autofocus)

Classical Autofocus

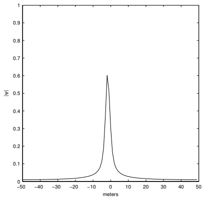
Classical (image based) autofocus assumes far field small aperture model

- System model $\sim$ fully determined and separable:

$$\mathbf{Y} = h(\mathbf{X})\,\mathrm{diag}\left\{e^{j\phi}\right\} \approx \mathbf{AX\Psi B}$$

- $\mathbf{A}$ and $\mathbf{B}$ $\sim$ Fourier
- Autofocus $\sim$ deconvolution

columns of $\mathbf{\Psi}$, due to
a quadratic phase error



- $\mathbf{X}$ is recovered from $\mathbf{X\Psi}$ using classical autofocus methods, e.g. Map Drift (MD) or Phase Gradient Autofocus (PGA)

Phase Calibration (Autofocus)

Undetermined System Model

$$\mathbf{Y} = \mathbf{A}'\mathbf{X}\boldsymbol{\Psi}$$

- $\mathbf{A}' \in \mathbb{C}^{N \times S}$ is undetermined, e.g. due to notching

Post-Reconstruction Autofocus

- Can $\mathbf{X}\boldsymbol{\Psi}$ be recovered from $\mathbf{Y}$ followed by a post-reconstruction autofocus?
  - CS Stable Sparse Recovery [Rudelson, Vershynin '08]:

$$S \geq C K_{\boldsymbol{\Psi}} K_{\mathbf{X}} \log^4(N)$$

with original sparsity $K_{\mathbf{X}}$ and blurring factor $K_{\boldsymbol{\Psi}}$

Reconstruction quality deteriorates as phase errors increases!

Phase Calibration (Autofocus)

Compressive Autofocus

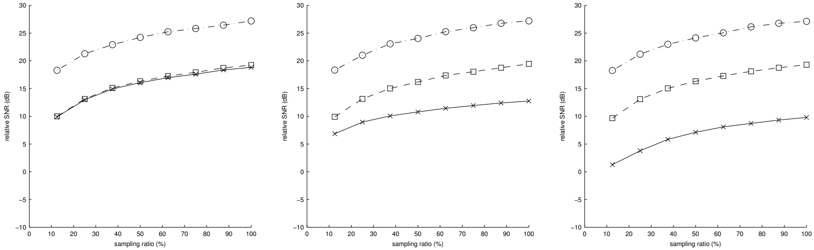
- Better Solution: perform joint reconstruction

$$\begin{aligned} \underset{\mathbf{X},\mathbf{d}}{\text{minimise}} \quad & \|\mathbf{X}\|_1 \\ \text{subject to} \quad & \|\mathbf{Y}\,\text{diag}\,\{\mathbf{d}\} - h(\mathbf{X})\|_F \leq \sigma \\ & d_n^* d_n = 1, \; n = 1, \ldots, N. \end{aligned}$$

- Fast Block-relaxation algorithms via majorisation-minimisation exist [Kelly et al 2012/14]
- No far field/small aperture assumptions
- Theoretical guarantees: open problem

Phase Calibration (Autofocus)

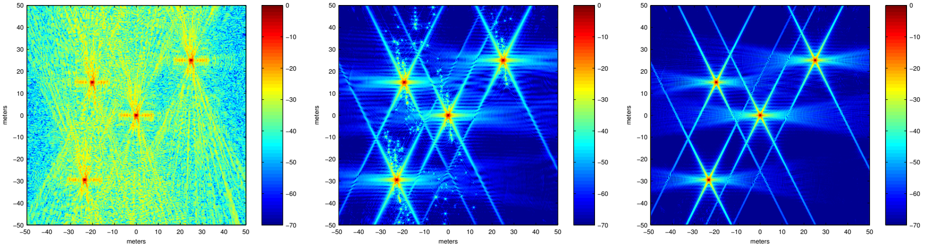
Reconstruction performance versus under-sampling ratio



⟶ increasing phase errors ⟶

'○' oracle reconstruction, '□' compressive auto-focus, '×' sparse image formation with post-processing autofocus.

Phase Calibration (Autofocus)



(a) (b) (c)

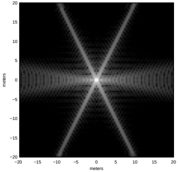
Figure: LF SAR image formations: (a) was formed using the BP algorithm; (b) was formed using sparse reconstruction (no autofocus); and (c) was formed using Compressive Autofocus.

Radio Frequency Interference

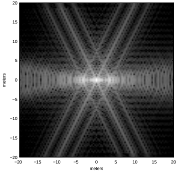
RFI suppression

- Strong interference from AM/FM transmitters.
- RFI pre-processing suppression methods:
  1. Estimate-and-subtract: estimate the frequencies and phases of the RFI and then abstract.
     Computationally expensive and approximation dependent.
  2. Linear filter: minimise RFI using linear filter, e.g. LMS filter and Wiener filter.
     Can produce large side lobes.

Unfiltered PSF



Filtered PSF

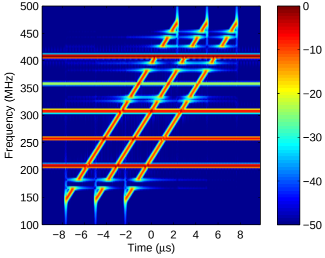


Radio Frequency Interference

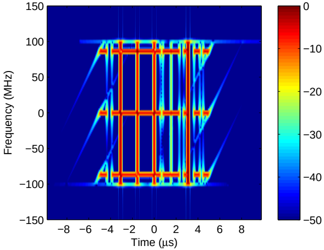
Dechirping

After dechirping and deskewing:

- narrowband interferes become concentrated in time and
- spectral notches become notches in time.



Before dechirping



After dechirping and deskewing

Radio Frequency Interference

Filter-based RFI suppression

Linear RFI Filtered Reconstruction:

$$\hat{\mathbf{X}} = g(\mathbf{H}\,\mathrm{vec}(\mathbf{Y}))$$

$$\mathbf{H} = \mathrm{diag}([\mathbf{H}_1, \cdots, \mathbf{H}_{N'}])$$

- $g(\cdot)$ is the filtered back-projection algorithm.
- $\mathbf{H}_{n'}$ are the Wiener filters for each slow-time position, i.e.

$$\mathbf{H}_{n'} = \mathbf{I} - \mathbf{Q}_{\mathbf{n}_{n'}}(\mathbf{Q}_{\tilde{\mathbf{y}}_{n'}} + \mathbf{Q}_{\mathbf{n}_{n'}})^{-1} \; \textit{for} \; \mathbf{Q}_{\mathbf{x}} = \mathrm{E}\left[\mathbf{x}\mathbf{x}^{\mathrm{H}}\right]$$

- $\mathbf{Q}_{\tilde{\mathbf{y}}_{n'}}$ are the covariance matrices of the received signal at each slow-time position
- $\mathbf{Q}_{\mathbf{n}_{n'}}$ are the covariance matrices of the RFI at each slow-time position

Radio Frequency Interference

RFI-aware Sparse Image Formation

Incorporate RFI into the Basis Pursuit Denoising:

$$\hat{\mathbf{X}} = \underset{\mathbf{X}}{\text{minimise}} \ \|\mathbf{X}\|_1 \quad \text{subject to } \|\mathbf{Y} - h(\mathbf{X})\|_{\mathbf{Q_N}^{-1}} \leq \epsilon,$$

$$\text{where, } \|\mathbf{A}\|_{\mathbf{Q}} = \text{vec}(\mathbf{A})^{\mathsf{H}}\mathbf{Q}\,\text{vec}(\mathbf{A})$$

- $\mathbf{Q_N}$ is full covariance matrix of the RFI and additive noise.
- $\mathbf{Q_N}$ is well approximated using a diagonal matrix so the data fidelity term becomes a weighted Frobenius norm.

Radio Frequency Interference

RFI-aware Sparse Image Formation Implementation

Estimate Noise Covariance:

Estimate $\mathbf{Q_N}$ using ten "dead-time" measurements.

Assume elements of $\mathbf{N}$ *are independent.*

Unconstrained Optimisation:

$$\hat{\mathbf{X}} = \underset{\mathbf{X}}{\text{minimise}} \ \|\mathbf{X}\|_1 + \lambda(\|\mathbf{Y} - \text{diag}\,(\mathbf{w})h(\mathbf{X})\|_{\mathbf{Q_N}^{-1}} - \epsilon)$$

Approximately solved using thirty iterations of a fast iterative shrinkage thresholding algorithm.

Project onto Domain of $h(\cdot)$*:*

$$\hat{\mathbf{X}} \leftarrow g(h(\hat{\mathbf{X}}))$$

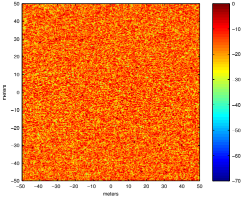
Radio Frequency Interference

VHF/UHF SAR simulation Parameters

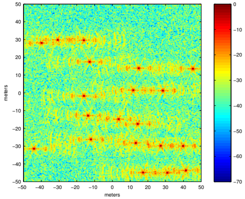
parameter	value
carrier frequency ($\omega_0/2\pi$)	308 MHz
altitude	7000 m
number of targets	20
chirp bandwidth ($\alpha T/\pi$)	324 MHz
stand-off distance	7000 m
number of interferes	80
IF bandwidth	60 MHz
aperture length	7000 m
signal to noise ratio (SNR)	60 dB
scene radius (L)	75 m
number of aperture samples	300
signal to interference ratio (SIR)	-30 dB
transmit notch centre frequencies	175, 330, 389, 416 and 448 MHz
transmit notch bandwidths	15, 7, 13, 20 and 10 MHz

Radio Frequency Interference

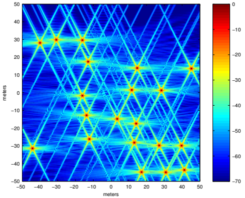
Reconstructed Images



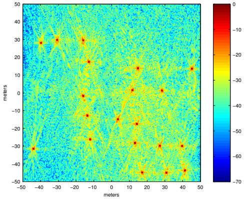
Filtered back-projection



Wiener filtered followed by filtered back-projection



RFI-aware sparse image formation



Range compression RFI-aware sparse image formation

Conclusions

Conclusions

- Iterative CS-based algorithms provide a good solution to LF SAR image formation with notch on transmit
- Compressive Autofocus can be performed simultaneously
- Receiver RFI suppression easily incorporated using a weighted Frobenius norm.
- The proposed technique is superior to previous approaches as it does not suffer from poor range side lobes and it can accommodate a wide range of RFI.

References

- S. I. Kelly, G. Rilling, M. Davies, and B. Mulgrew, 2011, "Iterative image formation using fast (re/back)-projection for spotlight-mode SAR," in Proc. IEEE Radar Conf. 2011, pp. 835-840.
- S. I. Kelly, C. Du, G. Rilling and M. Davies, 2012, "Advanced image formation and processing of partial synthetic aperture radar data," Signal Processing, IET 6 (5), pp. 511-520.
- S. I. Kelly, M. Yaghoobi and M. E. Davies, 2012, "Auto-focus for under-sampled synthetic aperture radar," in Sensor Signal Processing for Defence (SSPD 2012) pp. 1-5.
- S. I. Kelly and M. E. Davies, 2013, "RFI suppression and sparse image formation for UWB SAR," Radar Symposium (IRS), 14th International 2, pp. 655-660.
- S. I. Kelly, M. Yaghoobi and M. E. Davies, 2014, "Sparsity-based Autofocus for Under-sampled Synthetic Aperture Radar" to appear in IEEE Trans. Aerospace and Electronic Systems, 2014.
- S. I. Kelly and M. E. Davies, 2014, "A Fast Decimation-in-image Back-projection Algorithm for SAR," in Proc. IEEE Radar Conf. 2014.